Alternative approach of kinematic fitting for arbitrary resolution functions

Yu Kato, Tomohiko Tanabe, Masakazu Kurata,

Shogo Kajiwara, Takanori Mogi, Satoru Yamashita

The University of Tokyo, KEK, ICEPP

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Overview

- We are developing kinematic fitter based on the log-likelihood method.
  - in order to deal with non-Gaussian distributions
  - by implementation of numerical differentiation
  - previous related talk: Shogo Kajiwara, LCWS2019

- For proof of principle, we try to fit e+e- -> ZH -> μμbb at 250 GeV.
  - This process is selected for technical study of kinematic fitter.
  - No significant invisible particles -> good for ISR fit
  - B-jet has asymmetric energy distribution
  - We evaluated b-jet resolutions and compared fit result with uds-jet resolution.

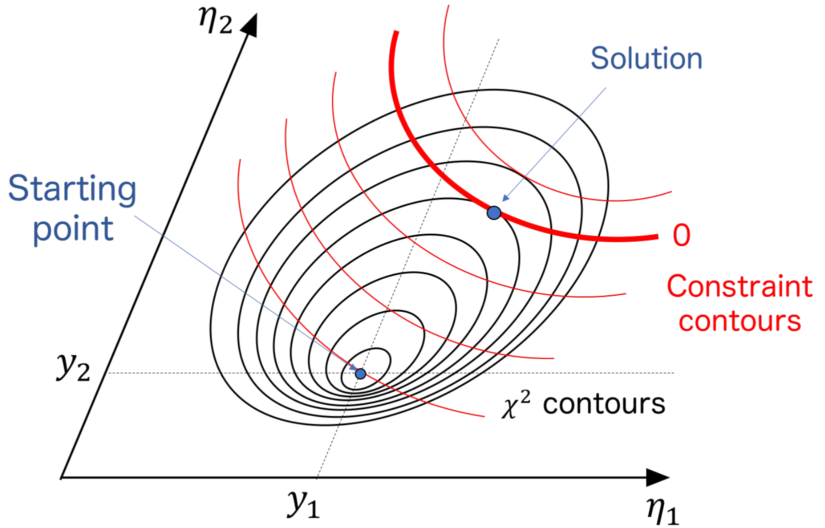
- Currently, we have confirmed a series of operations.

Introduction: Kinematic fit

- Kinematic fit:
  - one of the constrained optimization method
  - adjustment of measured kinematic parameters under certain constraints
    - distributions of parameters e.g. energy resolution
    - kinematic relations among the parameters e.g. energy conservation
- Purposes:
  - improve accuracy of measurements (reconstruction)
  - estimate how well a given event matches a signal model (event selection)
- Standard procedure: minimize χ^2

$$\chi^2(\boldsymbol{\eta}, \boldsymbol{\xi}, \boldsymbol{\lambda}) = (\boldsymbol{y} - \boldsymbol{\eta})^T \boldsymbol{V}^{-1} (\boldsymbol{y} - \boldsymbol{\eta}) - 2\boldsymbol{\lambda}^T \boldsymbol{h}(\boldsymbol{\eta}, \boldsymbol{\xi})$$

$\boldsymbol{y}$: measured variables
$\boldsymbol{\eta}$: fit parameters
$\boldsymbol{V}$: covariance matrix
$\boldsymbol{\xi}$: unmeasured parameters
$\boldsymbol{\lambda}$: Lagrange multipliers
$\boldsymbol{h}$: constraint functions



Our approach for non-Gaussian distributions

- The basic method assumes that the measured parameters would have Gaussian error against the true value.
- In order to treat arbitrary error distributions, the chi-square term is re-defined as the log-likelihood function;

$$\chi^2(\boldsymbol{\eta}, \boldsymbol{\xi}, \boldsymbol{\lambda}) = -2\ln L_{fo}(\boldsymbol{\eta}) - 2\boldsymbol{\lambda}^T \boldsymbol{h}(\boldsymbol{\eta}, \boldsymbol{\xi}) - 2\ln L_{sc}(\boldsymbol{\eta}, \boldsymbol{\xi})$$

$$L_{fo}(\boldsymbol{\eta}) = \prod_{i=1}^{n} f_i(y_i; \eta_i) \qquad L_{sc}(\boldsymbol{\eta}, \boldsymbol{\xi}) = \prod_{i=1}^{m} s_i(\boldsymbol{\eta}, \boldsymbol{\xi})$$

f_i : error distributions $\qquad$ s_i : soft constraint distributions

Note:

- The error distributions are normalized as the peak position returns 1.
- The soft constraint term is applied optionally.

- In the case of Gaussian distributions, the basic method is reproduced.

Notes on implementation

Requirements

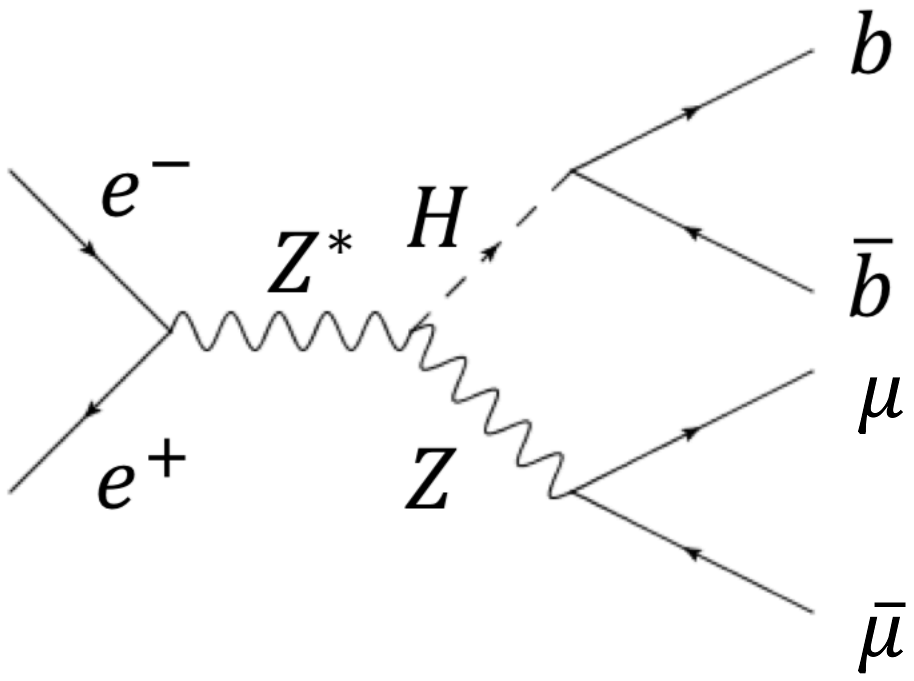
- Numerical differentiation
  - Although the Gaussian case can be solved analytically, the arbitrary case needs numerical calculation.
- Resolution information
  - It is necessary to prepare the error distribution functions for each measured parameters.

Fitter algorithm

- Based on Sequential Quadratic Programming (SQP) method
- Hessian matrix is approximated by damped-BFGS method. (quasi-Newton method)
- The size of the iteration step (α) is adjusted by Armijo condition.

Test process: e^+e^- -> ZH -> μμbb

- This process is selected for technical study of our kinematic fitter.



Simulation setup

- $\sqrt{s}$ = 250 GeV
- (P_{e^-}, P_{e^+}) = (-1, +1)
- ILD DBD sample, ~10k event
- Main background: 4f_ZZ_semileptonic

Event reconstruction

1. Particle reconstruction: PandoraPFA
2. Muon selection: IsolatedLeptonTaggingProcessor
3. Jet clustering & Flavor tagging : LCFIPlus
   Durham (forced 2 jets)

B-jet energy resolution

- The b-jet has asymmetric energy distribution due to neutrinos from semi-leptonic decay.
- We need to know the true energy distribution when a particular measured energy is obtained.
- The definition of the true jet:

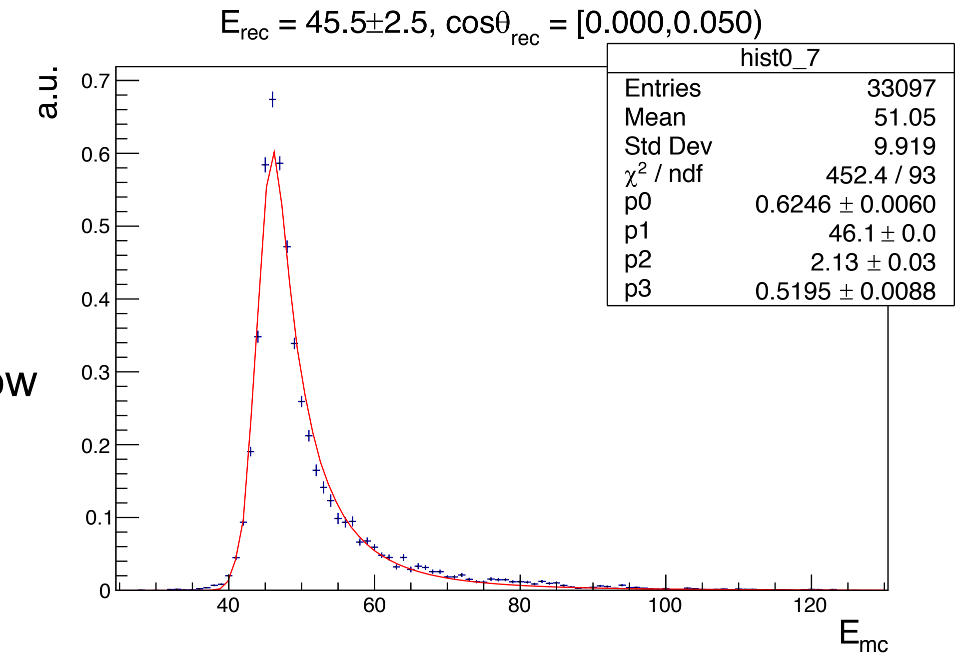
  Sum of the MCParticles which direction is close to reconstructed jet
  - Including neutrinos
- The resolutions are evaluated as the function of (E_{rec}, $\cos\theta_{rec}$) for each jet.

B-jet energy resolution: Evaluation setup

- Sample: b-jet pair
  - ILD DBD full simulation
  - E_{cm}: 20 - 240 GeV
  - PandoraPFA -> Durham jet clustering (LCFIPlus)

- Work flow:
  1. prepare data set of (E_{mc}, E_{rec}) in specific $\cos\theta_{rec}$ window
  2. generate E_{mc} histogram in specific E_{rec} window
     - normalized by all E_{rec} histogram
     - Each E_{mc} entry is shifted according as E_{rec} value.
  3. fit the spectrum

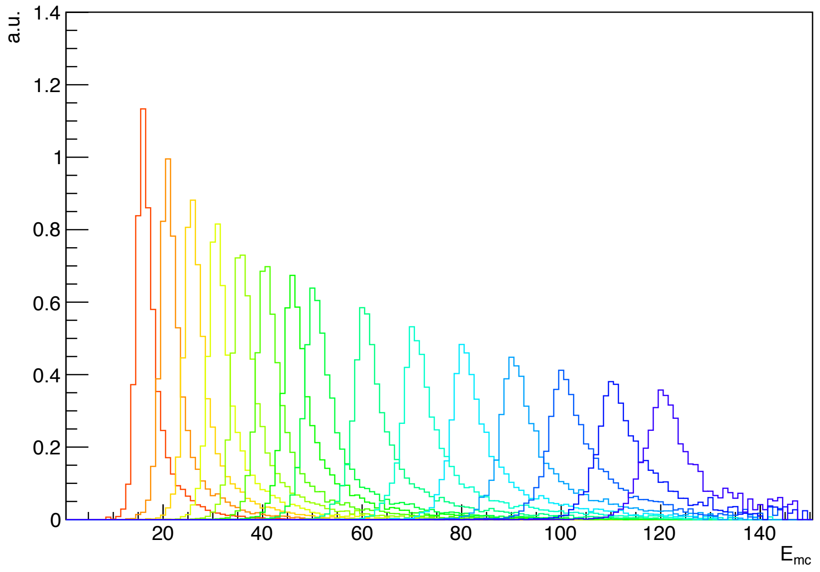
- Fit function: Crystal Ball (Gaussian & quartic polynomial)
  - p1: Gaussian mean
  - p2: Gaussian sigma
  - p3: Connection boundary in sigma unit

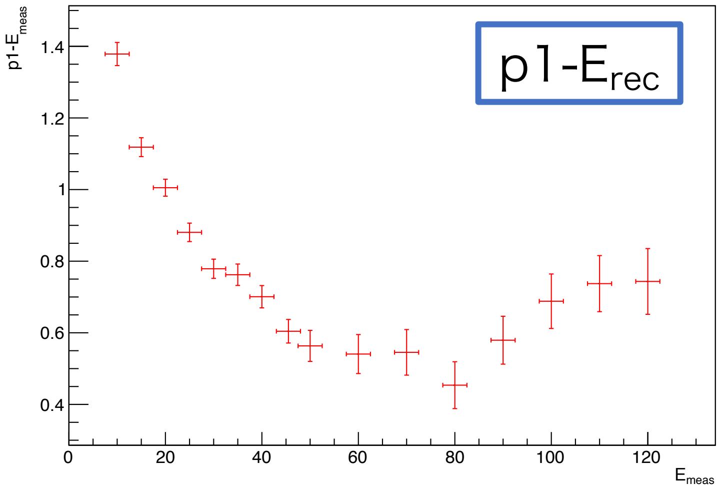


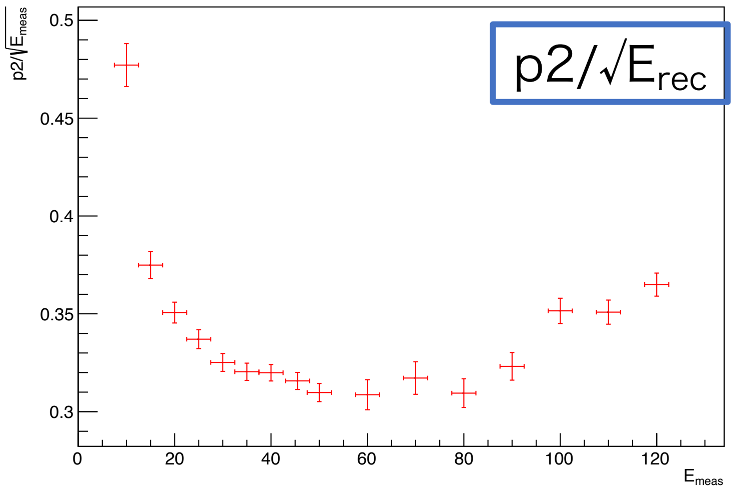
↑ True jet energy distribution for E_{rec} = 45.5 ± 2.5 GeV, $\cos\theta_{rec}$ = [0.,0.05)

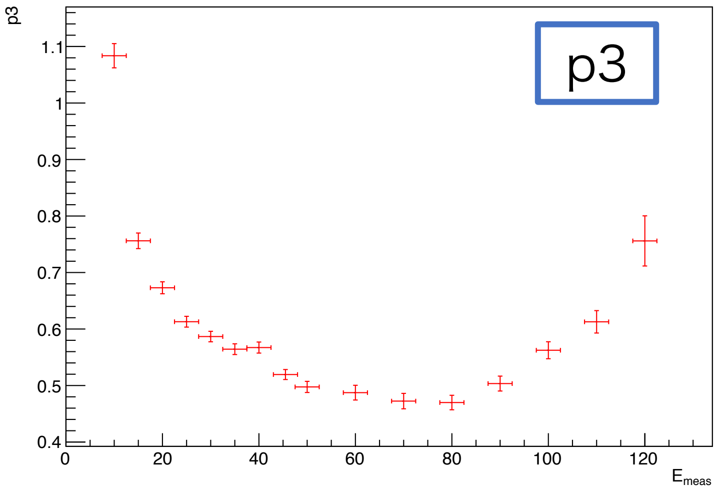
B-jet energy resolution: Energy dependence

- Energy scan in the barrel region
  - $\cos\theta_{rec}$ = [0.,0.05)
- In the higher edge the spectrum varies due to the lack of statistics.
- Parameters between points are interpolated.



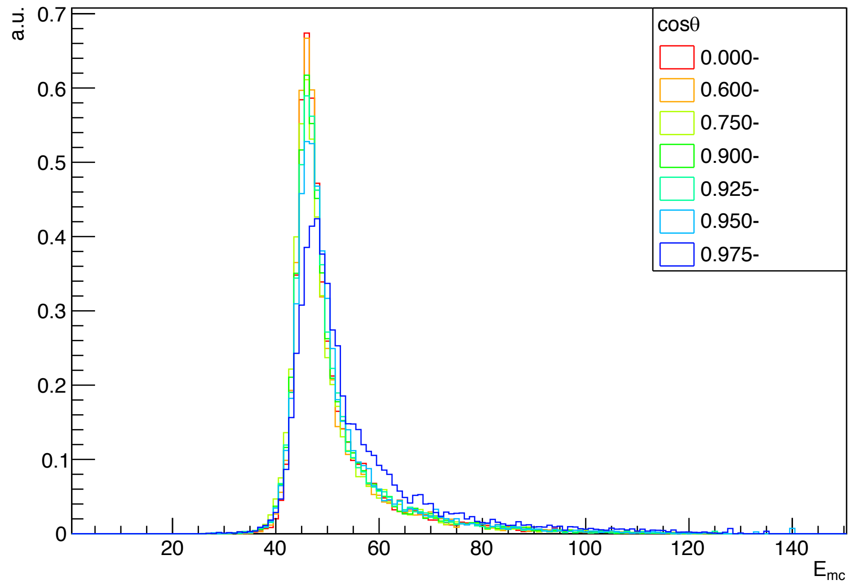


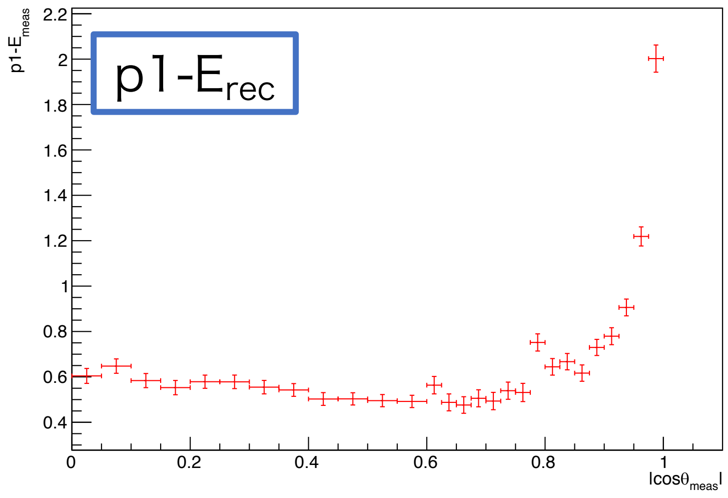


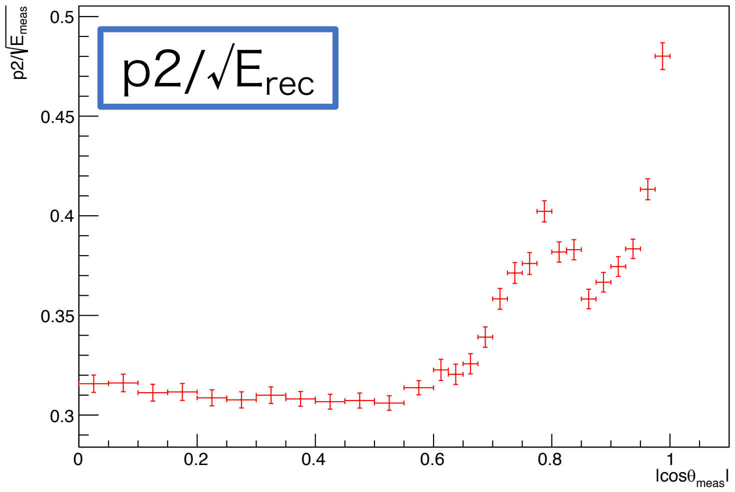


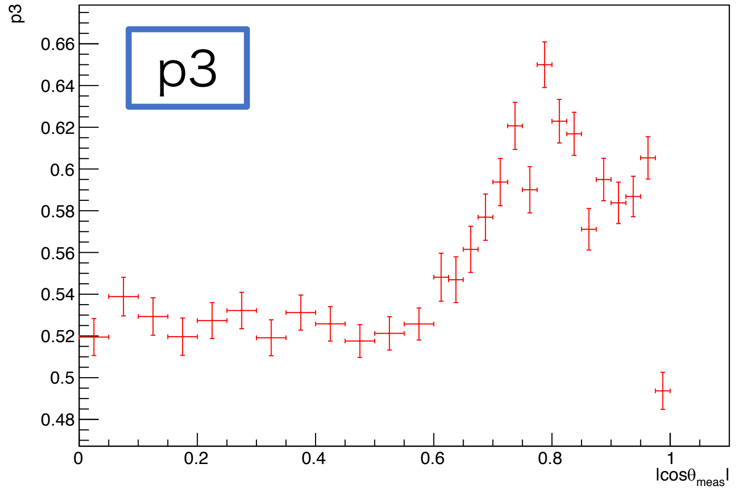
B-jet energy resolution: Angle dependence

- Angle scan at E_{rec} = 45.5 GeV
- JER is worse for forward jet as expected.









Setup of kinematic fit for e^+e^- -> ZH -> μμbb

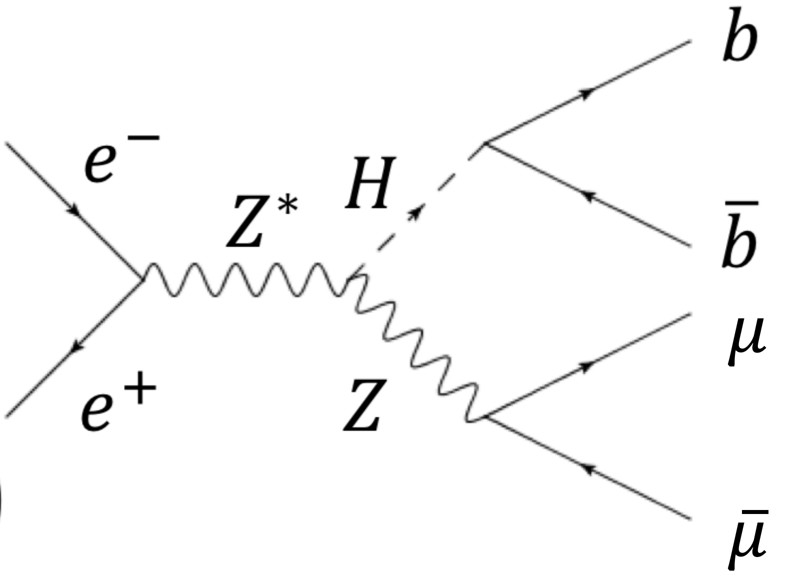
Fit Objects:

- JetFitObject (JFO) x 2
  - parameter: (E, θ, φ) with b-jet resolution
    E: Crystal Ball, θ: Gaus, φ: Gaus
  - $mass^{fit} \equiv E^{fit}/E^{meas.}$ x $mass^{meas.}$
  - Resolutions are adjusted by (E, cosθ) for each jet
- MuonFitObject (MFO) x 2
  - parameter: (Pt, θ, φ) with Gaussian error
    from track parameters
- ISRPhotonFitObject
  - parameter: Pz (E_{max} = 31.5 GeV)

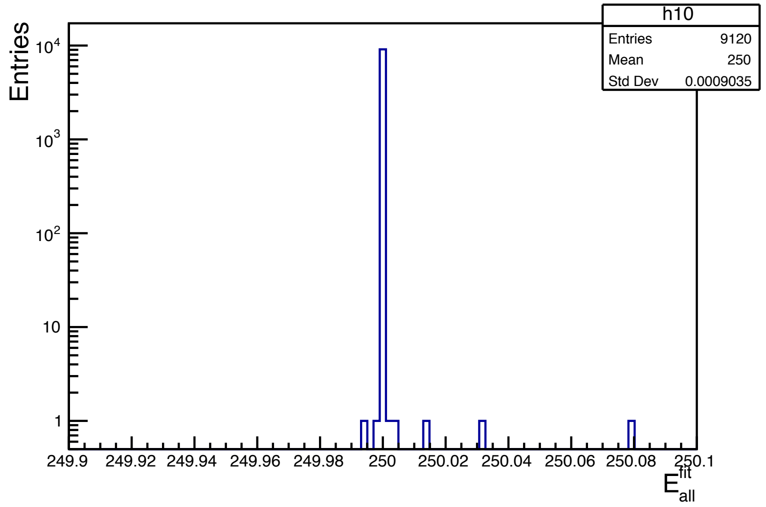
$$\mathcal{P}\left(p_{z,\gamma}\right) = \frac{\beta}{2E_{\max}} \cdot \left|\frac{p_{z,\gamma}}{E_{\max}}\right|^{\beta-1} \quad \beta = \frac{2\alpha}{\pi}\left(\ln\frac{s}{m_e^2} - 1\right)$$

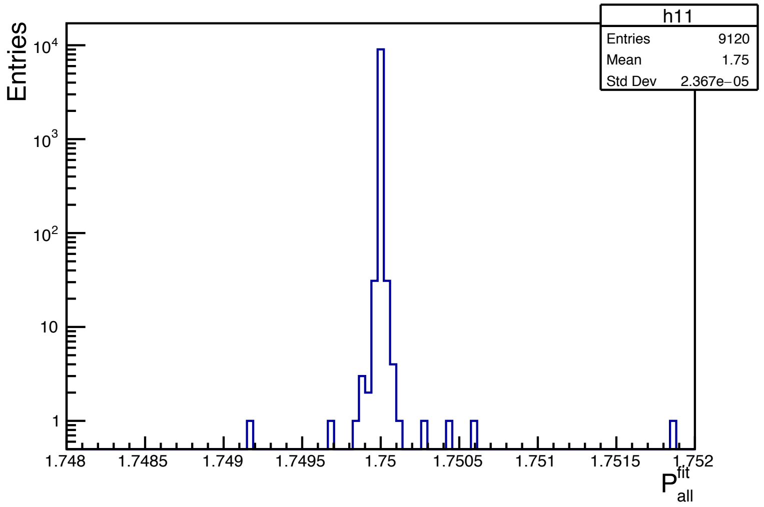
Constraints:

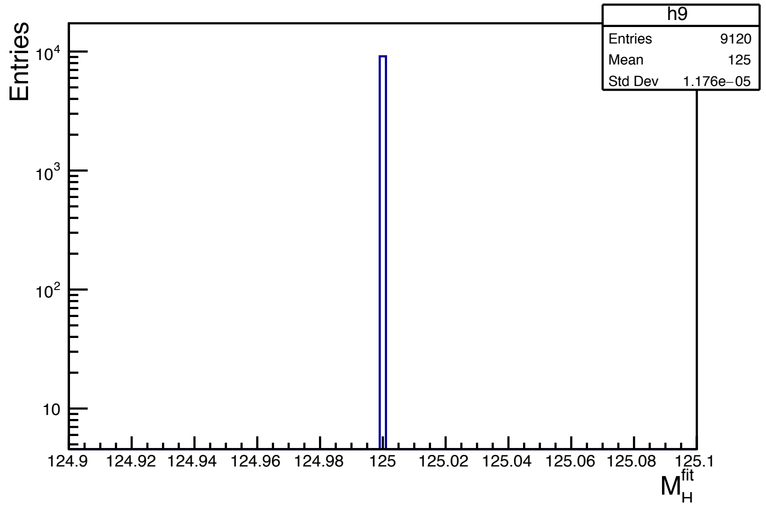
- Hard:
  - Total Energy/Px/Py/Pz for all FOs
  - Higgs mass = 125 GeV for 2 JFOs
- Soft:
  - Z mass w/ Breit-Wigner for 2 MFOs
    with mean 91.2 GeV and width 2.5 GeV



Fit Result: Hard constraint

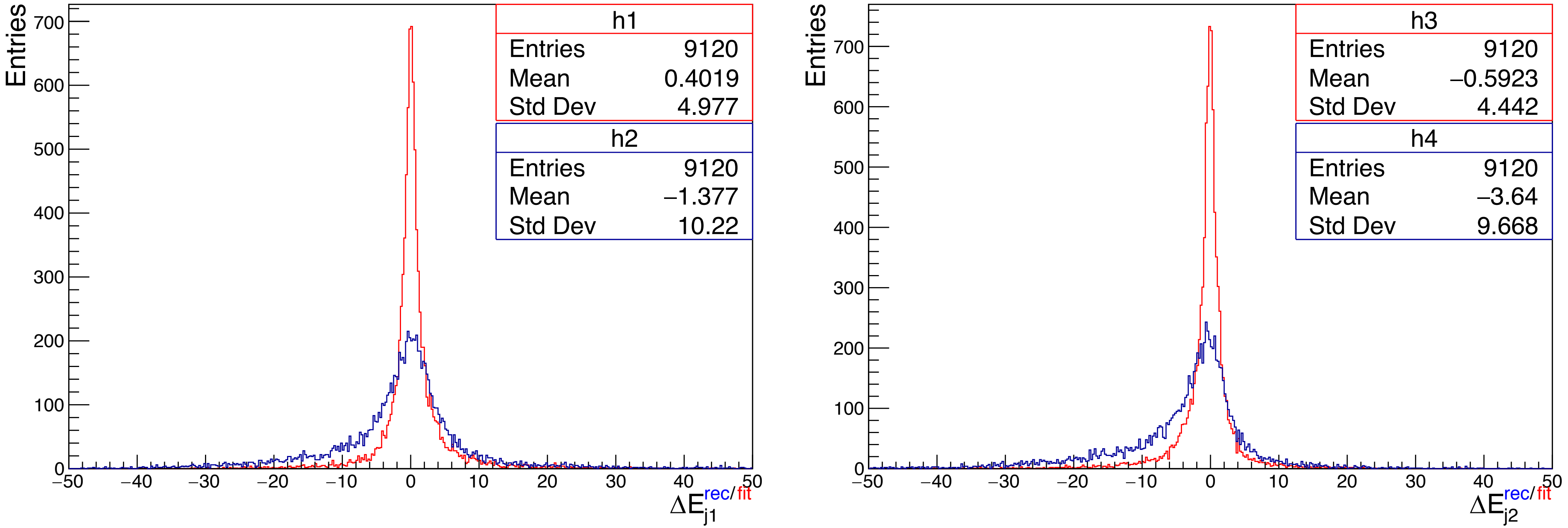






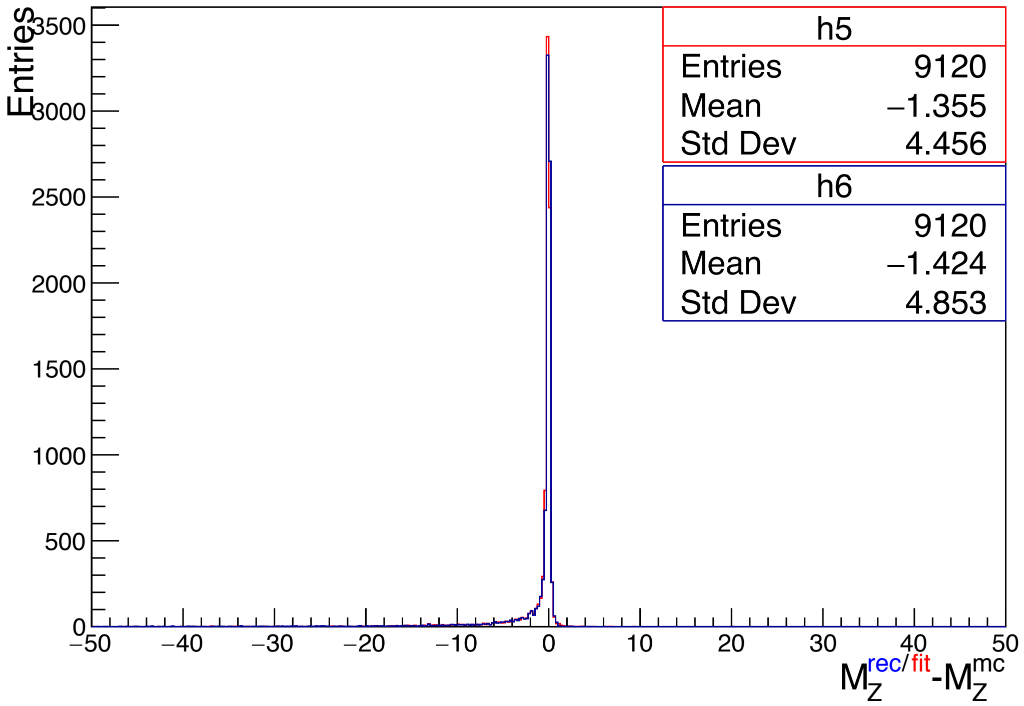
- 9120 / 9128 events are converged.
- All the hard constraints are working correctly.
  - Energy momentum constraint is slightly deviated.

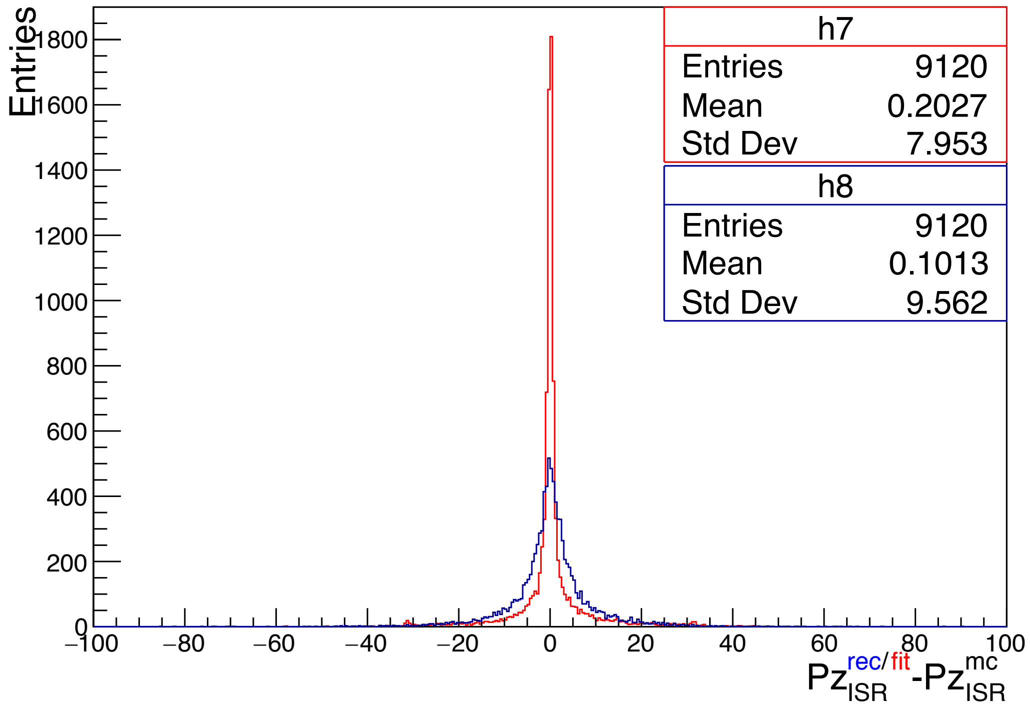
Fit Result: $\Delta Ej = Ej^{rec/fit} - Ej^{mc}$ $(Ej_1^{rec} > Ej_2^{rec})$



- The jet energies are significantly recovered!
- The error distributions after fit looks symmetry.

Fit Result: Soft constraint, ISR

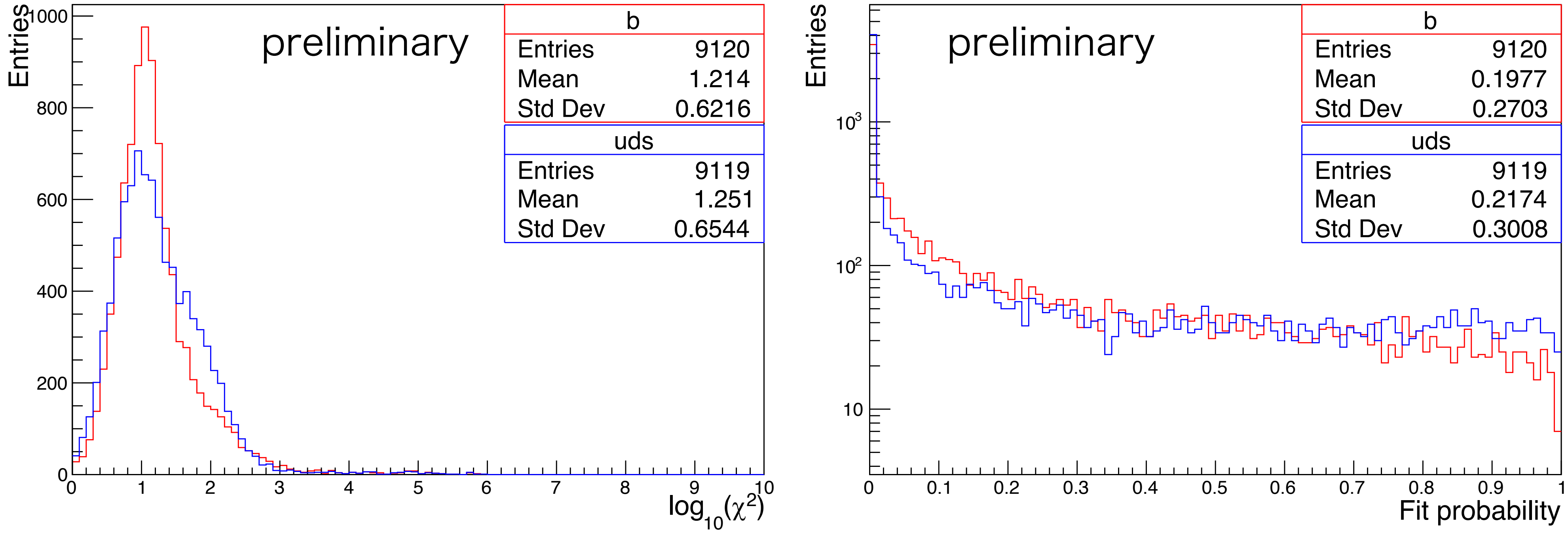




- ISR effect is also recovered!
- Mz hasn't changed so much. The lower tail is still remained.

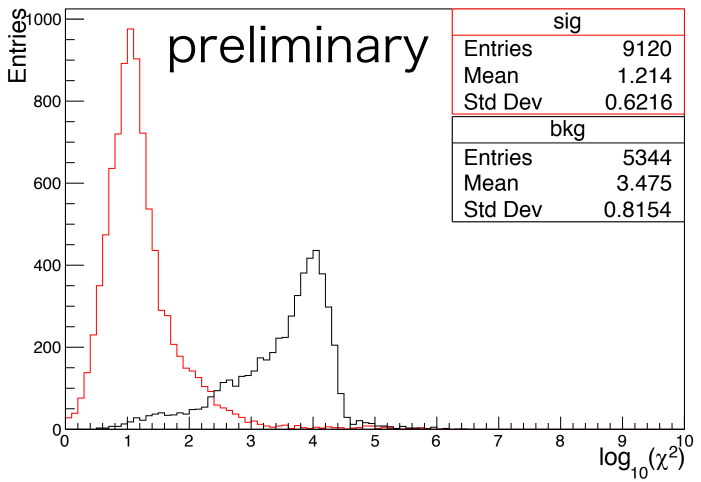
Fit Result: comparison of b/uds resolution

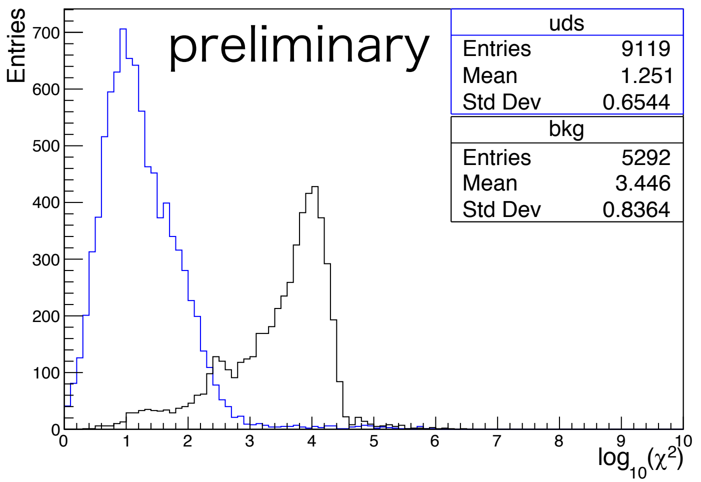
- We also obtained the result which use uds-jet resolutions.
- χ^2 is better for b-jet case.
- Fit probability flatness is slightly worse for b-jet case.
- A detailed check is in progress.



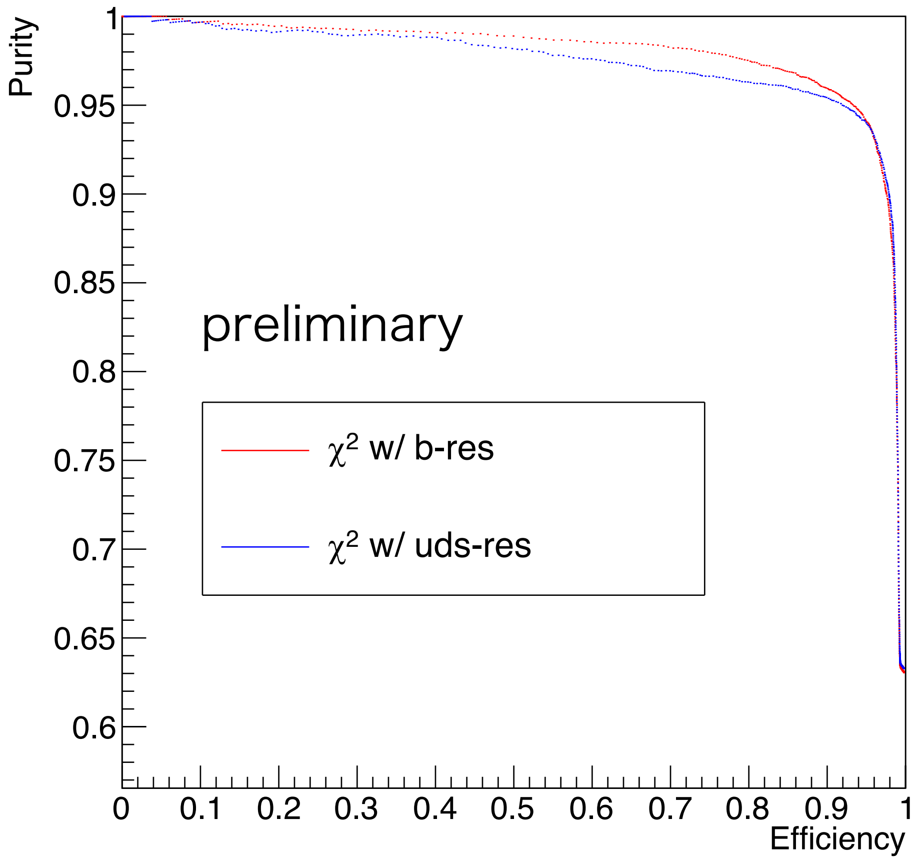
Fit Result: for Event Selection

- Using χ^2 distribution, event selection is tested.
- Event number is just sample number, not exact rate.
- This results are very preliminary, but b-jet case looks better than uds case.





Sig: μμh(→bb), Bkg: μμqq



Summary and Outlook

- We are developing kinematic fitter based on the log-likelihood method.
  - to deal with non-Gaussian distributions
  - by implementation of numerical differentiation

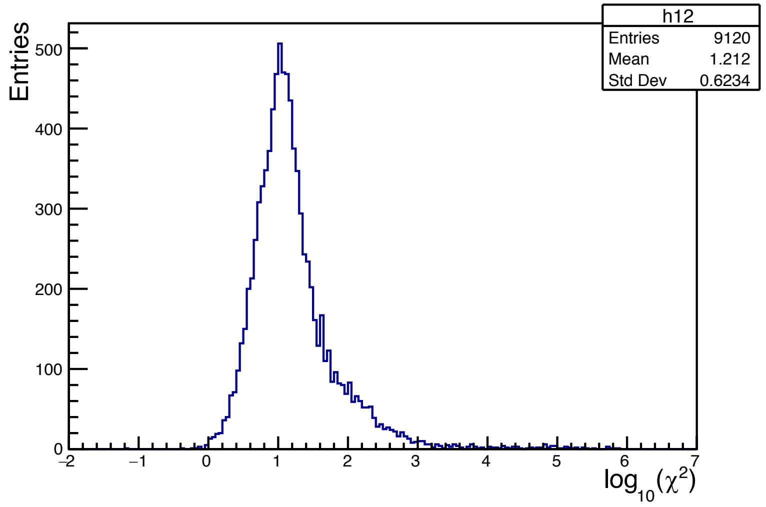
- For proof of principle, we try to fit e+e- -> ZH -> μμbb at 250 GeV.
  - This process is selected for technical study of kinematic fitter.
  - We evaluated b-jet resolutions using true jet definition.

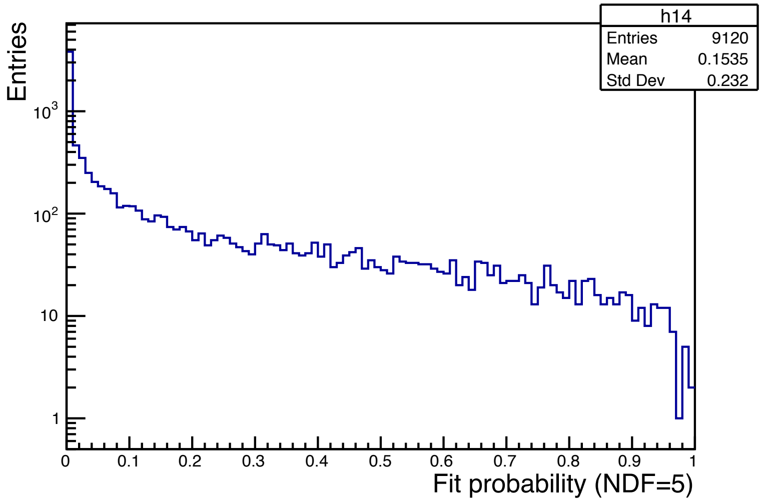
- Currently, we have confirmed a series of operations.
  - Fit results are obtained almost as we expected.
  - We also compared the results from b/uds resolutions and briefly obtained the benefit of b-resolution in event selection test.

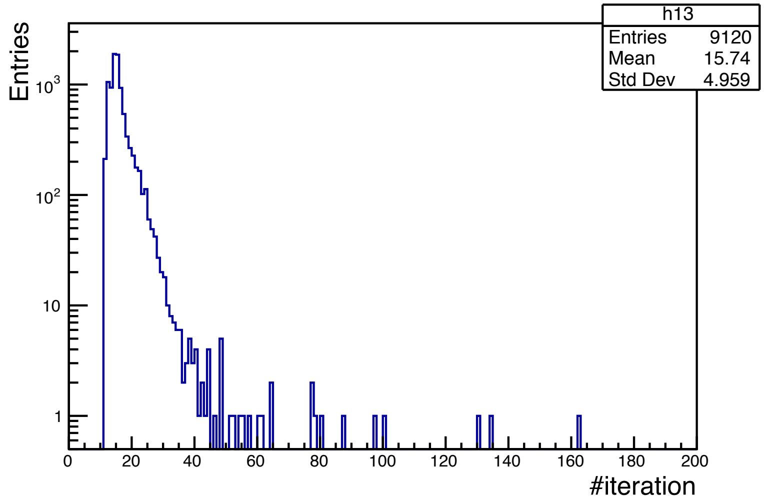
- We will move to next stage, physics study.
  - Next: ZH->qqbb, BSM(H->φφ->4b)

backup

Fit Result: $\log_{10}\chi^2$, Fit probability, #Interation





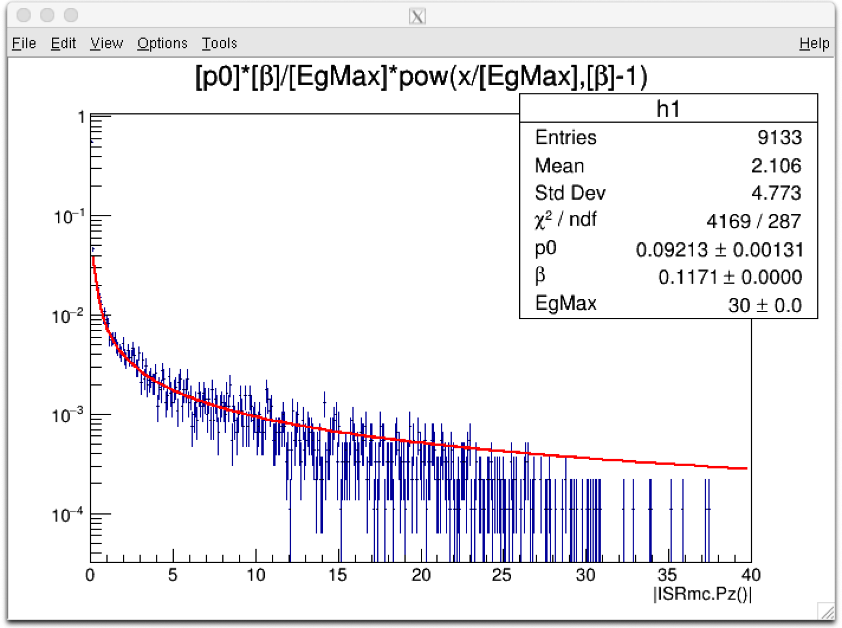


ISR spectrum

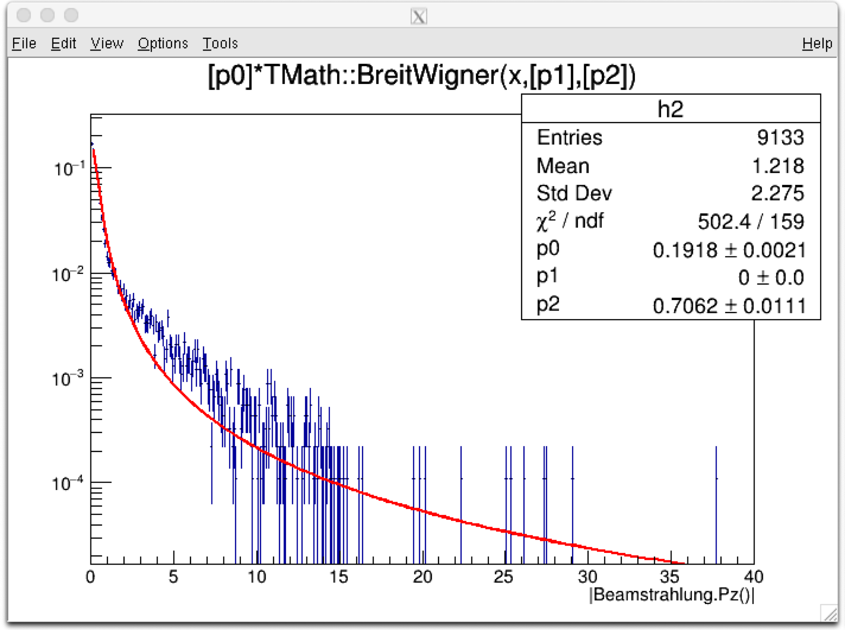
M. Beckmann, “Treatment of Photon Radiation in Kinematic Fits at Future e+e– Colliders”
F.A. Berends and R. Kleiss, Nucl. Phys. B177 (1981) 237

- ISR: $\mathcal{P}\left(p_{z,\gamma}\right) = \frac{\beta}{2E_{\max}} \cdot \left|\frac{p_{z,\gamma}}{E_{\max}}\right|^{\beta-1}$ $\beta = \frac{2\alpha}{\pi}\left(\ln\frac{s}{m_e^2} - 1\right)$
- beamstrahlung: ?

ISR



beamstrahlung



Ecm - Z - H