Current Status of Bhabha Luminosity at 0.01%

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Intro – lumi basics

Bhabha cross sect. depends on detector acceptance angles

$$\sigma_{Bh} \simeq \frac{16\pi\alpha^2}{s}\left(\frac{1}{\theta_{\rm min}^2} - \frac{1}{\theta_{\rm max}^2}\right), \quad t \simeq s\frac{\theta^2}{4}$$

at LEP: 28–50 mrad $\rightarrow |t|/s = 3 \times 10^{-4}$, $\sqrt{|t|}_{M_Z} = 1.68$ GeV

at FCCee: 64–86 mrad $\rightarrow |t|/s = 12 \times 10^{-4}$, $\sqrt{|t|}_{M_Z} = 3.25$ GeV

at ILC: 31–77 mrad $\rightarrow |t|/s = 6 \times 10^{-4}$, $\sqrt{|t|}_{500GeV} = 12.5$ GeV

at CLIC: 39–134 mrad $\rightarrow |t|/s = 13 \times 10^{-4}$, $\sqrt{|t|}_{3000GeV} = 108$ GeV

Luminosity at LEP

At LEP1 the experimental luminosity precision was < 0.05%.
The 1999 theoretical error was based on BHLUMI 4.04

	LEP1		LEP2	
Type of correction/error	1996	1999	1996	1999
(a) Missing photonic $\mathcal{O}(\alpha^2)$	0.10%	0.027%	0.20%	0.04%
(b) Missing photonic $\mathcal{O}(\alpha^3 L_e^3)$	0.015%	0.015%	0.03%	0.03%
(c) Vacuum polarization	0.04%	0.04%	0.10%	0.10%
(d) Light pairs	0.03%	0.03%	0.05%	0.05%
(e) Z and s-channel γ	0.015%	0.015%	0.0%	0.0%
Total	0.11%	0.061%	0.25%	0.12%

Table: Summary of the total (physical+technical) theoretical uncertainty for a typical calorimetric detector. For LEP1, the above estimate is valid for a generic angular range within 1°–3° (18–52 mrads), and for LEP2 energies up to 176 GeV and an angular range within 3°–6°. Total uncertainty is taken in quadrature. Technical precision is included in (a).

* OPAL declared 1999 precision of 0.054% due to improved treatment of fermion-pair calculation

Luminosity today (2020)

The 2019 update comes from P. Janot & S. Jadach Phys.Lett.B 803 (2020) 135319

Type of correction / Error	1999	Update 2019
(a) Photonic $\mathcal{O}(L_e\alpha^2)$	0.027%	0.027%
(b) Photonic $\mathcal{O}(L_e^3\alpha^3)$	0.015%	0.015%
(c) Vacuum polariz.	0.040%	0.011%
(d) Light pairs	0.030%	0.010%
(e) Z and s-channel γ exchange	0.015%	0.015%
(f) Up-down interference	0.0014%	0.0014%
(f) Technical Precision	–	(0.027)%
Total	0.061%	0.037%

Table: Summary of the total (physical+technical) theoretical uncertainty for a typical calorimetric LEP luminosity detector within the generic angular range of 18–52 mrad. Total error is summed in quadrature.

- Hadronic vacuum polarisation from F. Jegerlehner (fortran code `hadr5x.f`) 2019
- Light pairs: real – `FERMISV` MC by J. Hilgart et.al. 1993 and `KoralW` by S. Jadach et.al.; virtual – S. Actis et.al. 2008

Lumi at FCCee: QED $\mathcal{O}(\alpha^3 L^3)$ and $\mathcal{O}(\alpha^2 L^1)$

- Fixed order calculations are not enough, e.g. $\mathcal{O}(\alpha^2)$ is accurate to 0.5%. Exponentiation is mandatory
- QED corrs $\mathcal{O}(\alpha^2 L^1)$ (0.027%), $L = \ln(|t|/m_e^2)$, missing in BHLUMI 4.04, are known for long: real-real – 1985 (CALKUL), real-virtual – 1996 (Jadach et.al. up-down interf. neglected), virtual-virtual – 1988 (Berends et.al., in s channel) must be added to BHLUMI, preferably in CEEX style as in KKMC
- QED complete $\mathcal{O}(\alpha^2 L^0)$ terms, calculated in 2001-2009 (Bern et.al., Bonciani et.al., Czakon et.al., Pennin et.al., Kuhn et.al.), are of the order of 10^{-5}, and after independent checking will likely not be necessary in BHLUMI
- QED leading terms $\mathcal{O}(\alpha^3 L^3)$ (0.015%) are known (Jadach et.al., 1997) but not included in BHLUMI yet, must be added.
- The missing terms would be then $\mathcal{O}(\alpha^4 L^4)$ and $\mathcal{O}(\alpha^3 L^2)$: $\mathcal{O}(\alpha^4 L^4)$ are estimated as
$$\mathcal{O}(\alpha^3 L^3) \times (\alpha/\pi) L = 0.015\% \times 0.042 = 6 \times 10^{-6}$$
$\mathcal{O}(\alpha^3 L^2)$ are uncertain in estimate, generically $(\alpha/\pi)^3 \times L^2 \simeq 10^{-5}$

Lumi at ILC: QED $\mathcal{O}(\alpha^3 L^3)$ and $\mathcal{O}(\alpha^2 L^1)$

At 500 GeV ILC we have $(\alpha/\pi)L = 0.047$ instead of 0.042 for FCCee.

- Terms $\mathcal{O}(\alpha^2 L^1)$ increase by $0.027\% \times (0.047/0.042) = 0.03\%$.
- Terms $\mathcal{O}(\alpha^3 L^3)$ increase by $0.015\% \times (0.047/0.042)^3 = 0.021\%$.

Missing $\mathcal{O}(\alpha^4 L^4)$ and $\mathcal{O}(\alpha^3 L^2)$ terms are

- $\mathcal{O}(\alpha^4 L^4) \sim 2.1 \times 10^{-4} \times 0.047 = 0.1 \times 10^{-4}$
- $\mathcal{O}(\alpha^3 L^2) \sim 10^{-5}$

Lumi at FCCee: γZ interferences

- Z exchange and s-channel γ exchange.
  - At FCCee luminometer angles are twice bigger than at LEP: (28-50)→(64-86) mrad, i.e. $t/s \sim 4\times$ bigger,
  - So the s-channel Z and γ exch. is $4\times$ bigger – current BHLUMI error is 0.09%.
  - In BHLUMI only $(Z_s + \gamma_s) \otimes \gamma_t$ interferences are included and contribute 1%.

This is reducible already now with BHWIDE down to 10^{-5}:

  - BHWIDE includes all $(Z+\gamma)_{s+t}$ contribs up to $\mathcal{O}(\alpha)_{exp}$
  - The size of $(Z+\gamma)_{s+t} - \gamma_t$ contribs $(\mathcal{O}(\alpha^0 + \alpha^1)_{exp})$ is $\pm 0.7\%$ at $M_Z \mp 1$GeV for FCC angles (64-86 mrad) and LCAL-like setup. Pure $\mathcal{O}(\alpha)$ contrib is $\sim 0.4\%$
  - Missing higher order corrs in BHWIDE are estimated as $\frac{\alpha}{\pi} \ln \frac{|\bar{t}|}{m_e^2}$ smaller than the included ones $(\sim (8_{h.o.QED} + 4_{\mathcal{O}(\alpha)EW}) \times 10^{-4})$, i.e. $\sim 1 \times 10^{-4}$ (safety factor of 2 included).

γZ interferences at ILC

At ILC $|t|/s$ is twice bigger than at LEP, so is the γZ contribution (0.04%), which is missing in BHLUMI 4.04 and treated as an error.

γZ interferences at $\mathcal{O}(\alpha)$ at ILC

At higher energies missing higher order EW terms were estimated in [M. Battaglia, S. Jadach, D. Bardin, C010630 (2001) E3015.PDF] as 0.025% at 800 GeV and acceptance 50-100 mrad (0.1% at 3 TeV) with the help of DIZET EW library of ZFITTER. These results indicate, that at 500 GeV and 31–77 mrad acceptance, 0.012% is probable.

Missing QED higher orders are known and will be added to BHLUMI

Lumi at FCCee/ILC: QED up-down interference

- The up-down interference between e^+ and e^- lines in $\mathcal{O}(\alpha)$ known for long (1991, Jadach et.al.) to be roughly $\delta\sigma/\sigma \sim 0.07 \times |t|/s$.
- At LEP1 it is negligible: $1.4 \cdot 10^{-5}$.
- At FCCee angular range of Lumi detectors planned to be 64-86 mrad (narrow) as compared to LEP's 28-50 mrad. Transfer $|t|$ increases 4 times at FCCee and $\delta\sigma/\sigma \sim 9 \cdot 10^{-5}$.
- At ILC angular range of Lumi detectors planned to be 31-77 mrad (narrow) as compared to LEP's 28-50 mrad. $|t|/s$ increases 2 times at ILC and $\delta\sigma/\sigma \sim 4 \cdot 10^{-5}$.
- Known $\mathcal{O}(\alpha)$ up-down interference must be added to BHLUMI. Higher order corrs. are roughly of the size $\mathcal{O}(\alpha) \times 2(\alpha/\pi)L$ i.e. negligible, below $1 \cdot 10^{-5}$.

Lumi at FCCee/ILC: vacuum polarisation

- Vacuum polarisation uncertainty is simply $\delta_{VP}\sigma/\sigma = 2\delta\alpha_{eff}(\bar{t})/\alpha_{eff}(\bar{t})$ with $\sqrt{|\bar{t}|} \sim 3.5$ GeV at Z-peak and 13 GeV at 350 GeV
- At LEP this uncertainty was 4×10^{-4}.
- The uncertainty is mainly due to experimental cross section $e^- e^+ \rightarrow hadrons$ below 10 GeV (as an input to dispersion relations)
- Today we find for FCCee transfer $\delta_{VP}\sigma/\sigma \simeq 1.3 \cdot 10^{-4}$ at the Z-peak (based on results for $t = -4$ GeV^2 of Jegerlehner, arXiv:1711.06089)
- By FCCee time, anticipating improvements in data for $\sigma_{had} \leq 2.5$ GeV we expect another factor of 2 improvement and $\delta_{VP}\sigma/\sigma \simeq 0.6 \cdot 10^{-4}$ at Z-peak
- At $\sqrt{|\bar{t}|} \sim 12.5$ GeV (FCCee at 380 GeV,ILC at 500 GeV) we find $\delta_{VP}\sigma/\sigma \simeq 2.4 \cdot 10^{-4}$ with today's α and $1.2 \cdot 10^{-4}$ in 20+ years.

Lumi at FCCee: Light pairs

- Current precision of $1 \cdot 10^{-4}$ due to light pairs is taken from Montagna et.al., 1999, based on calculation of $e^+e^- \to e^+e^-e^+e^-$ plus virtual corrections from Barbieri et.al. 1972, Burgers 1985.
- Second approach is based on semianalytical LO calculation of Jadach et.al., 1993, and LO+NLO Arbuzov et.al., 1995-97, the latter with claimed precision of $0.6 \cdot 10^{-4}$.
- Third approach is based on extension of YFS formalism to emission of soft pairs, Jadach et.al., 1997, with precision of $2 \cdot 10^{-4}$, implemented in unpublished version 2.30 of BHLUMI code

Lumi at FCCee: Light pairs – missing corrs now

- The missing pair corrections include:
  - emission of $\mu^+\mu^-$-pairs: naive rescaling of the logarithm gives $\sigma_\mu \sim \ln^2(|t|/m_\mu^2)/\ln^2(|t|/m_e^2) \times \sigma_e \simeq 0.16 \times 5 \cdot 10^{-4}\sigma_0 \simeq 8 \cdot 10^{-5}\sigma_0$
  - emission of $\tau^+\tau^-$-pairs is suppressed by $\ln^2(|t|/m_\tau^2)/\ln^2(|t|/m_e^2) \simeq 0.005$ w.r.t. the electron pair and completely negligible
  - emission of light-quark-pairs we estimate as $R_{had} \times \ln^2(|t|/(0.5GeV)^2)/\ln^2(|t|/m_\mu^2) \sim 0.9$ of the muonic contribution, where $R_{had} = \sigma_{had}/\sigma_\mu \sim 3$
  - emission of two e^+e^--pairs is suppressed by another $(\alpha/\pi)^2\ln^2(|t|/m_e^2) \sim 10^{-3}$ and is negligible
  - leading emission of e^+e^--pair and photon, according to Arbuzov et.al., 1995, contributes less than $8 \cdot 10^{-5}$ for $z < 0.7, z = 1 - s'/s$ ($3 \cdot 10^{-5}$ for $z < 0.5$). Nonleading contribs are suppressed by additional $1/\ln(|t|/m_e^2) \sim 0.06$ and negligible.

Lumi at FCCee: Light pairs – missing corrs in the future

Uncertainty on light lepton pairs is now of the order of 1×10^{-4}.
At FCCee we can hope for 0.5×10^{-4}, provided one includes:

- muon pairs ($\sim 0.8 \times 10^{-4}$) with 10% accuracy, i.e. 0.1×10^{-4}. It is possible in all approaches: 4-fermion MC, YFS MC, LO semianal.
- quark pairs ($\sim 0.8 \times 10^{-4}$) are more difficult, we assume 25% accuracy, i.e. 0.2×10^{-4}
- $\mathcal{O}(\alpha)$ photonic correction to *eeee* final state, of the size at most 0.5 – 0.7 of the *eeee*, i.e. $1.5 - 2 \times 10^{-4}$. Can be calculated with the help of the 4-fermion $\mathcal{O}(\alpha)$ calculation by [Denner *et.al.*]. Possible in exponentiated soft approx. in BHLUMI (unpubl.). Missing non-soft, non-leading $\mathcal{O}(\alpha^2)$ photonic corrs to *eeee* are suppressed by $1/\ln(|t|/m_e^2)$ with respect to $\mathcal{O}(\alpha)$, i.e. of the order of 0.3×10^{-4}.

Two options are already available: dedicated version of BHLUMI 2.30 with soft pairs or 4-fermion codes of the LEP2 type. Virtual $\mathcal{O}(\alpha^2)$ calculations for $ee \to ee$ are also available [Gluza *et.al.*].

Lumi at ILC: Light pairs – missing corrs

Additional pair emission is ruled by $L_m^2 = \ln^2(|t|/m^2)$.
With change of $\sqrt{|t|}$: 3.25 → 12.5 GeV, L_e^2 increases by factor 1.25.
Similarly increase all precision estimates:

- current precision: 1.25×10^{-4}
- future precision: 0.6×10^{-4}

Lumi at FCCee: Technical precision

- At LEP BHLUMI technical prec. was tested in two ways:
  - Comparison with semian. integration of $\mathcal{O}(\alpha^2)_{exp}$ matrix el. of BHLUMI: agreement 2.7×10^{-4}
  - Comparison with LUMLOG+OLDBIS hybrid MC and with SABSPV MC. All of these MCs have incomplete soft resummation: agreement 2.7×10^{-4} (for sharp photon energy cut-offs 1.7×10^{-3})
- Now another MC code BabaYaga [Balossini et.al.] with complete soft-photon resummation is available. After upgrade to NNLO in hard process it could be ideal for technical comparison with BHLUMI

Lumi at FCCee

Type of correction / Error	Update 2019	FCC-ee forecast
(a) Photonic $[\mathcal{O}(L_e\alpha^2)]$ $\mathcal{O}(L_e^2\alpha^3)$	0.027%	0.1×10^{-4}
(b) Photonic $[\mathcal{O}(L_e^3\alpha^3)]$ $\mathcal{O}(L_e^4\alpha^4)$	0.015%	0.06×10^{-4}
(c) Vacuum polariz.	0.011%	0.6×10^{-4}
(d) Light pairs	0.010%	0.5×10^{-4}
(e) Z and s-channel γ exchange	0.090%	0.1×10^{-4}
(f) Up-down interference	0.009%	0.1×10^{-4}
(f) Technical Precision	(0.027)%	0.1×10^{-4}
Total	0.097%	1.0×10^{-4}

Table: Anticipated total (physical+technical) theoretical uncertainty for a FCC-ee luminosity calorimetric detector with the angular range being 64–86 mrad (narrow), near the Z peak. Description of photonic corrections in square brackets is related to the 2nd column. The total error is summed in quadrature. Based on BHLUMI 4.04 error budget.

Precision 1.0×10^{-4} or better within the reach

Lumi at ILC 500 GeV

Type of correction / Error	Update 2019	ILC 500 GeV forecast
(a) Photonic $[\mathcal{O}(L_e\alpha^2)]$ $\mathcal{O}(L_e^2\alpha^3)$	0.03%	10^{-5}
(b) Photonic $[\mathcal{O}(L_e^3\alpha^3)]$ $\mathcal{O}(L_e^4\alpha^4)$	0.021%	0.1×10^{-4}
(c) Vacuum polariz.	0.024%	1.2×10^{-4}
(d) Light pairs	0.013%	0.6×10^{-4}
(e) Z and s-channel γ exchange	0.04%	1.2×10^{-4}
(f) Up-down interference	0.004%	0.04×10^{-4}
(f) Technical Precision	(0.027)%	0.1×10^{-4}
Total	0.061%	2.0×10^{-4}

Table: Anticipated total (physical+technical) theoretical uncertainty for a ILC luminosity calorimetric detector with the angular range being 31–77 mrad, at 500 GeV. Description of photonic corrections in square brackets is related to the 2nd column. The total error is summed in quadrature. Based on BHLUMI 4.04 error budget.

Precision 2.0×10^{-4} within the reach

2019 development

In [P. Janot, S. Jadach, 1912.02067] LEP lumi error is reduced from 0.061% to 0.037%. Two components are responsible for this result: vacuum polarisation and fermion pairs.

- The hadronic component of vac.pol. taken from [F. Jegerlenher, Dec. 2019, `hadr5x.f`], has been implemented in BHLUMI 4.04 and 1×10^9 events were generated with realistic selection criteria. For comparison two independent codes, from KNT and DHMZ groups, were used for cross-checks. This delivered
  → new precision of vac.pol. component: 0.009%
- The light fermion pairs have been generated outside of BHLUMI:
  - Real pairs from the FERMISV MC [R. Kleiss et.al. 1993] and KORALW [S. Jadach et.al. 1999]
  - virtual corrs. due to pair production from [J. Gluza et.al. 0807.4691] implemented in simple MC
  - Error budget: 0.6×10^{-4} technical prec. of FERMISV and KORALW for $4e$ final state (includes 0.2×10^{-4} techn. prec. of the method); 20% for higher order terms, i.e. 0.8×10^{-4}
    → 0.01% total precision of pair corrs.

Can we go below 0.01% at FCCee?

Two items are crucial for further reduction:

- Vacuum pol. with current precision of 9×10^{-5} (2020, realistic selection criteria, Janot&Jadach). If factor of 2 improvement would be achieved, we could obtain 4.5×10^{-5} precision.
- Pair production error must be reduced below 5×10^{-5}
  - Leptonic real pairs can be generated with LEP2 MC tools. Control over precision of $4e$ matrix el. in collinear configurations is crucial. Quadruple precision is needed. 1×10^{-5} feasible.
  - Leptonic virtual pairs – exact calculations exist.
  - Higher orders: $\mathcal{O}(\alpha)$ to $4ferm$ – calculations exist;
    Choosing $z_{cut} = 0.5$, [Arbuzov et.al. NPB485, 457 (1997), tab. 1] calculated $e^+e^-\gamma$ correction to be 3×10^{-5}. Missing $e^+e^-2\gamma$ corrs. would then be negligible (smaller by $\ln(|t|/m_e^2)$)
    $\mathcal{O}(\alpha^3)_{virt}$ to born – (LL+soft) calculations exist. Must be precise to 20%.
  - Role of exponentiation not explored yet.

Precision 0.5×10^{-4} **may be feasible**

This would make lumi error subdominant

Do we need to go below 0.01%

Cross-section and ν number:

- σ_0 will be measured to 1×10^{-4} (rel.). Theoretical error with lumi error of 1×10^{-4} will dominate over exp. one. 0.5×10^{-4} for lumi would be welcomed!
- Neutrino number is now known to ± 0.0074. At FCC it will be measured with 0.001 precision, corresponding lumi error is $\delta N_\nu = 7.5 \frac{\delta \mathcal{L}}{\mathcal{L}} + \ldots$, i.e. 0.00075 for lumi error of 0.01%. Further reduction of lumi error below 0.01% would improve result, especially if exp. error improves over time.

Summary FCCee

- Our starting point is BHLUMI 4.04 with the inherited from LEP precision of 0.06%
- 2019 development of Janot&Jadach reduced this error to 0.037%
- On the way to 0.01% one must upgrade matrix el. of BHLUMI with known higher orders as well as with known $(\gamma, Z)_{s,t}$ and up-down interferences (BHWIDE)
- Vacuum polarisation is already at the level of 0.009%, further reduction possible in the future
- Fermion pairs must be reduced to sub-dominant 0.005%. Possible but needs new MC tools.
- Technical precision requires second MC code, e.g. BABAYAGA

0.01% precision looks realistic

0.005% feasible provided fermion pairs are further reduced ?

Summary ILC

- With the same improvements as discussed for FCCee

0.02% precision is possible at 500 GeV

- Further reduction is blocked by
  - hadronic vacuum polarisation (0.012%)
  - EW higher order corrs.(0.012%).
    There are however unpublished indications
    [https://jadach.web.cern.ch/jadach/public/LumLCslac.pdf]
    that this error is overestimated by factor 3–5.