Forward-backward asymmetries of the heavy quark pair productions in e^+e^- collisions at $\mathcal{O}(\alpha_s^2)$

Long Chen

Institute for Theoretical Particle Physics and Cosmology,
RWTH Aachen University

LCWS2021, March 15

based on JHEP 1612 (2016) (098) [arXiv:1610.07897]
JHEP 1701 (2017) (053) [arXiv:1611.07942]
Eur. Phys. J. C80 (2020) 649 [arXiv:2003.13941]

Motivation and Background

e^+e^- collisions offer a clean environment for studying properties of heavy quarks, e.g. masses and electroweak couplings of the top and bottom quarks.

- $b\bar{b}$ pair production at Z-pole:
  analysing the long-standing $2.9\,(2.4)\,\sigma$ tension between the direct determination of A^b_{FB} and its global SM fit;
- $t\bar{t}$ pair production:
  important for precision physics at future e^+e^- colliders.

NLO EW and QCD corrections with (massive) quarks known since long time ago $\cdots$

NNLO QCD corrections in the *massless* quark limit [Altarelli, Lampe 93; Ravindran, Neerven 98; Catani, Seymour 99; Weinzierl 07]

$e^+e^- \to t\bar{t}$ @ $NNLO\ QCD$ [Gao, Zhu, PRL (2014)]

$e^+e^- \to t\bar{t}$ near-threshold @ $NNNLO$ [M. Beneke *et al.*, PRL (2015)] ($\sigma^{t\bar{t}}_{tot}$ only)

Our aim was a stable fully differential NNLO QCD calculation of **massive** $Q\bar{Q}$ productions in the antenna subtraction framework for $e^+e^- \to Q\bar{Q} + X$ at $\mathcal{O}\left(\alpha_s^2\right)$ in the continuum.

Why is the calculation not straightforward ?

Schematically, beyond leading-order

$$\sigma_{\textbf{NLO}} = \int_{d\Phi_{n+1}} \sigma^{\mathcal{R}} + \int_{d\Phi_{n}} \sigma^{\mathcal{V}}$$

Divergences

Ultraviolet divergences $\Longleftarrow$ *UV-renormalization*

Soft and Collinear divergences (IR-divergences) $\Longleftarrow$ *KLN-theorem*

Finite Observables

All intermediate divergences should cancel in a physical observable

Factorization

Universal factorization structure of IR-singularity

Generally applicable methods: *Phase-space slicing*, *Subtraction*, $\cdots\cdots$

Antenna-subtraction method

General idea of any subtr.method: insert an *identity* $0 = \int d\sigma^{\mathcal{S}} - \int d\sigma^{\mathcal{S}}$ into $\sigma_{\mathbf{NLO}}$.

A (differential) cross section at LO: ($F_J{}^{(n)}$ defines the observable)

$$\sigma_{\text{LO}} = \int_{d\Phi_n} |\mathcal{M}_n|^2 \, F_J{}^{(n)}$$

The NLO contribution:

$$\begin{aligned}
\sigma_{\text{NLO}} &= \int_{d\Phi_{n+1}} d\sigma^{\mathcal{R}}_{\text{NLO}} + \int_{d\Phi_n} d\sigma^{\mathcal{V}}_{\text{NLO}} \\
&= \int_{d\Phi_{n+1}} |\mathcal{M}^{\mathcal{R}}_{n+1}|^2 \, F_J{}^{(n+1)} + \left(\int_{d\Phi_{n+1}} d\sigma^{\mathcal{S}} - \int_{d\Phi_{n+1}} d\sigma^{\mathcal{S}} \right) + \int_{d\Phi_n} |\mathcal{M}^{\mathcal{V}}_n|^2 \, F_J{}^{(n)} \\
&= \int_{d\Phi_{n+1}} \left[\left(|\mathcal{M}^{\mathcal{R}}_{n+1}|^2 \, F_J{}^{(n+1)} \right)_{\epsilon=0} - \left(d\sigma^{\mathcal{S}} \right)_{\epsilon=0} \right] + \int_{d\Phi_n} \left[|\mathcal{M}^{\mathcal{V}}_n|^2 \, F_J{}^{(n)} + \int_1 d\sigma^{\mathcal{S}} \right]_{\epsilon=0}
\end{aligned}$$

- $d\sigma^{\mathcal{S}}$ should approach $d\sigma^{\mathcal{R}}_{\text{NLO}}$ locally in all IR-soft limit.
- ***d*-dimensional (analytic) integrablity** of $\int_1 d\sigma^{\mathcal{S}}$

Antenna-subtraction method

The idea of IR-subtraction methods: insert an *identity* $0 = \int d\sigma^{\mathcal{S}} - \int d\sigma^{\mathcal{S}}$.

Exploiting the *universal IR-factorization property* of *color-ordered* partial QCD-amplitudes,

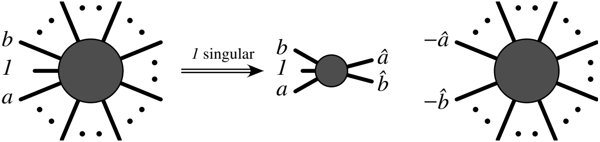


Figure: The Antenna-factorization of a *color-ordered* partial amplitude (PRD 71, 045016)

$$d\sigma^{\mathcal{S}}_{\mathrm{NLO}} \propto \sum \mathbf{A}_{a1b}\left(p_a, p_1, p_b\right) \otimes \left|\mathcal{M}^{\mathcal{R}}_{n}\left(\cdots, P_{\hat{a}}, P_{\hat{b}}, \cdots, p_{n+1}\right)\right|^2$$

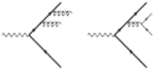
where the $\mathbf{A}_{a1b}$ is the *antenna-function*.

Ingredients for $e^+e^- \to Q\bar{Q} + X$ $(Q = t, b)$

Schematically, the NNLO corrections with IR subtraction terms:

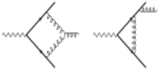
$$\begin{aligned} d\sigma_{\mathrm{NNLO}} &= \int_{d\Phi_4} \left(d\sigma^{RR}_{\mathrm{NNLO}} - d\sigma^{S}_{\mathrm{NNLO}} \right) \\ &+ \int_{d\Phi_3} \left(d\sigma^{RV}_{\mathrm{NNLO}} - d\sigma^{T}_{\mathrm{NNLO}} \right) \\ &+ \int_{d\Phi_2} d\sigma^{VV}_{\mathrm{NNLO}} + \int_{d\Phi_3} d\sigma^{T}_{\mathrm{NNLO}} + \int_{d\Phi_4} d\sigma^{S}_{\mathrm{NNLO}} \end{aligned}$$

- RR: Tree-level double real radiation correction: $S \to Q\bar{Q}Q\bar{Q}$, $Q\bar{Q}gg$, and $Q\bar{Q}q\bar{q}$



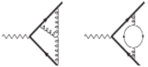
implicit IR-singularity removed by $d\sigma^{S}_{\mathrm{NNLO}}$.

- RV: One-loop correction to $S \to Q\bar{Q}g$



explicit and implicit IR-singularity removed by $d\sigma^{T}_{\mathrm{NNLO}}$

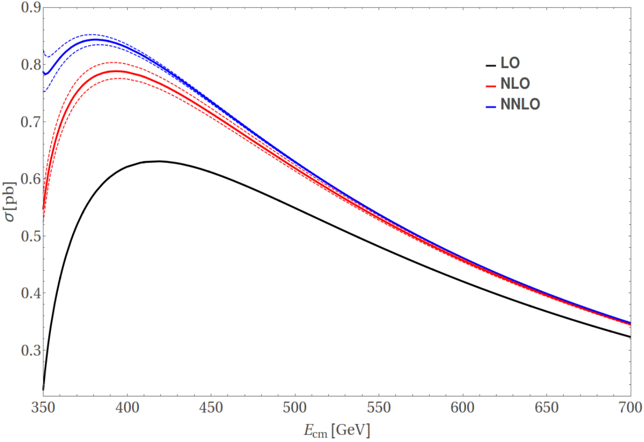
- VV: Two-loop corrections to $S \to Q\bar{Q}$



explicit IR-poles removed by $\int_{d\Phi_3} d\sigma^{T}_{\mathrm{NNLO}} + \int_{d\Phi_4} d\sigma^{S}_{\mathrm{NNLO}}$

Results: $t\bar{t}$ total cross section (above threshold)

The total cross section $\sigma\left(e^+e^- \to \gamma, Z \to t\bar{t} + X\right)$ with $m_t = 173.34$ GeV at $\mathcal{O}\left(\alpha_s^2\right)$ with scale variations indicated by dashed lines, $\mu_R = \in \left[\frac{E_{cms}}{2}, 2E_{cms}\right]$ [JHEP 1612 (098)]

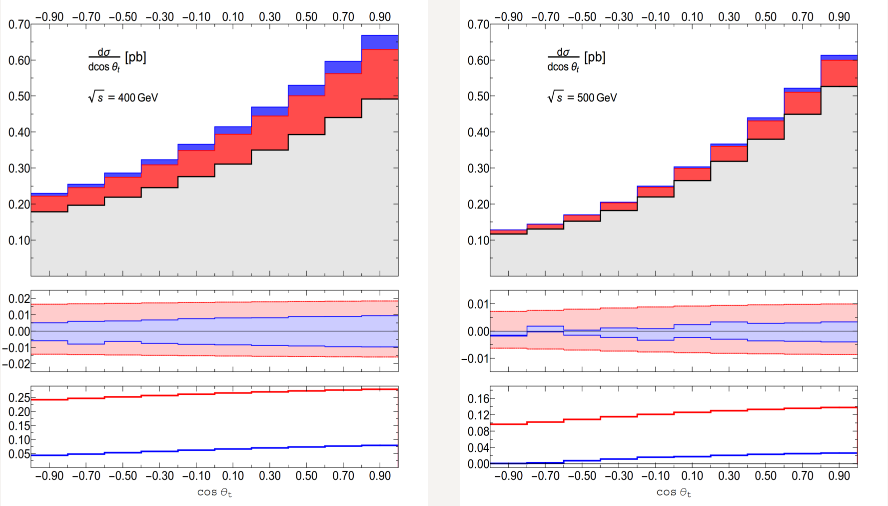


$$\sigma_{\text{NNLO}} = \sigma_{\text{LO}}\left(1 + \Delta_1 + \Delta_2\right)$$

$\sqrt{s}$ [GeV]	360	381.3	400	500
Δ_1	0.627	0.352	0.266	0.127
Δ_2	0.281	0.110	0.070	0.020

Results: The $\cos\theta_t$ distribution

$\theta_t = \angle(e^-, t)$ scattering angle between t and e^- beam (LO: black NLO: red NNLO: blue)



Results: Forward-Backward Asymmetry of t quark

$$A_{FB} \equiv \frac{N_F - N_B}{N_F + N_B} = \frac{\sigma\left(\cos\theta_t > 0\right) - \sigma\left(\cos\theta_t > 0\right)}{\sigma\left(\cos\theta_t > 0\right) + \sigma\left(\cos\theta_t > 0\right)}$$

The **unexpanded** A_{FB} of top-quark (in percent) [JHEP 1612 (098)]

$\sqrt{s}$ [GeV]	A_{FB}^{LO} [%]	A_{FB}^{NLO} [%]	A_{FB}^{NNLO} [%]
360	14.94	$15.31^{+0.02}_{-0.02}$	$15.82^{+0.08}_{-0.06}$
400	28.02	$28.77^{+0.05}_{-0.04}$	$29.42^{+0.10}_{-0.09}$
500	41.48	$42.32^{+0.06}_{-0.05}$	$42.83^{+0.08}_{-0.07}$
700	51.34	$51.78^{+0.03}_{-0.03}$	$52.03^{+0.04}_{-0.04}$

The numbers in superscript (subscript) refer to the changes if μ_R is set to $2\sqrt{s}$ ($\sqrt{s}/2$) compared to **the central value at** $\mu_R = \sqrt{s}$.

Agreement with [Gao, Zhu, PRL (2014)] was found.

Results: Forward-Backward Asymmetry of t quark

The A_{FB} of top-quark **expanded** in α_s:

$$\begin{aligned} A_{FB}^{\mathrm{NLO}} &= A_{FB}^{\mathrm{LO}}(1 + A_1) \\ A_{FB}^{\mathrm{NNLO}} &= A_{FB}^{\mathrm{LO}}(1 + A_1 + A_2) \end{aligned}$$

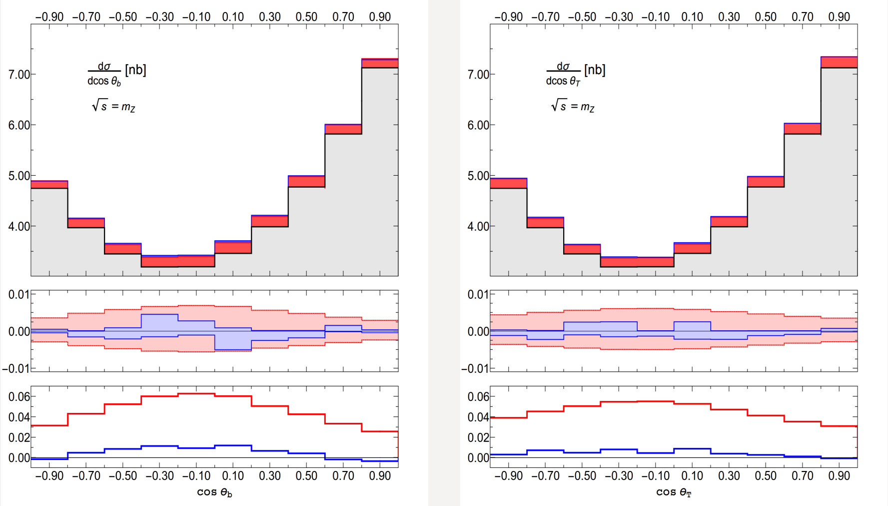
$\sqrt{s}$ [GeV]	A_{FB}^{LO} [%]	A_{FB}^{NLO} [%]	A_{FB}^{NNLO} [%]	A_1 [%]	A_2 [%]	$\delta A_{FB}^{\mathrm{NNLO}}$ [%]
360	14.94	$15.54^{+0.05}_{-0.04}$	$16.23^{+0.12}_{-0.10}$	$4.01^{+0.35}_{-0.29}$	$4.58^{+0.46}_{-0.38}$	± 0.59
400	28.02	$28.97^{+0.08}_{-0.07}$	$29.63^{+0.11}_{-0.10}$	$3.41^{+0.29}_{-0.25}$	$2.36^{+0.11}_{-0.11}$	± 0.27
500	41.48	$42.42^{+0.08}_{-0.07}$	$42.91^{+0.08}_{-0.07}$	$2.28^{+0.19}_{-0.16}$	$1.18^{+0.01}_{-0.01}$	± 0.13
700	51.34	$51.81^{+0.04}_{-0.03}$	$52.05^{+0.04}_{-0.04}$	$0.91^{+0.07}_{-0.06}$	$0.47^{+0.01}_{-0.01}$	± 0.06

$\delta A_{FB}^{\mathrm{NNLO}}$ signifies the uncertainties due to $\delta m_t = \pm 0.5$ GeV (around 173.34 GeV).

[JHEP 1612 (098)]

Results: $b\bar{b}$-production at Z-pole with $m_b \neq 0$

θ_b (θ_T): scattering angle between b-quark (thrust-axis) and e^- beam (LO: black NLO: red NNLO: blue)



Results: $A_{FB}^{0,b}$ at Z-pole for massive b-quarks

The α_s and α_s^2 QCD correction factors to LO A_{FB}^{b} at Z-pole ($\mu_R = m_z$)

	$1 + A_1$	$1 + A_1 + A_2$	A_1	A_2
thrust axis:	$0.9713^{+0.0027}_{-0.0026}$	$0.9608^{+0.0022}_{-0.0025}$	-0.0287	-0.0105

$$\begin{aligned} A_{\text{FB,exp}}^{b,T} &= [1 + A_1 + A_2] \left(A_{\text{FB}}^{0,b}\right)_{\text{exp}} \equiv \left(1 - C_{\text{QCD}}^{T}\right) \left(A_{\text{FB}}^{0,b}\right)_{\text{exp}} \\ A_{\text{FB}}^{0,b} &= \left(A_{\text{FB}}^{0,b}\right)_{\text{exp}} + \delta A_{\text{FB}}^{b} \,. \end{aligned}$$

$$\left(A_{\text{FB}}^{0,b}\right)_{\text{pre}} = 0.0992 \pm 0.0016$$ [arXiv:1012.2367]

$$\left(A_{\text{FB}}^{0,b}\right)_{\text{SM-fit}} = 0.1038 \pm 0.0007$$ [Phys. Rept. 427 ('06)]

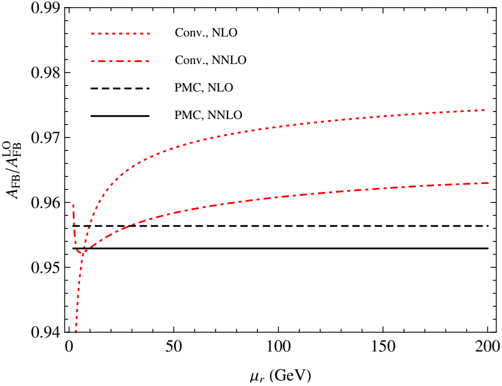
$$\left(A_{\text{FB}}^{0,b}\right)_{\text{new}} = 0.0996 \pm 0.0016$$ [JHEP 1701 (053)]

The pull between $A_{\text{FB}}^{0,b}$ and the SM fit is slightly reduced from 2.9σ to 2.6σ [JHEP 1701 (053)]

(See [d'Enterria, Yan, arXiv:2011.00530] for an updated analysis of the theoretical uncertainties)

$A_{FB}^{o,b}$ with Principle of Maximum Conformality (PMC)

Improvement by applying the PMC: resum all explicit $\ln\mu_R^2$ terms in A_{FB} that are related the QCD $\beta = \mathrm{d}\ln\alpha_s/\mathrm{d}\ln\mu_R^2$ function, absorbed into a new effective coupling $\alpha_s^{eff} = \alpha_s(\mu_r^{\mathrm{PMC}})$ [Eur. Phys. J. C80 (649)]



	μ_r	LO	NLO	NNLO	Total	$A_{FB}^{o,b}$
Conv.	M_Z	1	-0.0287	-0.0105	0.9608	0.0996
PMC	$\mu_r^{\mathrm{PMC}} = 9.7$ GeV	1	-0.0436	-0.0035	0.9529	0.1004

Discussion: $A_{FB}^{b,\text{fit}}$ v.s. $A_{FB}^{o,b}$

The tension between $A_{FB}^{b,\text{fit}}$ and $A_{FB}^{o,b}$ seems to decrease over the years...

[JHEP 1701 (053), Eur. Phys. J. C80 (649)]

$A_{FB}^{b,\text{fit}}$ v.s. $A_{FB}^{o,b}$	$A_{FB}^{o,b}$ ("massless") 0.0992	$A_{FB}^{o,b}$ (massive) 0.0996	$A_{FB}^{o,b}$ (massive+PMC) 1.004
0.1038 [Phys. Rept. 427 ('06)]	2.9 σ	2.6 σ	2.1 σ
0.1032 [Eur. Phys. J. C 74 ('14)]	2.5 σ	2.25 σ	1.8 σ
0.1030 [Phys. Rev. D 98 ('18)]	2.4 σ	2.1 σ	1.6 σ

On the other hand, the experimental uncertainties (currently, $\sim 1.6\%$), is expected to be greatly reduced at future lepton colliders!

(See [d'Enterria, Yan, arXiv:2011.00530] for an updated analysis of the theoretical uncertainties)

Summary and Outlook

- A complete fully differential NNLO QCD calculation of **massive** $Q\bar{Q}$ productions at order $\mathcal{O}\left(\alpha_s^2\right)$ in e^+e^- collisions (including γ and Z) was done using the antenna subtraction method.

- Predictions for the cross section, for the A_{FB} of the top-quark, as well as several distributions at order $\mathcal{O}\left(\alpha_s^2\right)$ in e^+e^- collisions have been made.

- Application to A_{FB} for massive b-quarks at Z pole to $\mathcal{O}\left(\alpha_s^2\right)$:
  a preliminary analysis shows that the tension between the $A_{\mathrm{FB}}^{0,b}$ and the SM fit $A_{FB}^{b,\mathrm{fit}} = 0.1038$ (0.1030) is slightly reduced from 2.9σ (2.4σ) to 2.6σ (2.1σ), and further down to 2.1σ (1.6σ) with the PMC optimization procedure.

- The calculation can be generalized straightforwardly to the cases of **polarized top quarks**, and with polarized e^+e^- beams.

Summary and Outlook

- A complete fully differential NNLO QCD calculation of **massive** $Q\bar{Q}$ productions at order $\mathcal{O}\left(\alpha_s^2\right)$ in e^+e^- collisions (including γ and Z) was done using the antenna subtraction method.
- Predictions for the cross section, for the A_{FB} of the top-quark, as well as several distributions at order $\mathcal{O}\left(\alpha_s^2\right)$ in e^+e^- collisions have been made.
- Application to A_{FB} for massive b-quarks at Z pole to $\mathcal{O}\left(\alpha_s^2\right)$: a preliminary analysis shows that the tension between the $A_{\mathrm{FB}}^{0,b}$ and the SM fit $A_{FB}^{b,\mathrm{fit}} = 0.1038$ (0.1030) is slightly reduced from 2.9σ (2.4σ) to 2.6σ (2.1σ), and further down to 2.1σ (1.6σ) with the PMC optimization procedure.
- The calculation can be generalized straightforwardly to the cases of **polarized top quarks**, and with polarized e^+e^- beams.

THANK YOU