

Scalar and fermion on-shell amplitudes in generalized HEFT

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In collaboration with

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Based on [arXiv:2102.08519](https://arxiv.org/abs/2102.08519)

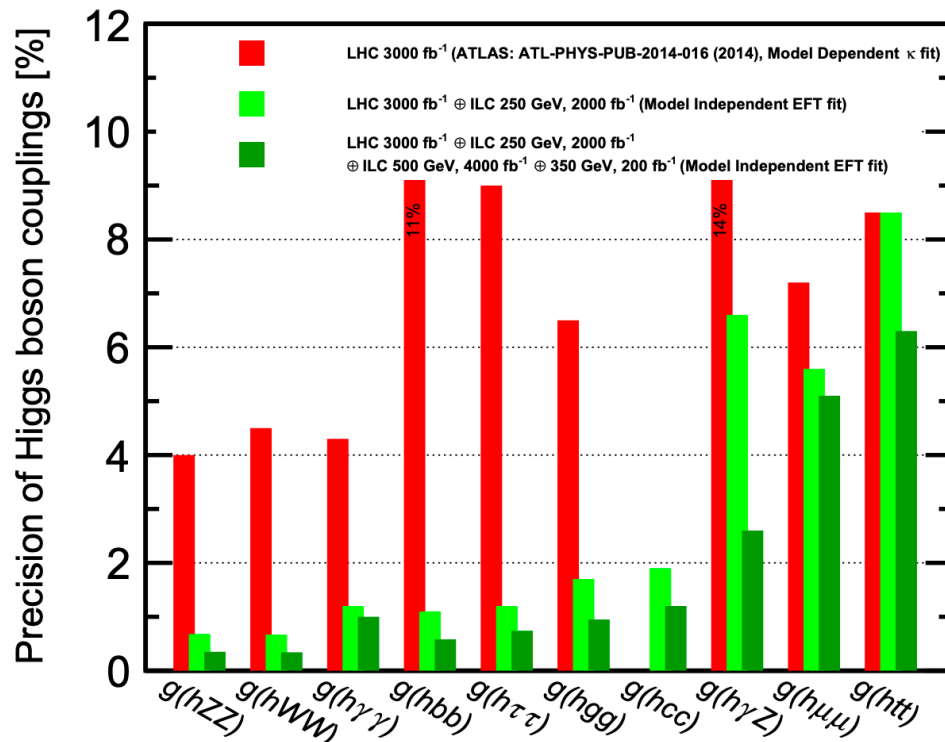
International Workshop on Future Linear Colliders, LCWS2021

ONLINE

Introduction

Overview

Linear collider experiments will enable us to test the SM precisely

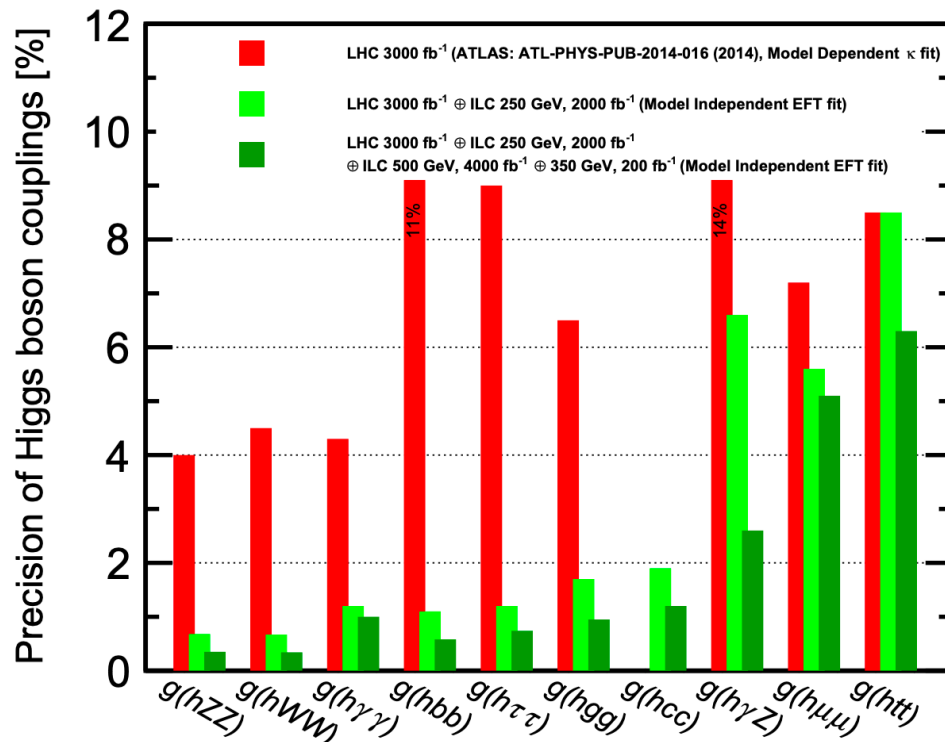


K. Fujii, et. al., arXiv:1710.07621

Question : “if we find the deviations from the SM in the future, can we extract the new physics properties from the deviation patterns?”

Overview

Linear collider experiments will enable us to test the SM precisely



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production cross sections of new particles

Goal

extract the new particles' **production cross sections** from the deviation patterns

Goal

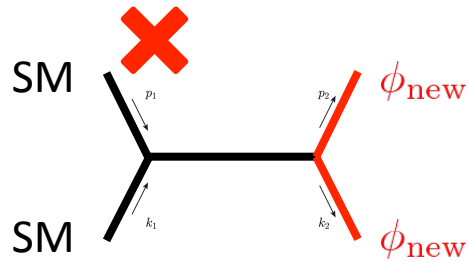
extract the new particles' production cross sections from the deviation patterns

Heavy new particles are integrated out in EFT framework

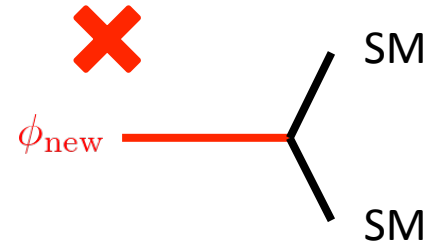
EFT approach is useful

➡ EFT approach cannot deal with ...

• production process



• decay process



Goal

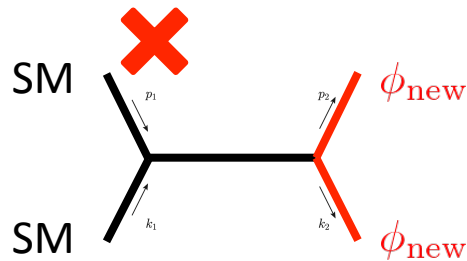
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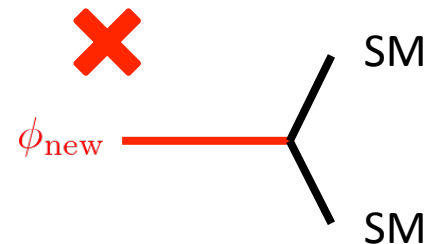
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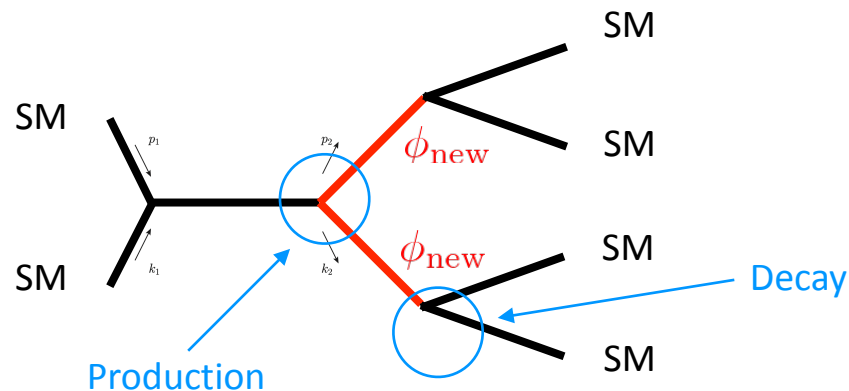
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• decay process



These are essential quantities for direct search



Goal

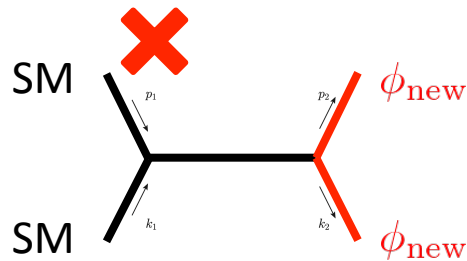
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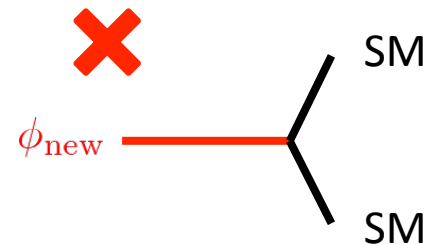
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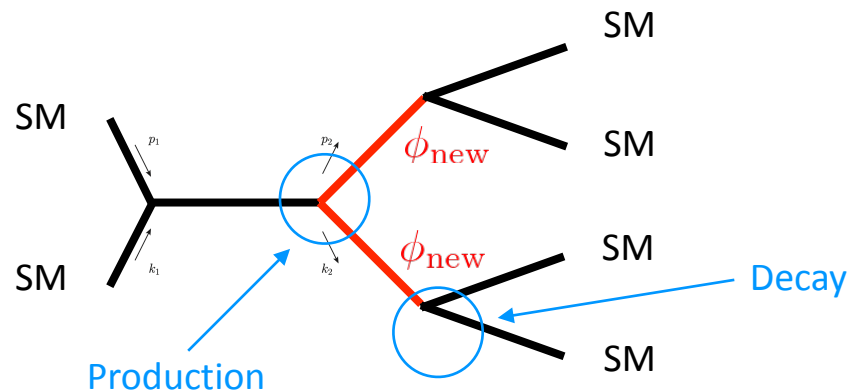
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These are essential quantities for direct search



To achieve **Goal**, we must extend EFT framework so that **it includes new particles**

Strategy

Add new particles to the HEFT in a consistent manner with (nonlinear) $SU(2)_L \times U(1)_Y$

HEFT : Higgs Effective Field Theory

$$\mathcal{L}_{\text{HEFT}} = \frac{v^2}{4} F(h) \text{Tr}[(D_\mu U)^\dagger D^\mu U] + \frac{1}{2} \partial_\mu h \partial^\mu h - V(h) + \dots$$

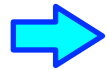
$$F(h) = 1 + \kappa_1 \frac{h}{v} + \kappa_2 \left(\frac{h}{v}\right)^2 + \dots \quad U = \exp\left(\frac{i\pi^a \tau^a}{v}\right)$$

$$V(h) = m_h^2 h^2 + \lambda_3 h^3 + \dots$$

- **Symmetry** : $SU(3)_C \times SU(2)_L \times U(1)_Y$
- **Matter contents** : $h, Z_\mu, W_\mu^\pm \dots$ classified under $U(1)_{\text{em}}$
- 125 GeV scalar h is not necessarily included in H

Strategy

Add new particles to the HEFT in a consistent manner with (nonlinear) $SU(2)_L \times U(1)_Y$



We can make model-independent prediction on new particles

HEFT : Higgs Effective Field Theory

1409.1709 R. Nagai, M. Tanabashi, K. Tsumura

1904.07618 R. Nagai, M. Tanabashi, K. Tsumura, Y.U.

✓ 2102.08519 R. Nagai, M. Tanabashi, K. Tsumura, Y.U.

Today's talk

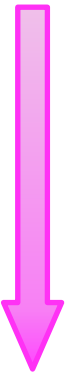
Matter Contents

SM

SM + neutral scalar

SM + charged scalar

SM + charged scalar
+ Weyl fermion



Extension of
HEFT

Extension of the HEFT

Generalized HEFT (GHEFT)

Symmetry : $SU(3)_C \times SU(2)_L \times U(1)_Y$

Matter contents :

w^\pm, z = would-be Goldstone bosons

$\phi^I = (h, H^+, H^-, \dots)$ ← New scalars

$\psi^{\hat{i}} = (q_L, u_R, d_R, \dots, \Psi_{\text{new}}, \dots)$ ← New fermions

$$\begin{aligned} \mathcal{L}_{\text{GHEFT}} = & \frac{1}{2} G_{ab}(\phi) \alpha_{\perp\mu}^a \alpha_{\perp}^{a\mu} \\ & + G_{aI}(\phi) \alpha_{\perp\mu}^a (\mathcal{D}^\mu \phi^I) \\ & + \frac{1}{2} G_{IJ}(\phi) (\mathcal{D}^\mu \phi^I) (\mathcal{D}_\mu \phi^J) - V(\phi) \\ & + \frac{i}{2} G_{\hat{i}\hat{j}^*}(\phi) \left(\psi^{\dagger\hat{j}^*} \bar{\sigma}^\mu (\mathcal{D}_\mu \psi^{\hat{i}}) - (\mathcal{D}_\mu \psi^{\dagger\hat{j}^*}) \bar{\sigma}^\mu \psi^{\hat{i}} \right) \\ & + V_{\hat{i}\hat{j}^*a}(\phi) \psi^{\dagger\hat{j}^*} \bar{\sigma}^\mu \psi^{\hat{i}} \alpha_{\perp\mu}^a \\ & + V_{\hat{i}\hat{j}^*I}(\phi) \psi^{\dagger\hat{j}^*} \bar{\sigma}^\mu \psi^{\hat{i}} (\mathcal{D}_\mu \phi^I) \\ & - \frac{1}{2} M_{\hat{i}\hat{j}}(\phi) \psi^{\hat{i}} \psi^{\hat{j}} - \frac{1}{2} M_{\hat{i}^*\hat{j}^*}(\phi) \psi^{\dagger\hat{i}^*} \psi^{\dagger\hat{j}^*} \\ & + \frac{1}{8} S_{\hat{i}\hat{j}\hat{k}\hat{l}}(\phi) (\psi^{\hat{i}} \psi^{\hat{j}}) (\psi^{\hat{k}} \psi^{\hat{l}}) \\ & + \frac{1}{8} S_{\hat{i}^*\hat{j}^*\hat{k}^*\hat{l}^*}(\phi) (\psi^{\dagger\hat{i}^*} \psi^{\dagger\hat{j}^*}) (\psi^{\dagger\hat{k}^*} \psi^{\dagger\hat{l}^*}) \\ & + \frac{1}{4} S_{\hat{i}\hat{j}\hat{k}^*\hat{l}^*}(\phi) (\psi^{\hat{i}} \psi^{\hat{j}}) (\psi^{\dagger\hat{k}^*} \psi^{\dagger\hat{l}^*}), \end{aligned}$$

- $\alpha_{\perp\mu}^a = \text{tr} \left[\frac{1}{i} \xi_W^\dagger (\partial_\mu \xi_W) \tau^a \right], \quad (a = 1, 2)$

- $\alpha_{\perp\mu}^3 = \text{tr} \left[\frac{1}{i} \xi_W^\dagger (\partial_\mu \xi_W) \tau^3 \right] + \text{tr} \left[\frac{1}{i} (\partial_\mu \xi_Y) \xi_Y^\dagger \tau^3 \right]$

ξ_W and ξ_Y are defined as

$$\xi_W := \exp \left(\frac{i}{2v} \sum_{\alpha=1,2} w^\alpha \tau^\alpha \right), \quad \xi_Y := \exp \left(\frac{i}{2v_Z} z \tau^3 \right)$$

- $(\mathcal{D}_\mu \phi)^I = \partial_\mu \phi^I + i \nu_\mu^3 [Q_\phi]^I{}_J \phi^J$

with ν_μ^3 defined by $\nu_\mu^3 = -\text{tr} \left[\frac{1}{i} (\partial_\mu \xi_Y) \xi_Y^\dagger \tau^3 \right] + c \alpha_{\perp\mu}^3$

Q_ϕ : charged matrix

$$Q_\phi = \begin{pmatrix} -q_1 \sigma_2 & & & & \\ & \dots & & & \\ & & -q_{n_C} \sigma_2 & & \\ & & & 0 & \\ & & & & \dots \\ & & & & & 0 \end{pmatrix}$$

charged

neutral

Please visit [2102.08519](https://arxiv.org/abs/2102.08519)

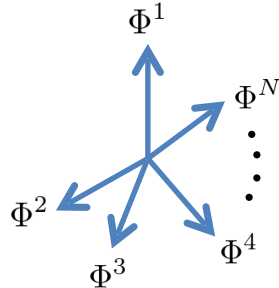
Difficulties in EFT analysis

Difficulty : non-uniqueness of parametrization methods

➡ Difficult to compare results computed in different field basis

Solution : Focus on target space of the scalar Lagrangian

Alonso et.al., 1511.00724 and 1605.03602




A diagram showing a set of axes in field space. The axes are labeled Φ^1 , Φ^2 , Φ^3 , Φ^4 , and Φ^N . The axes are represented by blue arrows originating from a central point. Φ^1 is vertical, Φ^2 is horizontal to the left, Φ^3 is diagonal down-left, Φ^4 is diagonal down-right, and Φ^N is diagonal up-right. There are three vertical dots between Φ^4 and Φ^N .

$$\mathcal{L}_{\text{GHEFT}} = \frac{1}{2} g_{ij}(\Phi) \partial_\mu \Phi^i \partial^\mu \Phi^j - V$$

➡ Amplitudes can be expressed by covariant tensors in field space

ex.) $\phi^1 \phi^2 \rightarrow \phi^3 \phi^4$



A diagram showing two dashed lines crossing each other. The top-left end is labeled '1', the top-right end is labeled '3', the bottom-left end is labeled '2', and the bottom-right end is labeled '4'.

$$\simeq s R_{1423} + s(1 + \cos \theta) R_{1234} \quad 1904.07618$$

Application to GHEFT

Symmetry : $SU(3)_C \times SU(2)_L \times U(1)_Y$

Matter contents : SM + new scalars + new Weyl fermions

ϕ^i : scalar (SM + NP) , $\psi^{\hat{i}}$: Weyl fermion (SM + NP)

$$\begin{aligned}
 \mathcal{L}_{\text{GHEFT}} = & \frac{1}{2} g_{ij}(\phi) (\partial_\mu \phi^i) (\partial^\mu \phi^j) - V(\phi) \\
 & + \frac{i}{2} g_{i\hat{j}^*}(\phi) \left(\psi^{\dagger\hat{j}^*} \bar{\sigma}^\mu (\partial_\mu \psi^{\hat{i}}) - (\partial_\mu \psi^{\dagger\hat{j}^*}) \bar{\sigma}^\mu \psi^{\hat{i}} \right) \\
 & + v_{i\hat{j}^*}(\phi) \psi^{\dagger\hat{j}^*} \bar{\sigma}^\mu \psi^{\hat{i}} (\partial_\mu \phi^i) \\
 & - \frac{1}{2} M_{\hat{i}\hat{j}}(\phi) \psi^{\hat{i}} \psi^{\hat{j}} + \text{h.c.} \\
 & + \frac{1}{8} S_{\hat{i}\hat{j}\hat{k}\hat{l}}(\phi) (\psi^{\hat{i}} \psi^{\hat{j}}) (\psi^{\hat{k}} \psi^{\hat{l}}) + \text{h.c.} \\
 & + \frac{1}{4} S_{\hat{i}\hat{j}\hat{k}^*\hat{l}^*}(\phi) (\psi^{\hat{i}} \psi^{\hat{j}}) (\psi^{\dagger\hat{k}^*} \psi^{\dagger\hat{l}^*})
 \end{aligned}$$

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 & + v_{i\hat{j}^*}(\phi) \psi^{\dagger\hat{j}^*} \bar{\sigma}^\mu \psi^{\hat{i}} (\partial_\mu \phi^i) \longrightarrow v_{i\hat{j}^*}(\phi) = \frac{i}{2} \left(g_{i\hat{j}^*,\hat{i}}(\phi) - g_{i\hat{i},\hat{j}^*}(\phi) \right) \\
 & - \frac{1}{2} M_{i\hat{j}}(\phi) \psi^{\hat{i}} \psi^{\hat{j}} + \text{h.c.} \\
 & + \frac{1}{8} S_{i\hat{j}\hat{k}\hat{l}}(\phi) (\psi^{\hat{i}} \psi^{\hat{j}}) (\psi^{\hat{k}} \psi^{\hat{l}}) + \text{h.c.} \\
 & + \frac{1}{4} S_{i\hat{j}\hat{k}^*\hat{l}^*}(\phi) (\psi^{\hat{i}} \psi^{\hat{j}}) (\psi^{\dagger\hat{k}^*} \psi^{\dagger\hat{l}^*})
 \end{aligned}$$

We define Affine connection: $\Gamma_{j\hat{k}}^{\hat{i}} := \frac{1}{2} g^{\hat{i}\hat{l}^*} \left[g_{\hat{k}\hat{l}^*,j} + g_{j\hat{l}^*,\hat{k}} - g_{j\hat{k},\hat{l}^*} \right]$

High Energy Behavior

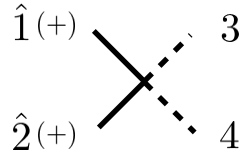
$$\phi^1 \phi^2 \rightarrow \phi^3 \phi^4$$



$$\simeq s R_{1423} + s(1 + \cos \theta) R_{1234}$$

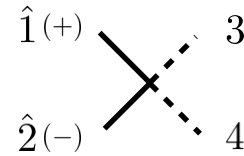
1904.07618

$$\psi^{\hat{1}} \psi^{\hat{2}} \rightarrow \phi^3 \phi^4$$



$$\simeq \sqrt{s} \left(\mathcal{D}_4 \mathcal{D}_3 M_{\hat{1}\hat{2}} \right) - \sqrt{s} \cos \theta \left(M_{\hat{2}} R_{\hat{1}\hat{2}34} + M_{\hat{1}} R_{\hat{1}\hat{2}34} \right)$$

2102.08519

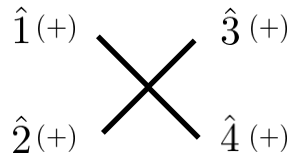


$$\simeq s \sin \theta R_{\hat{1}\hat{2}34}$$

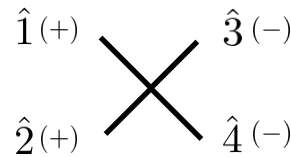
\hat{i} : flavor index of Weyl fermion

i : flavor index of scalar

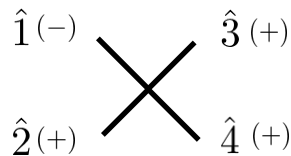
$$\psi^{\hat{1}} \psi^{\hat{2}} \rightarrow \psi^{\hat{3}} \psi^{\hat{4}}$$



$$\simeq s R_{\hat{1}\hat{4}\hat{2}\hat{3}} + s(1 + \cos \theta) R_{\hat{1}\hat{2}\hat{3}\hat{4}}$$



$$\simeq s R_{\hat{1}\hat{3}\hat{2}\hat{4}}$$



$$\simeq \sqrt{s} \sin \theta \left(M_{\hat{1}} R_{\hat{1}\hat{3}\hat{2}\hat{4}} - M_{\hat{3}} R_{\hat{2}\hat{1}\hat{4}\hat{3}} + M_{\hat{4}} R_{\hat{2}\hat{1}\hat{3}\hat{4}} \right)$$

Take Home Message & Summary

- We construct the generalized HEFT incorporating the new scalars and Weyl fermions.
- We find that the cancellation of the energy growing term in the scalar fermion scattering amplitudes require the flatness of the Riemann curvature tensors evaluated at the vacuum.

$$\begin{aligned} R_{1234} \Big|_0 &= R_{\hat{1}\hat{2}^*34} \Big|_0 = R_{\hat{1}\hat{2}\hat{3}\hat{4}} \Big|_0 \\ &= R_{\hat{1}^*\hat{2}^*\hat{3}^*\hat{4}^*} \Big|_0 = R_{\hat{1}\hat{2}^*\hat{3}\hat{4}^*} \Big|_0 = 0 \end{aligned}$$