

Tests of EFT positivity constraints at future colliders

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(\Rightarrow Fudan U. this Fall)

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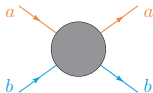
[arXiv:2011.03055] JG, Lian-Tao Wang, Cen Zhang

Introduction

- ▶ **Can all EFTs be UV completed?**
- ▶ Dispersion relations of forward elastic amplitudes suggest that certain operator coefficients can only be positive.
 - ▶ Assuming the UV physics is consistent with the fundamental principles of QFT (analyticity, locality, unitarity, Lorentz invariance).
 - ▶ [hep-th/0602178] Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi, ... many papers...
[1902.08977, 2005.03047] Zhang, Zhou et al.,
[1908.09845, 2004.02885] Remmen, Rodd, ...

- ▶ **These positivity bounds only exist for certain Dimension-8 (or higher) operators!**

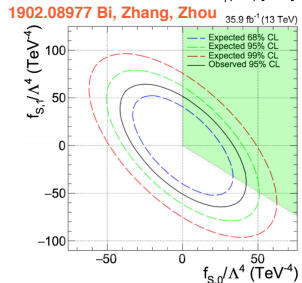
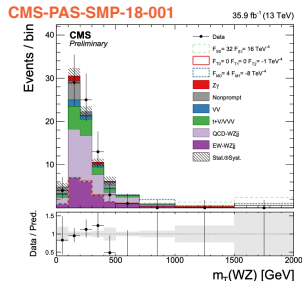
$$\frac{d^2}{ds^2} \mathcal{A}(ab \rightarrow ab)_{t \rightarrow 0} |_{s=0} \geq 0.$$



- ▶ By measuring these dim-8 operator coefficients, we can test whether the underlying new physics is consistent with the fundamental principles of QFT.
- ▶ **Can we do it?**

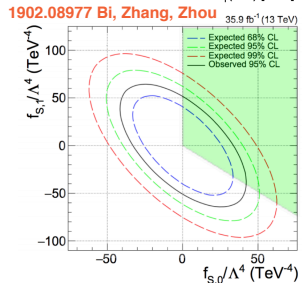
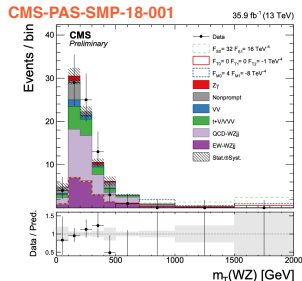
Probing positivity bounds on dimension-8 operators

- ▶ The dimension-8 contribution has a large energy enhancement ($\sim E^4/\Lambda^4$)!
- ▶ It is difficult for LHC to probe these bounds.
 - ▶ Low statistics in the high energy bins.
 - ▶ Example: Vector boson scattering.
 - ▶ $\Lambda \lesssim \sqrt{s}$, the EFT expansion breaks down!

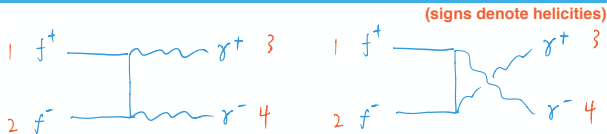


Probing positivity bounds on dimension-8 operators

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 - ▶ Example: Vector boson scattering.
 - ▶ $\Lambda \lesssim \sqrt{s}$, the EFT expansion breaks down!
- ▶ Can we separate the dim-8 and dim-6 effects?
 - ▶ Precision measurements at several different \sqrt{s} ?
(A **very** high energy lepton collider?)
 - ▶ Or find some special process where dim-8 gives the leading new physics contribution?



The diphoton channel [arXiv:2011.03055] JG, Lian-Tao Wang, Cen Zhang



- ▶ $e^+e^- \rightarrow \gamma\gamma$ (or $\mu^+\mu^- \rightarrow \gamma\gamma$), SM, non-resonant.
 - ▶ Tree level SM: the only helicity configuration is $\mathcal{A}(f^+f^-\gamma^+\gamma^-)$.

- ▶ Leading order contribution: **dimension-8 contact interaction**.
 $(f^+f^- \rightarrow \bar{e}_L e_L \text{ or } e_R \bar{e}_R)$

$$\mathcal{A}(f^+f^-\gamma^+\gamma^-)_{\text{SM+d8}} = 2e^2 \frac{\langle 24 \rangle^2}{\langle 13 \rangle \langle 23 \rangle} + \frac{a}{v^4} [13][23] \langle 24 \rangle^2.$$

- ▶ Operators: Also contributes to ZZ/Z γ final states with opposite helicities.

[1806.09640] Bellazzini, Riva, see also d8 basis in

[2005.00008] Shu et al., [2005.00059] Murphy

$$a_L = \frac{v^4}{\Lambda^4} \left(\cos^2 \theta_W c_{\ell B}^{(8)} - \cos \theta_W \sin \theta_W c_{\ell BW}^{(8)} + \sin^2 \theta_W c_{\ell W}^{(8)} \right),$$

$$a_R = \frac{v^4}{\Lambda^4} \left(\cos^2 \theta_W c_{eB}^{(8)} + \sin^2 \theta_W c_{eW}^{(8)} \right),$$

$$\mathcal{O}_{\ell B}^{(8)} = -\frac{1}{4} (i \bar{\ell}_L \gamma^{\{\rho} D^{\nu\}} \ell_L + \text{h.c.}) B_{\mu\nu} B_{\rho}^{\mu},$$

$$\mathcal{O}_{eB}^{(8)} = -\frac{1}{4} (i \bar{e}_R \gamma^{\{\rho} D^{\nu\}} e_R + \text{h.c.}) B_{\mu\nu} B_{\rho}^{\mu},$$

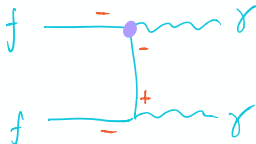
$$\mathcal{O}_{\ell W}^{(8)} = -\frac{1}{4} (i \bar{\ell}_L \gamma^{\{\rho} D^{\nu\}} \ell_L + \text{h.c.}) W_{\mu\nu}^a W_{\rho}^{a\mu},$$

$$\mathcal{O}_{eW}^{(8)} = -\frac{1}{4} (i \bar{e}_R \gamma^{\{\rho} D^{\nu\}} e_R + \text{h.c.}) W_{\mu\nu}^a W_{\rho}^{a\mu},$$

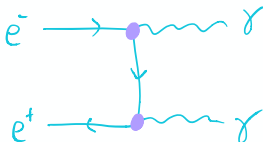
$$\mathcal{O}_{\ell BW}^{(8)} = -\frac{1}{4} (i \bar{\ell}_L \sigma^a \gamma^{\{\rho} D^{\nu\}} \ell_L + \text{h.c.}) B_{\mu\nu} W_{\rho}^{a\mu},$$

All other contributions are either vanishing or suppressed!

- ▶ The only tree-level $d6$ contribution are from dipole operators and have different fermion helicities.



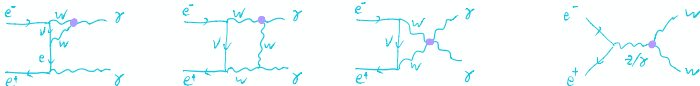
- ▶ $SM \times d6$ at tree level: no interference.
- ▶ $d6^2$: Dipole operators are very well constrained by $g - 2$ and EDM measurements.



All other contributions are either vanishing or suppressed!

- ▶ **SM \times d6 at 1-loop:** are either very-well constrained by other measurements with tree-level contributions, or forbidden by selection rules.

- ▶ O_{3W} is very well constrained by $e^+e^- \rightarrow WW$ measurements.



- ▶ Other contributions are constrained by Z-pole measurements or suppressed by the small y_e .
- ▶ Contribution from the $eetf$ 4f operator is forbidden by angular momentum selection rules. ([2001.04481] Shu et al.)

$$= 0 \quad (\text{for } e^+e^- \text{ with opposite helicities})$$

- ▶ **other d8:** They have different helicities and do not interfere with SM.

Positivity bounds

- ▶ Leading BSM contribution:

$$\mathcal{A}(\bar{e}_L e_L \gamma^+ \gamma^-)_{d8} = \frac{a_L}{v^4} [13][23] \langle 24 \rangle^2, \quad \mathcal{A}(e_R \bar{e}_R \gamma^+ \gamma^-)_{d8} = \frac{a_R}{v^4} [13][23] \langle 24 \rangle^2.$$

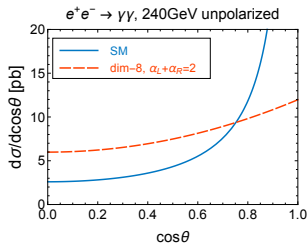
- ▶ Positivity bounds are obtained from the forward elastic amplitude $e\gamma \rightarrow e\gamma$:

$$\frac{d^2}{ds^2} \mathcal{A}(e\gamma \rightarrow e\gamma)|_{t \rightarrow 0} \geq 0,$$

- ▶ which implies

$$a_L \geq 0, \quad a_R \geq 0.$$

The diphoton cross section



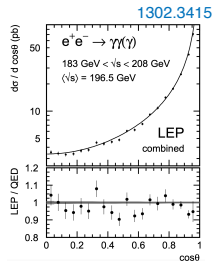
- **Differential cross section** (The production polar angle θ is “folded” since the photon polarizations are not measured.)

$$\begin{aligned} & \frac{d\sigma(e^+e^- \rightarrow \gamma\gamma)}{d\cos\theta} \\ &= \frac{(1 - P_{e^-})(1 + P_{e^+})}{4} \frac{e^4}{4\pi s} \left(\frac{1 + c_\theta^2}{1 - c_\theta^2} + a_L \frac{s^2(1 + c_\theta^2)}{4e^2v^4} \right) \\ &+ \frac{(1 + P_{e^-})(1 - P_{e^+})}{4} \frac{e^4}{4\pi s} \left(\frac{1 + c_\theta^2}{1 - c_\theta^2} + a_R \frac{s^2(1 + c_\theta^2)}{4e^2v^4} \right), \end{aligned}$$

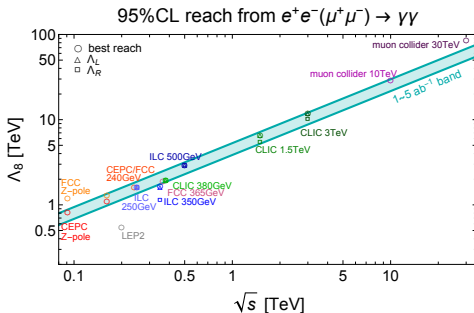
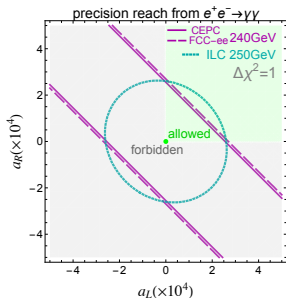
- **Positivity bounds:** $a_L \geq 0$, $a_R \geq 0$.
- **Positivity bound directly on the cross section!**

$$\sigma(e^+e^- \rightarrow \gamma\gamma) \geq \sigma_{\text{SM}}(e^+e^- \rightarrow \gamma\gamma).$$

- The LEP measurement was $\sim 1.5\sigma$ below the SM prediction.



Future projections



- ▶ χ^2 fit to the binned distribution
 - ▶ Statistics only, 19 bins in $\cos\theta \in [0, 0.95]$.
 - ▶ Agrees reasonably well with LEP result ($\lesssim 10\%$ in the reach on Λ).
- ▶ Beam polarization: **Good for testing positivity!**
 - ▶ One could measure σ_L and σ_R simultaneously.
- ▶ High energy still wins!

$$\frac{\Lambda_2}{\Lambda_1} = \left(\frac{E_2}{E_1}\right)^{\frac{3}{4}} \left(\frac{L_2}{L_1}\right)^{\frac{1}{8}}.$$

Combined $\gamma\gamma/Z\gamma/ZZ$ analysis at high energy

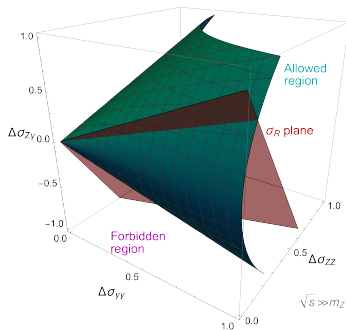
- ▶ $Z\gamma$, ZZ processes are more complicated due to the massive Z .
 - ▶ Other helicity states contribute in both SM and BSM (e.g. nTGCs).
 - ▶ In the high energy limit, $\mathcal{A}(f^+ f^- V^+ V^-)$ dominates in SM.
- ▶ In the $\sqrt{s} \gg m_Z$ limit,

$$\sigma(e^+ e^- \rightarrow ZZ) \geq \sigma_{\text{SM}}(e^+ e^- \rightarrow ZZ).$$

- ▶ Consider the elastic amplitude of $eV \rightarrow eV$,
 - ▶ V is an arbitrary mixing state of γ and Z ,
 - ▶ scan over the mixing angle to obtain the strongest bound ($\Delta\sigma \equiv \sigma - \sigma_{\text{SM}}$),

$$(\Delta\sigma_{Z\gamma})^2 \leq 4\Delta\sigma_{\gamma\gamma}\Delta\sigma_{ZZ}.$$

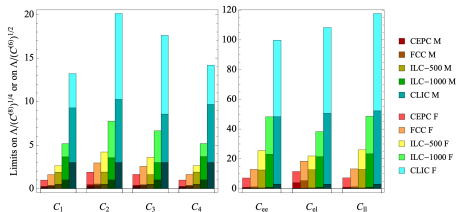
- ▶ σ_R s only occupy a plane in the 3d parameter space.
 - ▶ 3 operators with ℓ_L , 2 operators with e_R .



- ▶ Many operators, many positivity bounds...

$$\begin{aligned}
 O_{ee} &= (\bar{e}\gamma^\mu e) (\bar{e}\gamma_\mu e), & O_1 &= \partial^\alpha (\bar{e}\gamma^\mu e) \partial_\alpha (\bar{e}\gamma_\mu e), & C_1 &\leq 0, \\
 O_{el} &= (\bar{e}\gamma^\mu e) (\bar{l}\gamma_\mu l), & O_2 &= \partial^\alpha (\bar{e}\gamma^\mu e) \partial_\alpha (\bar{l}\gamma_\mu l), & C_4 + C_5 &\leq 0, \\
 O_{ll} &= (\bar{l}\gamma^\mu l) (\bar{l}\gamma_\mu l), & O_3 &= D^\alpha (\bar{e}l) D_\alpha (\bar{l}e), & C_5 &\leq 0, \\
 & & O_4 &= \partial^\alpha (\bar{l}\gamma^\mu l) \partial_\alpha (\bar{l}\gamma_\mu l), & C_3 &\geq 0, \\
 & & O_5 &= D^\alpha (\bar{l}\gamma^\mu \tau^I l) D_\alpha (\bar{l}\gamma_\mu \tau^I l), & 2\sqrt{C_1(C_4 + C_5)} &\geq C_2, \\
 & & & & 2\sqrt{C_1(C_4 + C_5)} &\geq -(C_2 + C_3).
 \end{aligned}$$

- ▶ Multiple runs with different energies & beam polarizations are very useful! Angular distributions also help.



- ▶ A high energy lepton collider can do more tests on positivity!

What if positivity bound is violated?

- ▶ Statistical fluctuation, systematic error, ...
 - ▶ Even 5σ can go away in the diphoton channel.
- ▶ EFT is not valid?
 - ▶ An s -channel light ($m \lesssim \sqrt{s}$) spin-2 particle?
 - ▶ Very well probed by resonance searches $e^+e^- \rightarrow X\gamma/XZ$, $X \rightarrow \gamma\gamma/e^+e^-$.
(see e.g. ILC 750 GeV study [1607.03829])
 - ▶ By measuring $e^+e^- \rightarrow \gamma\gamma$ at several energies (e.g. 250 and 500 GeV) we can check whether the deviation comes from d8 operators ($\sim s^2$) or something else.
- ▶ Can QFT really break down at the TeV scale?
 - ▶ Example from history: Nobody expected classical physics to break down, nobody expected parity to be violated, ...

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 - ▶ Example from history: Nobody expected classical physics to break down, nobody expected parity to be violated, ...



- ▶ **It's important to do the experiment!**

Conclusion

- ▶ Measurements of the diphoton process at lepton colliders ($e^+e^- \rightarrow \gamma\gamma$ or $\mu^+\mu^- \rightarrow \gamma\gamma$) offer a unique opportunity to directly probe dimension-8 operators and their positivity bounds.
- ▶ We can do it at a lepton collider with $\sqrt{s} \sim 250$ GeV (but higher energy is always better).
 - ▶ **Build a Higgs factory, get a positivity test for free!**

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“When you have excluded the impossible, whatever remains, however improbable, must be the truth.”

– Sherlock Holmes

backup slides

Dispersion relations

- ▶ Consider a forward ($t \rightarrow 0$) elastic amplitude
($s + t + u = 4m^2$)

$$\tilde{\mathcal{A}}_{ab}(s) = \sum_n c_n (s - \mu^2)^n,$$

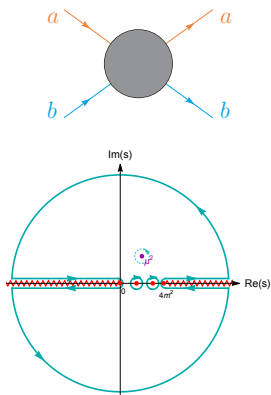
$$c_n = \frac{1}{2\pi i} \oint_{s=\mu^2} ds \frac{\tilde{\mathcal{A}}_{ab}(s)}{(s - \mu^2)^{n+1}},$$

- ▶ Applying the fundamental principles of QFT

- ▶ Analyticity (Cauchy's theorem applies)
- ▶ Locality (poles from tree-level factorization, branch cuts from loops, Froissart Bound)
- ▶ Unitarity (Optical theorem, $\text{Im}\mathcal{A} \sim \sigma_{\text{tot}}$)
- ▶ Lorentz invariance (Crossing symmetry)

- ▶ Dispersion relation tells us that

$$c_n = \int_{4m^2}^{\infty} \frac{ds}{\pi} s \sqrt{1 - \frac{4m^2}{s}} \left(\frac{\sigma_{\text{tot}}^{ab}}{(s - \mu^2)^{n+1}} + (-1)^n \frac{\sigma_{\text{tot}}^{a\bar{b}}}{(s - 4m^2 + \mu^2)^{n+1}} \right) + c_n^{\infty},$$



Sum rules and positivity bounds

- ▶ Sum rule:

$$c_n = \int_{4m^2}^{\infty} \frac{ds}{\pi} s \sqrt{1 - \frac{4m^2}{s}} \left(\frac{\sigma_{\text{tot}}^{ab}}{(s - \mu^2)^{n+1}} + (-1)^n \frac{\sigma_{\text{tot}}^{a\bar{b}}}{(s - 4m^2 + \mu^2)^{n+1}} \right) + c_n^{\infty},$$

- ▶ Froissart bound: $\mathcal{A} < \text{const} \cdot s \log^2 s \Rightarrow c_n^{\infty} = 0$ for $n > 1$.
- ▶ For even n , the two terms with cross sections are both positive, so $c_n > 0$.
- ▶ Consider the limit $m^2 \ll \mu^2 \ll \Lambda^2$ (massless SMEFT).

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{c_i^{(6)}}{\Lambda^2} \mathcal{O}_i^{(6)} + \sum_j \frac{c_j^{(8)}}{\Lambda^4} \mathcal{O}_j^{(8)} + \dots$$

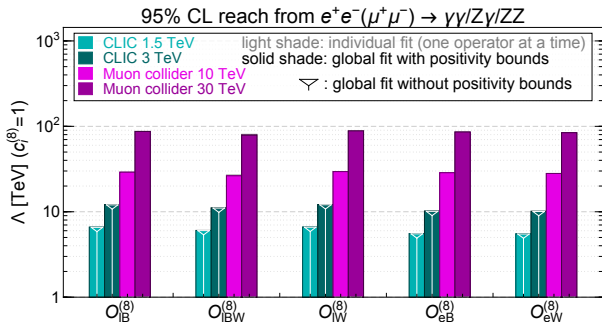
$$\mathcal{A}_4 = g_{[0]} \mathcal{A}_4^{[0]} + g_{[-2]} \mathcal{A}_4^{[2]} + g_{[-4]} \mathcal{A}_4^{[4]} + \dots$$

- ▶ $c_{n=1} \Leftrightarrow$ dimension-6 (no positivity bounds, boundary can be nonzero),
- ▶ $c_{n=2} \Leftrightarrow$ dimension-8 (or $d6^2$) (has positivity bounds),
- ▶ ...
- ▶ See e.g. [2011.00037] Bellazzini, Miró, Rattazzi, Riembau, Riva, [2012.15849] Arkani-Hamed, Huang, Huang for more general positivity bounds also for non-forward amplitudes.

Collider scenarios

$\int \mathcal{L} dt$ [ab^{-1}]				
unpolarized	91 GeV	161 GeV	240 GeV	365 GeV
CEPC	8	2.6	5.6	
FCC-ee	150	10	5	1.5
ILC	250 GeV	350 GeV	500 GeV	
(-0.8, +0.3)	0.9	0.135	1.6	
(+0.8, -0.3)	0.9	0.045	1.6	
($\pm 0.8, \pm 0.3$)	0.1	0.01	0.4	
CLIC	380 GeV	1.5 TeV	3 TeV	
(-0.8, 0)	0.5	2	4	
(+0.8, 0)	0.5	0.5	1	
muon collider	10 TeV	30 TeV		
unpolarized	10	90		

Combined $\gamma\gamma/Z\gamma/ZZ$ analysis at high energy



- ▶ Global fit of the $\gamma\gamma/Z\gamma/ZZ$ processes in the high energy limit.
- ▶ No beam polarizations \Rightarrow flat directions.
- ▶ Flat directions are lifted once the positivity bounds are imposed!

Sum rules for dimension-6 operators [arXiv:2008.07551] JG, L.-T. Wang

- ▶ Use helicity amplitudes to classify the sum rules.

elastic 4-point amplitudes $\mathcal{A}(12 \rightarrow 3_{=1} 4_{=2})$	spinor form of $\mathcal{A}_4^{[2]}$ (d6 operators)	spinor form of $\mathcal{A}_4^{[4]}$ (d8 or d6 ²)
$\phi_1 \phi_2 \phi_1^* \phi_2^*$	s_{ij}	$s_{ij} \times s_{kl}$
$\psi^- \phi \psi^+ \phi^*$	$\langle 12 \rangle [23]$	$\langle 12 \rangle [23] \times s_{ij}$
$\psi_1^- \psi_2^- \psi_1^+ \psi_2^+$	$\langle 12 \rangle [34]$	$\langle 12 \rangle [34] \times s_{ij}$
$V^- \phi V^+ \phi^*$	$\boldsymbol{\chi}$	$\langle 12 \rangle^2 [23]^2$
$V^- \psi^- V^+ \psi^+$	$\boldsymbol{\chi}$	$\langle 12 \rangle^2 [23] [34]$
$V_1^- V_2^- V_1^+ V_2^+$	$\boldsymbol{\chi}$	$\langle 12 \rangle^2 [34]^2, \langle 12 \rangle^2 [34]^2 \frac{t-u}{s}$

- ▶ Tree level dimension-6: only scalar-scalar, scalar-fermion and fermion-fermion amplitudes!
- ▶ Forward limit:

$$\tilde{\mathcal{A}}_4^{[2]} \equiv \mathcal{A}_4^{[2]}|_{t \rightarrow 0} \propto \mathbf{s}, \quad \tilde{\mathcal{A}}_4^{[4]} \equiv \mathcal{A}_4^{[4]}|_{t \rightarrow 0} \propto \mathbf{s}^2.$$

Sum rules

► scalar-scalar

$$\frac{c_H + 3c_T}{\Lambda^2} = \frac{d\bar{\mathcal{A}}_{\phi^+\phi^-}}{ds} \Big|_{s=0} = \int_0^\infty \frac{ds}{\pi s} \left(\sigma_{\text{tot}}^{\phi^+\phi^-} - \sigma_{\text{tot}}^{\phi^+\phi^+} \right) + c_\infty,$$

$$-\frac{2c_T}{\Lambda^2} = \frac{d\bar{\mathcal{A}}_{\phi^+\phi^0}}{ds} \Big|_{s=0} = \int_0^\infty \frac{ds}{\pi s} \left(\sigma_{\text{tot}}^{\phi^+\phi^0} - \sigma_{\text{tot}}^{\phi^+\phi^{0*}} \right) + c_\infty,$$

► fermion-fermion

only showing $\frac{c_{ee}}{\Lambda^2} (\bar{\theta}_R \gamma_\mu \theta_R) (\bar{\theta}_R \gamma^\mu \theta_R)$,

20 in total for 1 generation

$$-\frac{2c_{ee}}{\Lambda^2} = \frac{d\bar{\mathcal{A}}_{e_R \bar{e}_R}}{ds} \Big|_{s=0} = \int_0^\infty \frac{ds}{\pi s} \left(\sigma_{\text{tot}}^{e_R \bar{e}_R} - \sigma_{\text{tot}}^{e_R e_R} \right) + c_\infty.$$

$\mathcal{O}_H = \frac{1}{2}(\partial_\mu H ^2)^2$	$\mathcal{O}_T = \frac{1}{2}(H^\dagger \overleftrightarrow{D}_\mu H)^2$
$\mathcal{O}_{H\ell} = iH^\dagger \overleftrightarrow{D}_\mu H \bar{\ell}_L \gamma^\mu \ell_L$	$\mathcal{O}_{He} = iH^\dagger \overleftrightarrow{D}_\mu H \bar{e}_R \gamma^\mu e_R$
$\mathcal{O}'_{H\ell} = iH^\dagger \sigma^a \overleftrightarrow{D}_\mu H \bar{\ell}_L \sigma^a \gamma^\mu \ell_L$	$\mathcal{O}'_{He} = iH^\dagger \overleftrightarrow{D}_\mu H \bar{e}_R \sigma^a \gamma^\mu e_R$
$\mathcal{O}_{Hq} = iH^\dagger \overleftrightarrow{D}_\mu H \bar{q}_L \gamma^\mu q_L$	$\mathcal{O}_{Hu} = iH^\dagger \overleftrightarrow{D}_\mu H \bar{u}_R \gamma^\mu u_R$
$\mathcal{O}'_{Hq} = iH^\dagger \sigma^a \overleftrightarrow{D}_\mu H \bar{q}_L \sigma^a \gamma^\mu q_L$	$\mathcal{O}'_{Hd} = iH^\dagger \overleftrightarrow{D}_\mu H \bar{d}_R \gamma^\mu d_R$

► scalar-fermion

(also the same for leptons)

$$\frac{2(c_{Hq} - c'_{Hq})}{\Lambda^2} = \frac{d\bar{\mathcal{A}}_{u_L \phi^0}}{ds} \Big|_{s=0} = \int_0^\infty \frac{ds}{\pi s} \left(\sigma_{\text{tot}}^{u_L \phi^0} - \sigma_{\text{tot}}^{u_L \phi^{0*}} \right) + c_\infty,$$

$$\frac{2c_{Hu}}{\Lambda^2} = \frac{d\bar{\mathcal{A}}_{u_R \phi^0}}{ds} \Big|_{s=0} = \int_0^\infty \frac{ds}{\pi s} \left(\sigma_{\text{tot}}^{u_R \phi^0} - \sigma_{\text{tot}}^{u_R \phi^{0*}} \right) + c_\infty,$$

$$\frac{2(c_{Hq} + c'_{Hq})}{\Lambda^2} = \frac{d\bar{\mathcal{A}}_{d_L \phi^0}}{ds} \Big|_{s=0} = \int_0^\infty \frac{ds}{\pi s} \left(\sigma_{\text{tot}}^{d_L \phi^0} - \sigma_{\text{tot}}^{d_L \phi^{0*}} \right) + c_\infty,$$

$$\frac{2c_{Hd}}{\Lambda^2} = \frac{d\bar{\mathcal{A}}_{d_R \phi^0}}{ds} \Big|_{s=0} = \int_0^\infty \frac{ds}{\pi s} \left(\sigma_{\text{tot}}^{d_R \phi^0} - \sigma_{\text{tot}}^{d_R \phi^{0*}} \right) + c_\infty,$$

Example: $Zb\bar{b}$ Custodial symmetry

- How the $Zb_L\bar{b}_L$ couplings is related to heavy quarks.

$$\frac{4 \delta g_{Lb}}{v^2} = -\frac{2(c_{Hq} + c'_{Hq})}{\Lambda^2} = \frac{d\tilde{A}_{tL} \phi^-}{ds} \Big|_{s=0} = \int_0^\infty \frac{ds}{\pi s} \left(\sigma_{tL} \phi^- \rightarrow F^{-\frac{1}{3}} - \sigma_{tL} \phi^+ \rightarrow F^{\frac{5}{3}} \right) + c_\infty,$$

- We can impose some symmetry to ensure the cancellation of the two cross section terms. ($Zb\bar{b}$ custodial symmetry, [hep-ph/0605341] Agashe et al.)

