ILC prospects for bottom quark mass measurement from three-jet rates

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Background of This Study

b quark mass and New Physics

- $\overline{\mathrm{MS}}$ running mass : **Mass which has energy dependence**

QCD Renormalized Group Equation : $\mu^2 \frac{\partial m_q(\mu)}{\partial \mu^2} = -\gamma\left(\alpha_s(\mu)\right) m_q(\mu)$

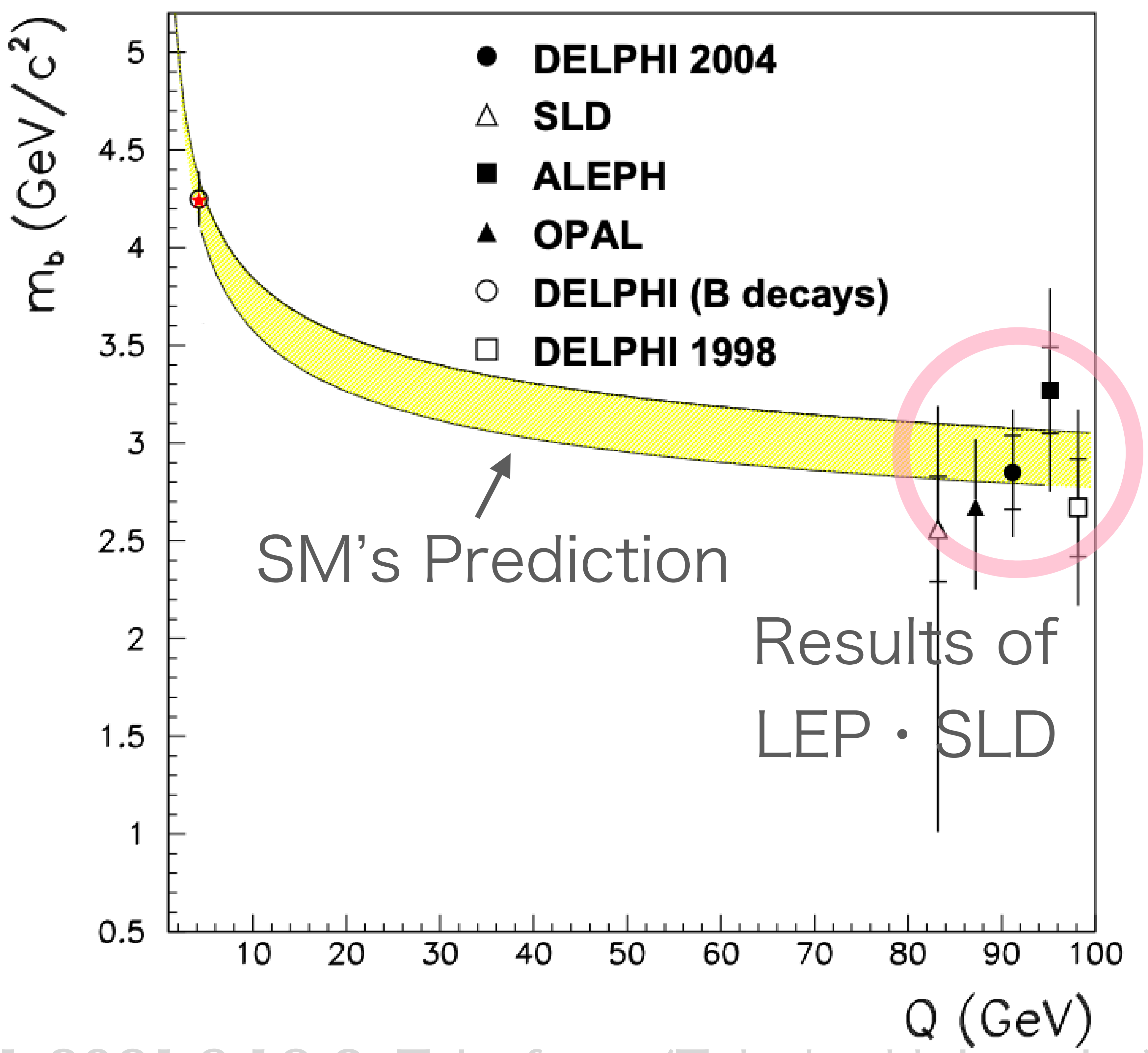
$\gamma(\alpha_s(\mu))$: Perturbative function

μ : energy scale of experiments

- Energy dependence deviates from SM's expectation by new particles's effect (SUSY etc).

- No indication of new physics in b quark mass at Z-pole (∼91GeV).

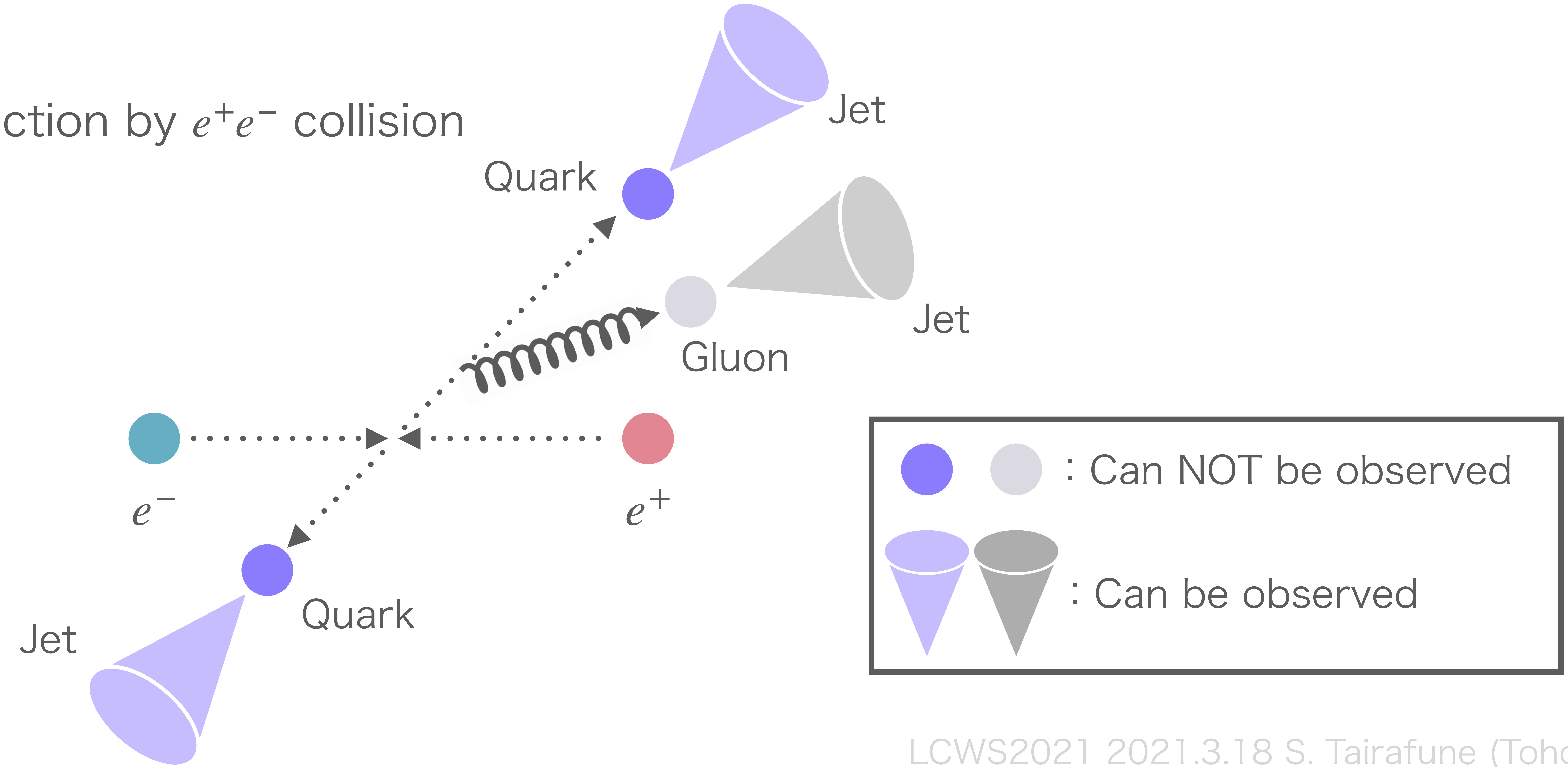
→ **b quark mass at higher energy scale above Z-pole can be SM's test and a probe of new physics.**



Purpose of This study

- **Estimate b quark mass at 250 GeV ILC through simulation.**

- $q\bar{q}$ production by e^+e^- collision



Definition of Observable

- **Heavier quark tends to be difficult to emit gluon $q \to q + g$.**

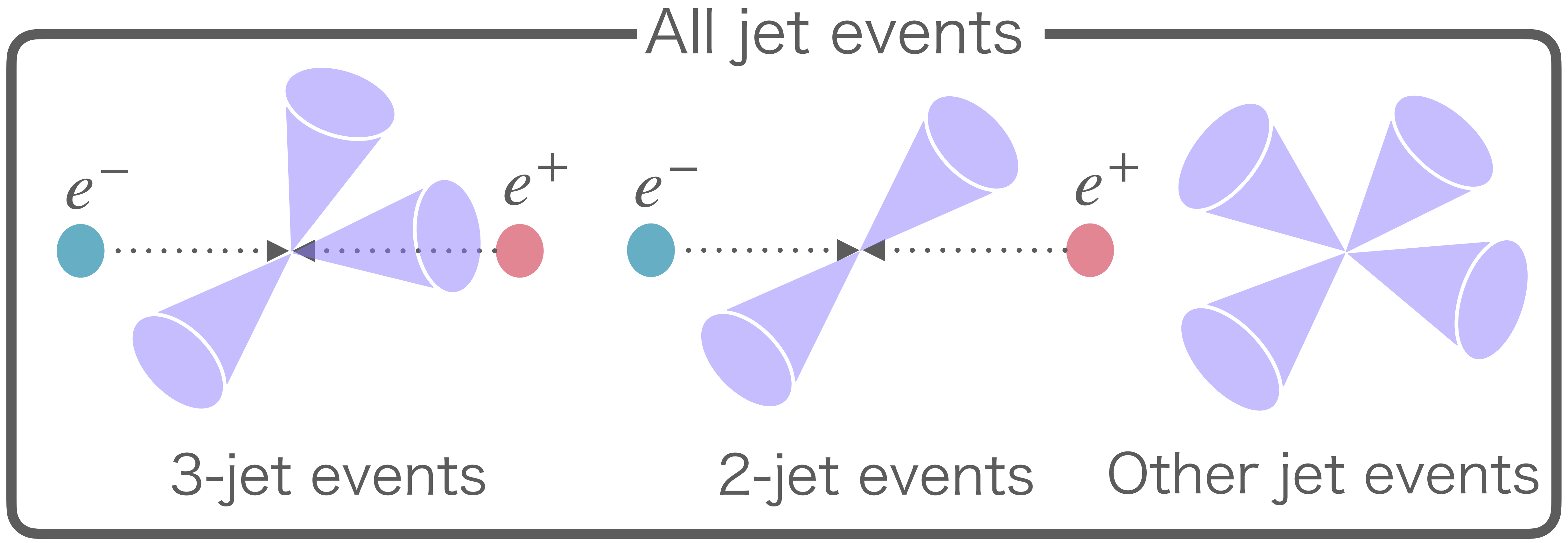
→ b quark mass sensitivity appears on 3-jet events after gluon radiation.

- Define **the double ratio of 3-jet fractions** for $e^+e^- \to q\bar{q}$.

$$R_3^{bl} = \frac{N_{3b}/N_b}{N_{3l}/N_l}$$
$$= 1 + \frac{\alpha_s}{\pi} a_{LO} + \frac{m_b^2}{s}\left(b_{LO}(m_b) + \frac{\alpha_s}{\pi} b_{NLO}(m_b)\right)$$

a_{LO} , b_{LO} : LO corrections b_{NLO} : NLO correction

$\sqrt{s}$: CM energy



N_b : $e^+e^- \to b\bar{b} \to$ all jet events number

N_{3b} : $e^+e^- \to b\bar{b} \to$ 3-jet events number

N_l : $e^+e^- \to l\bar{l} \to$ all jet events number $(l = u \text{ or } d \text{ or } s)$

N_{3l} : $e^+e^- \to l\bar{l} \to$ 3-jet events number $(l = u \text{ or } d \text{ or } s)$

Sensitivity of b quark mass

- For higher CM energy experiment, b quark mass becomes smaller relatively.

  → **b quark mass sensitivity be lower at higher energy scale.**

$$R_3^{bl} = \frac{N_{3b}/N_b}{N_{3l}/N_l} = 1 + \frac{\alpha_s}{\pi} a_{LO} + \frac{m_b^2}{s} \left(b_{LO}(m_b) + \frac{\alpha_s}{\pi} b_{NLO}(m_b) \right)$$

b quark mass sensitivity on R_3^{bl} : $\Delta R_3^{bl} = 2\left(1 - R_3^{bl}\right)\frac{\Delta m_b}{m_b}$

CM energy	Z-pole	250GeV
Necessary precision of R_3^{bl} for $\Delta m_b = 0.4\mathrm{GeV}$	∼1%	∼0.1%

- Precisely measurement of R_3^{bl} is needed for 250GeV measurement.

Flow of Simulation and Analysis

Flow of Simulation & Analysis

1. Generate Signal・Background events ･････････▶ Measure R_3^{bl} @ Parton, R_3^{bl} @ Hadronization

   Signal : $e^+e^- \to q\bar{q} \to Jets,$ Backgrounds : explain after

2. Detector Simulation・Event reconstruction

3. Cut of Background events ･･･････････････････▶ Measure R_3^{bl} @ Reconstructed

4. Estimate corrections of hadronization and detector

5. Estimate b quark mass precision

Samples in This Analysis

- Process : $e^+e^- \to q\bar{q} \quad (q = udscb)$
- CM energy : 250GeV
- 2 pure polarization configurations :

	e^-	e^+
Left components	100%	0%
Right components	0%	100%

Luminosity : $250\mathrm{fb}^{-1}$

	e^-	e^+
Left components	0%	100%
Right components	100%	0%

Luminosity : $250\mathrm{fb}^{-1}$

- Estimate b quark mass precision under the following polarizations by mixing above pure samples :

(-0.8,+0.3)

	e^-	e^+
Left components	90%	35%
Right components	10%	65%

Luminosity : $900\mathrm{fb}^{-1}$

(+0.8,-0.3)

	e^-	e^+
Left components	10%	65%
Right components	90%	35%

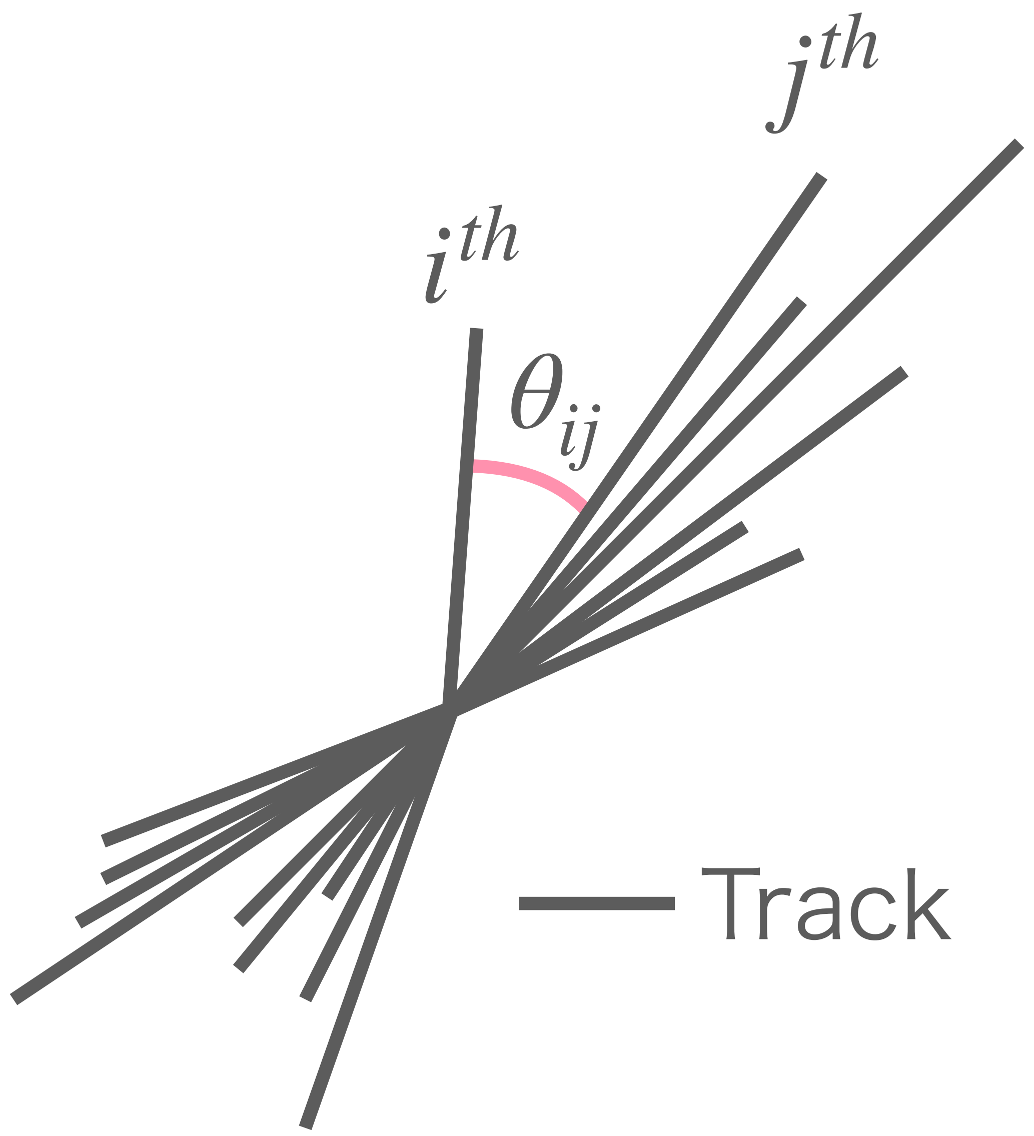
Luminosity : $900\mathrm{fb}^{-1}$

Reconstruction of Jets

- Define d_{ij} between i^{th} and j^{th} tracks (Cambridge algorithm) :

  $d_{ij} = 2E_i^2(1 - \cos\theta_{ij})$ E_i : Energy of i^{th} track

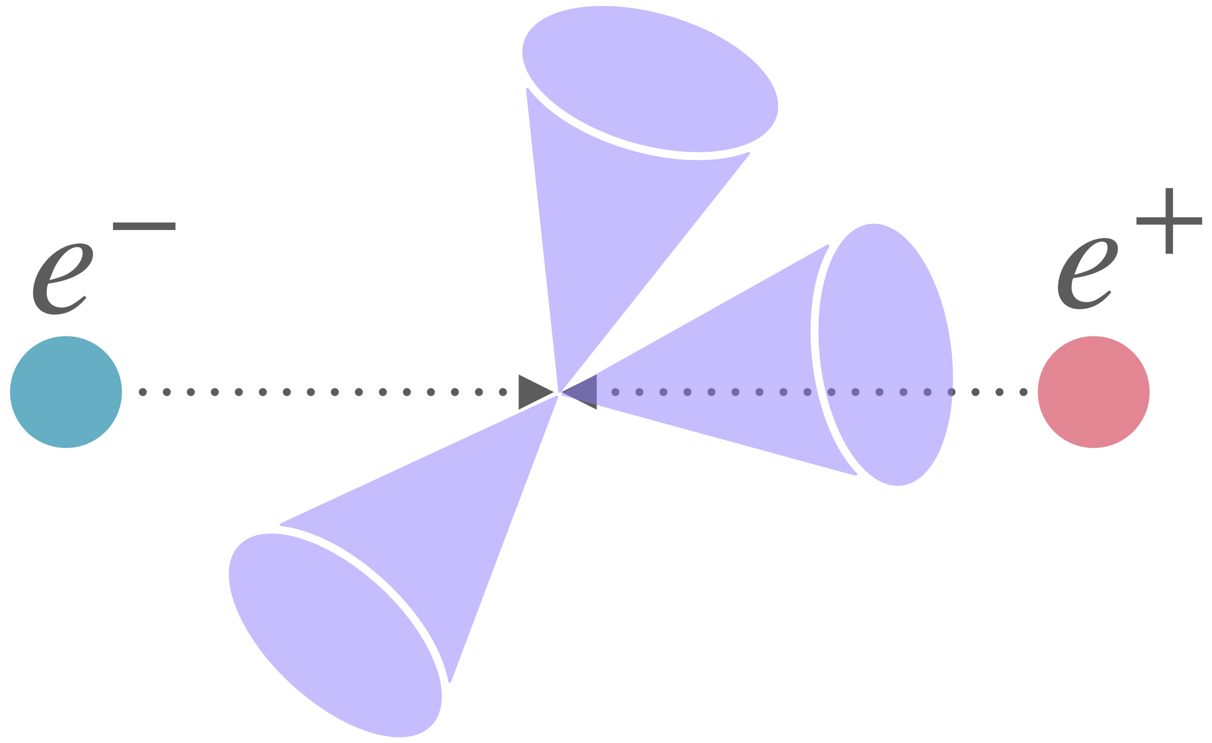
  If $d_{ij} < y_c$, these tracks are included in a same jet.



- R_3^{bl} has y_c dependence :

$$R_3^{bl} = \frac{N_{3b}/N_b}{N_{3l}/N_l}$$

$$= 1 + \frac{\alpha_s}{\pi}a_{LO}(y_c) + \frac{m_q^2}{s}\left(b_{LO}(m_q, y_c) + \frac{\alpha_s}{\pi}b_{NLO}(m_q, y_c)\right)$$



3-jet event

- Focuses on $y_c = 0.01$.

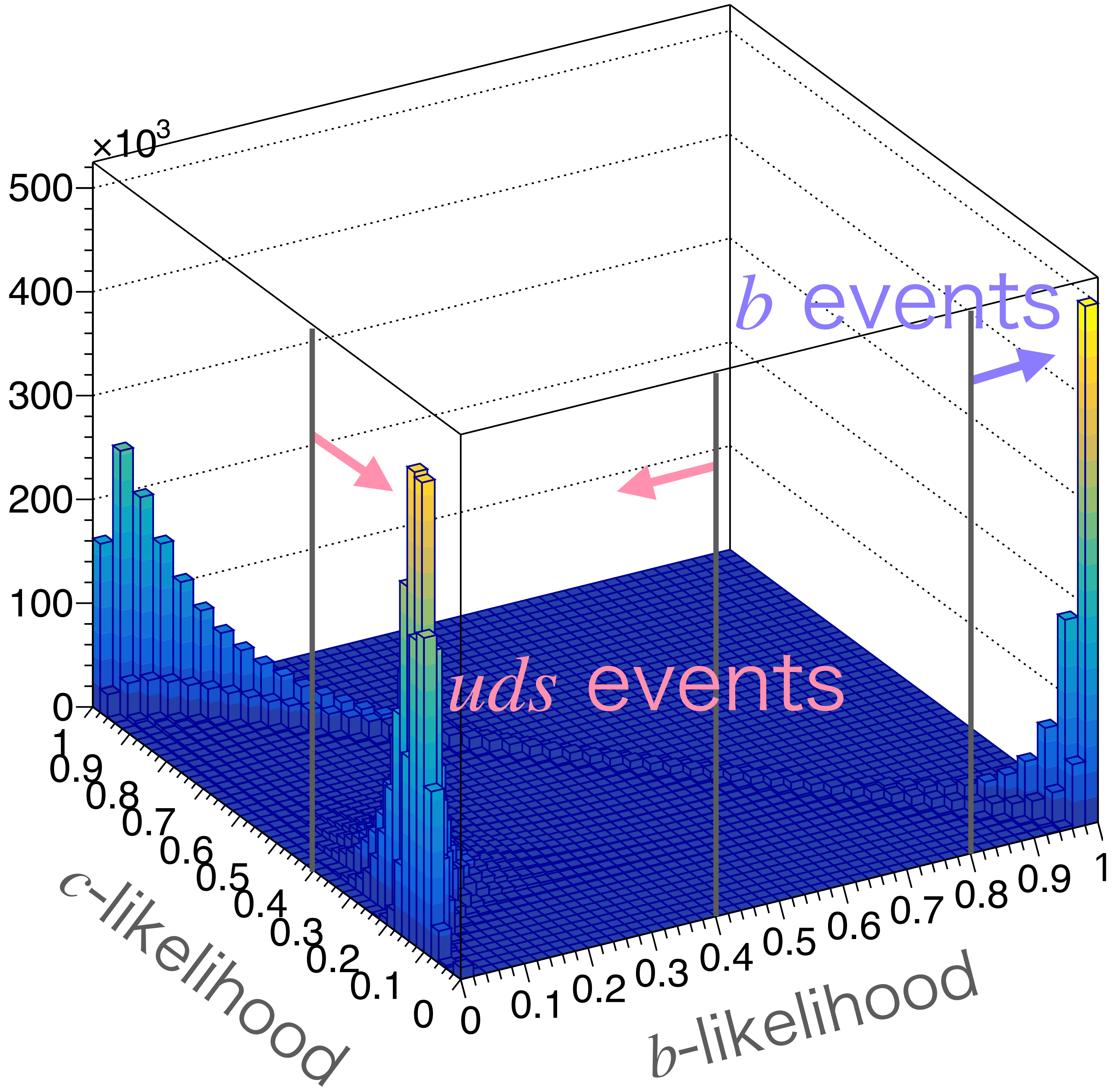
- Cluster until 2-jet events by loosing y_c for later analysis of backgrounds.

Flavor Identification

- Impose the following conditions to each jet

  b-likelihood > 0.8 → b events

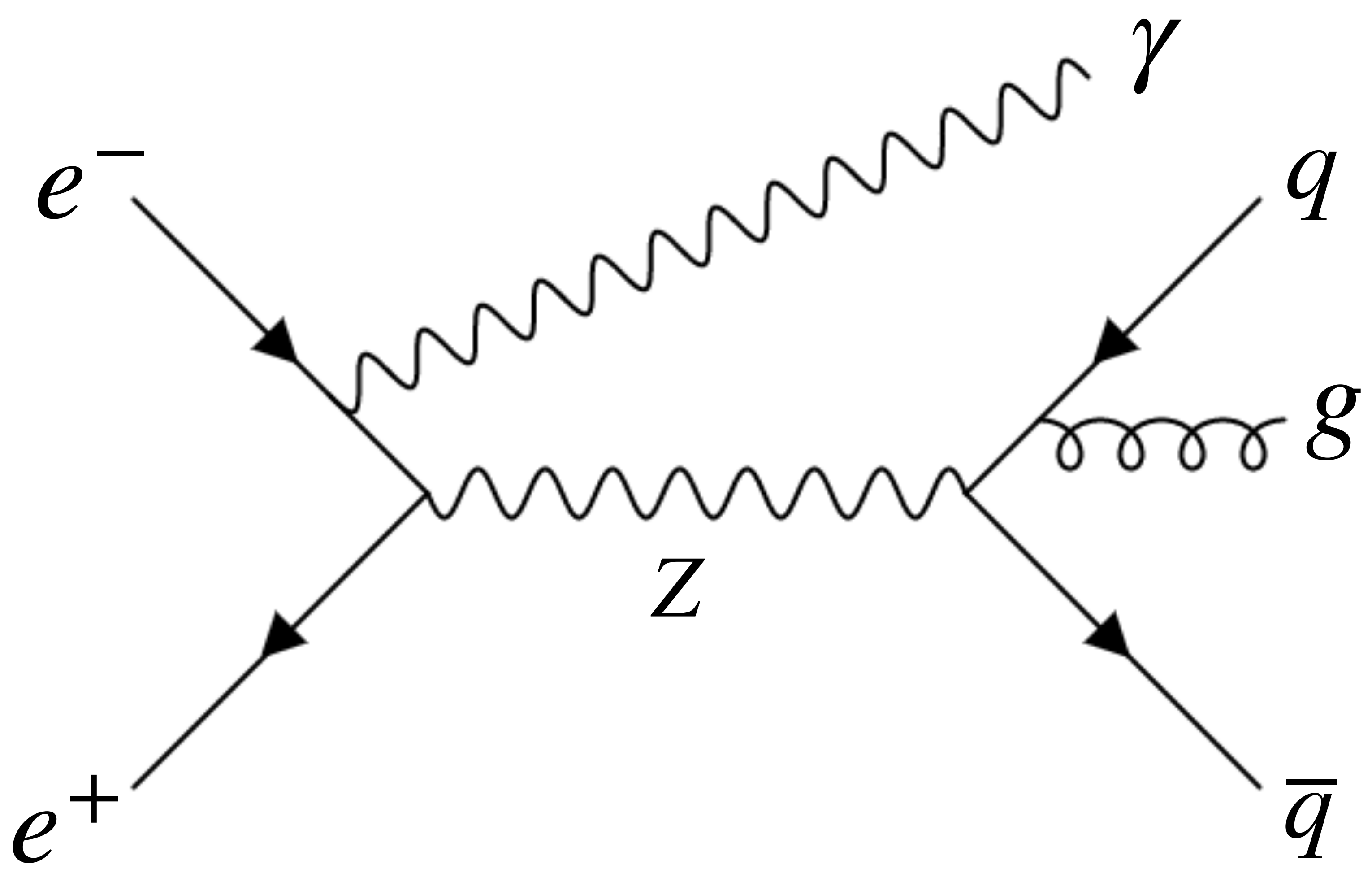
  b-likelihood < 0.4 & c-likelihood < 0.25 → uds events



	Efficiency of b-tag	Purity of b-tag	Efficiency of uds-tag	Purity of uds-tag
ILD	80%	98.7%	58%	96.1%
DELPHI	47%	86%	51%	82%

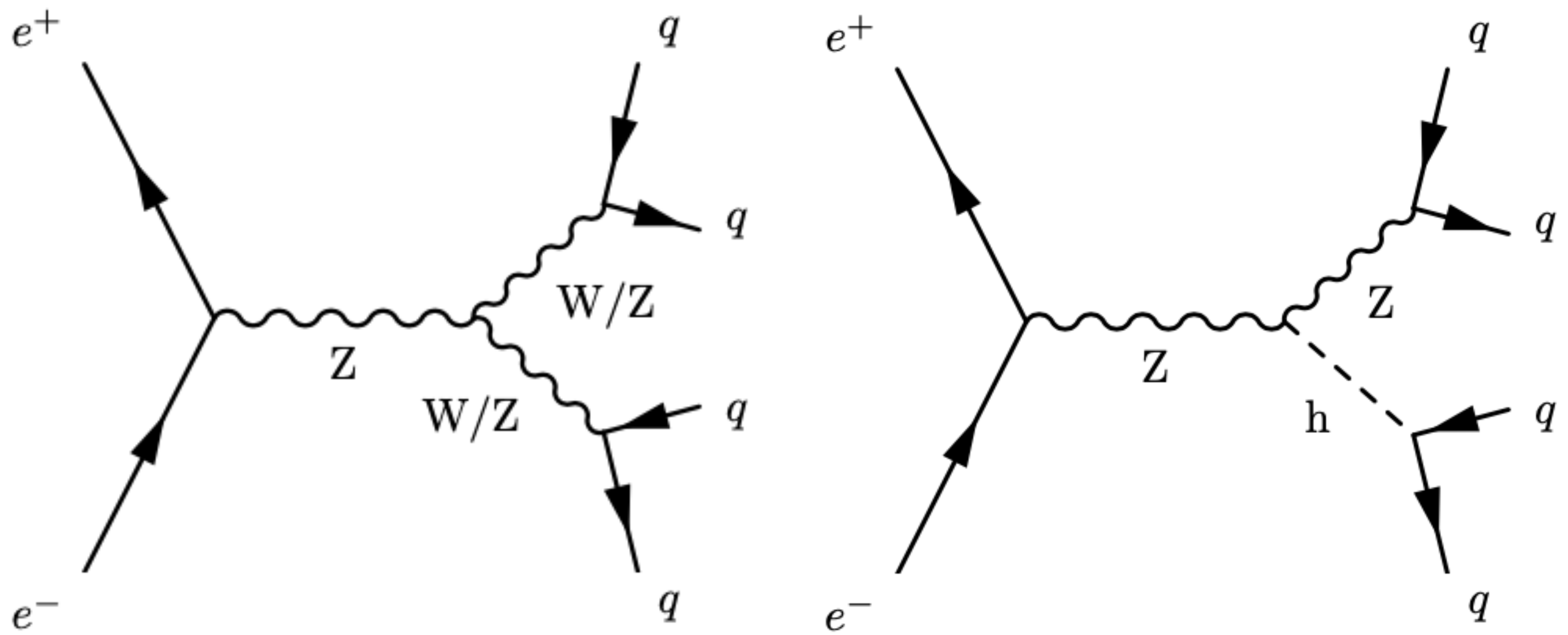
Cut of Background events

Types of Main Background events



Background event① : Radiative return

(Collision energy decreases by radiation)



Background event② : 4-jet events

Cut of Radiative return

- If radiation can not detect…

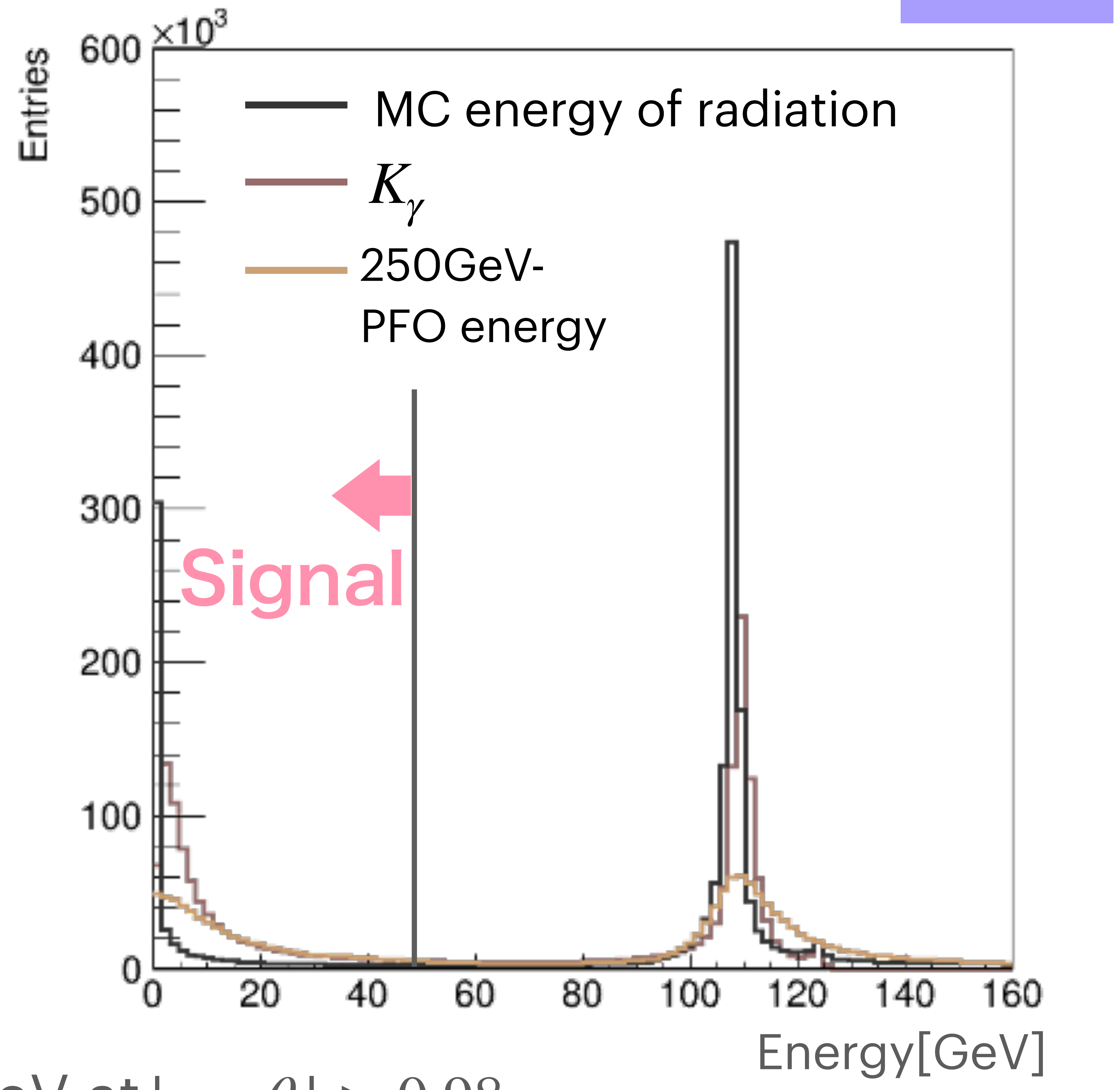
  →Construct energy of radiation K_γ from angles of jets,

  and cut $K_\gamma > 50\mathrm{GeV}$.

$$K_\gamma \equiv \frac{250GeV \cdot \sin\psi_{acol}}{\sin\psi_{acol} + \sin\theta_1 + \sin\theta_2}$$

ψ_{acol} : Angle between 2 jets

θ_1, θ_2 : Polar angles of each jet



- If radiation is detected, cut it by using neutral PFO.

1. Invariant mass of system less than 130GeV
2. Each jet should include particles more than 5
3. Jets include neutral particles which have energy of more than 50 GeV at $|\cos\theta| > 0.98$

100% Left polarized	(Remained BKG)/(Selected Signal)
b quark	1.2%
uds quarks	1.3%

Cut of 4-jet events

- If 4-jet events are forced to reconstruct as 2-jet events, shapes tend to be wider.
  → Cut by using Thrust T ($T < 0.85$)

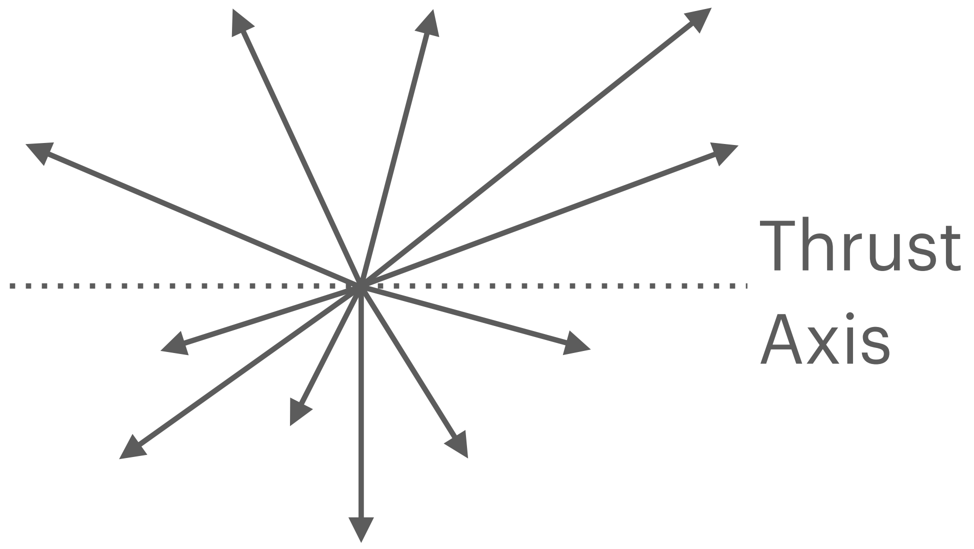
$$T \equiv \max_{\boldsymbol{n}} \frac{\sum_i^N |\boldsymbol{p}_i \cdot \boldsymbol{n}|}{\sum_i^N \boldsymbol{p}_i}$$

N : Total number of particles in jets

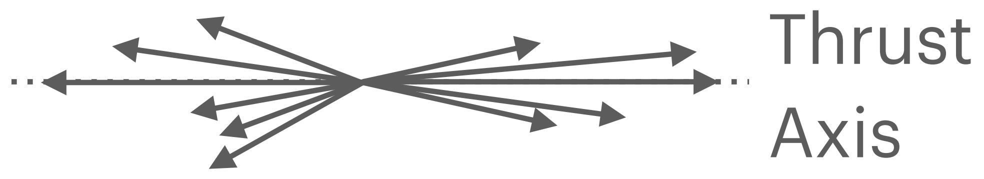
$\boldsymbol{p}_i$: momentum of each jet

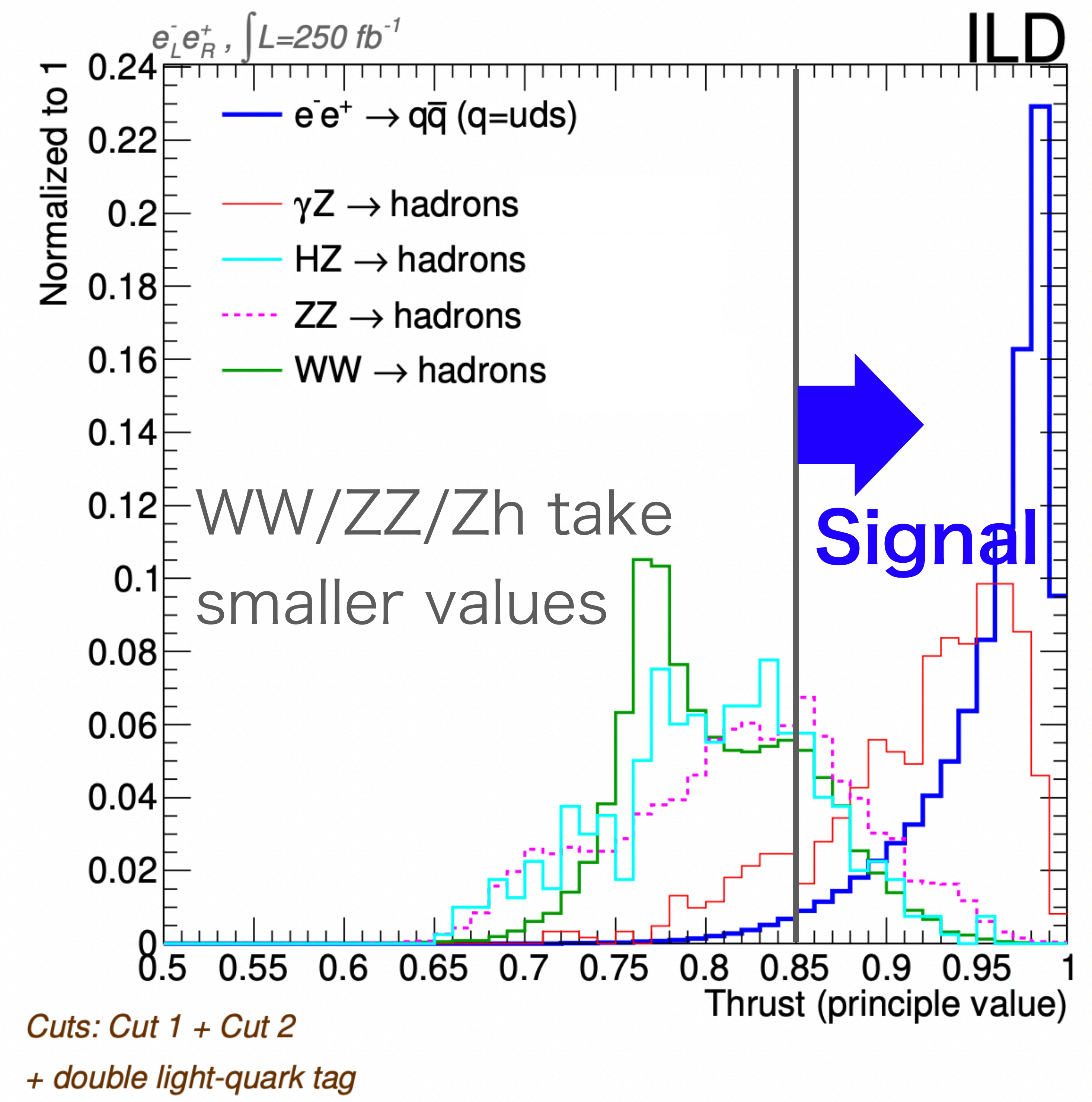
$\boldsymbol{n}$: unit vector of the thrust axis

$T \sim 0.5$ (Isotopic)



$T \sim 1$ (Sharper)





	(Remained BKG)/(Selected Signal)		
	WW	ZZ	Zh
100% Left polarized			
b quark	0.0%	0.3%	0.3%
uds quarks	2.3%	0.2%	0.0%
100% Right polarized			
b quark	0.0%	0.9%	1.0%
uds quarks	0.0%	0.3%	0.0%

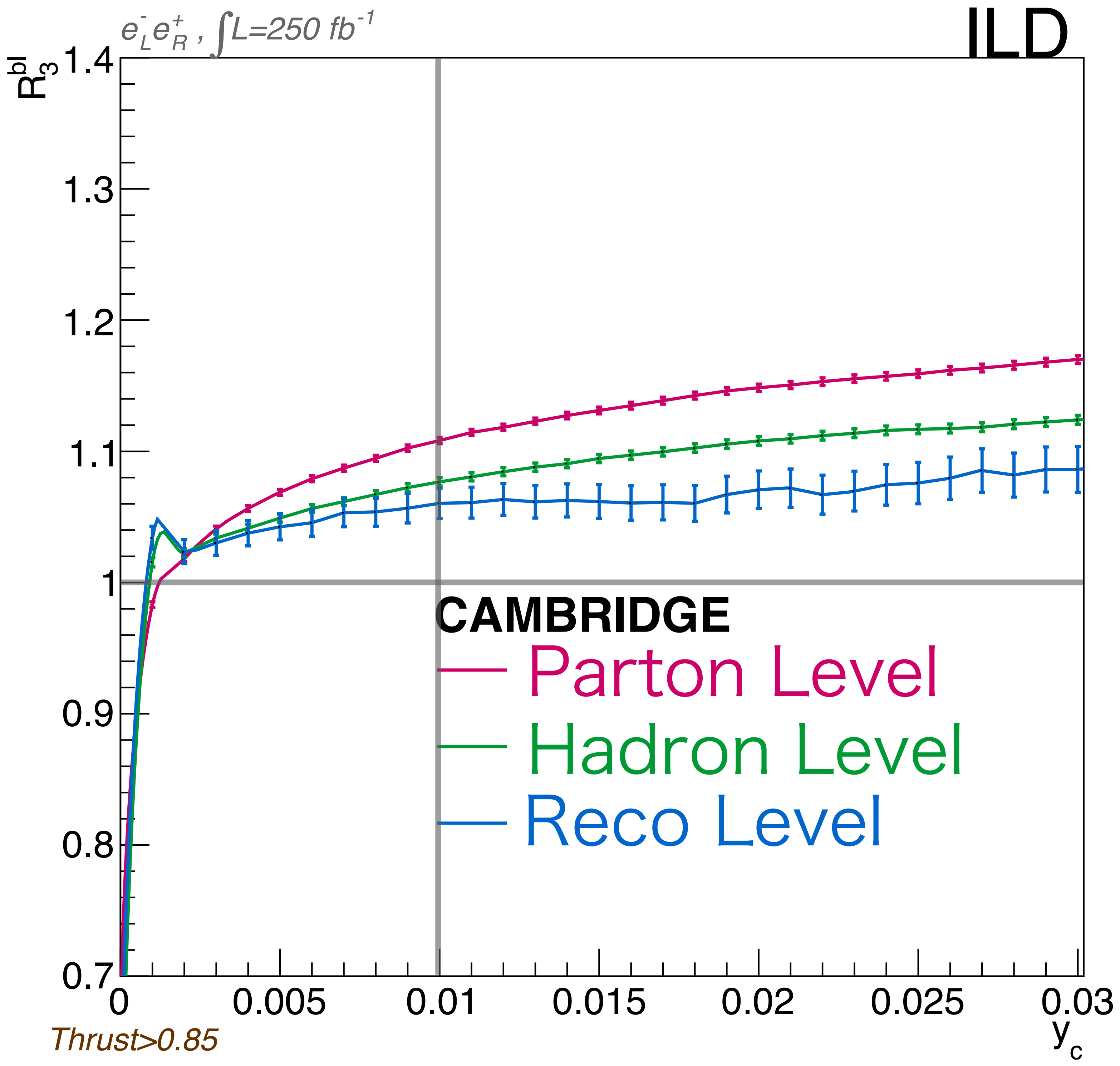
Measurement of the Observable

Measured Results of Observable

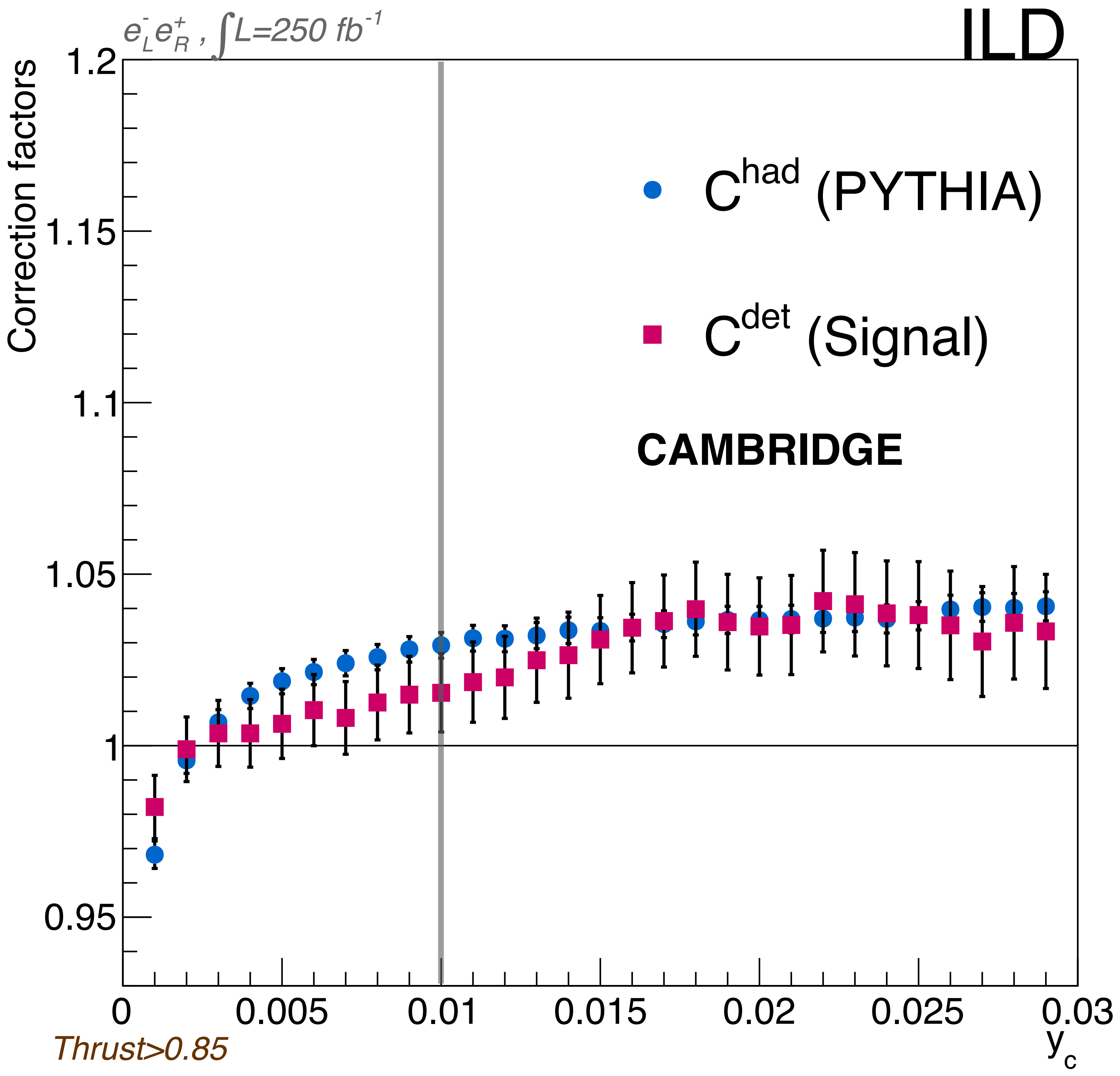
- Reco level is corrected to Parton level by corrections of hadronization and detector. Parton level can be compared to theoretical predictions.

$$R_3^{bl\ Par} = C^{had} C^{det} R_3^{bl\ Rec} \qquad C^{had} \equiv \frac{R_3^{bl\ Par}}{R_3^{bl\ Had}} \quad C^{det} \equiv \frac{R_3^{bl\ Had}}{R_3^{bl\ Rec}}$$

Each level R_3^{bl} by MC
(100% Left polarized)



Corrections C^{had}, C^{det}
(100% Left polarized)



Consideration of obtained results

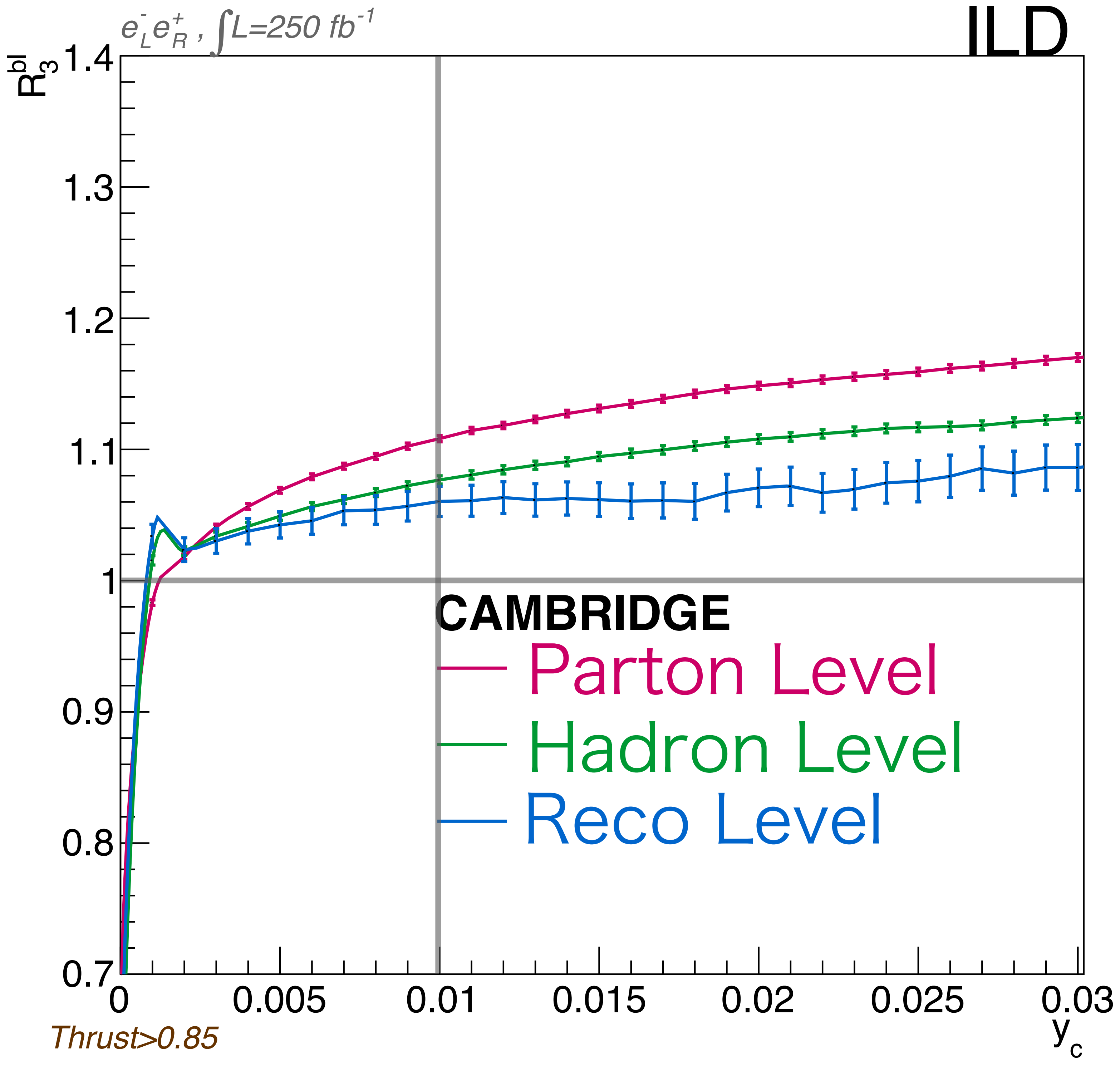
- b is heavier than uds → $R_3^{bl} = \dfrac{N_{3b}/N_b}{N_{3l}/N_l} < 1$

  But obtained result is $R_3^{bl} > 1$

- Numbers of 3-jet events are ~10 times less than the SM's expectation.
  → Is the MC sample adapted to this analysis?

- When examine MC ⋯
  $q\overline{q}$@LO+massless quarks, gluon radiation is only estimated by Parton Shower.
  → MC is updating to NLO+massive quarks now.

Each level R_3^{bl} by MC
(100% Left polarized)



Estimation of Errors

Estimation of Statistical error

- Statistical error on R_3^{bl} :

$$R_3^{bl} = \frac{N_{3b}/N_b}{N_{3l}/N_l} \rightarrow \frac{\Delta R_3^{bl}}{R_3^{bl}}(stat.) \sim \frac{1}{\sqrt{N_{3b}}} + \frac{1}{\sqrt{N_{3l}}} + \frac{1}{\sqrt{N_b}} + \frac{1}{\sqrt{N_l}}$$

N_{3q} : 3-jet events number
N_q : all jet events number

- Estimate under 2ab^{-1} of 250 GeV ILC (H20 scenario)

Polarization (P_{e^-}, P_{e^+})	(-0.8,+0.3)	(+0.8,-0.3)	(-0.8,-0.3)	(+0.8,+0.3)
Integrated luminosity	900fb^{-1}	900fb^{-1}	100fb^{-1}	100fb^{-1}

- Assumed the proportion of 3-jet events is 30% of the all jet events number.

	All jet events (MC)		3-jet events (Assumed)	
	N_b	N_l	$N_{3b} = 0.3N_b$	$N_{3l} = 0.3N_l$
(-0.8,+0.3)	1,210,601	1,659,628	363,180	497,888
(+0.8,-0.3)	341,956	596,210	102,587	178,863

30%

$$\frac{\Delta R_3^{bl}}{R_3^{bl}}(stat.) = 0.25\,\% \text{ for } (-0.8,+0.3),\ \ 0.45\,\% \text{ for } (+0.8,-0.3)$$

Types of Systematic errors

- Appear systematic errors on corrections C^{had}, C^{det}.

Parton Level

Reco Level

$$R_3^{bl\ Par} = C^{had} C^{det} R_3^{bl\ Rec}$$

Corrections of hadronization and detector

1. Hadronization error : Estimate the uncertainty on $C^{had} \equiv \dfrac{R_3^{bl\ Par}}{R_3^{bl\ Had}}$.

2. Detector error : Estimate the uncertainty on $C^{det} \equiv \dfrac{R_3^{bl\ Had}}{R_3^{bl\ Rec}}$.

Estimations of Systematic errors

- Hadronization model error

We expect hadronization error can be reduced with larger data samples and the larger momentum of hadrons in 250GeV

→ Assume the half of LEP's result : $\Delta C^{had}/C^{had} = 0.1\ \%$

- Detector error

Estimate appeared uncertainties on C^{det} from 3 element's uncertainties by Toy MC.

Common uncertainties for jet's number and flavor are cancelled.

	Each Uncertainty	Uncertainties on C^{det}	
		100% Left polarized	100% Right polarized
Tagging efficiency	0.5%	0.07%	0.06%
Signal selection efficiency	1%	0.06%	0.06%
Contaminations of BKG	1%	0.20%	0.10%
Total	—	0.22%	0.13%

Result and Summary

Result of This study

- Precision of R_3^{bl} :

$$\frac{\Delta R_3^{bl}}{R_3^{bl}} = 0.25(stat.) \pm 0.22(exp.) \pm 0.1(had.)[\%] \quad \text{for} \quad (-0.8, +0.3)$$

$$\frac{\Delta R_3^{bl}}{R_3^{bl}} = 0.45(stat.) \pm 0.13(exp.) \pm 0.1(had.)[\%] \quad \text{for} \quad (+0.8, -0.3)$$

(3-jet events number is assumed to be 30% of all jet events number.)

- Estimate b quark mass precision

b quark mass sensitivity on R_3^{bl} : $\frac{\Delta m_b}{m_b} = \frac{\Delta R_3^{bl}}{2(1 - R_3^{bl})}$

Precision for $R_3^{bl} = 0.996,\ m_b = 2.75\text{GeV}$:

$$\Delta m_b\left(250\right) = 0.76(stat.) \pm 0.59(exp.) \pm 0.34(had.) \pm 0.07(theo.)\ \text{GeV}$$

Theoretical uncertainty : come from renormalized scale and quark mass definition

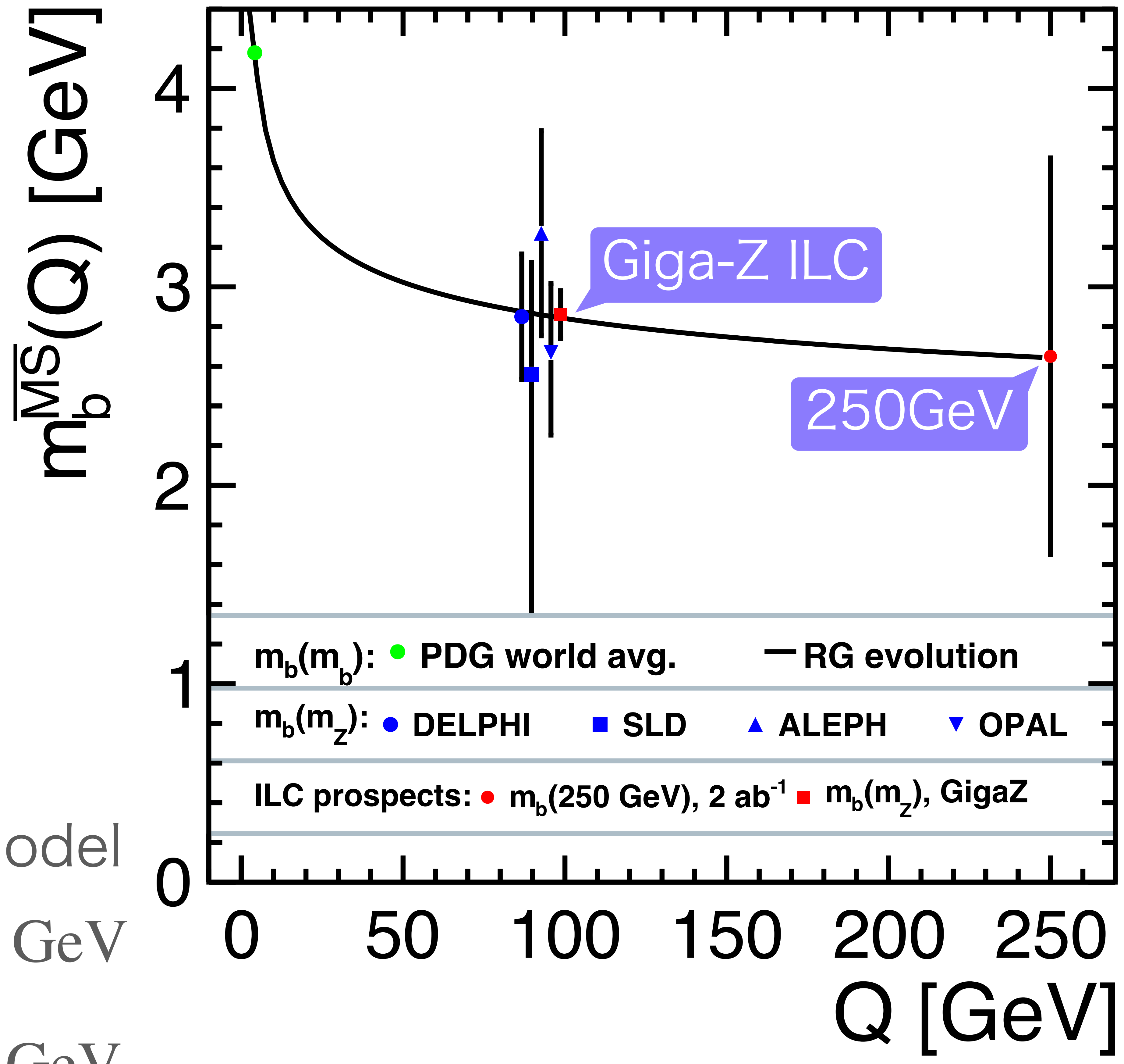
ILC Prospects

- 250GeV measurement is challenging, but it will provide an extra point at never probed energies.

- Giga-Z ILC (ILC@Z-pole) can measure b quark mass at the better precision.
  - Statistics is 100 times larger
  - ILD superior the performance of flavor tagging
  - Hadronization error will be half thanks for development of model

LEP : $\Delta m_b\left(M_Z\right) = 0.18(stat.) \pm 0.13(exp.) \pm 0.19(had.) \pm 0.12(theo.)\ \mathrm{GeV}$

ILC : $\Delta m_b\left(M_Z\right) = 0.02(stat.) \pm 0.02(exp.) \pm 0.09(had.) \pm 0.06(theo.)\ \mathrm{GeV}$


The running bottom quark mass, IFIC (U. Valencia/CSIC), U. Tohoku

- Check Marcel's talk in LCWS : The bottom quark mass and the Higgs boson (3.17th)

Summary

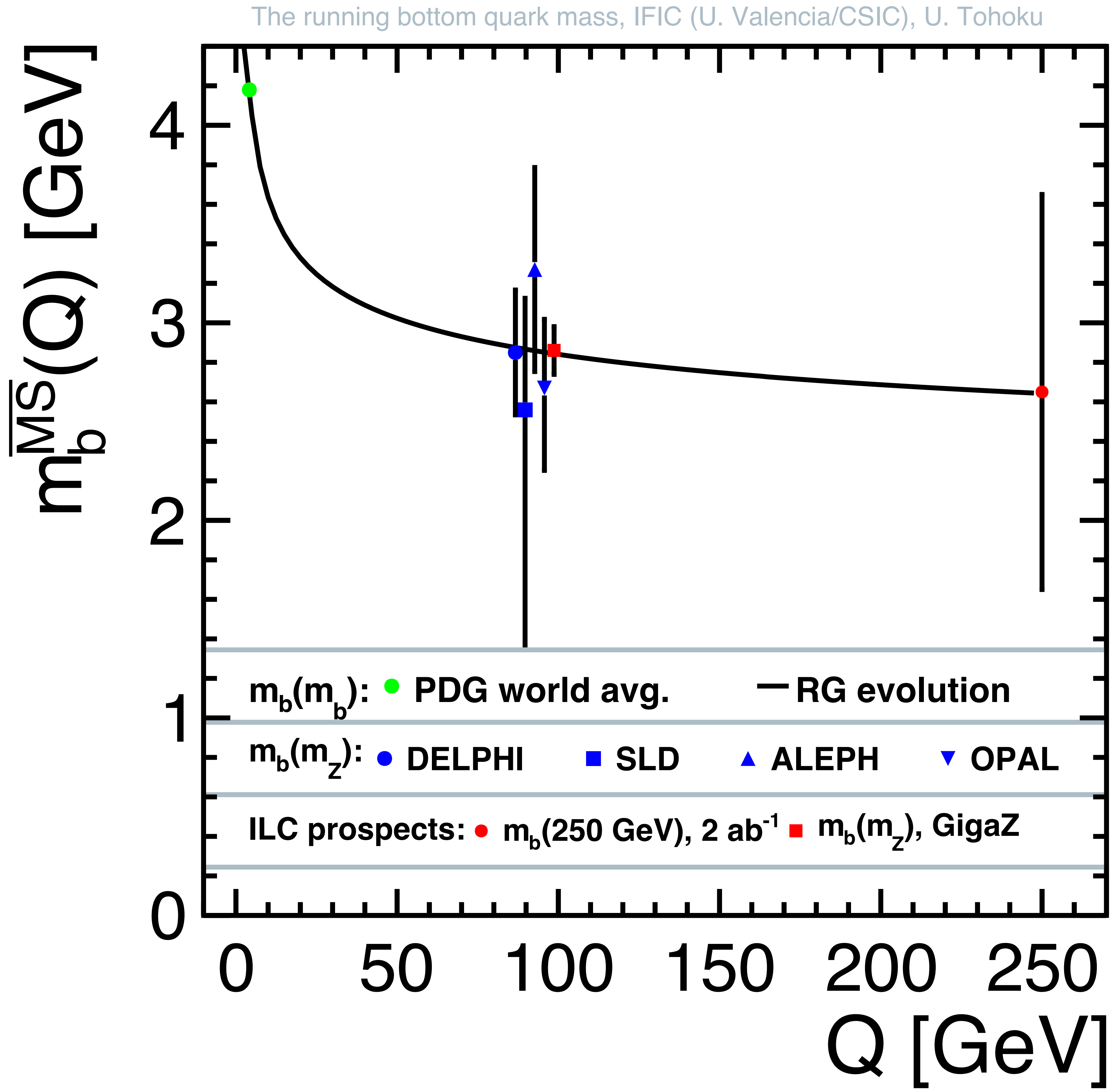
- The first Estimation of b quark mass precision at 250 GeV ILC :

$$\Delta m_b\left(250\right) = 0.76(stat.) \pm 0.59(exp.) \pm 0.34(had.) \pm 0.07(theo.)\ \mathrm{GeV}$$

- WHIZARD is updating to NLO+massive quarks.

- Giga-Z ILC provides a better measurement of b quark mass at Z-pole :

$$\Delta m_b\left(M_Z\right) = 0.02(stat.) \pm 0.02(exp.) \pm 0.09(had.) \pm 0.06(theo.)\ \mathrm{GeV}$$



Backup

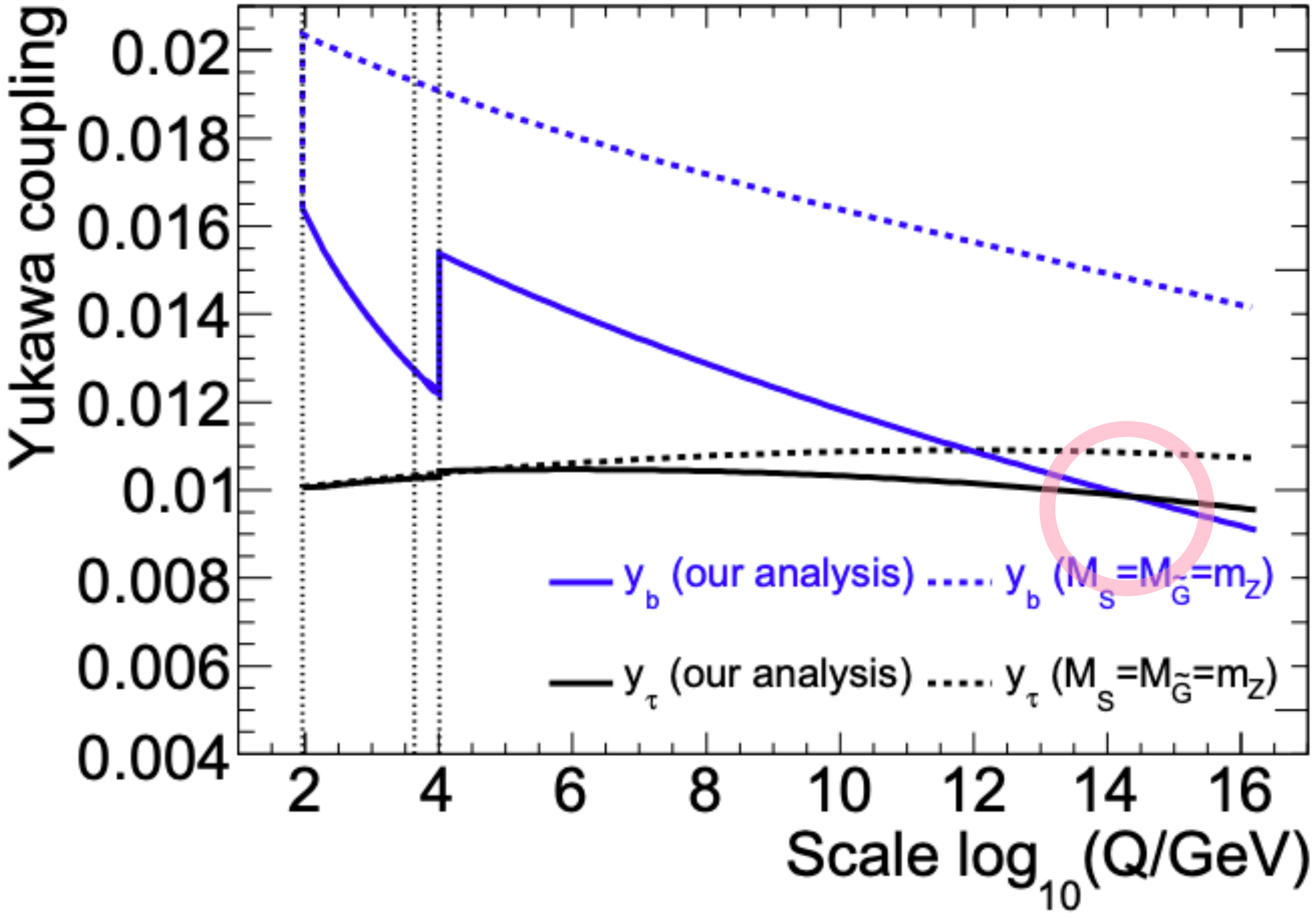
Unification of 3rd generation particles

- Scenario changes according to the ratio of vacuum expectation value of SUSY Higgs $\tan\beta$.

$\tan\beta \sim 3-11$: b-τ Yukawa coupling is unified

$\tan\beta \sim 35-60$: b-τ-t Yukawa coupling is unified

arXiv:1201.4412 [hep-ph]
arXiv:1604.02156 [hep-ph]



Polarization

- Polarization rate : $P_{e^{-(+)}} \equiv \dfrac{f_R - f_L}{f_R + f_L}$

f_L : proportion of left handed components

f_R : proportion of right handed components

(P_{e^-}, P_{e^+}) = (+80%,+30%)

	e^-	e^+
Left	10%	35%
Right	90%	65%

Luminosity : 100fb^{-1}

$e_L^- e_R^+$: 17fb^{-1}

(P_{e^-}, P_{e^+}) = (-80%,-30%)

	e^-	e^+
Left	90%	65%
Right	10%	35%

Luminosity : 100fb^{-1}

$e_L^- e_R^+$: 83fb^{-1}

(P_{e^-}, P_{e^+}) = (-80%,+30%)

	e^-	e^+
Left	90%	35%
Right	10%	65%

Luminosity : 900fb^{-1}

$e_L^- e_R^+$: 846fb^{-1}

(P_{e^-}, P_{e^+}) = (+80%,-30%)

	e^-	e^+
Left	10%	65%
Right	90%	35%

Luminosity : 900fb^{-1}

$e_L^- e_R^+$: 50fb^{-1}

……

- How to mix :

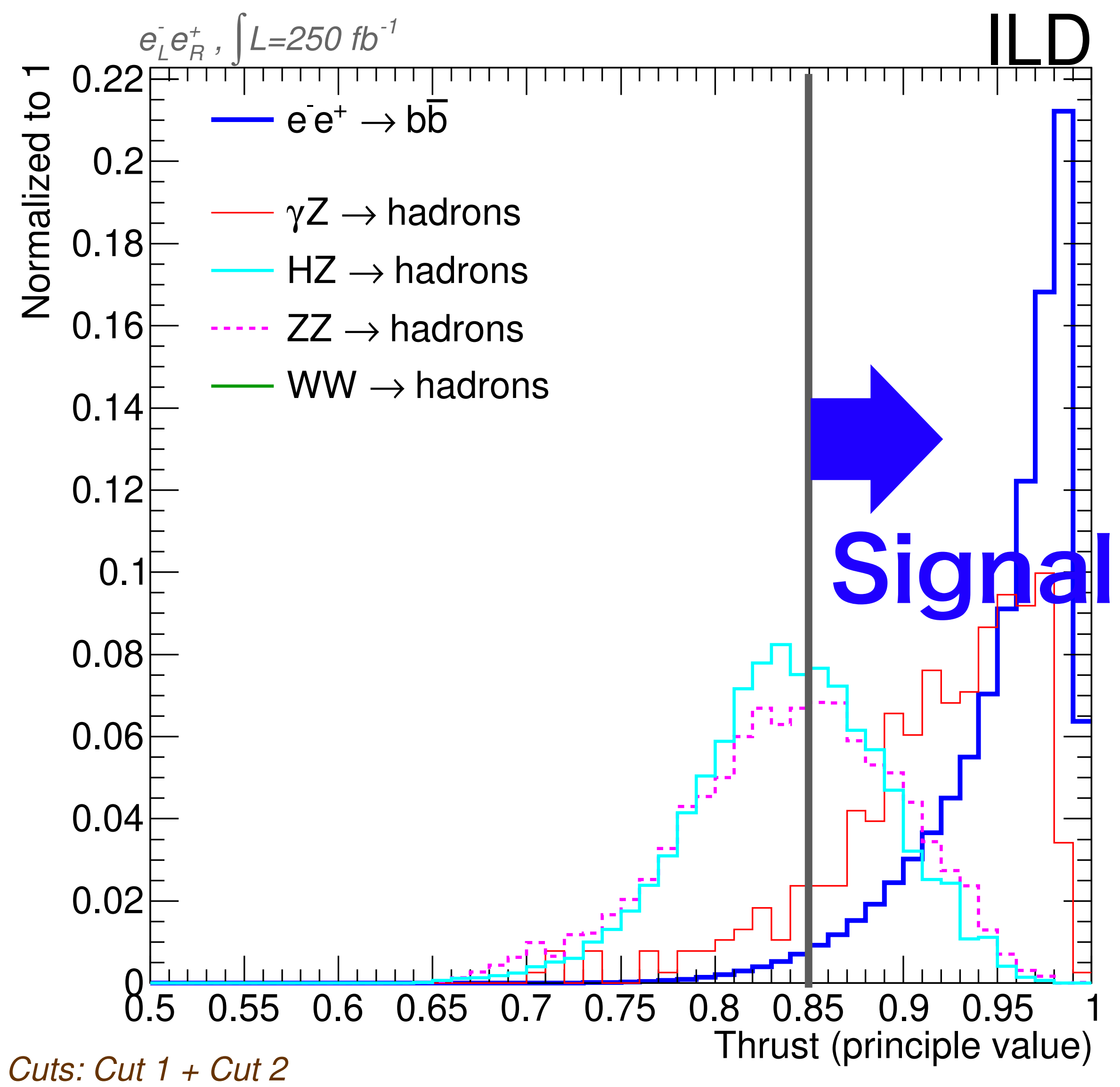
100% Left polarized

100% Right polarized

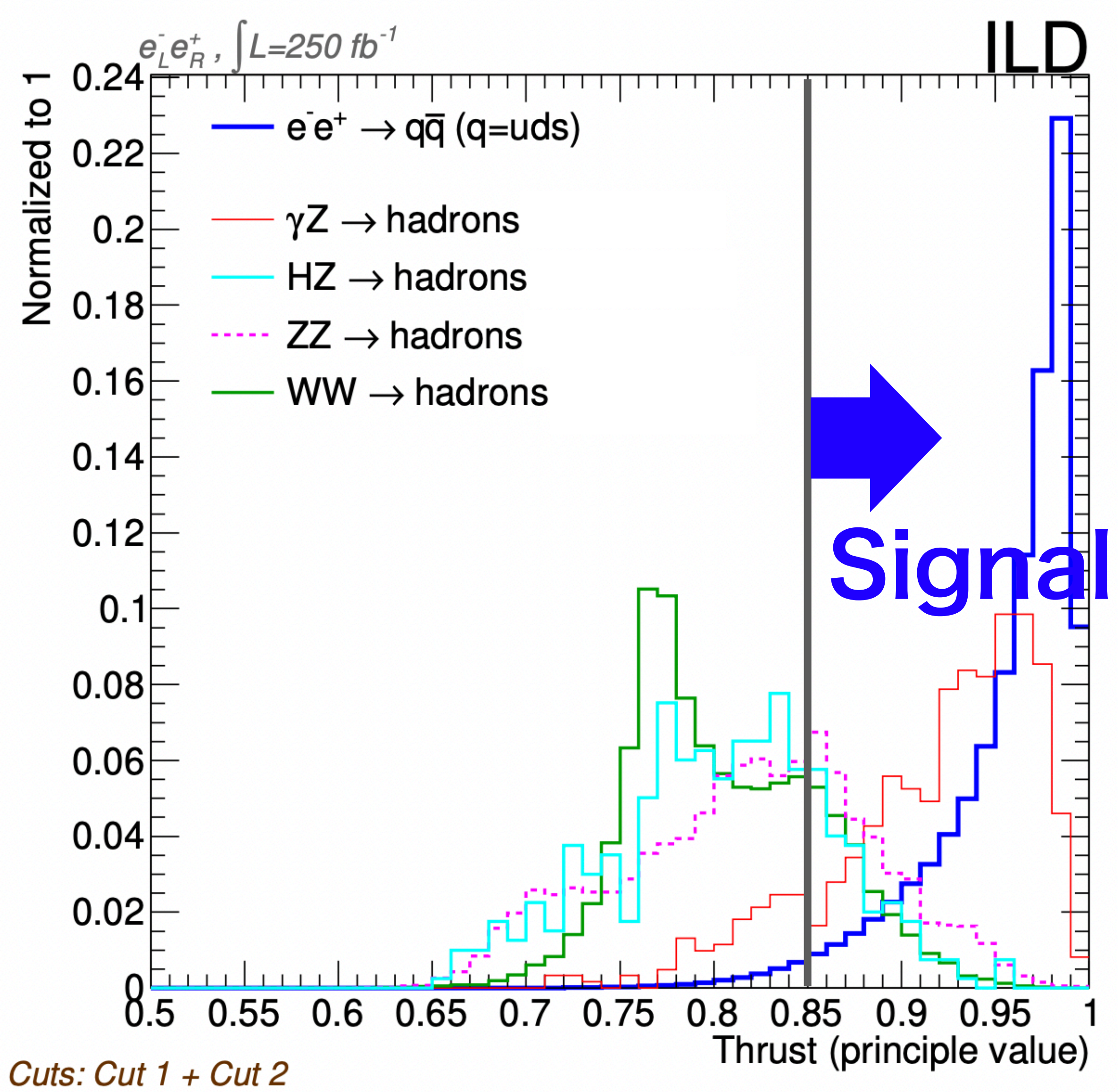
$$\sigma(P_{e^-}, P_{e^+}) = \frac{1}{4}\Big(\left(1 - P_{e^-}\right)\left(1 + P_{e^+}\right)\sigma_{LR} + \left(1 + P_{e^-}\right)\left(1 - P_{e^+}\right)\sigma_{RL}\Big)$$

$e_L^- e_L^+$ and $e_R^- e_R^+$ for quark pair production from e^+e^- are vanished by angular momentum conservation.

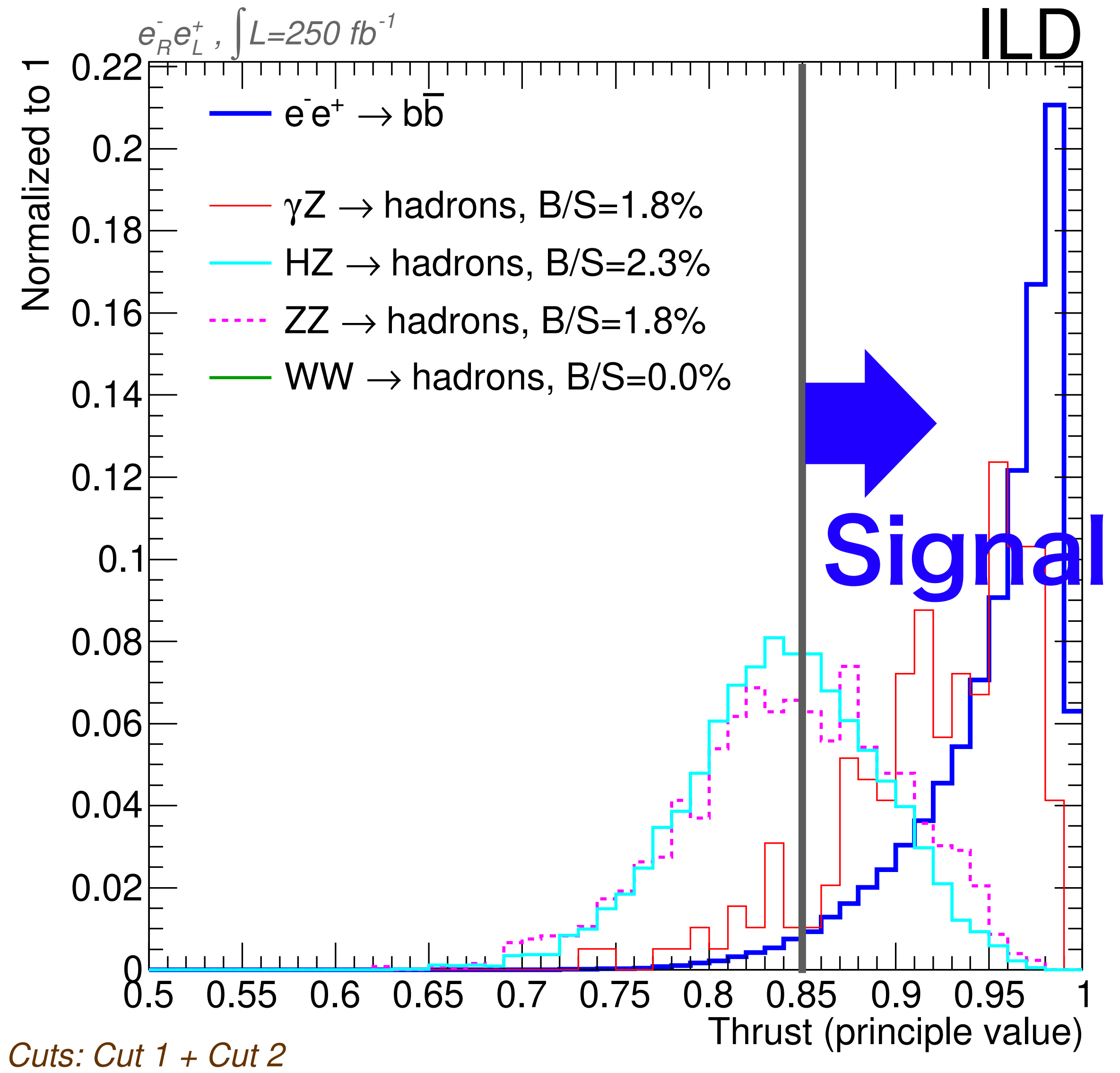
Thrust distributions



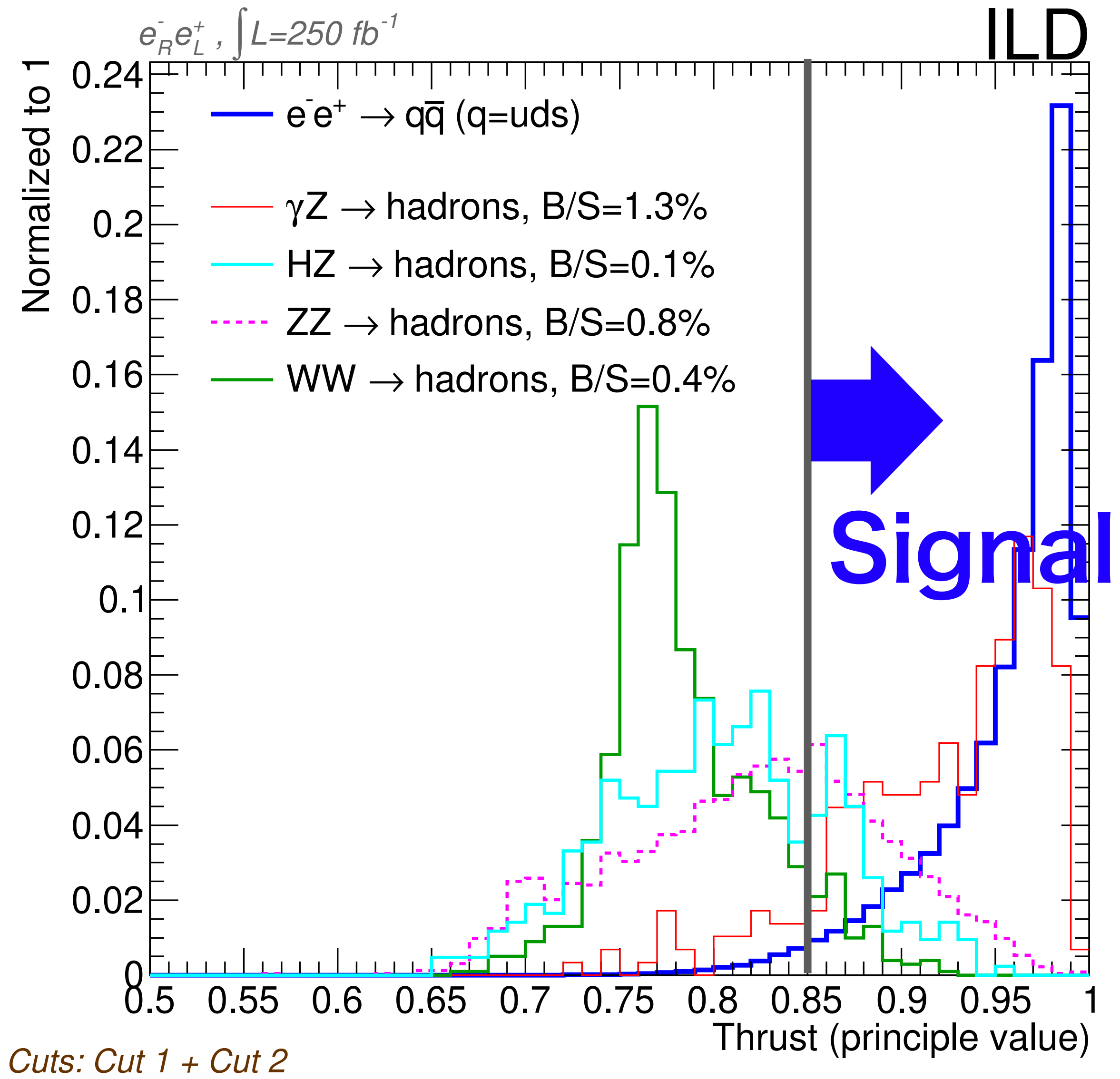
Cuts: Cut 1 + Cut 2
+ double b-quark tag



Cuts: Cut 1 + Cut 2
+ double light-quark tag



Cuts: Cut 1 + Cut 2
+ double b-quark tag



Cuts: Cut 1 + Cut 2
+ double light-quark tag

- Left polarized case suffers from more background contaminations than right polarized case.

Estimation of Detector errors

- Add fluctuations to each efficiency by Toy MC and estimate appeared uncertainties on C^{det}.

$$R_3^q \equiv \frac{N_{3q}}{N_q} = \frac{\varepsilon_{sel}\cdot\left[\varepsilon_q^2\sigma_q^{3jet} + \tilde{\varepsilon}_q^2\sigma_{q'}^{3jet}\right] + \varepsilon_{bkg}\sigma_{bkg}^{3jet}}{\varepsilon_{sel}\cdot\left[\varepsilon_q^2\sigma_q + \tilde{\varepsilon}_q^2\sigma_{q'}\right] + \varepsilon_{bkg}\sigma_{bkg}}$$

σ_q : X-sect for $e^+e^- \to q\bar{q}$

σ_q^{3jet} : X-sect for $e^+e^- \to q\bar{q} \to 3\ Jets$

ε_q : Tagging efficiency as q-flavor

$\tilde{\varepsilon}_q$: mis-tagging efficiency as q-flavor

- Selection efficiency ε_{sel} is given like this :

	Uncertainty	Flavor	100% Left polarized	100% Right polarized
Signal selection efficiency	1%	b quark	36.9%	36.5%
		uds quarks	16.2%	16.4%

- Flavor tagging efficiencies are estimated by double tagging methods, and add the fluctuation of 0.5%

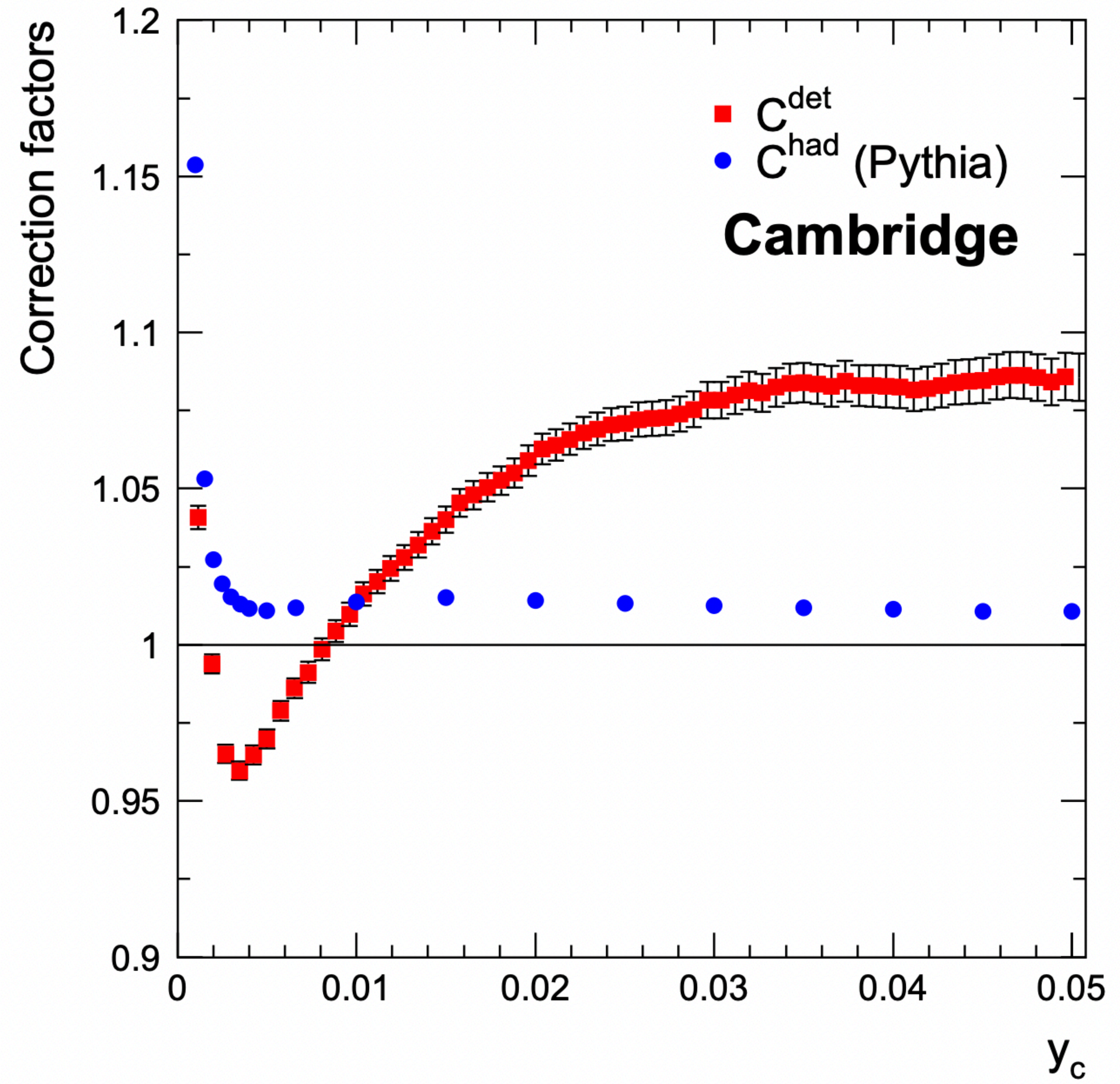
- Assume the fluctuation of 1% for BKG contaminations $\varepsilon_{bkg}\sigma_{bkg}^{(3-jet)}$.

- Total detector error : $\dfrac{\Delta R_3^{bl}}{R_3^{bl}}(exp.) = 0.22\,\%$ for $(-0.8, +0.3)$, $0.13\,\%$ for $(+0.8, -0.3)$ (See page.22)

Correction factors

$$R_3^{bl\ Par} = C^{had} C^{det} R_3^{bl\ Rec} \qquad C^{had} \equiv \frac{R_3^{bl\ Par}}{R_3^{bl\ Had}} \quad C^{det} \equiv \frac{R_3^{bl\ Had}}{R_3^{bl\ Rec}}$$

Corrections C^{had}, C^{det} (DELPHI)



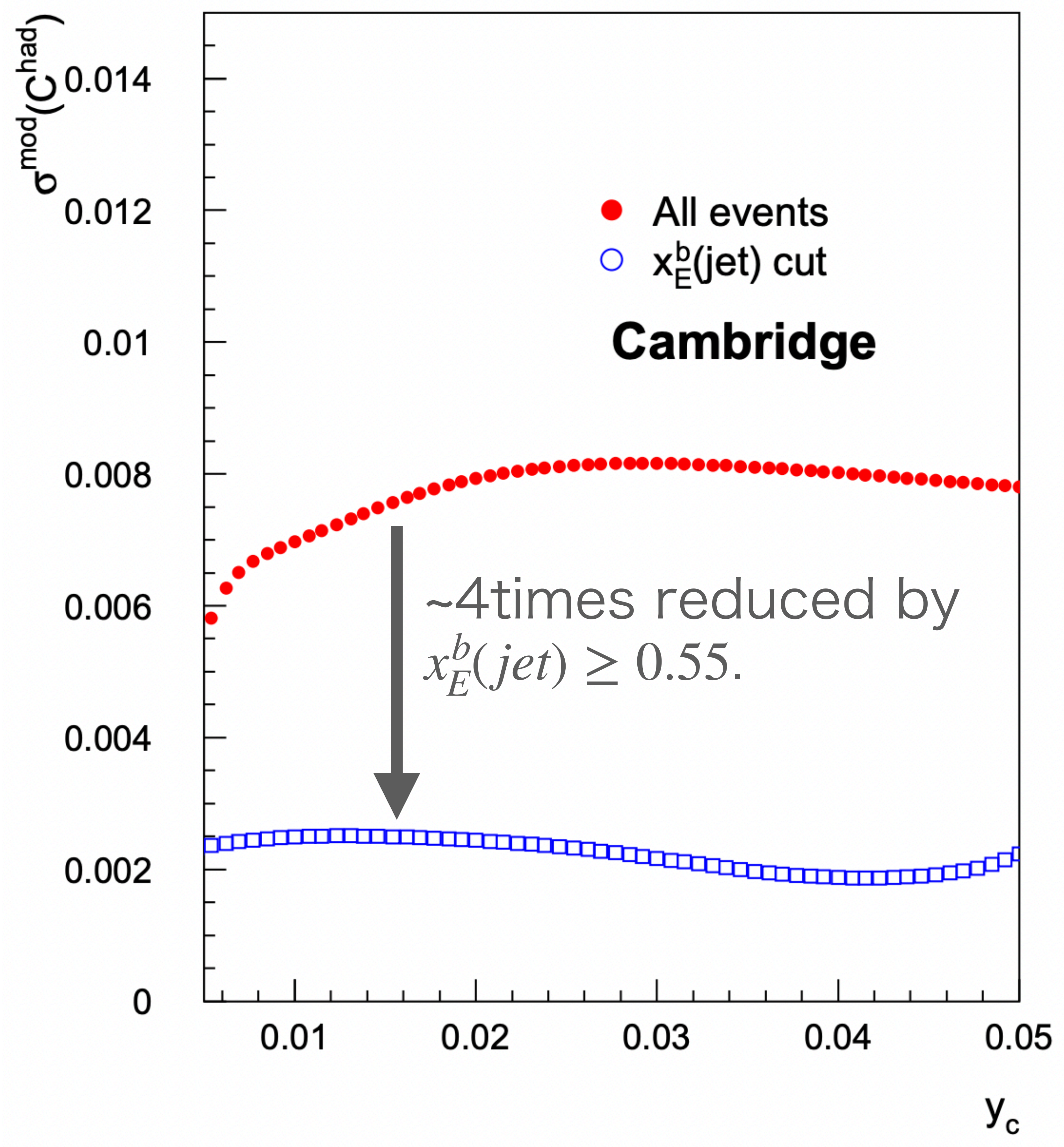
- DELPHI took $y_c = 0.0085$ in Cambridge algorithm, and corrections are close to 1.

  arXiv:hep-ex/0603046

Hadronization model error in DELPHI

$R_3^{bl\ Par} = C^{had} C^{det} R_3^{bl\ Rec}$ $\quad C^{had} \equiv \frac{R_3^{bl\ Par}}{R_3^{bl\ Had}}$ $\quad C^{det} \equiv \frac{R_3^{bl\ Had}}{R_3^{bl\ Rec}}$

Uncertainty on C^{had} (DELPHI)



- Estimate uncertainty by considering different models (PYTHIA, HERWIG…) and tuning parameters.
  arXiv:hep-ex/0603046
- The model uncertainty depends on $x_E^b(jet) \equiv 2E_{b-jet}/M_Z$, and if $x_E^b(jet) \geq 0.55$ is imposed, it is reduced by a factor of 4.
- Configuration of b quark mass at Parton Shower & hadronization is taken into account.

About the number of $m_b(m_b)$

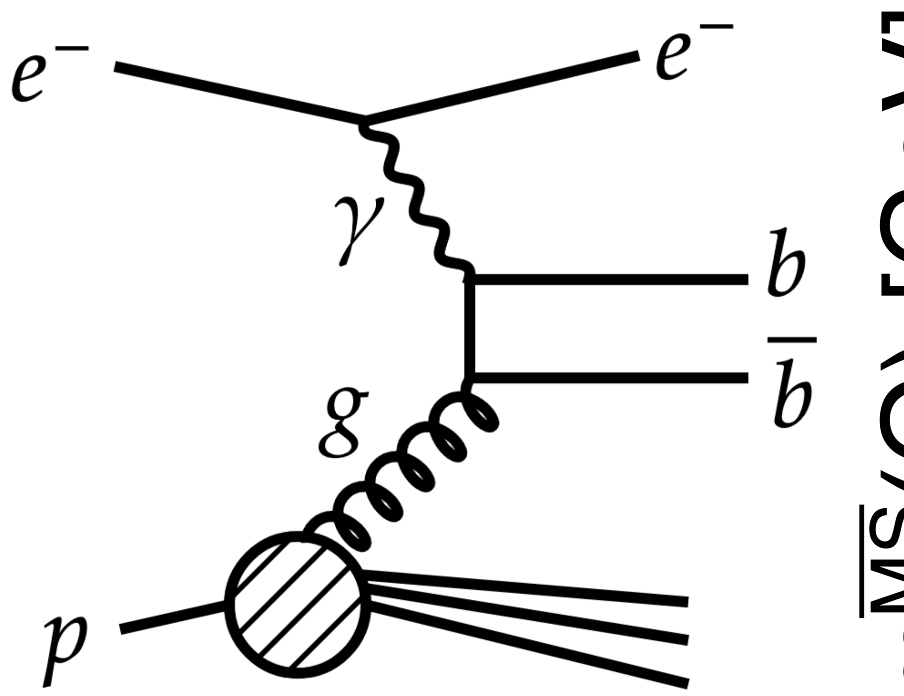
- Green marker is the word average of PDG

$$m_b(m_b) = 4.18^{+0.03}_{-0.02}\ \mathrm{GeV}$$

Main experiments：

ZEUS @ HERA (2018) arXiv:1804.01019 **[hep-ex]**

The e-p deep inelastic scattering depends on b mass.

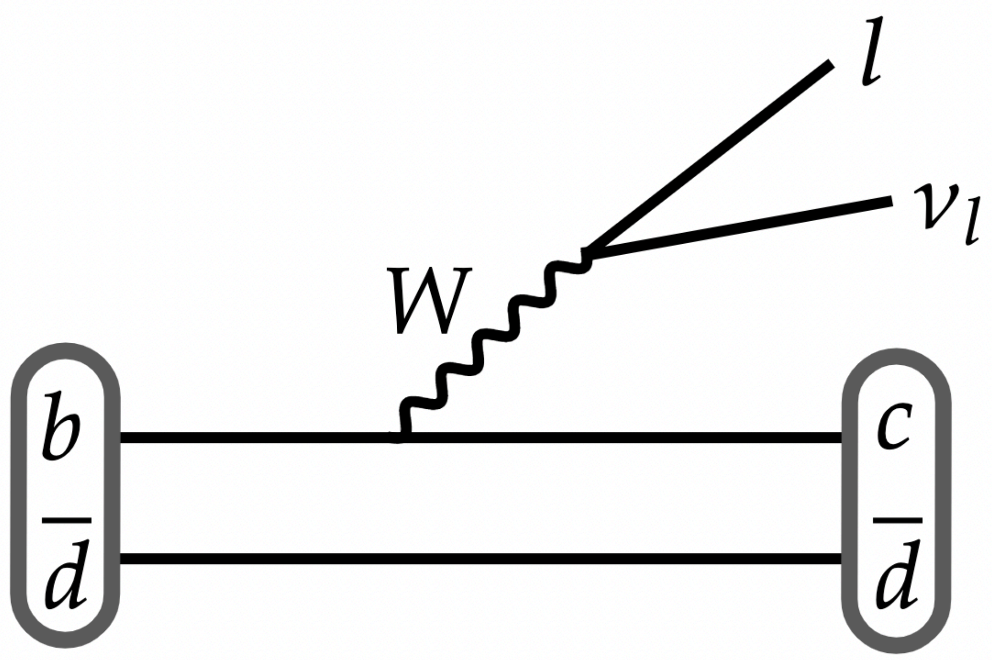


BABAR @ PEP-II (2009) arXiv:0908.0415 **[hep-ex]**

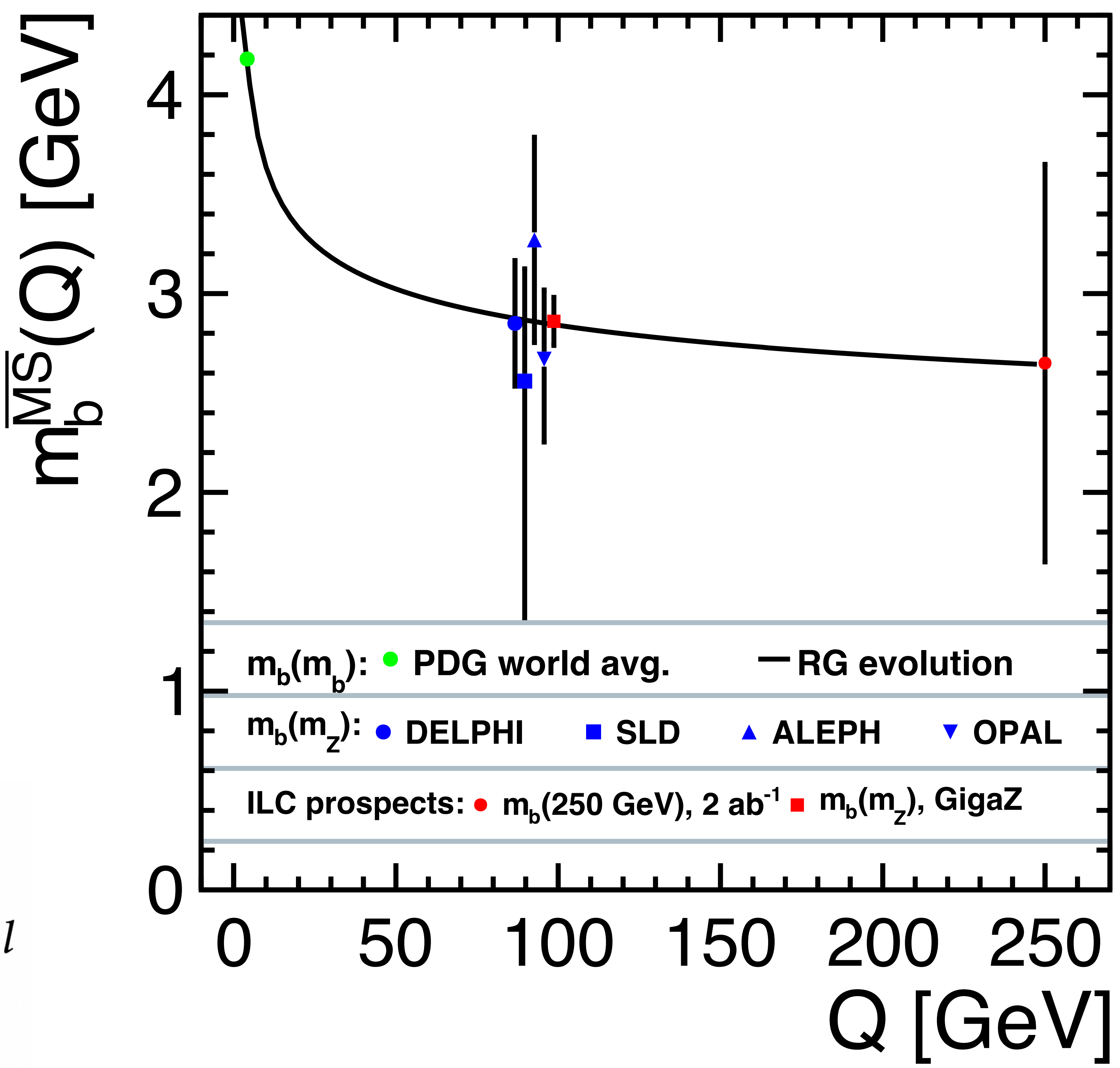
Belle @ KEKB (2008) arXiv:0803.2158 **[hep-ex]**

Semi-leptonic decay of B-hadron(e.g. $B \to X_c l \nu_l$)

depends on b mass.

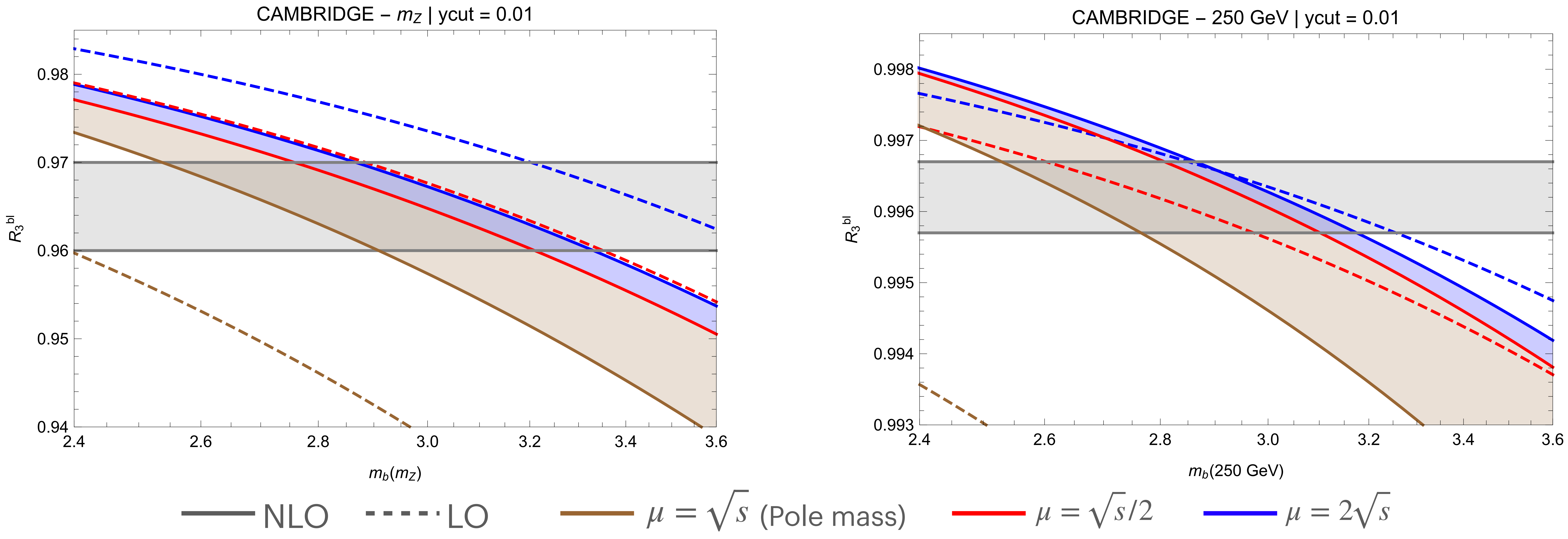


The running bottom quark mass, IFIC (U. Valencia/CSIC), U. Tohoku



b quark mass sensitivity on R_3^{bl}

- Following figures are visualized plots of b mass sensitivity $\frac{\Delta m_b}{m_b} = \frac{\Delta R_3^{bl}}{2(1 - R_3^{bl})}$ (See page.6) for both of Z-pole and 250GeV.
- Gray bands mean necessary precisions of R_3^{bl} to measure b quark mass at the precision of ~0.4GeV for both of Z-pole and 250GeV.
- The sensitivity at 250GeV ~5 times deteriorates.



About the number of $m_b(m_b)$

- QCD theoretical prediction of R_3^{bl} as a function of y_c is given below.