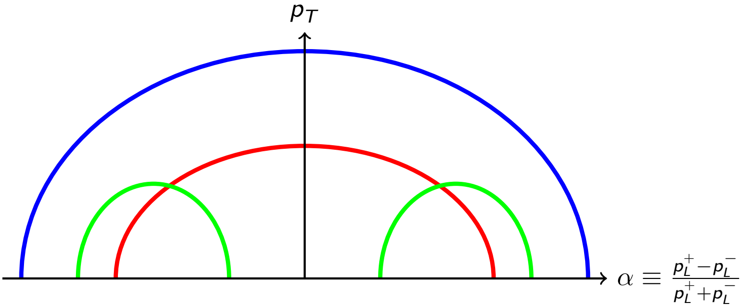
High Precision Tracker Momentum-Scale Calibration

with Particle Decays ($\mathrm{K^0_S}$, Λ, $\mathrm{D^0}$, J/ψ) at ILC

Graham W. Wilson (Univ. of Kansas)

March 17, 2021



Introduction

- ILC can and should make precision measurements of the **masses** of known fundamental particles (M_{H}, M_{t}, M_{W}, M_{Z}), and Γ_{Z}. Measure new ones, M_{X}.
- A primary issue for most determinations is the measurement of the **absolute center-of-mass energy scale**. A method, the so-called $\sqrt{s}_p$ method, has been proposed using only the momenta of muons in di-muon events.
- Critical issue for $\sqrt{s}_p$ method: calibrating the **tracker momentum scale**.
- Up to now, $J/\psi \to \mu^+\mu^-$ envisaged as the method of choice (mass known to 1.9 ppm). **Statistically limited** though in $\mathrm{e}^+\mathrm{e}^-$ collisions.

More details in older studies of $\sqrt{s}_p$ from DESY, ECFA LC2013, and of momentum-scale from Fermilab, AWLC 2014.

Today,

1. Explore method based on the Armenteros-Podolanski kinematic construction using mainly $\mathrm{K}^0_{\mathrm{S}}$, Λ (inspired by Rodríguez et al.). **Much higher statistics.**
2. If proven realistic, **enables precision Z program** (polarized lineshape scan).
3. Bonus: potential to also **improve masses** of parent and daughter particles.

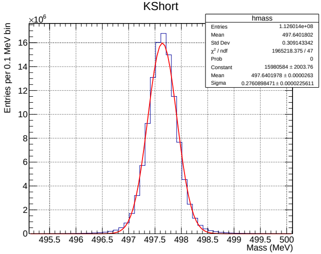
Particles of Interest

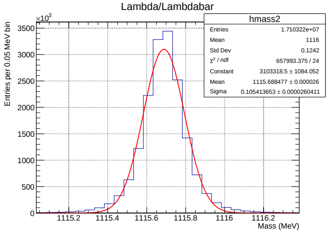
- Indeed J/ψ is the best measured parent particle, but other particles much more prevalent
- Daughter masses are known very well (except $K^{\pm}$)
- ppm = parts per million, ppb = parts per billion (10^9)

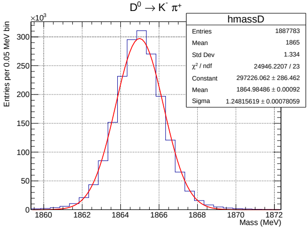
Particle	Mode	$\Delta M/M$ (PDG-2020)	$n_Z^{\mathrm{had}} B$
$\mathrm{K_S^0}$	$\pi^+\pi^-$	26 ppm	0.71
Λ	p π^-	5.4 ppm	0.25
D^0	$\mathrm{K}^-\pi^+$	27 ppm	0.018
J/ψ	$\mu^+\mu^-$	1.9 ppm	0.00031
Z	$\mu^+\mu^-$	23 ppm	(0.047)
μ	–	23 ppb	
$\pi^{\pm}$	–	1.3 ppm	
$K^{\pm}$	–	32 ppm	
p	–	6.4 ppb	

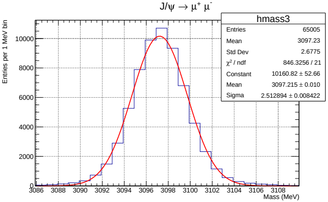
Study the first 4 particles using given decay modes including charge conjugates.

Simulated main decay mode mass distributions for 250 M hadronic Z's (91 GeV). Uses ILD momentum resolution parametrization. Hierarchy: $\mathrm{K^0_S}, \Lambda, \mathrm{D^0}, J/\psi$.







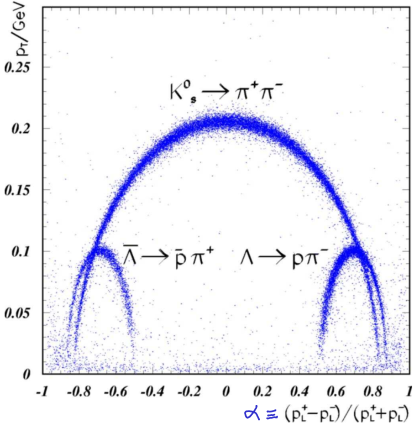


113M $\mathrm{K^0_S}$, 34M (Λ, $\bar{\Lambda}$) , 3.8M ($\mathrm{D^0}$,$\mathrm{\bar{D}^0}$), 65k J/ψ

Caveats

- Study uses lots of events. Sufficient that many of the known particle masses could be improved by factors of up to **75** for 250M hadronic Zs.
- Likely a real reach in terms of eventual **systematics**. Discussed more later.
- Study so far is like an estimate of the statistical uncertainty on the average B-field absolute scale (assuming perfect knowledge of the B-field map, but incorporating particle mass uncertainties)
- For now I decided to be somewhat cavalier for several reasons:
  1. High event counts are needed to uncover residual systematics in the fit procedure and figure out how to correct them
  2. Some channels need very high statistics to be viable especially in the context of the analysis method chosen
  3. A high statistics run at the Z for EW precision measurements with potentially 4000 M Z's would have 16 times this statistics (assumes 100 fb^{-1} with polarized beams). Consistent with L $= 4.2 \times 10^{33}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$.
  4. This study should give an idea of how well point-to-point systematics might be controllable for scan observables

J. Podolanski and R. Armenteros, "Analysis of V-events", Phil. Mag. 1954.
For a "V-decay", $M^0 \to m_1^+ m_2^-$, decompose the daughter momenta in the lab into components transverse and parallel to the parent momentum.
The distributions of (daughter p_T, $\alpha \equiv \frac{p_L^+ - p_L^-}{p_L^+ + p_L^-}$) are related by the CM decay angle, θ^*, and the underlying masses, (M, m_1, m_2), that determine, p^*, and β.



- K^0-decay

$$M = 0.498 \text{ GeV}$$
$$m_{1,2} = 0.140 \text{ GeV}$$
$$p_{cm} = 0.206 \text{ GeV}$$
$$\alpha_0 = 0$$
$$r_\alpha = 0.827$$

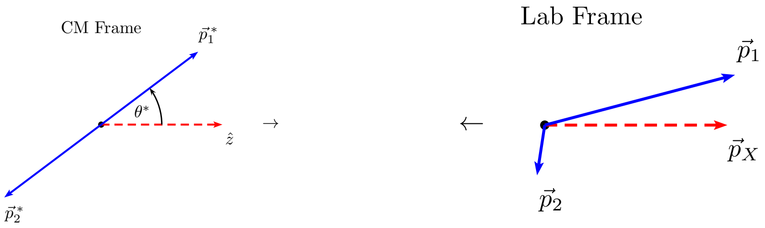
- Λ-decay

$$M = 1.116 \text{ GeV}$$
$$m_1 = 0.938 \text{ GeV}$$
$$m_2 = 0.140 \text{ GeV}$$
$$p_{cm} = 0.101 \text{ GeV}$$
$$\alpha_0 = \pm 0.691$$
$$r_\alpha = 0.181$$

plot from talk by M. Schmelling

Armenteros-Podolanski II

CM frame, so $p_1^* = p_2^* = p^* = \frac{\sqrt{[M^2-(m_1+m_2)^2][M^2-(m_1-m_2)^2]}}{2M}$.
Transverse momentum, $p_T^* = p^* \sin\theta^* = p_T$ in CM and lab.



One can derive, that $\alpha = \frac{2p^*}{\beta M}\cos\theta^* + \frac{m_+^2 - m_-^2}{M^2}$ and can rewrite as

$$\alpha = \alpha_0 + \frac{r_\alpha}{\beta}\cos\theta^* \quad \text{where } \alpha_0 \equiv \frac{m_+^2 - m_-^2}{M^2}, \quad r_\alpha \equiv \frac{2p^*}{M}$$

$$\text{AP Ellipse}: \quad \left(\frac{\alpha - \alpha_0}{(r_\alpha/\beta)}\right)^2 + \left(\frac{p_T}{p^*}\right)^2 = 1 \tag{1}$$

Elliptical Transformation à la Rodríguez et al.

Building on the recent preprint, arXiv:2012.03620, one can "flatten" the measured AP ellipse (α, p'_T) into variables (r, ϕ) defined using

$$\begin{cases} \alpha(r,\phi) = \widetilde{\alpha}_0 \quad + \quad \frac{\widetilde{r}_\alpha}{\beta} r\cos\phi \\ p'_T(r,\phi) = S_p \; p_T(r,\phi) = \widetilde{p}^* r\sin\phi \end{cases} , \tag{2}$$

where the $\sim$ variables assume some reference masses and p'_T is the measured p_T biased by a momentum scale factor, S_p. Straightforward solution is:

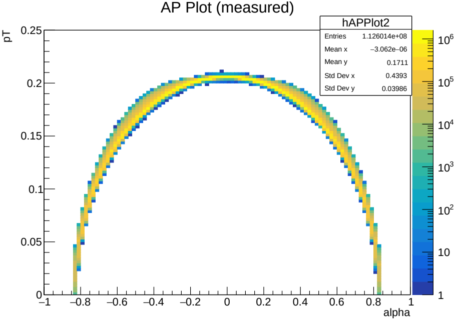
$$\begin{cases} r^2(\alpha,p'_T,\beta) = \left(\frac{\alpha-\widetilde{\alpha}_0}{(\widetilde{r}_\alpha/\beta)}\right)^2 + \left(\frac{p'_T}{\widetilde{p}^*}\right)^2 \\ \phi(\alpha,p'_T,\beta) = \mathrm{atan2}\left(\frac{p'_T}{\widetilde{p}^*}, \frac{\alpha-\widetilde{\alpha}_0}{(\widetilde{r}_\alpha/\beta)}\right) \end{cases} \tag{3}$$

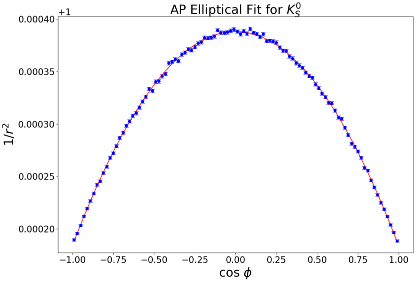
Eqn. 1 results in a quadratic in r (Eqn. 4) that depends on ϕ with coefficients defined by combinations of true masses, reference masses, S_p, and β:

$$\left(\frac{(\widetilde{\alpha}_0 - \alpha_0) + (\widetilde{r}_\alpha/\beta) r\cos\phi}{(r_\alpha/\beta)}\right)^2 + \left(\frac{\widetilde{p}^* r\sin\phi}{S_p \; p^*}\right)^2 = 1 \ , \tag{4}$$

If all masses are as assumed, and $S_p = 1$, expect $r(\phi) = 1$ for $m_1 = m_2$.

Simulate measurement of (p_T, α). For each decay, measure corresponding (r, ϕ) using Eqn. 3 with reference masses. For each bin in $\cos\phi$ (approximately the same as $\cos\theta^*$), find the mean value of $1/r^2$. For $S_p = 1.0001$.





Fit $1/r^2$ vs $\cos\phi$ for underlying parameters ($m_{\mathrm{K^0_S}}$ and S_p) with m_π assumed known perfectly. Uses reference (tilde) value of the PDG $\mathrm{K^0_S}$ mass + 0.1 MeV. Results: $\chi^2/\mathrm{dof} = 108/98$ and relative uncertainties on $(m_{\mathrm{K^0_S}}, S_p)$ of (0.3, 0.5) ppm with correlation of -97.9%.

Deviations are $(+2.6, -3.9)\sigma$ (residual systematics? fix...)

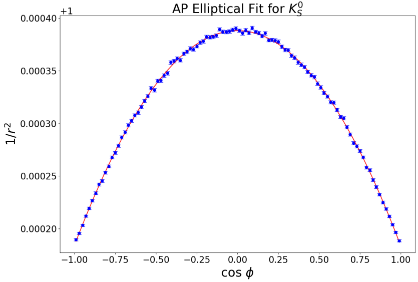
Parametric Form (for $m_1 = m_2 = m$)

For this simpler case, the solutions of the quadratic equation (with $B = 0$) lead to

$$\frac{1}{r^2} = -\frac{A}{C} = \left(\frac{\widetilde{r}_\alpha}{r_\alpha}\right)^2 \cos^2\phi + \left(\frac{\widetilde{p}^*}{S_p\, p^*}\right)^2 \sin^2\phi \ . \tag{5}$$

Note: no dependence on β. Define, $c_r = \frac{\widetilde{r}_\alpha}{r_\alpha}$, $c_p = \frac{\widetilde{p}^*}{S_p\, p^*}$. Eqn. 5 becomes

$$\frac{1}{r^2} = c_p^2 + (c_r^2 - c_p^2)\cos^2\phi \tag{6}$$



- Can measure both r_α and the product, $S_p\, p^*$.
- These depend on M, m, and S_p.
- When m is well known. Can measure M and S_p.
- $r^{-2}(\cos\phi = 0, \pm 1) = (c_p^2, c_r^2)$
- p'_T depends on S_p. α does not.

Experimental Methods Used (more details in backup)

Caveat: short-term goal to establish whether this method can work and get first estimates of precisions especially at the Z. Not meant to be an end-to-end study.

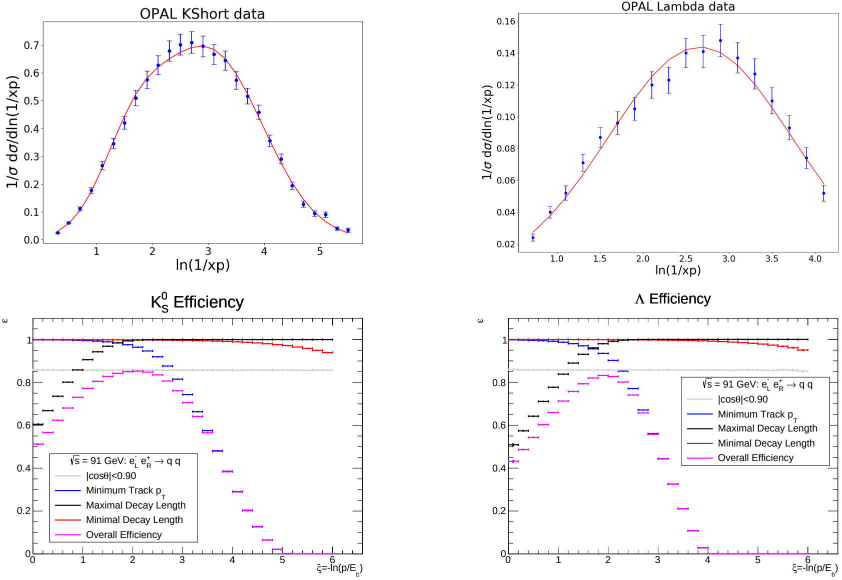
Methods

- Use toy MC generator with momentum spectrum from PYTHIA 6.
- Use parametrization of "normal" ILD momentum resolution (from fits to DBD curves) - checked with SGV.
- Scale a and b parameters for reduced lever arm (decays). Scale b by $1/\beta$.
- $\frac{d\sigma}{d\cos\theta} = 1 + \cos^2\theta$. Rates from PDG.
- Neglect angular resolution and backgrounds.

Cuts

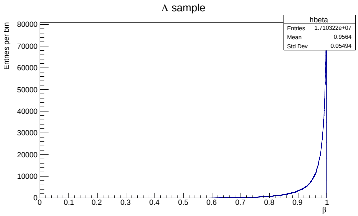
- Angular acceptance: $|\cos\theta| < 0.90$
- Minimum track detector p_T of 0.25 GeV
- Fiducial cut (decay vertex within 20 cm of outer edge of TPC in r and z)
- Require decay radius exceeds 250 μm (only for $\mathrm{K^0_S}$, Λ)
- Require momentum resolution of each track $< 1\%$ (for late decays ...)

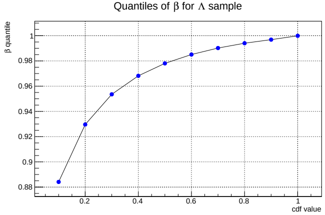
ξ Distributions and Efficiency



Note: efficiency plots neglect the 1% resolution cut (recent addition)

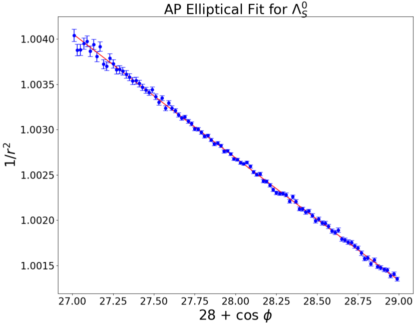
Lambda Fits (Eqn 4 with $\widetilde{\alpha}_0 \neq \alpha_0$)





Lowest decile ($\beta < 0.885$) is excluded from all fits for now. (suspect β-bite is currently too big.)

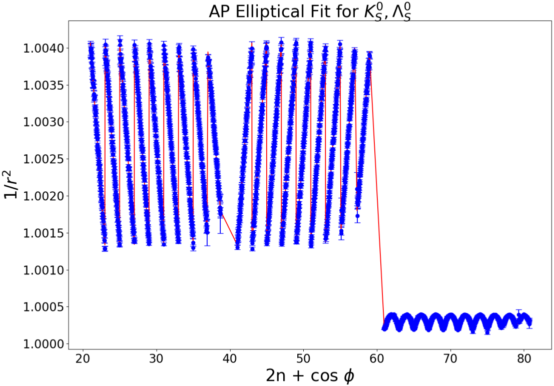
r vs ϕ dependence depends on β. So divide and conquer. Divide β range in tenths. Use $< \beta >$ in fit for now. Example with $\beta \in [0.9853, 0.9904]$.



Again for $S_p = 1.0$ and $M + 0.1$ MeV. Fit $\chi^2/\mathrm{dof} = 109/98$. Find (M, S_p) to (0.3, 5) ppm, $\rho = -0.962$. Uses 1.7M Λ's. Overall fit has $\times 18$ (for 250M Z).

Mega Fit for $\mathrm{K^0_S}$, Λ, $\overline{\Lambda}$ with 250M Z statistics

Fit deciles for each particle type (necessary for $\Lambda/\overline{\Lambda}$).
9 deciles of Λ, 9 deciles of $\overline{\Lambda}$, 10 deciles of $\mathrm{K^0_S}$.



$\chi^2/\nu = 3117/2714$ (NYGE)

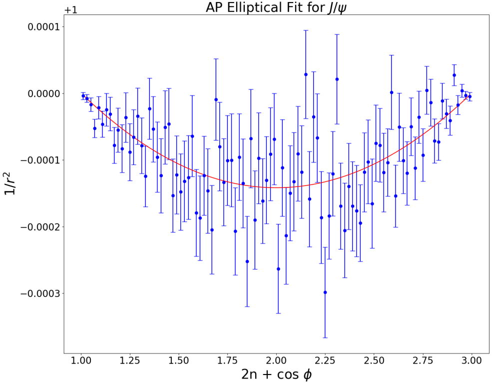
1. $m_{\mathrm{K^0_S}}$: 0.48 ppm
2. m_Λ: 0.072 ppm
3. m_π: 0.46 ppm
4. S_p: 0.57 ppm

	$M(\mathrm{K^0_S})$	$M(\Lambda)$	m_π	S_p
$M(\mathrm{K^0_S})$	1.0	0.934	0.814	−0.942
$M(\Lambda)$	–	1.0	0.914	−0.818
m_π	–	–	1.0	−0.601
S_p	–	–	–	1.0

Here m_p is fixed. Technically, fits are set up with χ^2 penalty terms that constrain the particles to the PDG masses within known uncertainties. So in essence the fitted masses are new world averages.

J/ψ stand-alone fit (250M Z's, 65k accepted J/ψ)

Fix m_μ.



1. $m_{J/\psi}$: 1.9 ppm (no improvement wrt PDG)
2. S_p: 4.4 ppm
3. Correlation: -0.44

- Consistent with prior estimate of 1.0 ppm statistical uncertainty on S_p from $J/\psi \to \mu^+\mu^-$ in 4 GZ hadronic (find 1.03 ppm here).
- Fit with $m(J/\psi)$ fixed gives 4.0 ppm uncertainty on S_p.

More realistic and improved methods / systematics?

Full Reconstruction

- Need to develop performant V0 finder and fitter for large IP
- With nano-beams - much potential
- Looper reconstruction
- Angular resolution
- Backgrounds
- Expect some degradation in resolution for Si-poor tracks

Systematics

- Field map precision
- Tracker alignment/survey
- Material distribution
- Energy loss corrections
- Radiative tails
- Variations with p, $\cos\theta$, decay point
- Smaller bins / better β treatment
- Understand $r, \cos\phi$ resolution

- Current measurements are based on the sample average from TProfile plots. Scope for improved measurement uncertainties simply from fits to the r distributions in each $\cos\phi$ bin and better use of errors.
- Note β estimate uses the measured P and measured mass of the two tracks.

$$M^2 = m_1^2 + m_2^2 + 2p_1p_2\left(\frac{1}{\beta_1\beta_2} - \cos\psi_{12}\right),\quad \beta = P/\sqrt{P^2 + M^2}$$

Conclusions

- Tremendous opportunity to target ppm type uncertainties on the momentum scale factor, S_p, at the Z.
- For 100 fb^{-1} Z run, can measure S_p to 2.5 ppm each 10M Z day. Assume 400 such measurements in 2.5 "Snowmass" years.
- Would open up precision measurements of the masses of lots of known particles at the ppm level. In particular: K^0_S, Λ, $\pi^{\pm}$, $\mathrm{K}^{\pm}$.
- Would enable similar precision in the center-of-mass energy scale which would open a high precision Z program.
- A few years ago I considered 10 ppm a sensible but quite challenging goal. Now one may argue plausibly to tighten substantially this goal.
- Convincing people that 1–10 ppm is feasible when typical experiments are at best at the 100 ppm level needs much work. Need to design this kind of functionality in from the start. See backup for remarks re B-field mapping.
- Now likely need to pay attention to evaluating better other systematics (besides S_p) that may limit the $\sqrt{s}_p$ method for $\sqrt{s}$.

Backup: Other modes

For example,

1. $\mathrm{D}^+ \to \mathrm{K}^0_\mathrm{S}\, \pi^+$ (1.56%)
2. $\mathrm{D}^+ \to \mathrm{K}^-\, \pi^+\, \pi^+$ (9.38%)
3. $\mathrm{D}^0 \to \mathrm{K}^0_\mathrm{S}\, \pi^+\, \pi^-$ (2.80%)

So far fits to D^0 with only $\mathrm{K}^-\pi^+$ (3.95%) with two free parameters, S_p and $m(K)$, (ie. $m(\mathrm{D}^0)$ fixed) give S_p to 0.69 ppm and $m(K)$ to 2.0 ppm for 250M Zs. $(\rho = +0.63)$.

Suggestions welcome

Backup: The Field Mapper's Design Criterion Fallacy

All the documents that I have read about how field maps have been measured by the major experiments say something like:

- In order to not have a significant effect on the momentum **resolution** of reconstructed tracks, the field must be mapped to better than xxx%
- Hall probes are used calibrated to 0.05% (CMS)
- The field is measured over the whole volume with a precision of 0.07% (CMS)
- NMR is used for monitoring

The design criterion should be more like:

- In order to not have a significant **bias** on the momentum **scale** of reconstructed tracks averaged over billions of tracks, it is required that the field is mapped with a precision better than yyy **ppm**
- Many NMR probes are used in the field mapping with intrinsic precision at the ppm level supplemented by Hall probes.

Commercial NMR has 5 ppm accuracy and 0.01 ppm resolution (eg. PT2026).

Backup: Intrinsic S_p sensitivity (mostly for reference)

For each particle decay separately, carry out fit with all particles set to correct masses, and fit only for S_p (again using 250M hadronic Zs).

Results:

Particle	Mode	S_p uncertainty	$n_Z^{\mathrm{had}} B$	S
$\mathrm{K_S^0}$	$\pi^+\pi^-$	0.104 ppm	0.71	0.00139
Λ	p π^-	0.297 ppm	0.25	0.00235
$\mathrm{D^0}$	$\mathrm{K}^-\pi^+$	0.538 ppm	0.018	0.00114
J/ψ	$\mu^+\mu^-$	3.98 ppm	0.00031	0.00111

Sensitivity, S, defined as S_p relative uncertainty per event, ie,

$$S = [\Delta(S_p)/S_p]\sqrt{250 \times 10^6 (n_Z^{\mathrm{had}} B)}$$

Note that the sensitivity to S_p differs quite a lot (due to different Q values).

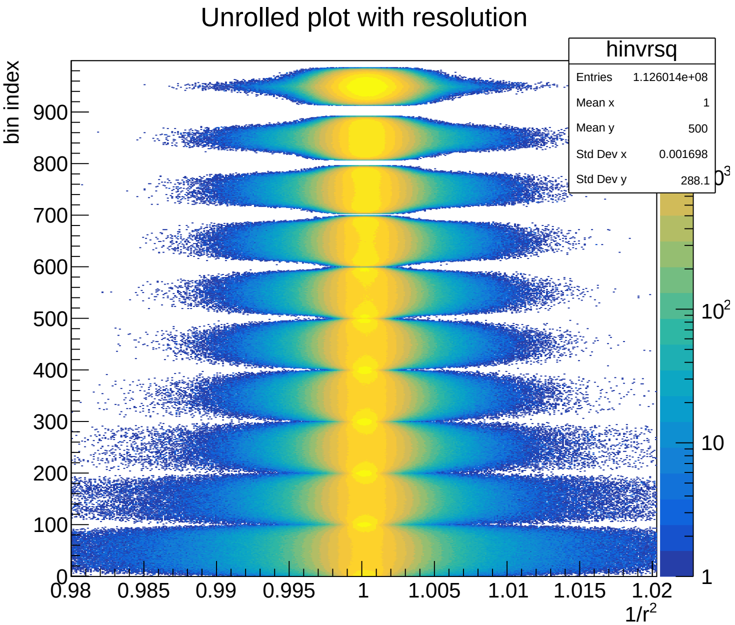
I am also looking into adding other decay modes into the mix. For now, adding $\mathrm{D^0}$ with only the $\mathrm{K}^-\pi^+$ decay mode would add uncertainty from both the $\mathrm{D^0}$ mass and the charged kaon mass. $\phi(1020)$ is not so interesting given the width.

Backup: Current simulation approach

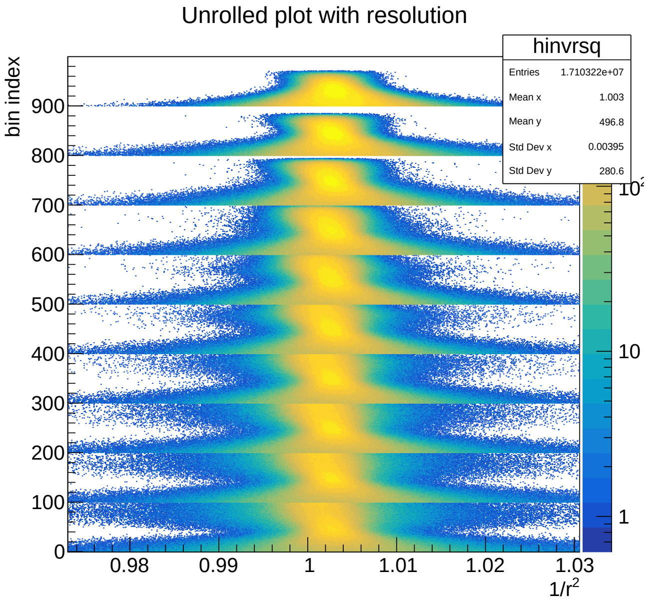
Some simplifications:

- Each of the decay tracks is assumed to be in the same direction as the parent particle in terms of detector, θ (should fix). This affects the simulated detector acceptance and the momentum resolution formula. (fixed recently but not yet incorporated in results shown here - main effect seems to be a modest reduction in efficiency).
- Each track's momentum magnitude along its momentum direction is smeared as detailed on p11. So no angular smearing.
- The AP variables are calculated using the components of the smeared decay particle momentum perpendicular and parallel to the parent particle's true flight direction.
- In particular the AP p_T variable is calculated as the average of the $|p_T|$ of the two tracks.
- Note that once the tracks are fitted to a common vertex and this neutral vertex is constrained to the nano beam spot, it is expected that the above assumptions are not unreasonable, especially in $r - \phi$.

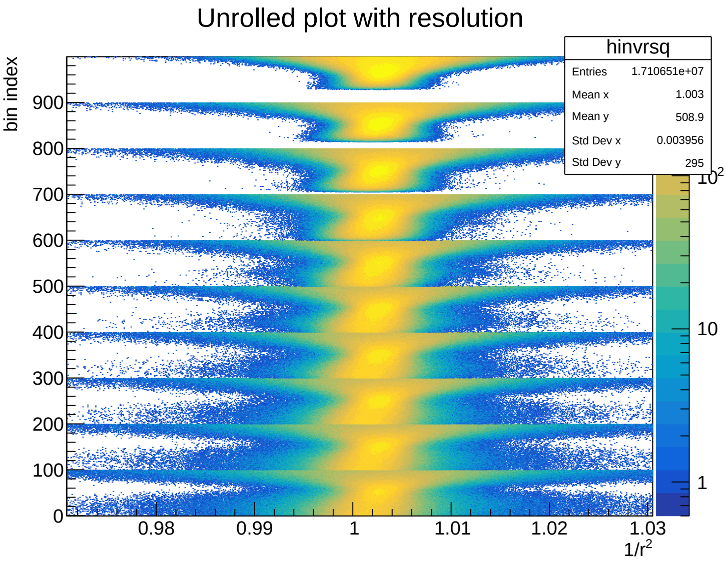
Backup: K_S^0 plot (β deciles and $\cos\phi$ bins)



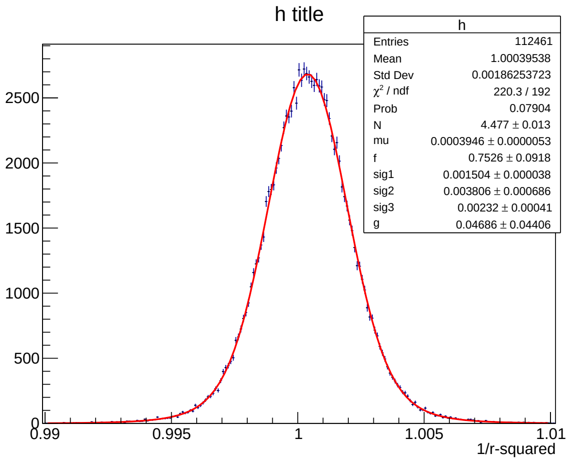
Backup: Λ plot (β deciles and $\cos\phi$ bins)



Backup: $\overline{\Lambda}$ plot (β deciles and $\cos\phi$ bins)

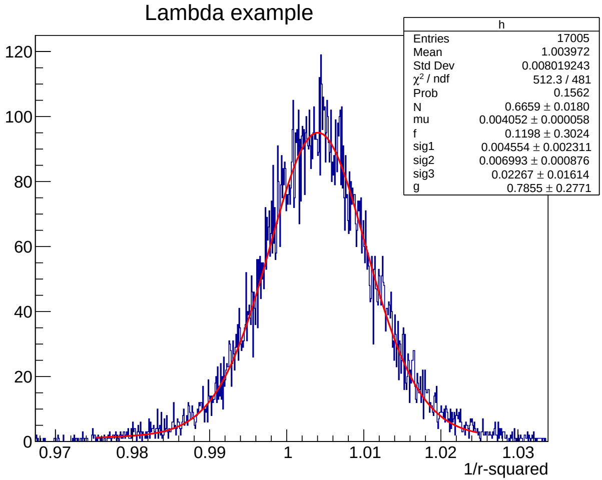


K_S^0, $\beta \in [0.99037, 0.99418]$, $\cos\phi \in [-0.02, 0.0]$



Triple Gaussian fit with common mean of $1 + \mu$. (Note that histogram mean is currently used - overly sensitive to far tails ...)

Λ, same β as slide 13, $\cos\phi \in [-1.0, -0.98]$



Triple Gaussian fit with common mean of $1+\mu$. (Note that histogram mean is currently used)

Backup: Plots from Rodríguez et al. Preprint

Note here ϕ definition differs by $\pi/2$.

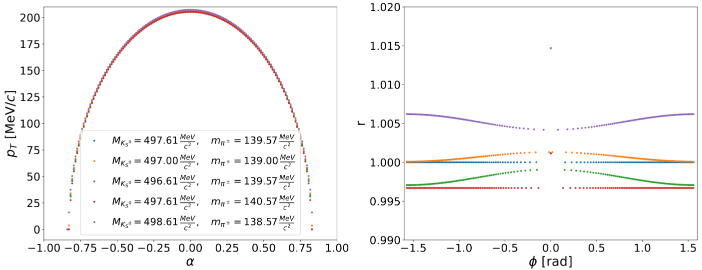


Figure 2: Functional form of a set of symmetric decays ($m_1 = m_2 = m$) in the Armenteros plot (left) and in the plot with elliptical coordinates (right).

In this case ($S_p = 1$). I included this mainly for illustration. It does not comport well with the plotting convention I used, nor the idea that there is a true AP ellipse, and a series of flattened elliptical coordinate plots for different reference values.

5 different (M, m) assumptions are simulated (left). Analysis based on the PDG as reference (leading to the blue set at $r = 1$) (right).