Dark Matter at future e^+e^- colliders

chances of detection vs DM spin

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based on

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Dark-matter-spin effects at future e^+e^- colliders

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International Workshop on Future Linear Colliders
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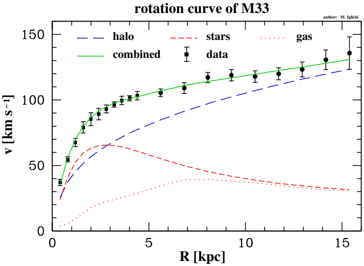
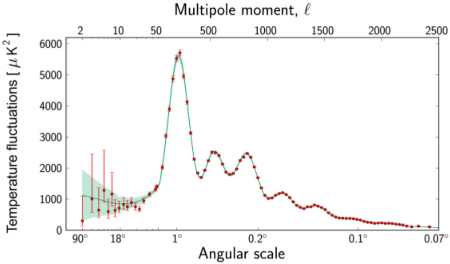
Evidence for dark matter

- rotation curves
- CMB fluctuations ⇒ ~~MOND~~
- gravitational lensing
- colliding clusters
- ...



source of the data: astro-ph/9909252



source: http://sci.esa.int/planck

source: https://apod.nasa.gov/apod

DM believed to be beyond the Standard Model

Introduction: DM at future e^+e^- colliders

Collider search for dark matter

- Experimental approach: missing-energy analysis
- Signal more clear at e^+e^- than at hadron colliders
- Near-future plans:
  - ILC (international)
  - CEPC (China)
  - FCC-ee (Europe)
  - CLIC (Europe)

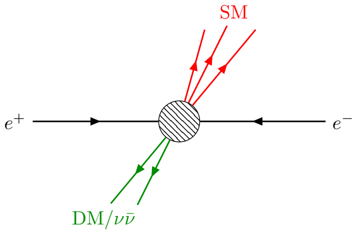
$\}$ $\sqrt{s} \approx 250$ GeV

ILC 1903.01629

CEPC 1811.10545

FCC CERN-ACC-2018-0057

CLIC 1812.06018



Introduction: our approach

- 3 simple (but not simplified) DM models of different spins: 0, 1, 1/2
- Consider a mechanism of DM production at e^+e^- colliders
- Take current experimental constraints into account
- Check...
  - ...what range of parameters is still allowed?
  - ...how many DM-production events can we expect at future colliders*?
  - ...whether it is possible to disentangle models of different spins?

*during calculations, collider parameters of ILC are used

Theoretical models

The main properties of the models:

- Simple, but renormalizable and QFT-consistent
- Common parameter space
- DM connected to SM by the Higgs portal: $\kappa|H|^2|S|^2$
- Mixing in the scalar sector

real-part fluctuations of a new singlet S and the Higgs doublet H

$$\downarrow \times \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

two Higgs-like mass eigenstates: h_1 ($m_1 = 125$ GeV) and h_2 ($m_2 =?$)

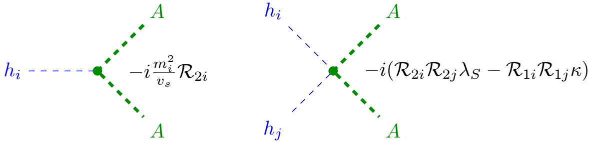
The models:

- pseudo-Goldstone (scalar) DM model ($s = 0$) → dark scalar A
- vector DM model ($s = 1$) → dark gauge vector X_μ
- fermion DM model ($s = 1/2$) → dark Majorana fermion ψ

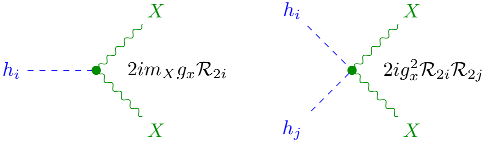
Theoretical models

DM interactions

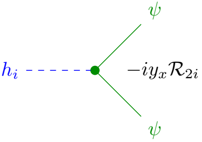
- pGDM



- VDM



- FDM

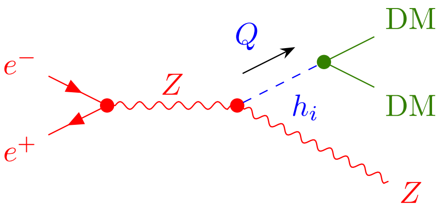


Input parameters: $\underbrace{v,\ m_1}_{\text{assumed to be SM-like}},\ v_s,\ m_2,\ \sin\alpha,\ m_{\text{DM}} \longrightarrow \kappa \equiv \frac{(m_1^2 - m_2^2)\sin 2\alpha}{2vv_s}$

$\Downarrow$

DM purely gravitational if $m_1 = m_2$

Considered process



$$Q^2 = s - 2E_Z\sqrt{s} + m_Z^2 \equiv m_{\text{rec}}^2$$

$$\text{DM} = A,\ X_\mu,\ \psi$$

note: in principle $\Gamma_{1,2}$ model-dependent

the differential cross-section:

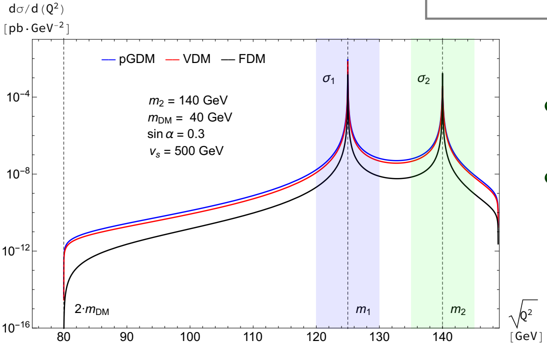
$$\frac{d\sigma}{dQ^2} \sim \sigma_{\text{SM}}(Q^2) \cdot \sin^2\alpha\cos^2\alpha \cdot \left|\frac{1}{Q^2 - m_1^2 + im_1\Gamma_1} - \frac{1}{Q^2 - m_2^2 + im_2\Gamma_2}\right|^2$$

$$\times \sqrt{1 - 4\frac{m_{\text{DM}}^2}{Q^2}} \cdot \begin{cases} 1 & \text{(pGDM)} \\ 1 - 4\frac{m_{\text{DM}}^2}{Q^2} + 12\left(\frac{m_{\text{DM}}^2}{Q^2}\right)^2 & \text{(VDM)} \\ 2\frac{m_{\text{DM}}^2}{Q^2}\left(1 - 4\frac{m_{\text{DM}}^2}{Q^2}\right) & \text{(FDM)} \end{cases}$$

Considered process: shape of the differential cross section

$$\frac{d\sigma}{dQ^2} \sim \sigma_{\mathsf{SM}}(Q^2)\cdot \sin^2\alpha\cos^2\alpha \cdot \left|\frac{1}{Q^2-m_1^2+im_1\Gamma_1}-\frac{1}{Q^2-m_2^2+im_2\Gamma_2}\right|^2$$

$$\times\sqrt{1-4\frac{m_{\mathsf{DM}}^2}{Q^2}}\cdot\begin{cases}1 & (\mathsf{pGDM})\\ 1-4\frac{m_{\mathsf{DM}}^2}{Q^2}+12\left(\frac{m_{\mathsf{DM}}^2}{Q^2}\right)^2 & (\mathsf{VDM})\\ 2\frac{m_{\mathsf{DM}}^2}{Q^2}\left(1-4\frac{m_{\mathsf{DM}}^2}{Q^2}\right) & (\mathsf{FDM})\end{cases}$$



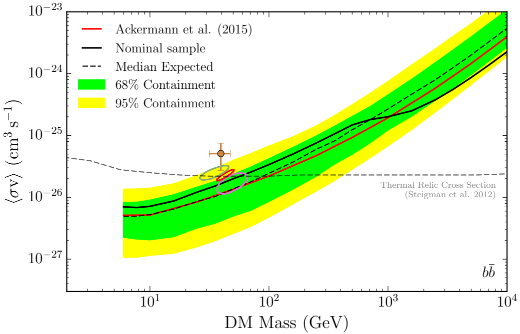
- Shape fitting
  $\Rightarrow$ spin hard to check
- Total σ as a function of $\sqrt{s}$
  ($\sqrt{Q^2} = 2m_{\mathsf{DM}}$ threshold)
  $\Rightarrow$ mass of DM possible to determine

after integration: $\sigma \approx \sigma_1 \cdot \mathbb{1}_{2\,m_{\mathsf{DM}}<m_1<\sqrt{s}-m_Z} + \sigma_2\cdot \mathbb{1}_{2\,m_{\mathsf{DM}}<m_2<\sqrt{s}-m_Z}$

$\sigma_1 \equiv \sigma_{\mathsf{SM}}(m_1)\cdot\cos^2\alpha\cdot \mathsf{BR}(h_1\to \mathsf{DM})$ $\sigma_2 \equiv \sigma_{\mathsf{SM}}(m_2)\cdot\sin^2\alpha\cdot \mathsf{BR}(h_2\to \mathsf{DM})$

Current limits and constraints

1. Perturbativity (h_i-DM): $g_x, y_x < 4\pi \;\Rightarrow\; v_s > m_{\text{DM}}/4\pi$
2. $h^2\,\Omega_0^{\text{DM}} = 0.12 \pm 0.0012 \;\Rightarrow$ constraint on $\langle \sigma v \rangle_{\text{ann}} \;\Rightarrow$ constraint on κ^2 — Planck 1807.06209
3. ID experiments $\Rightarrow$ limit on $\langle \sigma v \rangle_{\text{ann}} \Rightarrow$ limit on m_{DM} — Fermi-LAT 1611.03184



4. DD experiments and ν telescopes $\Rightarrow$ limit on $\sigma^{\text{DD}} \;\Rightarrow$ limit on κ^2 — XENON1T 1805.12562, IceCube 1601.00653
5. LHC $\Rightarrow$ $|\sin\alpha| \lesssim 0.3$ — Robens, Stefaniak 1601.07880
6. LHC $\Rightarrow$ BR$(h_1 \to$ DM$) < 19\%$ — CMS 1809.05937

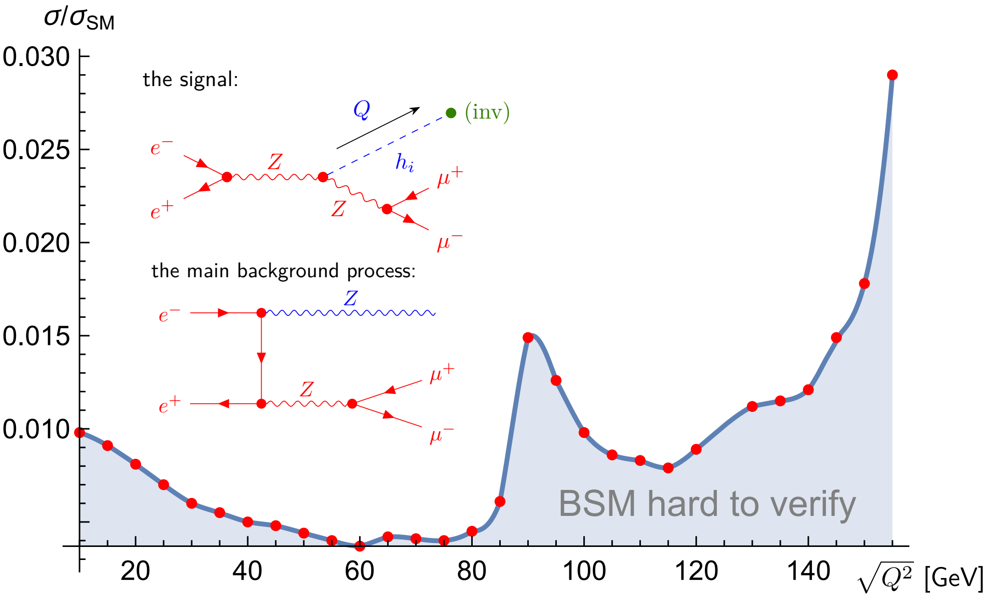
Results: methodology

- Constraints:
  - $|\sin\alpha| \lesssim 0.3$
  - relic density (Ω_0^{DM}) constraint on κ^2
  - $v_s < \frac{m_{\text{DM}}}{4\pi}$
  - direct detection (DD) limit on κ^2
  - $\text{BR}(h_1 \to \text{DM}) < 19\%$

  } taken into account numerically
- Free parameters: m_2, m_{DM}, $\sin\alpha$, $v_s \to \kappa^2$
- $\kappa^2 = \kappa^2(\Omega_0^{\text{DM}}) \Rightarrow$ 3 parameters left
- Cross section maximized with respect to $\sin\alpha$
  $\Rightarrow \sin\alpha \to \sin\alpha_{\max} = 0.3$, free parameters: m_2, m_{DM}
- Low $\sqrt{s}$ gives high cross section
  (ILC: $\sqrt{s} = 250$ GeV, 20-year luminosity: $\int \mathcal{L}\, dt = 2000\ \text{fb}^{-1}$)

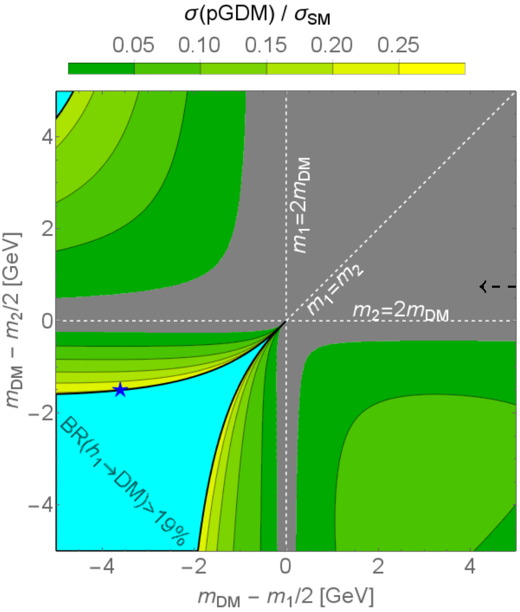
ILC: 1903.01629

The 95% CL sensitivity limit for ILC



(see also Krzysztof Mękała's talk: *CLIC sensitivity to invisible scalar decays*)

Results: maximal cross section for pGDM



parameters of ⋆

$m_2 = 120.8$ GeV

$m_{\text{DM}} = 58.9$ GeV

$\sin\alpha = 0.30$

$v_s = 646$ GeV

$\Gamma_1 = 7.4 \cdot 10^{-3}$ GeV

$\Gamma_2 = 9.8 \cdot 10^{-3}$ GeV

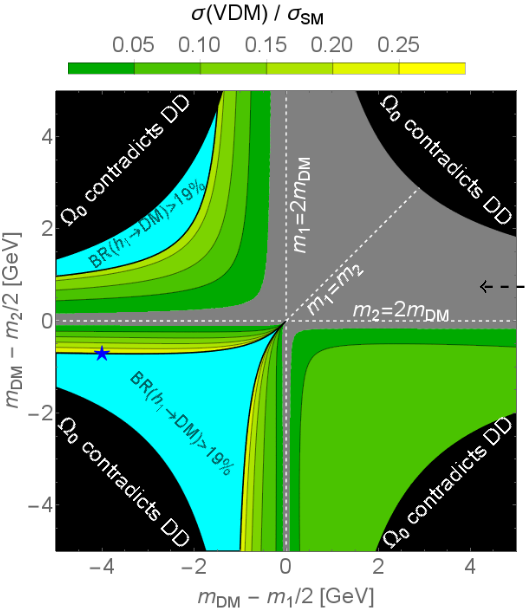
$\text{BR}(h_1 \to \text{DM}) = 19\%$

$\text{BR}(h_2 \to \text{DM}) = 95\%$

$\sigma = 62 \text{ fb} \to 1.24 \cdot 10^5$ events

below the 95% CL limit for the ILC

1903.01629



parameters of ⋆

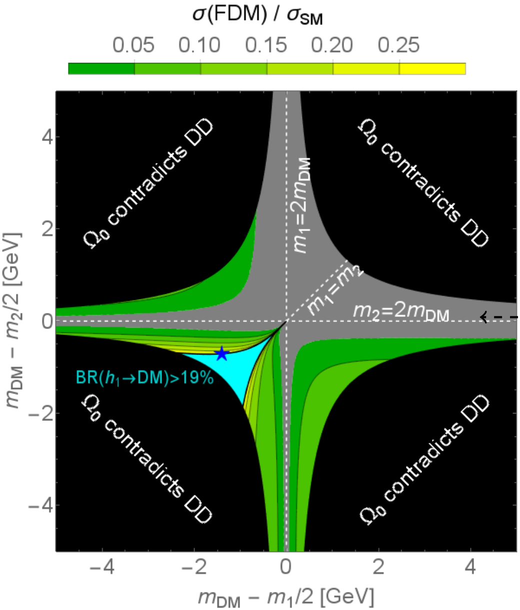
$m_2 = 118.4$ GeV
$m_{\text{DM}} = 58.5$ GeV
$\sin\alpha = 0.30$
$v_s = 561$ GeV

$\Gamma_1 = 7.4 \cdot 10^{-3}$ GeV
$\Gamma_2 = 6.4 \cdot 10^{-3}$ GeV
BR$(h_1 \to$ DM$) = 18\%$
BR$(h_2 \to$ DM$) = 92\%$

$\sigma = 61$ fb $\longrightarrow 1.22 \cdot 10^5$ events

below the 95% CL limit for the ILC
1903.01629

Results: maximal cross section for FDM



parameters of ⋆

$m_2 = 123.6$ GeV

$m_{\text{DM}} = 61.1$ GeV

$\sin\alpha = 0.30$

$v_s = 76$ GeV

$\Gamma_1 = 7.4 \cdot 10^{-3}$ GeV

$\Gamma_2 = 5.9 \cdot 10^{-3}$ GeV

$\text{BR}(h_1 \to \text{DM}) = 18\%$

$\text{BR}(h_2 \to \text{DM}) = 91\%$

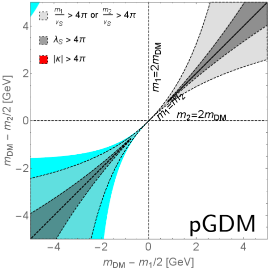
$\sigma = 59$ fb $\longrightarrow 1.18 \cdot 10^5$ events

below the 95% CL limit for the ILC

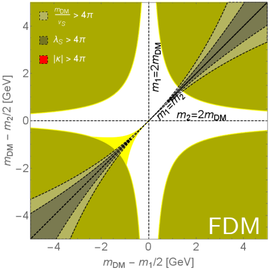
1903.01629

Results: allowed and forbidden regions

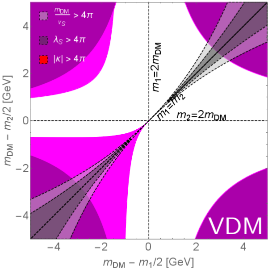
White: pGDM allowed,
Cyan: pGDM forbidden ($BR_{h_1 \to DM} > 19\%$)



White: FDM allowed, Yellow: FDM forbidden
(light: $BR_{h_1 \to DM}$>19%, dark: σ_{DD} too large)

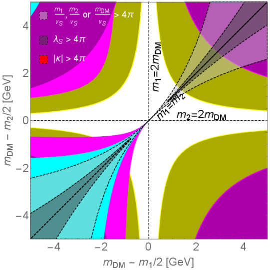


White: VDM allowed, Magenta: VDM forbidden
(light: $BR_{h_1 \to DM}$>19%, dark: σ_{DD} too large)



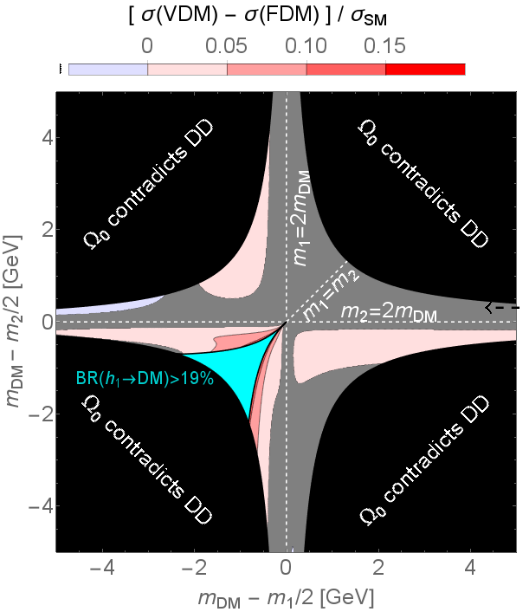
all plots combined:

Yellow: FDM forbidden, pGDM and VDM allowed
Magenta: FDM and VDM forbidden, pGDM allowed
Cyan: all forbidden, White: all allowed



$\Rightarrow$ there exist regions where

- only pGDM and VDM are allowed
- only pGDM is allowed
- nothing is allowed



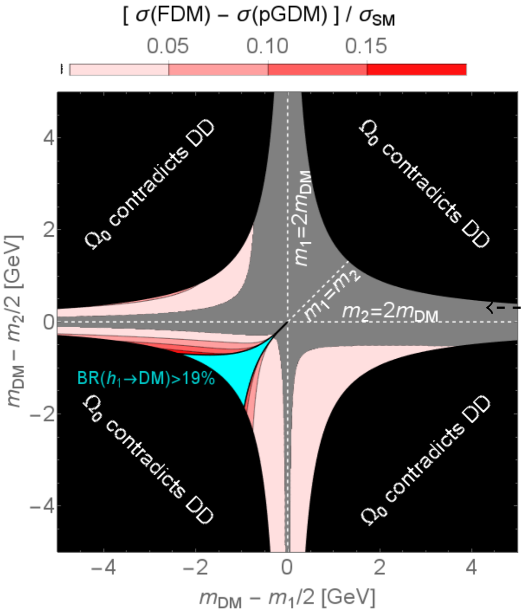
below the 95% CL limit for the ILC

1903.01629

⇓

models are indistinguishable

Results: pseudo-Goldstone vs fermion



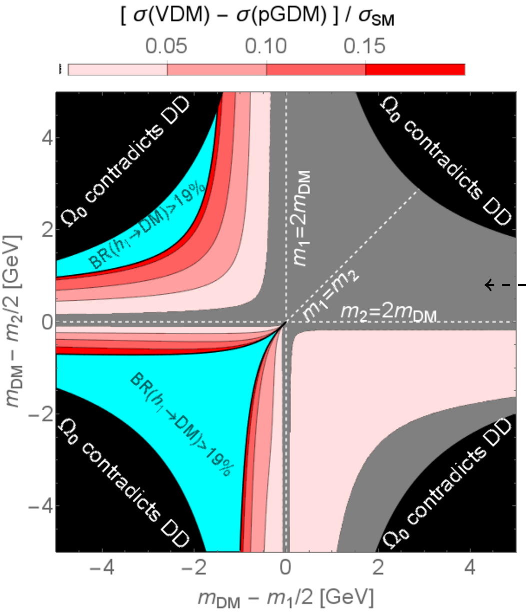
below the 95% CL limit for the ILC

1903.01629

⇓

models are indistinguishable

Results: pseudo-Goldstone vs vector



below the 95% CL limit for the ILC

1903.01629

⇓

models are indistinguishable

Summary

We introduce 3 simple models:

- pseudo-Goldstone dark matter model ($s = 0$)
- vector dark matter model ($s = 1$)
- fermion dark matter model ($s = 1/2$)

which are QFT-consistent and share common parameter space

Comparing models of various spins we conclude that...

- ...maximal cross section is similar in all 3 cases ($\sim 1.2 \cdot 10^5$ events / 20 y)
- ...signal-to-background ratio can be $\sim 10\%$
- ...allowed range of parameters is largest for pGDM
- ...there are regions where cases of different spins could be disentangled

But...

- ...if parameters far from optimal, DM of any spin hard to be detected

Summary

We introduce 3 simple models:

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But...

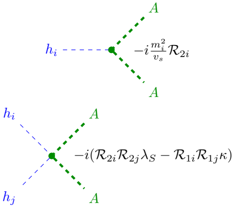
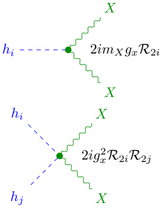
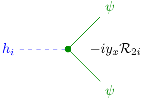
- ...if parameters far from optimal, DM of any spin hard to be detected

Thank you!

a lot of

BACKUP SLIDES

Theoretical models – comparison

	PSEUDO-GOLDSTONE DM MODEL	VECTOR DM MODEL	FERMION DM MODEL
symmetry group	$\mathcal{G} = \mathcal{G}_{\mathsf{SM}} \times \mathbb{Z}_2 \times U(1)_X$ (↰ global, softly broken)	$\mathcal{G} = \mathcal{G}_{\mathsf{SM}} \times U(1)_X$ (↰ local)	$\mathcal{G} = \mathcal{G}_{\mathsf{SM}} \times \mathbb{Z}_4$
new states ($\mathcal{G}_{\mathsf{SM}}$-even)	complex scalar S $\quad (q = (1,1))$	gauge vector X_μ complex scalar S $\quad (q = 1)$	LH fermion χ $\quad (q = 1)$ real scalar S $\quad (q = 2)$
Lagrangian	$\mathcal{L} = \mathcal{L}_{\mathsf{SM}} + \mathcal{L}_{\mathsf{DM}} - V(H,S)$		
scalar potential	$V(H,S) = -\mu_H^2\|H\|^2 + \lambda_H\|H\|^4 - \mu_S^2\|S\|^2 + \lambda_S\|S\|^4 + \kappa\|H\|^2\|S\|^2 + \mu^2(S^2 + S^{*2})$	$V(H,S) = -\mu_H^2\|H\|^2 + \lambda_H\|H\|^4 - \mu_S^2\|S\|^2 + \lambda_S\|S\|^4 + \kappa\|H\|^2\|S\|^2$	$V(H,S) = -\mu_H^2\|H\|^2 + \lambda_H\|H\|^4 - \frac{\mu_S^2}{2}S^2 + \frac{\lambda_S}{4}S^4 + \frac{\kappa}{2}\|H\|^2 S^2$
SSB	$H \to (\pi^+, v + h + i\pi^0)^T/\sqrt{2}$ $S \to (v_s + \phi + iA)/\sqrt{2}$	$H \to (\pi^+, v + h + i\pi^0)^T/\sqrt{2}$ $S \to (v_s + \phi + i\sigma)/\sqrt{2}$	$H \to (\pi^+, v + h + i\pi^0)^T/\sqrt{2}$ $S \to v_s + \phi$
Higgs sector mixing	$\begin{pmatrix} h \\ \phi \end{pmatrix} = \mathcal{R}\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}\begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$ $\quad \operatorname{tg} 2\alpha = \frac{\kappa v v_s}{\lambda_H v^2 - \lambda_S v_s^2}$		
dark state	$A \equiv \sqrt{2}\,\mathcal{I}m\,S \quad (m_A^2 = -4\mu^2)$	$X_\mu \quad (m_X^2 = g_x^2 v_s^2)$	$\psi \equiv \chi + \chi^c \quad (m_\psi^2 = y_x^2 v_s^2)$
dark matter interactions			

input parameters: $\underbrace{v,\ m_1,}_{\text{assumed to be SM-like}}$ v_s, m_2, $\sin\alpha$, m_{DM} $\longrightarrow$ $\kappa \equiv \frac{(m_1^2 - m_2^2)\sin 2\alpha}{2 v v_s}$ $\Rightarrow$ DM purely gravitational if $m_1 = m_2$

Pseudo-Goldstone DM model

- Gauge group: $\mathcal{G} = \underbrace{SU(3)_c \times SU(2)_L \times U(1)_Y}_{\text{Standard Model gauge group}} \times \mathbb{Z}_2 \times \overbrace{U(1)_X}^{\text{global symmetry}}$
- $U(1)_X$ softly broken to $\mathbb{Z}_2'$
- Complex scalar S introduced, $\mathbb{Z}_2 : S \to S^*$, $\mathbb{Z}_2' : S \to -S$
- Lagrangian

$$\mathcal{L} = \mathcal{L}_{\mathsf{SM}} + (\partial^\mu S)^*(\partial_\mu S) - V(H,S)$$

$$V(H,S) = -\mu_H^2|H|^2 + \lambda_H|H|^4 - \mu_S^2|S|^2 + \lambda_S|S|^4 + \underbrace{\kappa|H|^2|S|^2}_{\text{Higgs portal coupling}} + \overbrace{\mu^2(S^2 + S^{*2})}^{U(1)_X\text{-breaking term}}$$

$$V(H,S) = -\mu_H^2|H|^2 + \lambda_H|H|^4 - \mu_S^2|S|^2 + \lambda_S|S|^4 + \underbrace{\kappa|H|^2|S|^2}_{\text{Higgs portal coupling}} + \overbrace{\mu^2(S^2 + S^{*2})}^{U(1)_X\text{-breaking term}}$$

- SSB $\quad H \to (\pi^+, v + h + i\pi^0)^T/\sqrt{2} \qquad S \to (v_s + \phi + iA)/\sqrt{2}$
- We introduce h_1, h_2

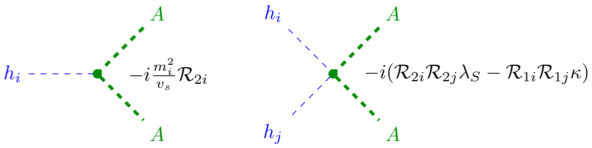
$$\mathrm{tg}\,2\alpha = \kappa v v_s/(\lambda_H v^2 - \lambda_S v_s^2)$$

$$\begin{pmatrix} h \\ \phi \end{pmatrix} = \underbrace{\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}}_{\mathcal{R}} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$$

$$m_1^2 = \lambda_H v^2(1 + \sec 2\alpha) + \lambda_S v_s^2(1 - \sec 2\alpha)$$
$$m_2^2 = \lambda_H v^2(1 - \sec 2\alpha) + \lambda_S v_s^2(1 + \sec 2\alpha)$$

$$V(H,S) \longrightarrow \frac{1}{2}m_1^2 h_1^2 + \frac{1}{2}m_2^2 h_2^2 - 2\mu^2 A^2 + \frac{m_1^2\sin^2\alpha + m_2^2\cos^2\alpha}{8v_s^2}A^4$$
$$- \sum_{i=1,2}\frac{1}{2}\frac{m_i^2\mathcal{R}_{2i}}{v_s}A^2 h_i + \sum_{i,j=1,2}\frac{1}{4}(\mathcal{R}_{2i}\mathcal{R}_{2j}\lambda_s - \mathcal{R}_{1i}\mathcal{R}_{1j}\kappa)A^2 h_i h_j$$
$$+ \text{const} + \mathcal{O}\left([h_1, h_2]^3\right)$$

- $A \equiv \sqrt{2}\,\mathcal{I}m\,(S)$ is a massive dark particle, $m_A^2 = -4\mu^2$



- Gauge group: $\mathcal{G} = \underbrace{SU(3)_c \times SU(2)_L \times U(1)_Y}_{\text{Standard Model gauge group}} \times \overbrace{U(1)_X}^{\text{gauge symmetry}}$
- $U(1)_X$ gauge vector X_μ and complex scalar S introduced
- Discrete $\mathbb{Z}_2$ symmetry: $X_\mu \to -X_\mu$, $S \to S^*$
  $\Rightarrow$ no kinetic mixing with SM fields
- Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{DM}} - V(H,S)$$

$$\mathcal{L}_{\text{DM}} = -\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + (\mathcal{D}^\mu S)^*(\mathcal{D}_\mu S)$$

$$V(H,S) = -\mu_H^2|H|^2 + \lambda_H|H|^4 - \mu_S^2|S|^2 + \lambda_S|S|^4 + \underbrace{\kappa|H|^2|S|^2}_{\text{Higgs portal coupling}}$$

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- SSB $\quad H \to (\pi^+, v + h + i\pi^0)^T/\sqrt{2} \qquad S \to (v_s + \phi + i\sigma)/\sqrt{2}$
- We introduce h_1, h_2

$$\begin{pmatrix} h \\ \phi \end{pmatrix} = \underbrace{\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}}_{\mathcal{R}} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$$

$$\mathrm{tg}\, 2\alpha = \kappa v v_s / (\lambda_H v^2 - \lambda_S v_s^2)$$

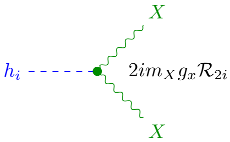
$$m_1^2 = \lambda_H v^2 (1 + \sec 2\alpha) + \lambda_S v_s^2 (1 - \sec 2\alpha)$$

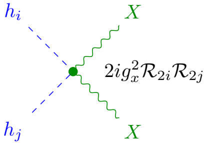
$$m_2^2 = \lambda_H v^2 (1 - \sec 2\alpha) + \lambda_S v_s^2 (1 + \sec 2\alpha)$$

$$\mathcal{L}_{\mathrm{DM}} \longrightarrow -\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + \frac{1}{2}\partial^{\mu}\phi\,\partial_{\mu}\phi + \frac{g_x^2 v_s^2}{2}X^{\mu}X_{\mu} + \frac{g_x^2}{2}X^{\mu}X_{\mu}\phi^2 + g_x^2 v_s X^{\mu}X_{\mu}\phi$$

$$V(H,S) \longrightarrow \frac{1}{2}m_1^2 h_1^2 + \frac{1}{2}m_2^2 h_2^2 + \text{const} + \mathcal{O}\left([\text{field}]^3\right)$$

- X_μ is a massive dark particle, $m_X^2 = g_x^2 v_s^2$





Fermion DM model

- Gauge group: $\mathcal{G} = \underbrace{SU(3)_c \times SU(2)_L \times U(1)_Y}_{\text{Standard Model gauge group}} \times \mathbb{Z}_4$
- Left-handed fermion χ and real scalar S introduced,
  $\mathbb{Z}_4 : \chi \to i\chi \,, \; S \to -S$
- Lagrangian

$$\mathcal{L} = \mathcal{L}_{\mathsf{SM}} + \mathcal{L}_{\mathsf{DM}} - V(H, S)$$

$$\mathcal{L}_{\mathsf{DM}} = i\bar{\chi}\partial\!\!\!/\chi + \frac{1}{2}\partial^\mu S\, \partial_\mu S - \frac{y_x}{2}(\bar{\chi^c}\chi + \bar{\chi}\chi^c)S$$

$$V(H, S) = -\mu_H^2 |H|^2 + \lambda_H |H|^4 - \frac{\mu_S^2}{2} S^2 + \frac{\lambda_S}{4} S^4 + \underbrace{\frac{\kappa}{2}|H|^2 S^2}_{\text{Higgs portal coupling}}$$

$$\mathcal{L}_{\mathsf{DM}} = i\bar{\chi}\partial\!\!\!/\chi + \frac{1}{2}\partial^\mu S\,\partial_\mu S - \frac{y_x}{2}(\bar{\chi^c}\chi + \bar{\chi}\chi^c)S$$

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- SSB $\quad H \to (\pi^+, v + h + i\pi^0)^T/\sqrt{2} \qquad S \to v_s + \phi$
- We introduce h_1, h_2

$$\begin{pmatrix} h \\ \phi \end{pmatrix} = \underbrace{\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}}_{\mathcal{R}} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$$

$$\mathrm{tg}\,2\alpha = \kappa v v_s/(\lambda_H v^2 - \lambda_S v_s^2)$$

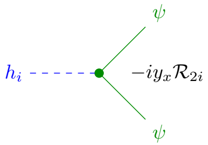
$$m_1^2 = \lambda_H v^2(1 + \sec 2\alpha) + \lambda_S v_s^2(1 - \sec 2\alpha)$$

$$m_2^2 = \lambda_H v^2(1 - \sec 2\alpha) + \lambda_S v_s^2(1 + \sec 2\alpha)$$

$$\mathcal{L}_{\mathsf{DM}} \longrightarrow \frac{i}{2}\bar{\psi}\partial\!\!\!/\psi + \frac{1}{2}\partial^\mu\phi\,\partial_\mu\phi - \frac{y_x v_s}{2}\bar{\psi}\psi - \frac{y_x}{2}\bar{\psi}\psi\phi$$

$$V(H,S) \longrightarrow \frac{1}{2}m_1^2h_1^2 + \frac{1}{2}m_2^2h_2^2 + \text{const} + \mathcal{O}\left([\text{field}]^3\right)$$

- Majorana fermion $\psi \equiv \chi + \chi^c$ is a massive dark particle, $m_\psi^2 = y_x^2 v_s^2$



Stability conditions

$$V_{\text{pGDM}}(H,S) = -\mu_H^2|H|^2 + \lambda_H|H|^4 - \mu_S^2|S|^2 + \lambda_S|S|^4 + \kappa|H|^2|S|^2 + \mu^2(S^2 + S^{*2})$$
$$V_{\text{VDM}}(H,S) = -\mu_H^2|H|^2 + \lambda_H|H|^4 - \mu_S^2|S|^2 + \lambda_S|S|^4 + \kappa|H|^2|S|^2$$
$$V_{\text{FDM}}(H,S) = -\mu_H^2|H|^2 + \lambda_H|H|^4 - \frac{\mu_S^2}{2}S^2 + \frac{\lambda_S}{4}S^4 + \frac{\kappa}{2}|H|^2S^2$$

- asymptotic positivity of the scalar potential
  - $\lambda_S > 0\,, \quad \lambda_H > 0$
  - $\kappa > -2\sqrt{\lambda_H \lambda_S}$ ← always satisfied:

$$\kappa = \frac{(m_1^2 - m_2^2)\sin 2\alpha}{2vv_s} > -\sqrt{\left[\frac{(m_1^2 - m_2^2)\sin 2\alpha}{2vv_s}\right]^2 + \frac{m_1^2 m_2^2}{v^2 v_s^2}} = -2\sqrt{\lambda_H \lambda_S}$$

- minimum at $v, v_s > 0$ (μ^2 only for pGDM):
  - $\kappa^2 < 4\lambda_H h \lambda_S$
  - $\mu^2 < 0$
  - $2\lambda_S \mu_H^2 - \kappa(\mu_S^2 - 2\mu^2) > 0$
  - $2\lambda_H(\mu_S^2 - 2\mu^2) - \kappa\mu_H^2 > 0$

Input parameters

$$V_{\text{pGDM}}(H,S) = -\mu_H^2|H|^2 + \lambda_H|H|^4 - \mu_S^2|S|^2 + \lambda_S|S|^4 + \kappa|H|^2|S|^2 + \mu^2(S^2 + S^{*2})$$

$$V_{\text{VDM}}(H,S) = -\mu_H^2|H|^2 + \lambda_H|H|^4 - \mu_S^2|S|^2 + \lambda_S|S|^4 + \kappa|H|^2|S|^2$$

$$V_{\text{FDM}}(H,S) = -\mu_H^2|H|^2 + \lambda_H|H|^4 - \frac{\mu_S^2}{2}S^2 + \frac{\lambda_S}{4}S^4 + \frac{\kappa}{2}|H|^2S^2$$

- Input parameters: $\underbrace{v,\ m_1,}_{\text{assumed to be SM-like}}$ v_s, m_2, $\sin\alpha$, m_{DM}
- Other parameters of the models in terms of the input parameters:

$$\mu^2 = -\frac{1}{4}m_{\text{DM}}^2 \text{ (pGDM)} \qquad g_x = \frac{m_{\text{DM}}}{v_s} \text{ (VDM)} \qquad y_x = \frac{m_{\text{DM}}}{v_s} \text{ (FDM)}$$

$$\lambda_H = \frac{m_1^2\cos^2\alpha + m_2^2\sin^2\alpha}{2v^2} \qquad \lambda_S = \frac{m_1^2\sin^2\alpha + m_2^2\cos^2\alpha}{2v_s^2}$$

$$\mu_H^2 = \frac{1}{2}m_1^2\cos^2\alpha + \frac{1}{2}m_2^2\sin^2\alpha + \frac{1}{4}\frac{v_s}{v}(m_1^2 - m_2^2)\sin 2\alpha$$

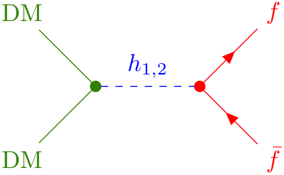
$$\mu_S^2 = \frac{1}{2}m_1^2\sin^2\alpha + \frac{1}{2}m_2^2\cos^2\alpha + \frac{1}{4}\frac{v}{v_s}(m_1^2 - m_2^2)\sin 2\alpha$$

$$\kappa = \frac{(m_1^2 - m_2^2)\sin 2\alpha}{2vv_s}$$

- $m_2 = m_1 \Rightarrow \kappa = 0 \Rightarrow$ no Higgs portal $\Rightarrow$ DM completely decoupled

Relic density constraint on indirect detection

Steigman et al. 1204.3622



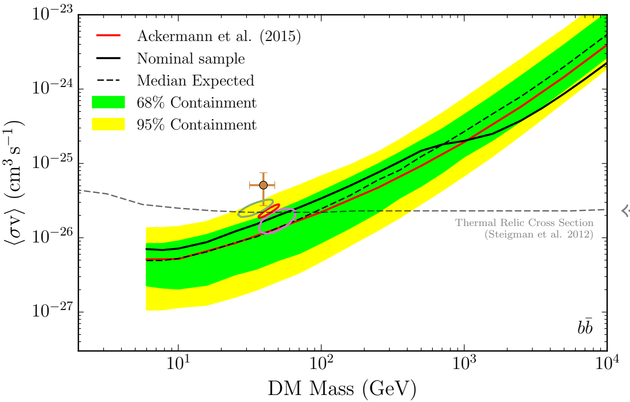
$$\langle\sigma v\rangle^{\text{ID}}_{f\bar{f}} = \frac{m_{\text{DM}} m_f^2}{\pi} \frac{\kappa^2 \cdot (m_{\text{DM}}^2 - m_f^2)^{3/2}}{D_1 D_2} \times \longleftarrow D_i = (4m_{\text{DM}}^2 - m_i)^2 + m_i^2\Gamma_i^2$$

$$\times \begin{cases} 12 & \text{pGDM} \\ 1 & \text{VDM} \\ \frac{9}{4}\left(\frac{m_{\text{DM}}}{T}\right)^{-1} & \text{FDM} \end{cases} + [\text{higher orders in } (m_{\text{DM}}/T)^{-1}]$$

m_f^2 factor $\Rightarrow$ $b\bar{b}$ contribution dominates.

- If $\langle\sigma v\rangle^{\text{ID}} = \sigma_0 (m_{\text{DM}}/T)^{-n}$ then $h^2\Omega_0^{\text{DM}} \sim (n+1)(m_{\text{DM}}/T_f)^{n+1}/\sigma_0$
- Correct $h^2\Omega_0^{\text{DM}} \longleftrightarrow \langle\sigma v\rangle\big|_{\text{now}} = (T_0/T_f)^n \cdot \underbrace{(n+1)\cdot 1.9\cdot 10^{-9}\ \text{GeV}^{-2}}_{\langle\sigma v\rangle|_{\text{freeze out}}}$
- $T_f \sim m_{\text{DM}}/25$, hence

$$\kappa^2 = 3.5\cdot 10^{-10}\ \text{GeV}^{-4} \frac{D_1 D_2}{m_{\text{DM}}(m_{\text{DM}}^2 - m_b^2)^{3/2}} \cdot \begin{cases} 1/12 & \text{pGDM} \\ 1 & \text{VDM} \\ \frac{8m_\psi}{9T_f} \approx 22 & \text{FDM} \end{cases}$$



value of $\langle\sigma v\rangle\big|_{\text{freeze out}}$ corresponding to correct Ω_0^{DM}

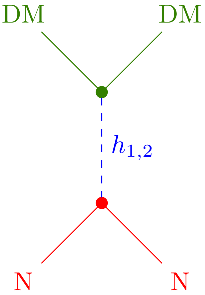
$$\langle\sigma v\rangle\big|_{\text{now}} = (T_0/T_f)^n \cdot \langle\sigma v\rangle\big|_{\text{freeze out}}$$

- pGDM, VDM: $n = 0 \;\Rightarrow\; m_{\text{DM}} \gtrsim 30$ GeV
- FDM: $n = 1 \;\Rightarrow\; \langle\sigma v\rangle\big|_{\text{now}}$ orders of magnitude lower than the limit

Direct detection limit

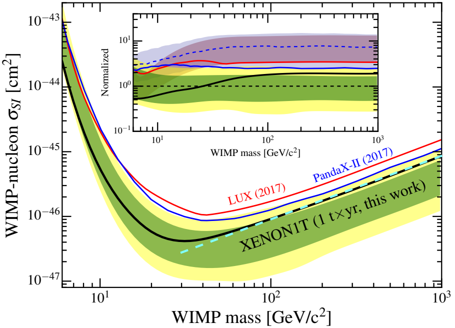
$\sigma_{\text{SD}} = 0$ (no axial couplings in our models)

$$\sigma_{\text{SI}} \sim \kappa^2 \cdot \frac{m_{\text{DM}}^2}{m_2^4} \begin{cases} \left[\frac{\mathcal{A}}{64\pi^2 v v_s^2}\right]^2 & \text{(pGDM)} \\ 1 & \text{(VDM), (FDM)} \end{cases}$$



$\mathcal{A}$ – combination of 1-loop integrals

(the tree level vanishes for pGDM)



$$\frac{\sigma_{\text{SI}}}{1\ \text{cm}^2} \lesssim \frac{m_{\text{DM}}}{1\ \text{GeV}} \cdot 10^{-48.05}$$

$$\Downarrow$$

$$\kappa^2 < \frac{m_2^4}{m_{\text{DM}}} \cdot 2.5 \cdot 10^{-11}\ \text{GeV}^{-3} \times \begin{cases} \left[\frac{\mathcal{A}}{64\pi^2 v v_s^2}\right]^{-2} & \text{(pGDM)} \\ 1 & \text{(VDM), (FDM)} \end{cases}$$

Direct detection limit

XENON1T 1805.12562

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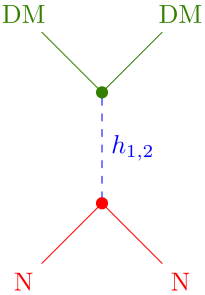

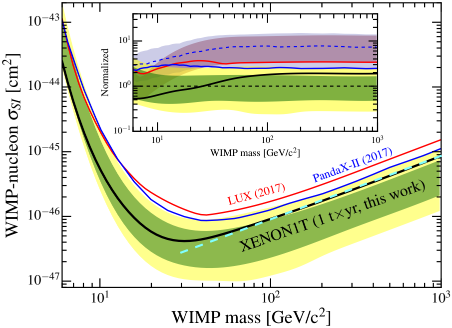

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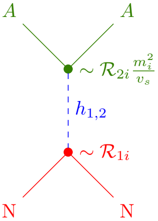
$$\times \begin{cases} \left[\frac{\mathcal{A}}{64\pi^2 v v_s^2}\right]^{-2} & \text{(pGDM)} \\ 1 & \text{(VDM), (FDM)} \end{cases}$$

huge!

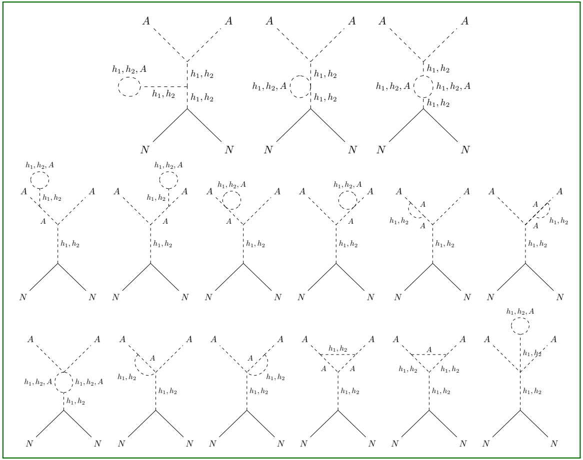


the tree level:

$$\sigma \sim \frac{(\cos\alpha\sin\alpha)^2}{v_s^2}\left|\frac{m_1^2}{Q^2-m_1^2}-\frac{m_2^2}{Q^2-m_2^2}\right|^2 \xrightarrow{Q^2\to 0} 0$$

the 1-loop level:

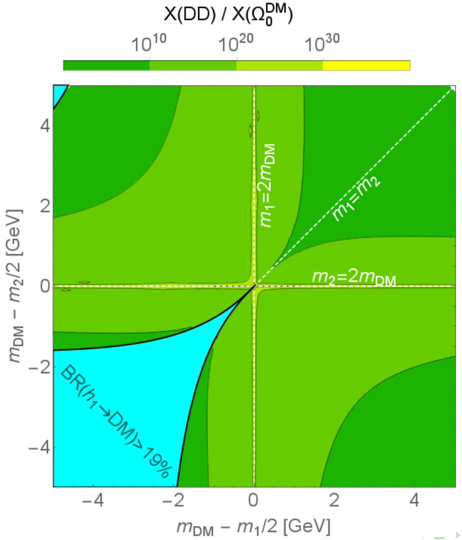
$$\sigma \sim \mathcal{A}^2 \longleftarrow$$



DD vs $\Omega_0^{\rm DM}$ limit for pGDM

huge!

$$\kappa^2(\text{DD}) \equiv \frac{m_2^4}{m_{\text{DM}}} \cdot 2.5 \cdot 10^{-11}\ \text{GeV}^{-3} \begin{cases} \left[\frac{\mathcal{A}}{64\pi^2 v v_s^2}\right]^{-2} & (\text{pGDM}) \\ 1 & (\text{VDM}), (\text{FDM}) \end{cases}$$



- DM annihilation rate in the Sun: Γ_A
- Capture rate: C_C
- Change of DM's amount N in the Sun

$$\dot{N} = C_C - C_A N^2$$

where $C_A \equiv 2 \cdot \Gamma_A / N^2 = \text{const}$

- Solution

$$N(t) = \sqrt{C_C/C_A} \cdot \tanh(t\sqrt{C_C C_A})$$

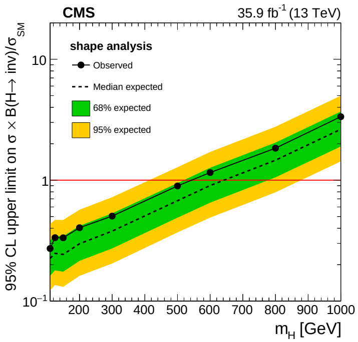
- For large t

$$N(t) \to \sqrt{C_C/C_A}$$

$$\Rightarrow \quad \boxed{\Gamma_A = \frac{1}{2} C_C}$$

- C_C expressible in terms of DM-nucleon cross section
- Γ_A calculable from muon flux measurements

Limits on the invisible branching ratio



$$\sigma \approx \sigma_1 \cdot \mathbb{1}_{2\,m_{\mathsf{DM}} < m_1 < \sqrt{s} - m_Z} + \sigma_2 \cdot \mathbb{1}_{2\,m_{\mathsf{DM}} < m_2 < \sqrt{s} - m_Z}$$

$$\sigma_1 \equiv \sigma_{\mathsf{SM}}(m_1) \cdot \cos^2\alpha \cdot \mathsf{BR}(h_1 \to \mathsf{DM})$$

$$\sigma_2 \equiv \sigma_{\mathsf{SM}}(m_2) \cdot \sin^2\alpha \cdot \mathsf{BR}(h_2 \to \mathsf{DM})$$

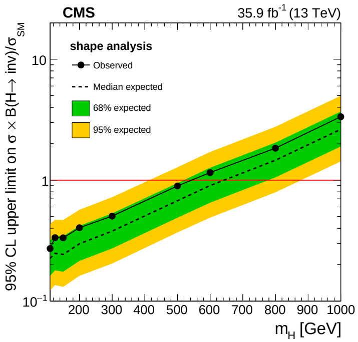
Conditions:

$$(1) \qquad \sigma_1 < 0.19\,\sigma_{\mathsf{SM}}(m_1)$$

$$(2) \qquad \log\left[\frac{\sigma_2}{\sigma_{\mathsf{SM}}(m_2)}\right] < 0.0011 \cdot \frac{m_2}{1\ \mathsf{GeV}} - 0.63 \qquad (\sqrt{s} = 13\ \mathsf{TeV})$$

For $\sin\alpha < 0.3$ condition (2) always satisfied:

$$\left.\frac{\sigma_2}{\sigma_{\mathsf{SM}}(m_2)}\right|_{\sqrt{s}=13\ \mathsf{TeV}} = \sin^2\alpha \cdot \mathsf{BR}(h_2 \to \mathsf{DM}) < 10^{-1} < 10^{\,0.0011\cdot\frac{m_2}{1\ \mathsf{GeV}} - 0.63}$$



$$\sigma \approx \sigma_1 \cdot \mathbb{1}_{2\,m_{\text{DM}}<m_1<\sqrt{s}-m_Z} + \sigma_2 \cdot \mathbb{1}_{2\,m_{\text{DM}}<m_2<\sqrt{s}-m_Z}$$

$$\sigma_1 \equiv \sigma_{\text{SM}}(m_1)\cdot\cos^2\alpha\cdot \text{BR}(h_1\to\text{DM})$$

$$\sigma_2 \equiv \sigma_{\text{SM}}(m_2)\cdot\sin^2\alpha\cdot \text{BR}(h_2\to\text{DM})$$

Conditions:

$$(1) \qquad \boxed{\sigma_1 < 0.19\,\sigma_{\text{SM}}(m_1)}$$

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Considered process: degenerated case

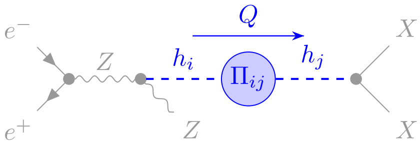
$$\frac{d\sigma}{dQ^2} = \frac{\sigma_{\mathsf{SM}}(Q^2)v^2}{32\pi^2}\frac{Q^4\cdot\mathcal{X}}{\left[(Q^2-m_1^2)^2+(m_1\Gamma_1)^2\right]\left[(Q^2-m_2^2)^2+(m_2\Gamma_2)^2\right]}\times$$

$$\times\sqrt{1-4\frac{m_{\mathsf{DM}}^2}{Q^2}}\cdot\begin{cases}1 & (\mathsf{pGDM})\\ 1-4\frac{m_{\mathsf{DM}}^2}{Q^2}+12\left(\frac{m_{\mathsf{DM}}^2}{Q^2}\right)^2 & (\mathsf{VDM})\\ 2\frac{m_{\mathsf{DM}}^2}{Q^2}\left(1-4\frac{m_{\mathsf{DM}}^2}{Q^2}\right) & (\mathsf{FDM})\end{cases}$$

$$\left.\begin{array}{l}\Gamma_1 \xrightarrow{m_2\to m_1} \cos^2\alpha\,\Gamma_{h\to\mathsf{SM}} + \sin^2\alpha\,\Gamma_{h\to\mathsf{DM}}\\ \Gamma_2 \xrightarrow{m_2\to m_1} \sin^2\alpha\,\Gamma_{h\to\mathsf{SM}} + \cos^2\alpha\,\Gamma_{h\to\mathsf{DM}}\end{array}\right\}$$

$$\mathcal{X}\equiv(\cos\alpha\sin\alpha)^2\frac{(m_1^2-m_2^2)^2+(m_1\Gamma_1-m_2\Gamma_2)^2}{v^2v_s^2}\Bigg|_{m_1=m_2}\neq 0$$

but $\kappa = \dfrac{(m_1^2-m_2^2)\sin 2\alpha}{2vv_s} \xrightarrow{m_2\to m_1} 0$!



$$\Pi_{ij} \equiv \Pi_{ij}(Q^2) \equiv i \cdot \mathcal{I}m\,[\text{self energy}]$$
$$\Pi_{ii}(m_i^2) = i m_i \Gamma_i$$

the matrix element:

$$\mathcal{M} = \mathcal{M}_{e^+e^- \to Zh_i}(Q^2) \cdot \Delta_{ij}(Q^2) \cdot \mathcal{M}_{h_j \to XX}(Q^2) =$$
$$= \mathcal{M}_{e^+e^- \to Zh}(Q^2) \cdot \underbrace{\mathcal{R}_{1i} \cdot \Delta_{ij}(Q^2) \cdot \mathcal{R}_{2j}}_{\widehat{\Delta}(Q^2)} \cdot \mathcal{M}_{h \to XX}(Q^2)\,,$$

the propagator:

$$\widehat{\Delta}(Q^2) = \mathcal{R}_{1i}\mathcal{R}_{2j} \cdot \frac{1}{\det D} \overbrace{\begin{bmatrix} Q^2 - m_2^2 + \Pi_{22} & -\Pi_{12} \\ -\Pi_{21} & s - m_1^2 + \Pi_{11} \end{bmatrix}}^{D}_{ij} =$$
$$= \sin\alpha\cos\alpha \cdot \frac{(m_1^2 - m_2^2) - [(\Pi_{11} - \Pi_{22}) - (\operatorname{tg}\alpha \cdot \Pi_{12} - \operatorname{ctg}\alpha \cdot \Pi_{21})]}{(Q^2 - m_1^2 + \Pi_{11})(Q^2 - m_2^2 + \Pi_{22}) - \Pi_{12}\Pi_{21}}$$

$$\widehat{\Delta}(Q^2) = \sin\alpha\cos\alpha \cdot \frac{(m_1^2 - m_2^2) - [(\Pi_{11} - \Pi_{22}) - (\mathrm{tg}\,\alpha \cdot \Pi_{12} - \mathrm{ctg}\,\alpha \cdot \Pi_{21})]}{(Q^2 - m_1^2 + \Pi_{11})(Q^2 - m_2^2 + \Pi_{22}) - \Pi_{12}\Pi_{21}}$$

- If $|m_1 - m_2| \gg \Gamma_1, \Gamma_2$, then $\longleftarrow$ in practice, only this case relevant

$$\begin{aligned} \widehat{\Delta}(Q^2) &\approx \sin\alpha\cos\alpha \cdot \frac{m_1^2 - m_2^2}{(Q^2 - m_1^2 + \Pi_{11})(Q^2 - m_2^2 + \Pi_{22})} \\ &\approx \sin\alpha\cos\alpha \cdot \left[\frac{1}{Q^2 - m_1^2 + im_1\Gamma_1} - \frac{1}{Q^2 - m_2^2 + im_2\Gamma_2}\right] \\ &\equiv \widehat{\Delta}^{(\mathrm{BW})}(Q^2) \end{aligned}$$

- If $m_1 \sim m_2$, an explicit calculation shows that

$$[(\Pi_{11} - \Pi_{22}) - (\mathrm{tg}\,\alpha \cdot \Pi_{12} - \mathrm{ctg}\,\alpha \cdot \Pi_{21})]\Big|_{m_1 = m_2} = 0$$

$$\Rightarrow \quad \widehat{\Delta}(Q^2)\Big|_{m_1 = m_2} = 0$$

$$\Pi_{ij} = \Pi_{ij}^{\text{DM}} + \Pi_{ij}^{W^+W^-} + \Pi_{ij}^{ZZ} + \sum_q \Pi_{ij}^{q\bar{q}} + \sum_l \Pi_{ij}^{l^+l^-} + \sum_{k,l} \Pi_{ij}^{h_k h_l}$$

(DM $= A, X, \psi$; q – SM quarks; l – SM leptons)

by straightforward calculations:

$$\left[\left(\Pi_{11}^{ab} - \Pi_{22}^{ab}\right) - \left(\operatorname{tg}\alpha \cdot \Pi_{12}^{ab} - \operatorname{ctg}\alpha \cdot \Pi_{21}^{ab}\right)\right]\Big|_{m_1=m_2} = 0$$

for $ab =$ DM, W^+W^-, ZZ, $q\bar{q}$, l^+l^-, $h_k h_l$ (sum over $k, l = 1, 2$)

$$\Rightarrow \left[(\Pi_{11} - \Pi_{22}) - (\operatorname{tg}\alpha \cdot \Pi_{12} - \operatorname{ctg}\alpha \cdot \Pi_{21})\right]\Big|_{m_1=m_2} = 0\,.$$

- $\Pi_{ij}^{\mathsf{DM}}(Q^2) = I(Q^2, m_{\mathsf{DM}}, m_{\mathsf{DM}}) \frac{\mathcal{R}_{2i}\mathcal{R}_{2j}}{32\pi^2 v_s^2}(m_i m_j)^2 \begin{cases} 1 & (\mathsf{pGDM}) \\ 1 - 2\, m_{\mathsf{DM}}^2 \frac{4Q^2 - m_i^2 - m_j^2}{(m_i m_j)^2} + 12 \left(\frac{m_{\mathsf{DM}}^2}{(m_i m_j)^2}\right)^2 & (\mathsf{VDM}) \\ 2 \frac{m_{\mathsf{DM}}^2 Q^2}{(m_i m_j)^2} \left(1 - 4 \frac{m_{\mathsf{DM}}^2}{Q^2}\right) & (\mathsf{FDM}) \end{cases}$
- $\Pi_{ij}^{W^+W^-}(Q^2) = I(Q^2, m_W, m_W) \frac{\mathcal{R}_{1i}\mathcal{R}_{1j}}{16\pi^2 v^2}(m_i m_j)^2 \times \left[1 - 2m_W^2 \frac{4Q^2 - m_i^2 - m_j^2}{(m_i m_j)^2} + 12 \frac{m_W^4}{(m_i m_j)^2}\right]$
- $\Pi_{ij}^{ZZ}(Q^2) = I(Q^2, m_Z, m_Z) \frac{\mathcal{R}_{1i}\mathcal{R}_{1j}}{32\pi^2 v^2}(m_i m_j)^2 \times \left[1 - 2m_Z^2 \frac{4Q^2 - m_i^2 - m_j^2}{(m_i m_j)^2} + 12 \frac{m_Z^4}{(m_i m_j)^2}\right]$
- $\Pi_{ij}^{q\bar{q}}(Q^2) = I(Q^2, m_q, m_q) \cdot \frac{3\mathcal{R}_{1i}\mathcal{R}_{1j}}{8\pi^2 v^2} m_q^2 Q^2 \left(1 - 4\frac{m_q^2}{Q^2}\right)$
- $\Pi_{ij}^{l^+l^-}(Q^2) = I(Q^2, m_l, m_l) \cdot \frac{\mathcal{R}_{1i}\mathcal{R}_{1j}}{8\pi^2 v^2} m_l^2 Q^2 \left(1 - 4\frac{m_l^2}{Q^2}\right)$
- $\Pi_{ij}^{h_k h_l}(Q^2) = I(Q^2, m_k, m_l) \cdot \frac{V_{ikl}V_{jkl}}{32\pi^2}$

  - $V_{111} \equiv 3m_1^2 \left(\frac{\sin^3\alpha}{v_s} + \frac{\cos^3\alpha}{v}\right)$
  - $V_{222} \equiv 3m_2^2 \left(\frac{\cos^3\alpha}{v_s} - \frac{\sin^3\alpha}{v}\right)$
  - $V_{112} = V_{121} = V_{211} \equiv (2m_1^2 + m_2^2)\sin\alpha\cos\alpha \left(\frac{\sin\alpha}{v_s} - \frac{\cos\alpha}{v}\right)$
  - $V_{221} = V_{212} = V_{122} \equiv (m_1^2 + 2m_2^2)\sin\alpha\cos\alpha \left(\frac{\cos\alpha}{v_s} + \frac{\sin\alpha}{v}\right)$

- $I(Q^2, m_a, m_b) \equiv i \cdot \mathcal{Im}\left[\frac{1}{i\pi^2}\int \frac{d^4 l}{(l^2 - m_a^2)[(l+Q)^2 - m_b^2]}\right] = i\pi \cdot \frac{\lambda^{1/2}(Q^2, m_a^2, m_b^2)}{Q^2} \cdot \mathbb{1}_{Q^2 > (m_a+m_b)^2}$

$$\Rightarrow \quad I(Q^2, m, m) = i\pi \cdot \sqrt{1 - \frac{4m^2}{Q^2}} \cdot \mathbb{1}_{Q^2 > 4m^2} \qquad (\ \lambda(a,b,c) \equiv a^2 + b^2 + c^2 - 2(ab + bc + ca)\ )$$

The h_1's and h_2's decay widths

$$\Gamma_{h_i \to ab} \equiv \frac{\Pi^{ab}_{ii}(m_i^2)}{i m_i}$$

$$\Downarrow$$

- $\Gamma_{h_i \to \mathrm{DM}} = \frac{\mathcal{R}_{2i}^2}{v_s^2} \frac{m_i^3}{32\pi} \sqrt{1 - \frac{4 m_{\mathrm{DM}}^2}{m_i^2}} \begin{cases} 1 & \text{(pGDM)} \\ 1 - 4\frac{m_{\mathrm{DM}}^2}{m_i^2} + 12\left(\frac{m_{\mathrm{DM}}^2}{m_i^2}\right)^2 & \text{(VDM)} \\ 2\frac{m_{\mathrm{DM}}^2}{m_i^2}\left(1 - 4\frac{m_{\mathrm{DM}}^2}{m_i^2}\right) & \text{(FDM)} \end{cases}$
- $\Gamma_{h_i \to \mathrm{SM}} = \mathcal{R}_{1i}^2 \cdot \gamma(m_i)$ (γ – decay width of SM Higgs particle of given mass)
- $\Gamma_{h_1 \to h_2 h_2} = \sin^2\alpha \, \cos^2\alpha \, (m_1^2 + 2m_2^2)^2 \left(\frac{\cos\alpha}{v_s} + \frac{\sin\alpha}{v}\right)^2 \frac{\sqrt{m_1^2 - 4m_2^2}}{32\pi m_1^2} \approx$
  $\approx \frac{\sin^2\alpha \, \cos^4\alpha}{v_s^2} (m_1^2 + 2m_2^2)^2 \frac{\sqrt{m_1^2 - 4m_2^2}}{32\pi m_1^2}$
- $\Gamma_{h_2 \to h_1 h_1} = \sin^2\alpha \, \cos^2\alpha \, (2m_1^2 + m_2^2)^2 \left(\frac{\sin\alpha}{v_s} - \frac{\cos\alpha}{v}\right)^2 \frac{\sqrt{m_2^2 - 4m_1^2}}{32\pi m_2^2} \approx$
  $\approx \frac{\sin^2\alpha \, \cos^4\alpha}{v^2} (2m_1^2 + m_2^2)^2 \frac{\sqrt{m_2^2 - 4m_1^2}}{32\pi m_2^2}$