Precision Top Mass Measurement

LCWS 2021

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in collaboration with

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Outline

1. Overview of top mass measurements
2. Boosted top quarks in the threshold region
3. Results for massive 2-jettiness at NNLO + N^3LL accuracy
4. Soft drop groomed top jet mass

What does top mass measurement entail?

Top mass is *not* a physical observable, but a Lagrangian parameter and has to be defined through a well defined theoretical prescription: *a renormalization scheme*

[Hoang AH. 2020. What is the top quark mass? Annu. Rev. Nucl. Part. Sci. 70]

Top mass measurement

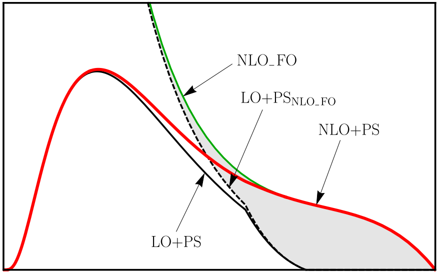
$$\sigma^{\rm exp} = \hat{\sigma}(Q, m_t^X, \alpha_s(\mu), \mu; \delta m^X) + \sigma^{\rm NP}(Q, \Lambda_{\rm QCD})$$

Mass Scheme	**Comments**
Short distance mass: $m_t^{\overline{\rm MS}}$, $m_t^{\rm PS}$, $m_t^{\rm MSR}$	Involves a nonzero counter term that compensates for the linear IR sensitivity in $\hat{\sigma}$
Pole mass, $m_t^{\rm pole}$	leaves unphysical $\mathcal{O}(\Lambda_{\rm QCD})$ corrections uncancelled
Monte Carlo Mass $m_t^{\rm MC}$	A parameter in the MC simulation that is not $m_t^{\rm pole}$ [Hoang et al. 1807.06617]

Direct vs. indirect measurements

Direct top mass meausrements

$m_t^{\mathrm{MC}} = 172.26 \pm 0.61$ GeV [CMS, 1812.10534]
$m_t^{\mathrm{MC}} = 172.69 \pm 0.48$ GeV [ATLAS, 1810.01772]
$m_t^{\mathrm{MC}} = 174.34 \pm 0.64$ GeV [Tevatron, 1407.2682]



[Figure from Hoang 2004.12915]

Total cross section measurements

$m_t^{\mathrm{pole}} = 172.9^{+2.5}_{-2.6}$ GeV [ATLAS, 1406.5375]
$m_t^{\mathrm{pole}} = 172.7^{+2.4}_{-2.7}$ GeV [CMS, 1701.06228]

- Based on inclusive or exclusive differential $t\bar{t}$ cross sections [Czakon et al. 1303.6254; Alioli et al. 1303.6415]
- Away from kinematic thresholds and dominated by normalization uncertainty and relatively weaker dependence on m_t
- Here NLO and higher order FO calculations *are* reliable

Kinematic top mass extraction is more precise but at a cost

Direct top mass meausrements

$m_t^{\mathrm{MC}} = 172.26 \pm 0.61$ GeV [CMS, 1812.10534]
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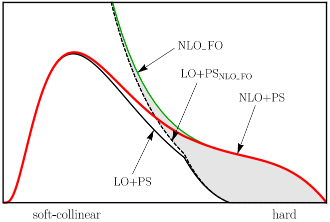


[Figure from Hoang 2004.12915]

Current MC's not adequate!

- PS not precise enough $\lesssim$ NLL
- σ^{NP} and $\hat{\sigma}$ not separately consistent with QCD
- massive PS assume boosted limit $m_t \ll p_T$
- m_t^{MC} shown to be dependent on the shower-cut Q_0 [Hoang et al. 1807.06617]

What are our goals?

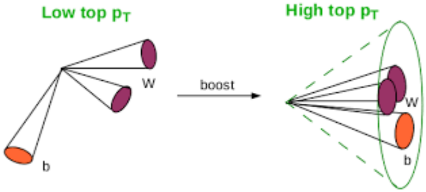
- **Direct measurements** alone leave us in darkness as to what the relation of $m_t^{\rm MC}$ to any well defined top mass scheme is. [Hoang, Plätzer, Samitz 1807.06617]
- **Alternative top mass measurements** rely on MC to perform resummation and/or correct for hadronization effects in the kinematic threshold region and hence suffer from the same problem [Lester, Summers hep-ph/9906349; CMS 1304.5783; Agashe et al 1603.03445; CMS 1608.03560]
- **Indirect measurements** are less precise than other methods that make use of kinematic information of top quark decay products

Goals of this program

- focus on observables[a] in the kinematic threshold region where resummation and hadronization effects can be described in a field theoretic framework.
- Employ these observables to calibrate $m_t^{\rm MC}$ and for direct comparison with data

[a] Threshold energy scan is a viable candidate but not the focus of this talk

Boosted top quarks in the threshold region

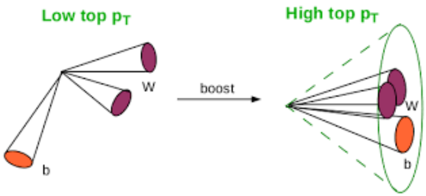


Boosted top quark jets in the threshold region

Why boosted?

- Isolate (factorize) the soft-collinear dynamics in the threshold region from the underlying hard process
- Isolate the top quarks from rest of the event activity
- Allow inclusive treatment of the collimated top-decay products

  [Fleming et al. hep-ph/0703207, 0711.2079]

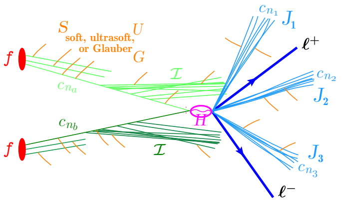


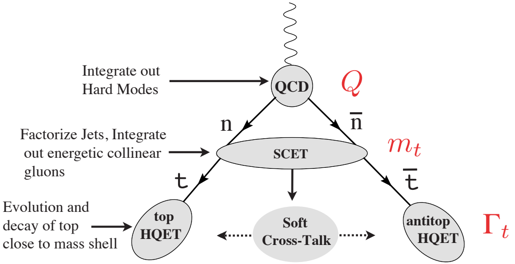
Study jet mass in the peak region

- **Jet mass** (and other variants) of boosted top jets as a benchmark observable:

$$M_t^2 = \Big(\sum_{i\in J} p_i^\mu\Big)^2, \qquad \tau_2 \equiv 1 - \max_{\hat{n}_t} \frac{\sum_i |\hat{n}_t \cdot \vec{p}_i|}{Q}$$

- **Soft collinear effective theory (SCET)** to resum logarithms in the region $M_J^2 \sim m_t^2 \ll Q^2$
- **Heavy quark effective theory (HQET)** to describe the threshold region $M_J^2 - m_t^2 \sim m_t \Gamma_t \ll m_t^2$
- **Nonperturbative corrections** accounted using the EFT framework in a field-theoretic well defined way





Factorization formula for massive 2-jettiness in e^+e^- collisions

In the threshold region 2-jettiness in boosted limit $\tau_2 - 2m_t^2/Q^2 \ll 1$ behaves as the sum of two hemisphere jet masses: [Bachu, Hoang, AP, Mateu, Stewart 2012.12304; Butenschoen et al. 1608.01318]

$$\tau_2 = \frac{M_t^2 + M_{\bar{t}}^2}{Q^2} + \mathcal{O}(\tau_2^2)$$

with the following factorization formula valid:[1] [Fleming et al. hep-ph/0703207, 0711.2079]

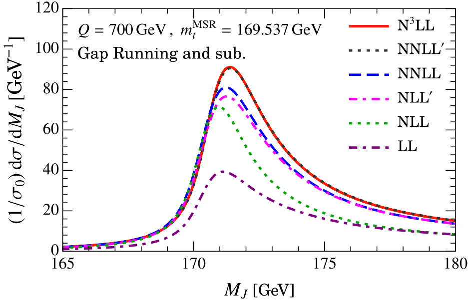
$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau_2} = m_t\, Q^2 H_{\rm evol}^{(5,6)}(Q, m_t, \varrho, \mu) \int \mathrm{d}\ell\, \mathrm{d}k\, J_{B,\tau_2}^{(5)}(\hat{s}, \Gamma_t, \delta m, \mu)\, \hat{S}_{\tau_2}^{(5)}(\ell - k, \bar{\delta}, \mu)\, F(k - 2\Delta)$$

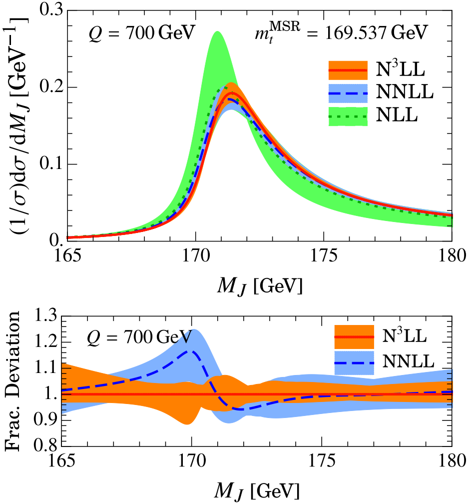
[1] Our analysis focuses on the peak region and systematically drops $\mathcal{O}(\Gamma_t/m_t)$ power corrections

Results at N^{3}LL accuracy

Results at N^{3}LL+$\mathcal{O}(\alpha_s^2)$ accuracy show good convergence and improved stability of the peak position

[Bachu, Hoang, AP, Mateu, Stewart 2012.12304]





Renormalons and power corrections

$$\sigma^{\text{exp}} = \hat{\sigma}(Q, m_t^X, \alpha_s(\mu), \mu; \delta m^X) + \sigma^{\text{NP}}(Q, \Lambda_{\text{QCD}})$$

QCD series are asymptotic and adopt divergent patterns at higher orders:

$$\hat{\sigma} = \sum_{n=0}^{\infty} r_n \alpha_s^n \quad \Rightarrow \quad B[\hat{\sigma}](u) = \sum_{n=0}^{\infty} r_n \frac{u^n}{n!} \sim \frac{\#}{n-2u} + \ldots \quad \Rightarrow \quad \sigma^{\text{NP}}(Q, \Lambda_{\text{QCD}}) \sim \left(\frac{\Lambda_{\text{QCD}}}{Q}\right)^n$$

Perturbative series	**Scaling of power corrections**
$\sigma(e^+e^- \to \gamma, Z \to t\bar{t})$	$\sigma^{\text{NP}} \sim \Lambda_{\text{QCD}}^4$
Pole-$\overline{\text{MS}}$ mass relation,	$\delta\bar{m} = m_t^{\text{pole}} - \bar{m}_t = \bar{m}_t \sum_n a_n \left(\frac{\alpha_s(\bar{m}_t)}{4\pi}\right)^n + \mathcal{O}(\Lambda_{\text{QCD}})$
Color reconnection	$\sigma^{\text{NP}} \sim \Lambda_{\text{QCD}}$

Expressing a cross section in terms of pole mass introduces unphysical linear Λ_{QCD} sensitivity!

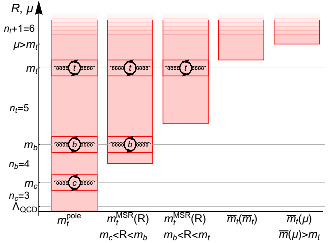
[Beneke et al. 1605.03609; Hoang et al. 1706.08526]

Renormalon in the pole mass

MSR mass defined using $\overline{\mathrm{MS}}$ coefficients

$$\delta m(R) = m_t^{\mathrm{pole}} - m_t^{\mathrm{MSR},(5)}(R) = R \sum_{i=1} \left[\frac{\alpha_s^{(5)}(R)}{4\pi} \right]^i a_i^{(n_\ell=5, n_h=0)}$$

[Hoang et al. 0803.4214, 1704.01580, 1706.08526]



$$[m_t^{\mathrm{pole}} - m_t^{\mathrm{MSR}}]|_{\beta_0/\mathrm{LL}} = \frac{a_1}{2\beta_0} R \sum_{n=0}^{\infty} \left(\frac{\beta_0 \alpha_s(R)}{2\pi} \right)^{n+1} n!$$

When using MSR mass scheme, this divergent piece cancels against *other non-self energy corrections* in the cross section. [Jain et al. 0801.0743; Fleming et al. 0711.2079; Hoang et al. 1807.06617]

$$J_{B,\tau_2}^{(5)}(\hat{s}_\tau, \Gamma_t, \delta m, \mu_B) = \int \frac{\mathrm{d}\hat{s}'}{\pi} \frac{2\Gamma_t}{(2\Gamma_t)^2 + (\hat{s}_\tau - \hat{s}')^2} J_{B,\tau_2}^{(5)}(\hat{s}' - 4\delta m, \Gamma_t = 0, \mu_B)$$

Renormalon in the soft function

The leading nonperturbative corrections occur in the soft sector

$$S_{\tau_2}(\ell, \mu_S) = \int_0^{\ell} \mathrm{d}k \, \hat{S}^{(5)}_{\tau_2}(\ell - k, \bar{\delta} = 0, \mu_S) F(k - 2\Delta)$$

OPE in the tail region:

$$S_{\tau_2}(\ell \gg \Lambda_{\mathrm{QCD}}, \mu_S) = \hat{S}^{(5)}_{\tau_2}(\ell, \bar{\delta} = 0, \mu_S) - 2\overline{\Omega}_1 \, \hat{S}^{(5)\prime}_{\tau_2}(\ell, \bar{\delta} = 0, \mu_S) + \ldots$$

Partonic Soft function and $\overline{\Omega}_1$ have $\mathcal{O}(\Lambda_{\mathrm{QCD}})$ renormalon in $\overline{\mathrm{MS}}$ scheme: [hep-ph/0003179; 0709.3519]

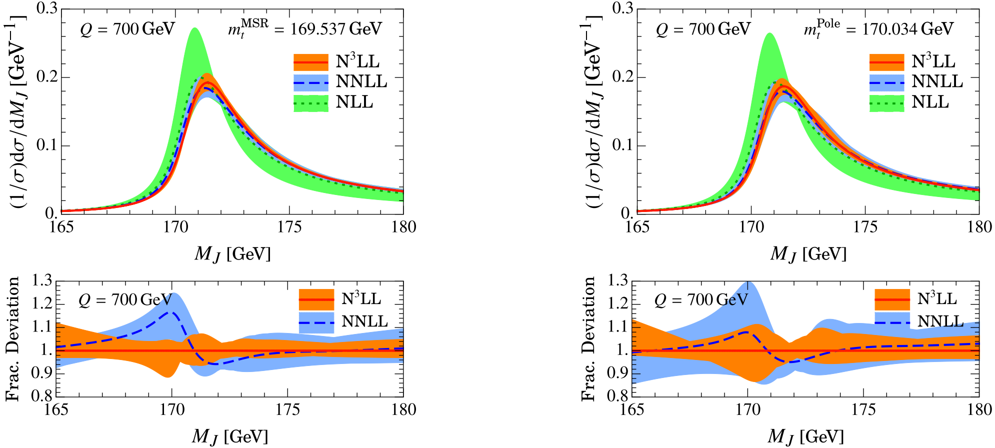
$$B\left[\hat{S}^{(5)}_{\tau_2}(\ell, \mu)\right]\left(u \simeq \frac{1}{2}\right) = \mu \frac{16 C_F e^{-5/6}}{\pi \beta_0 \left(u - \frac{1}{2}\right)} \frac{\partial}{\partial \ell} S^{(5)}_{\tau_2}(\ell, \mu)$$

Cure this renormalon by adopting a new scheme

$$\Omega_1(R_s) = \overline{\Omega}_1 - \bar{\delta}(R_s)\,, \qquad \bar{\delta}(R_s) \equiv \frac{R_s}{2} \log\left[\tilde{S}^{(5)}_{\tau_2}\left(\frac{1}{R_s}, \bar{\delta} = 0, R_s\right)\right], \qquad \overline{\Delta}(R_s) = \Delta - \bar{\delta}(R_s)$$

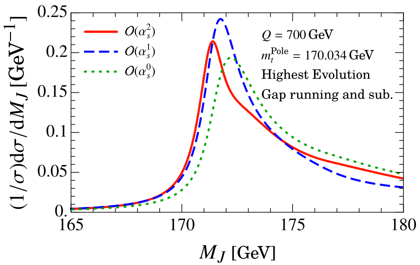
A rather surprising result

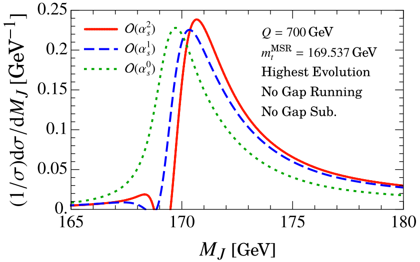
A first comparison of fully gap+mass subtracted result shows comparable uncertainties as the unsubtracted cross section



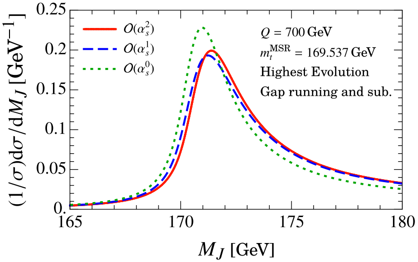
Does that mean renormalon subtractions play no significant role?

Behind the scenes cancellation of two renormalon effects





The two renormalons go in opposite directions and the unsubtracted result involves partial cancellation



Peak position and uncertainties

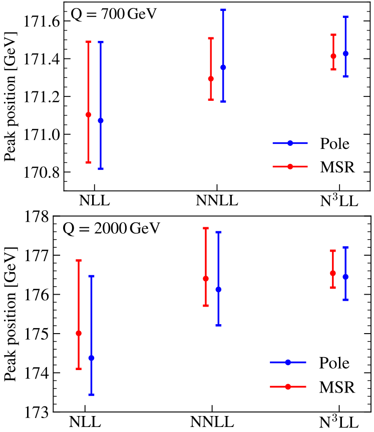
Peak position directly sensitive to the top mass.

[Bachu, Hoang, AP, Mateu, Stewart 2012.12304]

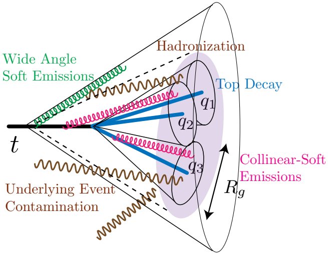
Uncertainties at $Q = 700$ GeV

- MSR + gap subtracted result: $\pm$ 85 MeV
- Pole mass + no gap sub: $\pm$ 150 MeV

- MSR + gap subtracted results are more precise.
- Shows that we need higher orders to estimate pole mass below the ambiguity $\sim$ 160 MeV.



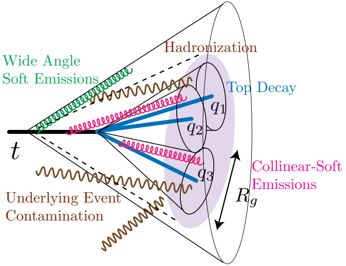
Groomed top jets at the LHC



Grooming boosted top quark jets

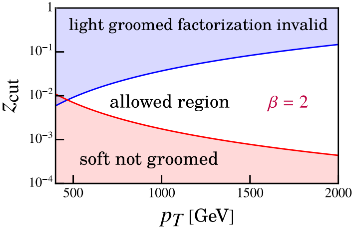
Soft drop criteria: [Larkoski et al. 1402.2657]

$$\frac{\min(p_{T_i}, p_{T_j})}{p_{T_i} + p_{T_j}} > z_{\rm cut} \left(\frac{\Delta R_{ij}}{R_0}\right)^{\beta}$$



Jet grooming for fat top jets

- Reduces effects of underlying event, pile up and hadronization
- *Decouples* the jet from the rest of the event
- Complicates considerations of the inclusive treatment
  [Hoang, Mantry, AP, Stewart 1708.02586]



We aim for a direct comparison with data.

[Aparisi Pozo, Hoang, Leblanc, Mantry, AP, Roloff, Stewart, Vos, ATLAS (in progress)]

Factorization for groomed top quark jets

Factorization at NLL

[Hoang, Mantry, AP, Stewart 1708.02586]

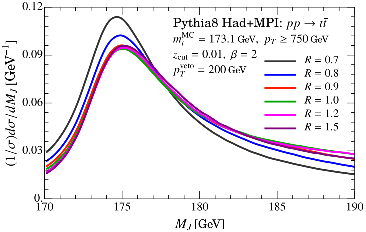
$$\frac{d\sigma^{\rm NLL}(\Phi_J)}{dM_J} = N(\Phi_J, z_{\rm cut}, \beta, \mu)\int d\tilde{h}\; P\Big(\tilde{h}, \frac{m_t}{Q}\Big)\int d\ell^+\, J_B\Big(\hat{s}_t - \frac{Q\ell^+}{m_t}, \delta m, \Gamma_t, \mu\Big)$$
$$\times \int dk^+\, S_c^q\bigg[\Big(\ell^+ - \max\Big\{C_1^{q(pp)}(m_t\hat{s}_t), \frac{m_t\tilde{h}}{Q}\Big\}k^+\Big)Q_{\rm cut}^{\frac{1}{1+\beta}}, \beta, \mu\bigg]$$
$$\times F_{\otimes}^q(k^+)\bigg\{1 - \Theta\Big(C_1^{q(pp)}(m_t\hat{s}_t) - \frac{m_t\tilde{h}}{Q}\Big)\frac{Qk^+}{m_t}\frac{dC_1^{q(pp)}(m_t\hat{s}_t)}{d\hat{s}_t}\bigg\}$$

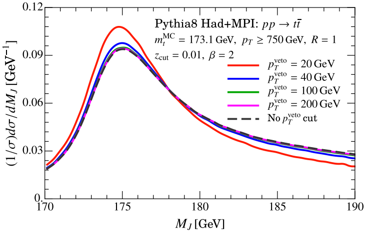
Work in progress on including NLL + $\mathcal{O}(\alpha_s)$ corrections

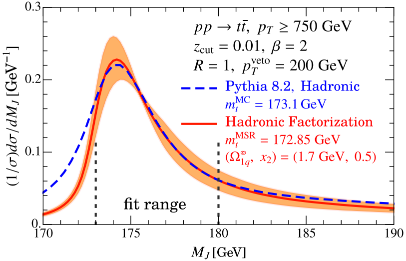
[Hoang, AP, Mantry, Michel, Stewart]

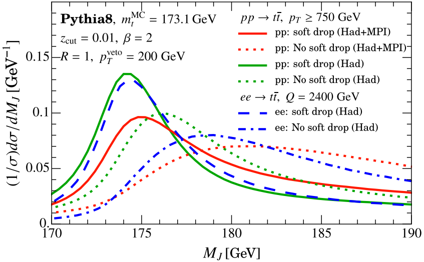
Soft drop jet mass is robust and enables analytical calculations

[Hoang, Mantry, AP, Stewart 1708.02586]







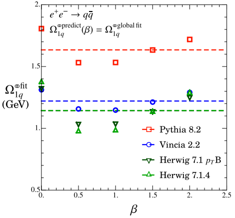


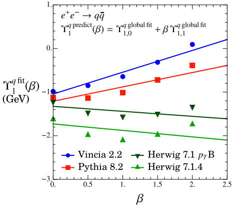
Field theoretic treatment of nonperturbative corrections

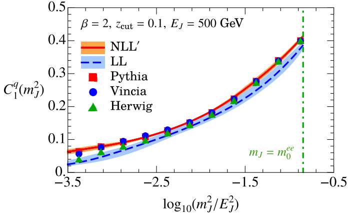
[Hoang, AP, Mantry, Stewart 1906.11843; AP, Stewart, Vaidya, Zoppi 2012.15568]

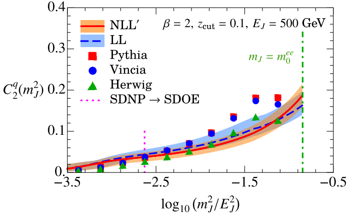
NP effects in soft drop jet mass

$$\frac{d\sigma_\kappa^{\rm had}}{dm_J^2\, d\Phi_J} = \frac{d\hat\sigma^\kappa}{dm_J^2\, d\Phi_J} - Q\,\Omega_{1\kappa}^{\circledcirc}\,\frac{d}{dm_J^2}\left(C_1^\kappa(m_J^2, Q, z_{\rm cut}, \beta, R)\,\frac{d\hat\sigma^\kappa}{dm_J^2\, d\Phi_J}\right)$$
$$+ \frac{Q\left(\Upsilon_{1,0}^\kappa + \beta\,\Upsilon_{1,1}^\kappa\right)}{m_J^2}\, C_2^\kappa(m_J^2, Q, z_{\rm cut}, \beta, R)\,\frac{d\hat\sigma^\kappa}{dm_J^2\, d\Phi_J}$$









Underlying Event can also be accounted for

[J. Aparisi Pozo, AP, M. Vos]

Hadronization corrections

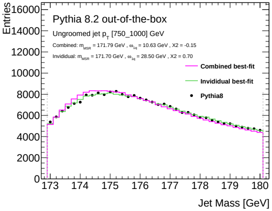
$$\frac{d\sigma_\kappa^{\rm had}}{dm_J^2} = \frac{d\hat{\sigma}_\kappa}{dm_J^2} - Q\,\Omega_{1\kappa}^{\infty}\,\frac{d}{dm_J^2}\left(C_1^\kappa(m_J^2)\,\frac{d\hat{\sigma}_\kappa}{dm_J^2}\right)$$

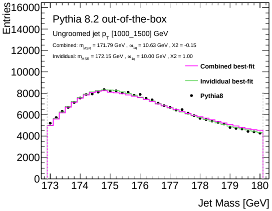
$$C_1^\kappa(m_J^2) \sim \langle \theta_g(m_J^2)/2 \rangle$$

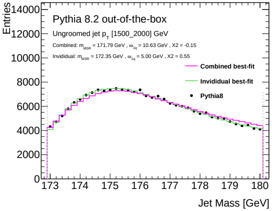
Underlying event contribution

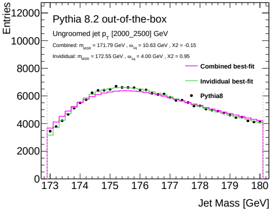
$$\frac{d\Delta\sigma^{\rm UE}}{dm_J^2} = \frac{-Q\Omega_1^{\infty\,\rm UE}}{m_J^2}\,\frac{d}{dm_J^2}\left(C_1^{\kappa(2)}(m_J^2)\,\frac{d\hat{\sigma}_\kappa}{dm_J^2}\right)$$

$$C_1^{\kappa(2)}(m_J^2) \sim \langle \theta_g^2(m_J^2)/4 \rangle$$









Conclusions

1. Direct measurements and other measurements using top kinematic threshold structures rely on MC for resummation and hadronization, leading to top mass interpretation problem.
2. Using EFT's in boosted limit allow for accounting for subleading QCD quantum corrections in the threshold region.
3. Our N^3LL study in the peak region of the 2-jettiness distribution underscores importance of using short distance mass scheme and renormalon subtractions.
4. A field theory based treatment of groomed top jet cross section enables direct comparison with data and a kinematic extraction of the top mass in a well defined scheme.

Thank you