Next-to-leading-power factorization in QED: a diagrammatic approach

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Outline

- Next-to-leading-power (NLP) factorization
  Brief introduction

- Diagrammatic NLP factorization of QED amplitudes
  Keeping technical details to a minimum

- Is NLP resummation numerically relevant?
  A couple of plots borrowed from [van Beekveld, Laenen, Sinninghe Damsté, Vernazza '21]

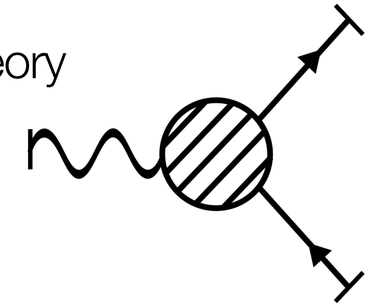
- Applications to linear colliders
  Cases where NLP resummation could be relevant

Factorization

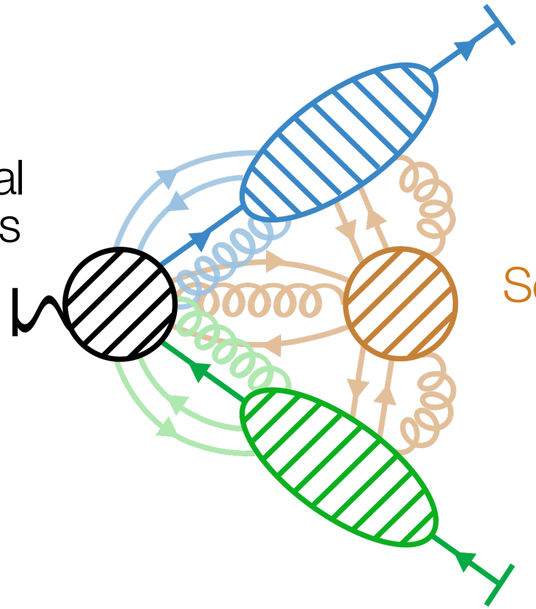
Systematic analysis of the on-shell singularities of Feynman diagrams shows that amplitudes factorize

[Collins, Soper, Sterman '83]

General gauge-theory process



Hard virtual corrections



$$\mathcal{M}^{\mathrm{LP}} = H \times S \times \frac{J}{\mathcal{J}} \times \frac{\bar{J}}{\bar{\mathcal{J}}}$$

(Schematic) factorized amplitude

Factorization extends from amplitudes to cross sections.
This has crucial consequences on phenomenology!

See previous talks in this session!

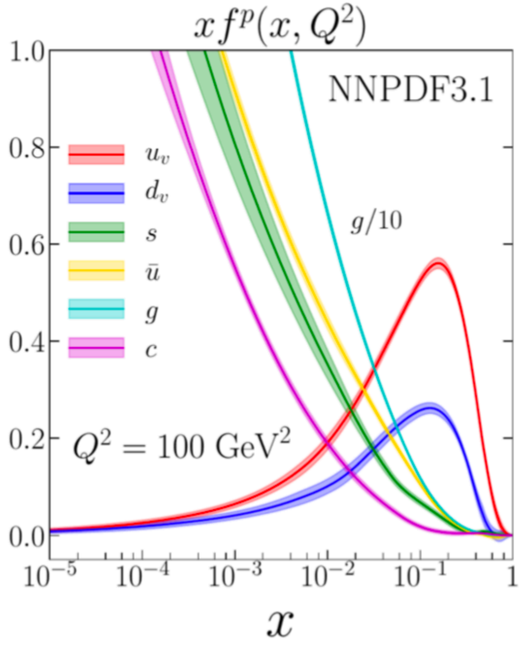
$$\frac{\mathrm{d}\sigma_{pp\to\ell\bar{\ell}X}}{\mathrm{d}Q^2} = \sum_f \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2\, f_{p\to f}(x_1,\mu) f_{p\to \bar{f}}(x_2,\mu) H_{f\bar{f}\to\ell\bar{\ell}}(x_1 x_2 s, Q^2, \mu)$$

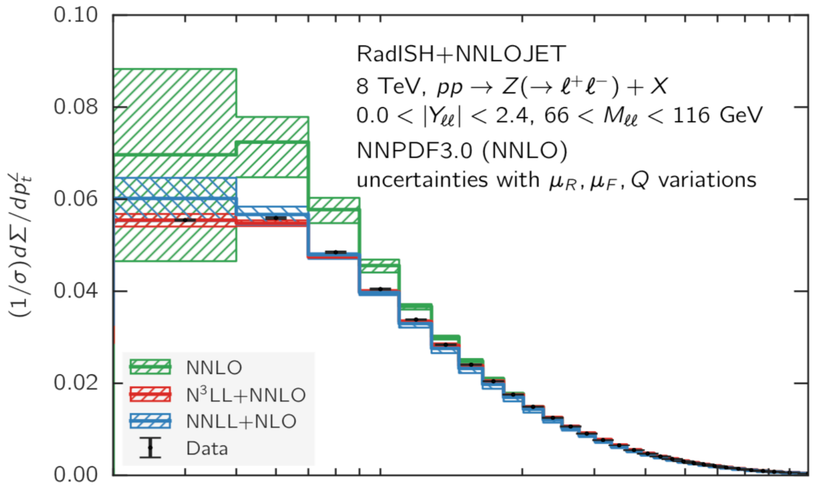
e.g. Drell-Yan cross section

(Collinear) PDFs: non-perturbative, universal

Hard scattering: perturbative, calculable

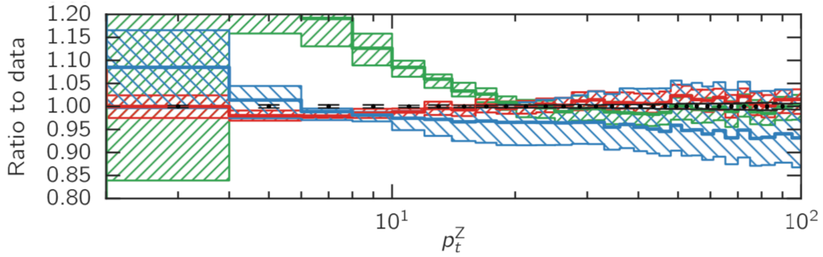
Plot from [Ethier, Nocera '20]





Resummation is often fundamental to get agreement with experimental data

(here Z pT spectrum / ATLAS data)
Plot from [Bizoń et al.,'18]



Subleading power factorization

Factorization needs expanding observables in a small parameter λ (ratio of scales/threshold variable)

Thrust $\lambda = \sqrt{\tau} \equiv \sqrt{1-T}$

Drell-Yan at threshold $\lambda = \sqrt{1 - \frac{Q^2}{s}}$

Transverse Momentum Dependent factorization $\lambda = \frac{k_T}{Q}$

Standard factorization: retain leading power (LP) in the expansion

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\xi}\right)_{\mathrm{LP}} = \sum_{n=0}^{\infty} \alpha_s^n \left[\underbrace{\sum_{m=0}^{2n-1} a_{nm} \left(\frac{\ln^m \xi}{\xi}\right)_+}_{\text{LP}} + b_n \delta(\xi) \right] = \sum_{n=0}^{\infty} \alpha_s^n \left[\underbrace{a_{n,2n-1} \left(\frac{\ln^{2n-1} \xi}{\xi}\right)_+}_{\text{LP Leading Logs}} + \underbrace{a_{n,2n-2} \left(\frac{\ln^{2n-2} \xi}{\xi}\right)_+}_{\text{LP Next-to-Leading Logs}} + \dots \right]$$

Subleading power factorization: include more terms in the expansion

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\xi}\right)_{\mathrm{NLP}} = \sum_{n=0}^{\infty} \alpha_s^n \left[\underbrace{\sum_{m=0}^{2n-1} a_{nm} \left(\frac{\ln^m \xi}{\xi}\right)_+}_{\text{LP}} + b_n \delta(\xi) + \underbrace{\sum_{m=0}^{2n-1} d_{nm} \ln^m \xi}_{\text{NLP}} \right]$$

Suppressed by one power of the threshold variable

Subleading power factorization is very active topic!

(Very partial compilation of results from the last few years)

[Larkoski, Neill, Stewart '14]
[Bonocore, Laenen, Magnea, (Melville), Vernazza, White, ('15),'16]
[van Beekveld, Beenakker, Laenen, White '16]
[del Duca, Laenen, Magnea, Vernazza, White '17]
[Bahjat-Abbas, Sinninghe Damsté, Vernazza, White '18]
[Moult, Stewart, Vita '19]
[Beneke, Broggio, Jaskiewicz, Vernazza '19]

Soft theorems / radiative jets

[Moult, Stewart, Vita '17]
[Feige, Kolodrubetz, Moult, Stewart '17]
[Beneke, Garny, Szafron, Wang '17,'18,'19]
[Moult, Stewart, Vita, Zhu '19]
[Moult, Rothen, Stewart, Tackmann, Zhu,'16,'17]
[Boughezal, Liu, Petriello,'16]
[Boughezal, Isgrò, Petriello, '18,'19]
[Ebert, Moult, Stewart, Tackmann, Vita, Zhu '18,'19]

Operator bases / anomalous dimensions / N-Jettiness subtractions

NLP resummed predictions available for highly relevant collider processes

Event shapes (thrust)

[Moult, Stewart, Vita, Zhu '18]

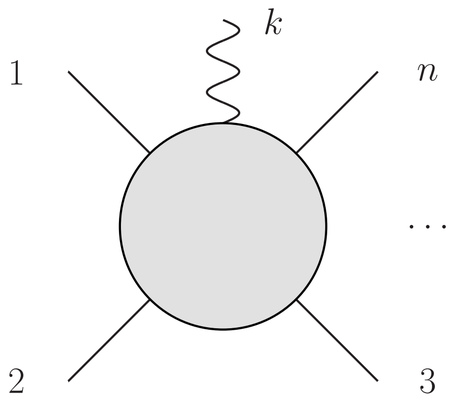
Drell-Yan scattering / Higgs production at threshold

[Beneke, Broggio, Garny, Jaskiewicz, Szafron, Vernazza, Wang '18]
[Bahjat-Abbas, Bonocore, Sinninghe Damsté, Laenen, Magnea, Vernazza, White '19]

NLP factorization in QED

[Laenen, Sinninghe Damsté, Vernazza, Waalewijn, LZ '20]

We focused on soft theorems



Soft emission, $E \equiv k^0$

Determined by $\mathcal{M}_n$

$$\mathcal{M}_{n+1} = \mathcal{M}_{n+1}^{\mathrm{LP}} + \mathcal{M}_{n+1}^{\mathrm{NLP}} + \mathcal{O}(E)$$

Low-Burnett-Kroll theorem relates the radiative amplitude to the elastic amplitude up to NLP

[Del Duca '90] LBK theorem valid only for $E \ll m^2/Q$
(ignores subsequent collinear emissions!)
Extensions require radiative jet functions

By power counting, we proposed a NLP factorization formula in two regimes

- parametrically small fermion masses $\lambda = \dfrac{m}{Q}$
- purely massless, $\lambda^2 = \dfrac{E}{Q}$ set by external radiation

NLP complications

We focused on the collinear sector

Several terms, corresponding to additional legs connecting jets and hard function

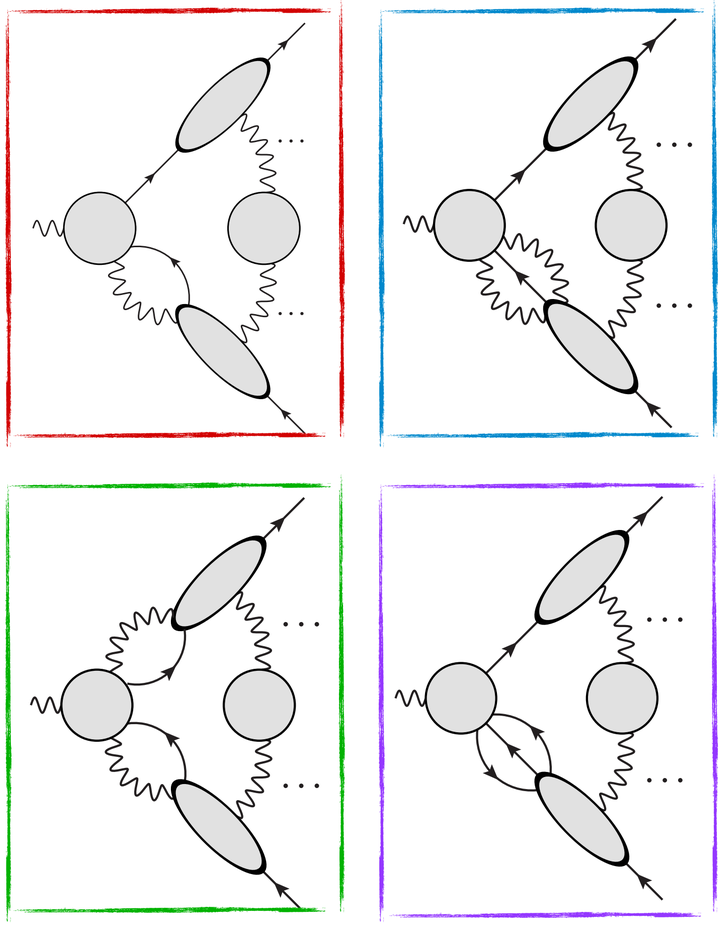
$$\mathcal{M}^{\rm NLP}_{\rm coll} = \sum_{i=1}^{n}\left(\prod_{j\neq i} J^j_{(f)}\right)\left[J^i_{(f\gamma)}\otimes H^i_{(f\gamma)} + J^i_{(f\partial\gamma)}\otimes H^i_{(f\partial\gamma)}\right] S$$

$$+\sum_{i=1}^{n}\left(\prod_{j\neq i} J^j_{(f)}\right) J^i_{(f\gamma\gamma)}\otimes H^i_{(f\gamma\gamma)}\, S$$

$$+\sum_{1\leq i\leq j\leq n}\left(\prod_{k\neq i,j} J^k_{(f)}\right) J^i_{(f\gamma)} J^j_{(f\gamma)}\otimes H^{ij}_{(f\gamma)(f\gamma)}\, S$$

$$+\sum_{i=1}^{n}\left(\prod_{j\neq i} J^j_{(f)}\right) J^i_{(fff)}\otimes H^i_{(fff)}\, S$$

See also earlier SCET work in QCD
[Larkoski, Neill, Stewart '14]
[Moult, Stewart, Vita '19]
[Beneke, Broggio, Jaskievicz, Vernazza '19]



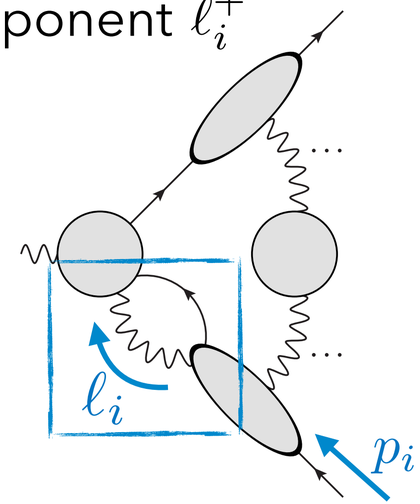
NLP complications

$$\left(\prod_{j\neq i} J^j_{(f)}\right)\left[J^i_{(f\gamma)}\otimes H^i_{(f\gamma)} + J^i_{(f\partial\gamma)}\otimes H^i_{(f\partial\gamma)}\right] S$$

$$\equiv \left(\prod_{j\neq i} J_{(f)}(p_j)\right) \int_0^{p_i^+} d\ell_i^+ \left[J^\nu_{(f\gamma)}(p_i-\hat{\ell}_i,\hat{\ell}_i)\, H_{(f\gamma)\nu}(p_1\ldots;p_i-\hat{\ell}_i,\hat{\ell}_i;\ldots p_n)\right.$$

$$\left. + \; J^{\nu\rho}_{(f\partial\gamma)}(p_i-\hat{\ell}_i,\hat{\ell}_i)\, H_{(f\partial\gamma)\nu\rho}(p_1\ldots;p_i-\hat{\ell}_i,\hat{\ell}_i;\ldots p_n)\right] S(\hat{p}_i\cdot\hat{p}_j)$$

Convolution: how the two legs share out the large momentum component ℓ_i^+



Results in endpoint singularities:

Poles in the dimensional regulator ε not captured by J, H, but generated from the convolution

[Beneke, Garny, Szafron, Wang '19]
[Moult, Stewart, Vita, Zhu '19]
[Beneke, Broggio, Jaskiewicz, Vernazza '19]
[Liu, Mecaj, Neubert, Wang '20]

- Complicates resummation
- May require additional rapidity regulator

NLP complications

$$\left(\prod_{j\neq i} J^j_{(f)}\right)\left[\underline{J^i_{(f\gamma)}\otimes H^i_{(f\gamma)}} + \underline{J^i_{(f\partial\gamma)}\otimes H^i_{(f\partial\gamma)}}\right] S$$

Presence of additional derivative terms

$$\widetilde{H}^i_{(f\gamma)\nu}(p_1\ldots p_n;\ell_i) = \widetilde{H}^i_{(f\gamma)\nu}(p_1\ldots p_n;\hat{\ell}_i) + \ell^\rho_\perp\left[\frac{\partial}{\partial\ell^\rho_\perp}\widetilde{H}^i_{(f\gamma)\nu}(p_1\ldots p_n;\ell_i)\right]_{\ell_\perp=0}$$

$$\equiv \underline{H_{(f\gamma)\nu}(p_1\ldots;p_i-\hat{\ell}_i,\hat{\ell}_i;\ldots p_n)} + \underline{\ell^\rho_\perp H_{(f\partial\gamma)\nu\rho}(p_1\ldots;p_i-\hat{\ell}_i,\hat{\ell}_i;\ldots p_n)}$$

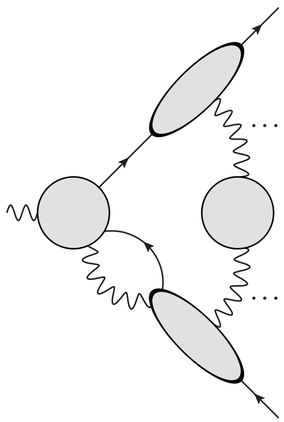
Leading power: safely set $\ell_i = \hat{\ell}_i \equiv \ell_i^+ n$

Next-to-leading power: $\mathcal{O}(\lambda^2)$ can also arise from $\ell_{i\perp}\cdot\ell_{i\perp}$

Taylor expand the hard matrix element, absorb $\ell^\rho_{i\perp}$ in the $J^{i\,\rho}_{(f\partial\gamma)}$ jet function

we followed [Gervais '17]

Results

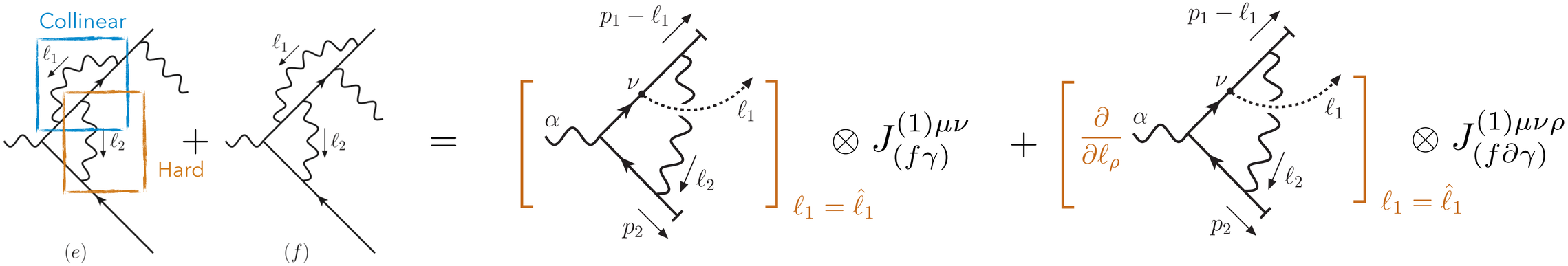


We computed the $(f\gamma)$ and $(f\partial\gamma)$ jets at one loop in the massless radiative / massive case

Extends [Beneke, Bobeth, Szafron '19] to $\mathcal{O}(\lambda^2)$

Includes a term first missed by [Bonocore, Laenen, Magnea, Melville, Vernazza, White '15]

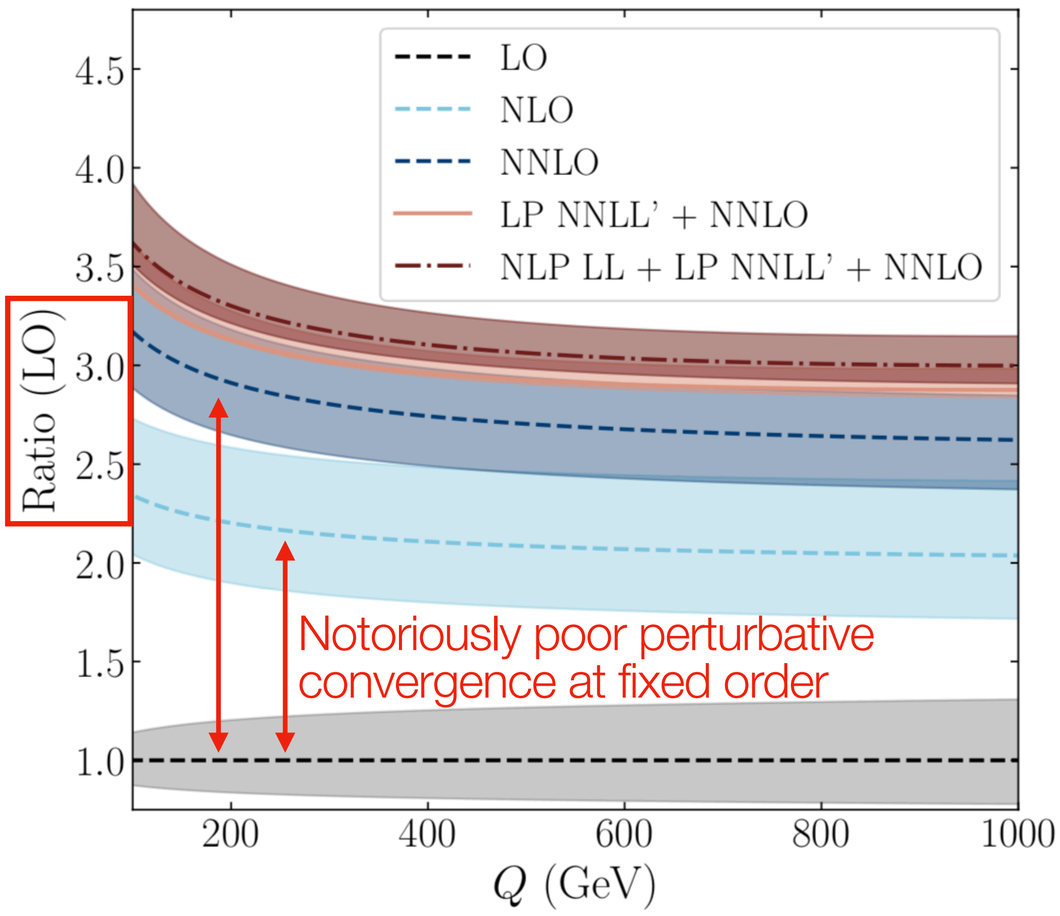
We performed two-loop checks using the method of regions [Beneke, Smirnov '98] [Janzen '11]

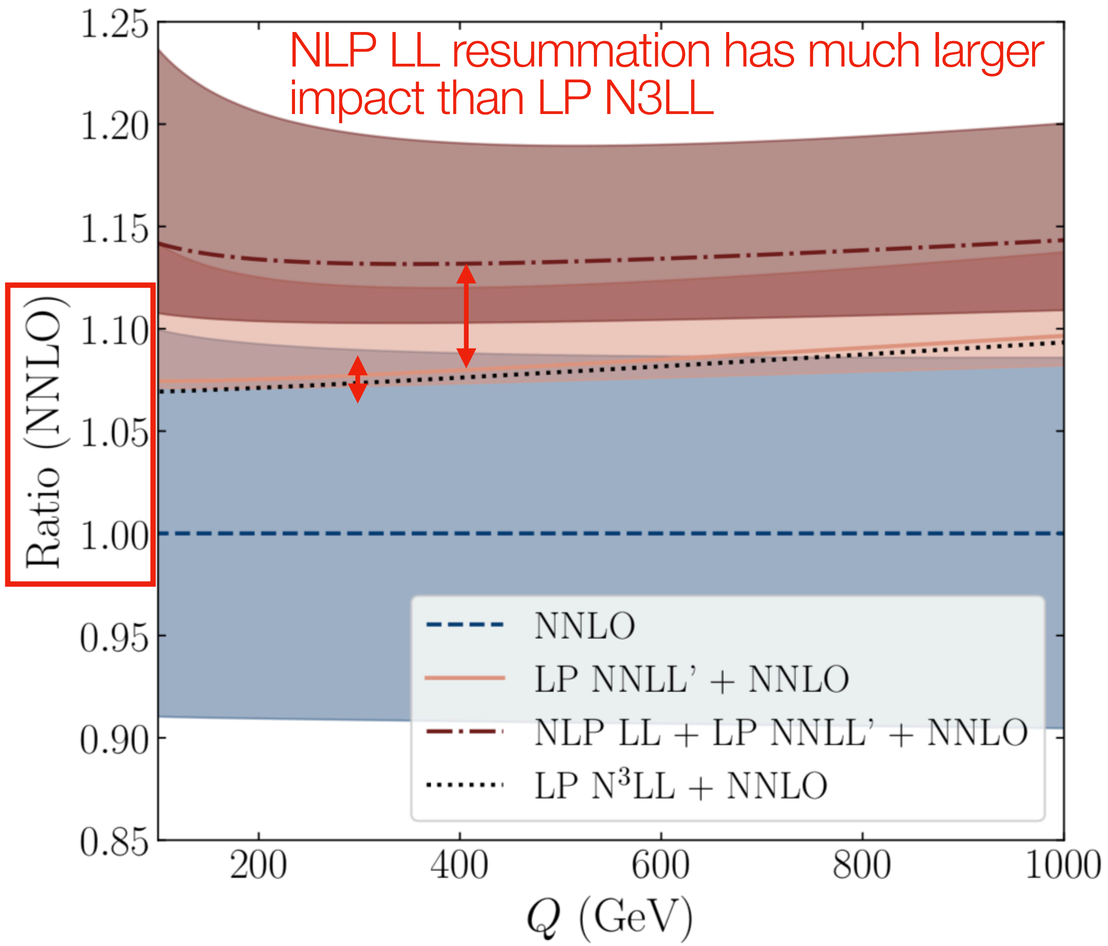


Numerical impact of NLP resummation

[Plots from van Beekveld, Laenen, Sinninghe Damsté, Vernazza '21]

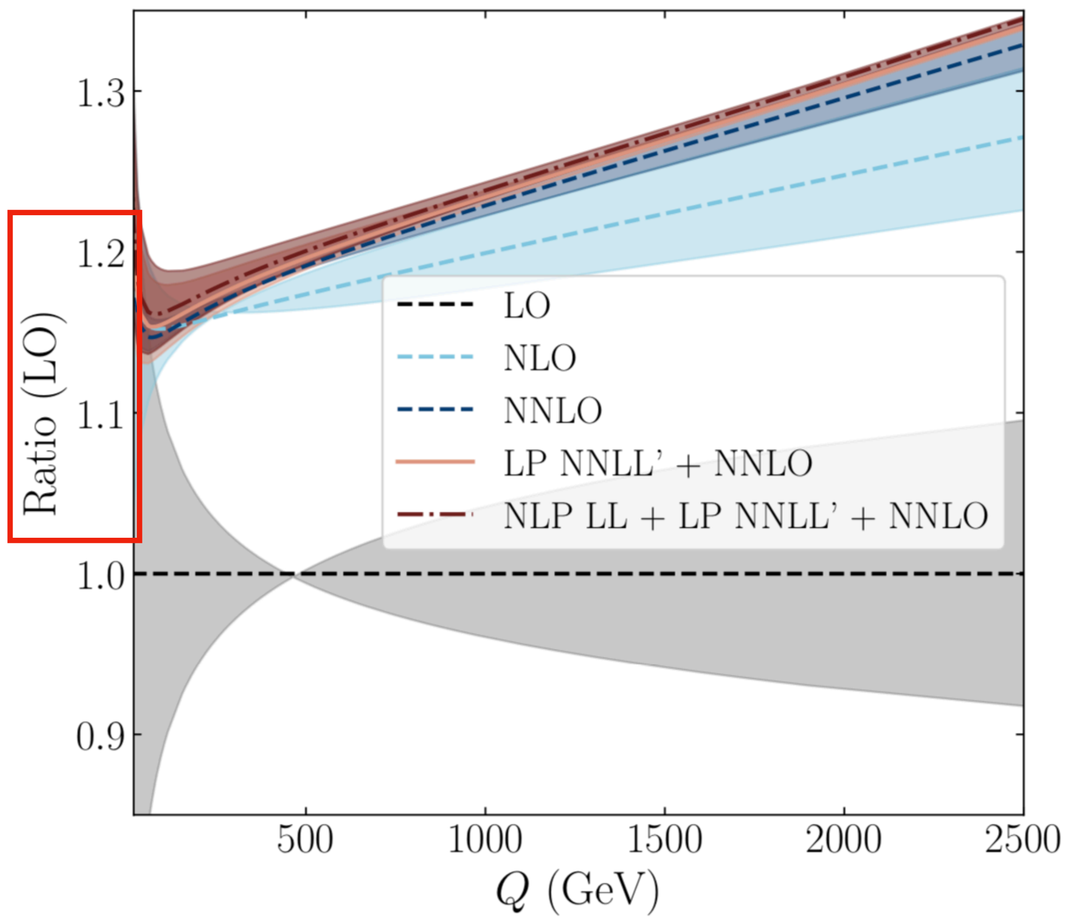
Single Higgs production (pp collisions, gluon fusion), total cross section

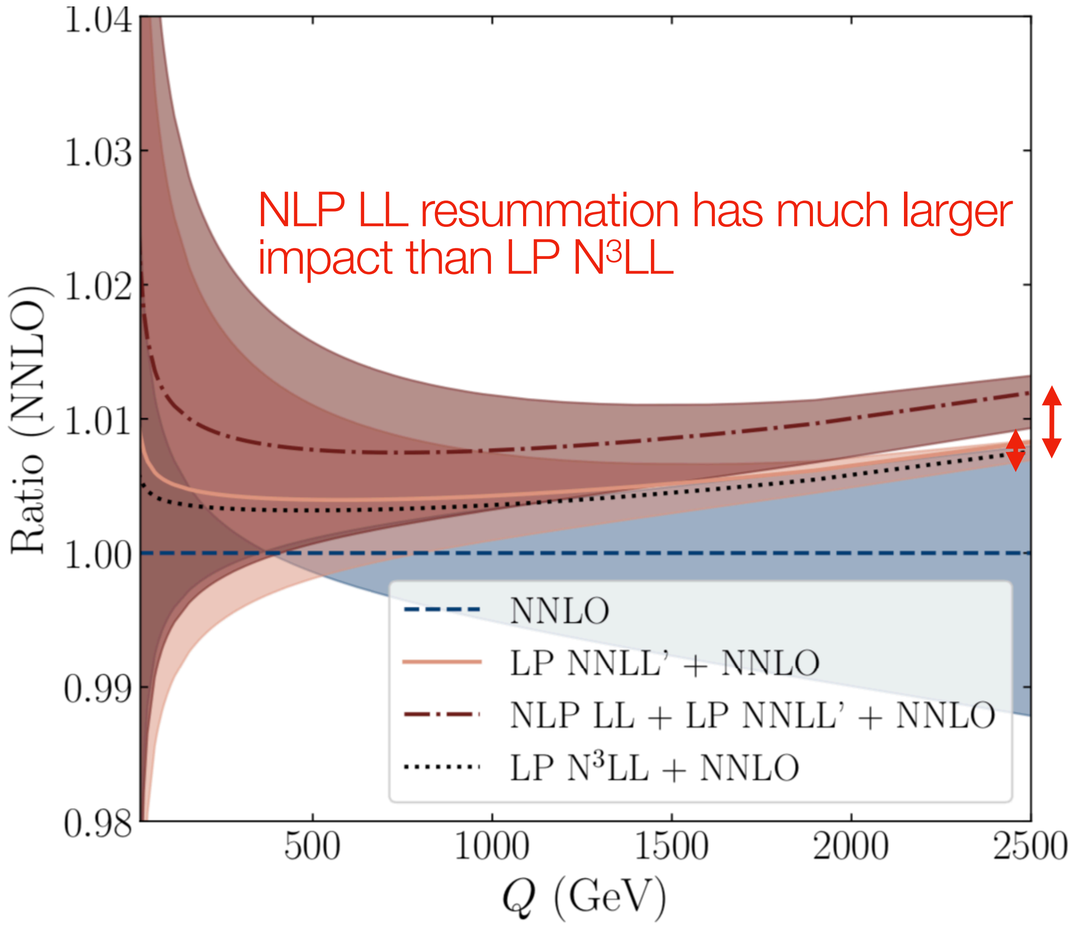




[Plots from van Beekveld, Laenen, Sinninghe Damsté, Vernazza '21]

Drell-Yan scattering at threshold





Considerations for an e^+e^- linear collider

- We want precision physics. Wherever LP resummation is relevant, we expect NLP resummation to also become important when pushing calculations to high enough precision

- In e^+e^- collisions, QCD resummation is still relevant in the final state
  Event shapes like thrust are used for precise determination of α_s
  NLP resummation achieved in [Moult, Stewart, Vita, Zhu '18]

- An interesting case is top pair production at threshold, where $\beta = \sqrt{1 - \frac{4m_t^2}{M_{t\bar{t}}^2}}$
  Subleading power logarithms were shown relevant (pp collisions) in [Ju, Wang, Wang, Xu, Xu, Yang '20]

- QED resummation is relevant for initial state radiation
  subleading logarithms considered in [Ablinger, Blümlein, De Freitas, Schönwald '20]
  See Kay Schönwald's talk on Monday

Conclusions

- Next-to-leading-power factorization is a mature field

- Our contribution:
  - proposed a diagrammatic approach to NLP factorization valid for QED
  - computed one class of jet functions at NLP
  - performed two-loop checks with the method of regions

- For various processes, subleading power resummation gives sizable effects
  (comparable with state-of-the art LP resummation)
  Should keep in mind when pushing LP resummation to higher accuracy

- NLP resummation will play a role at future linear colliders
  (e.g. event shapes, top pair production, initial-state QED radiation)