Importance of Z-pole and WW running

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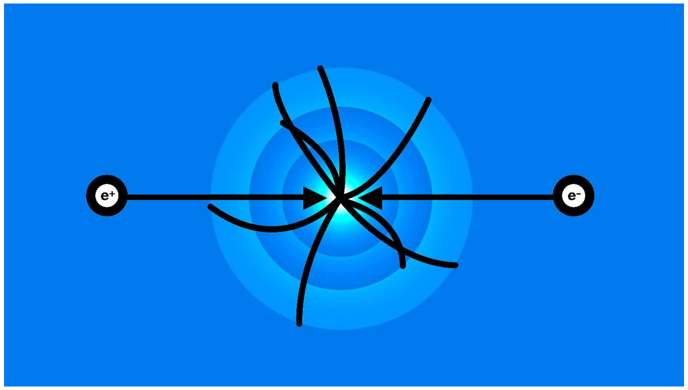
- Electroweak precision at Z pole & WW
- Electroweak precision at $\sqrt{s} = 250$ GeV
- α_s and $\alpha(s)$ measurements

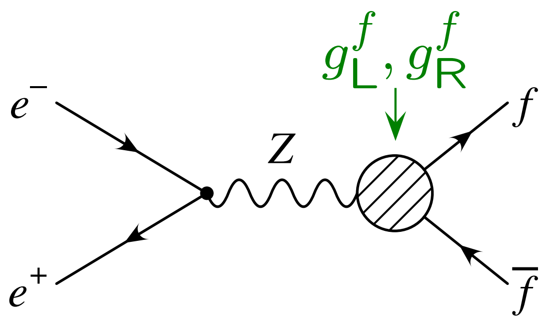


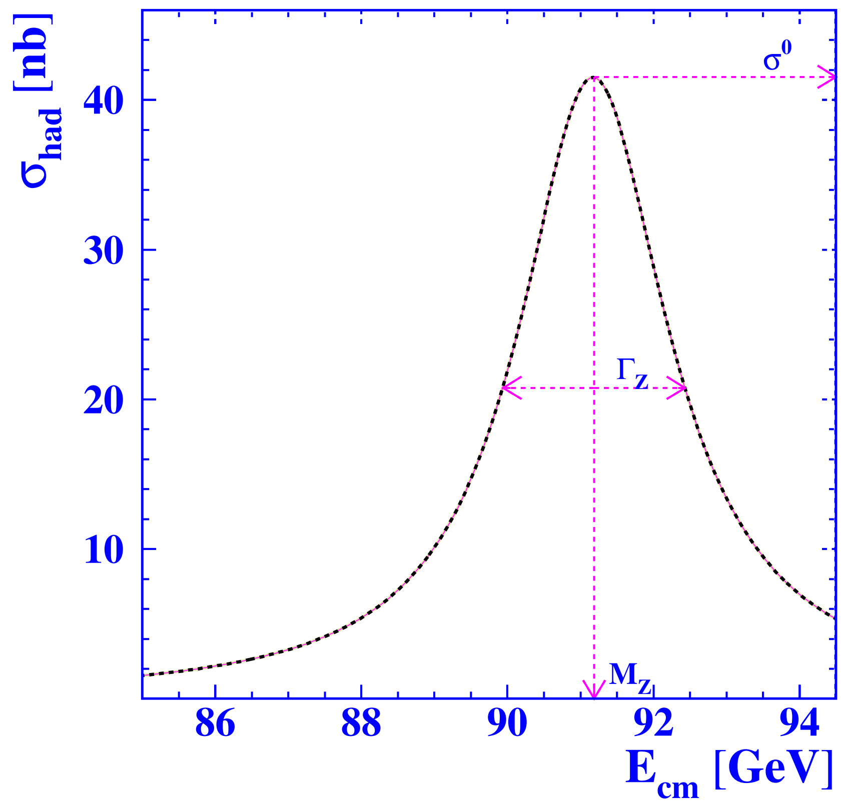
Z cross section and branching fractions

$e^+e^- \to f\bar{f}$ for $E_{CM} \sim M_Z$:

- Mass M_Z
- Width $\Gamma_Z = \sum_f \Gamma_{ff}$
- Braching ratio $R_f = \Gamma_{ff}/\Gamma_Z$
- $\sigma^0 \approx \frac{12\pi\,\Gamma_{ee}\Gamma_{ff}}{(s-M_Z^2)^2+M_Z^2\Gamma_Z^2} = \frac{12\pi}{M_Z^2} R_e R_f$

$$\Gamma_{ff} = C\left[(g_L^f)^2 + (g_R^f)^2\right]$$

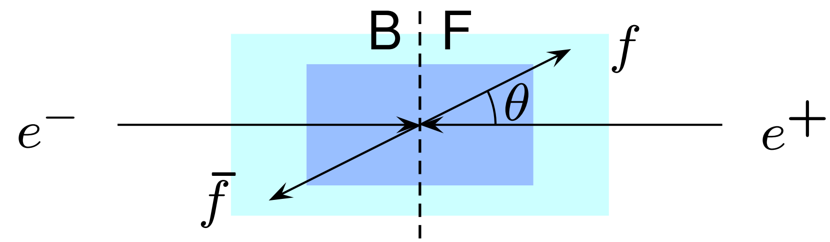




Z-pole asymmetries

Forward-backward asymmetry:

$$A_{\text{FB}} \equiv \frac{\sigma_{\text{F}} - \sigma_{\text{B}}}{\sigma_{\text{F}} + \sigma_{\text{B}}} = \frac{3}{4}\mathcal{A}_e\mathcal{A}_f$$



$$\mathcal{A}_f = \frac{2(1 - 4\sin^2\theta^f_{\text{eff}})}{1 + (1 - 4\sin^2\theta^f_{\text{eff}})^2} \qquad \sin^2\theta^f_{\text{eff}} = \frac{g^f_R}{2|Q_f|(g^f_R - g^f_L)}$$

Left-right asymmetry:

With polarized e^- beam: $$A_{\text{LR}} \equiv \frac{\sigma_{\text{L}} - \sigma_{\text{R}}}{\sigma_{\text{L}} + \sigma_{\text{R}}} = \mathcal{A}_e$$

Polarization asymmetry:

Average τ pol. in $e^+e^- \to \tau^+\tau^-$: $$\langle\mathcal{P}_\tau\rangle = -\mathcal{A}_\tau$$

Z lineshape

- Deconvolution of initial-state QED radiation:

$$\sigma[e^+e^- \to f\bar{f}] = \mathcal{R}_{\mathrm{ini}}(s,s') \otimes \sigma_{\mathrm{hard}}(s')$$

- Subtraction of γ-exchange, γ–Z interference, box contributions:

$$\sigma_{\mathrm{hard}} = \sigma_{\mathrm{Z}} + \sigma_{\gamma} + \sigma_{\gamma\mathrm{Z}} + \sigma_{\mathrm{box}}$$

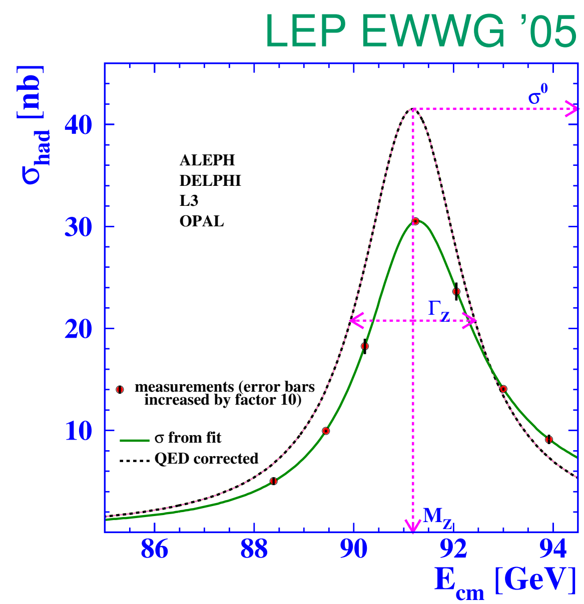
- Z-pole contribution:

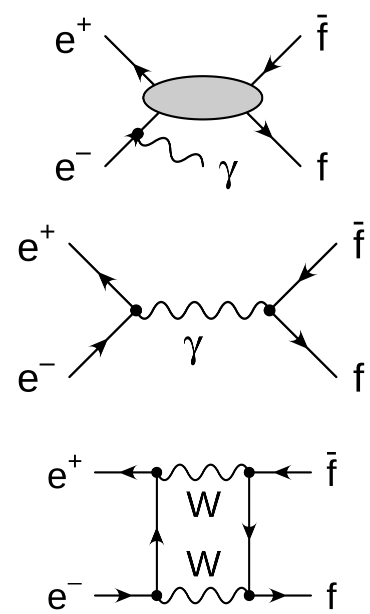
$$\sigma_{\mathrm{Z}} = \frac{R}{(s-M_{\mathrm{Z}}^2)^2 + M_{\mathrm{Z}}^2\Gamma_{\mathrm{Z}}^2} + \sigma_{\mathrm{non-res}}$$

σ_{γ}, $\sigma_{\gamma\mathrm{Z}}$, σ_{box}, $\sigma_{\mathrm{non-res}}$ known at NLO

$\longrightarrow$ need consistent pole expansion framework

$\longrightarrow$ leading NNLO may be needed for future e^+e^-





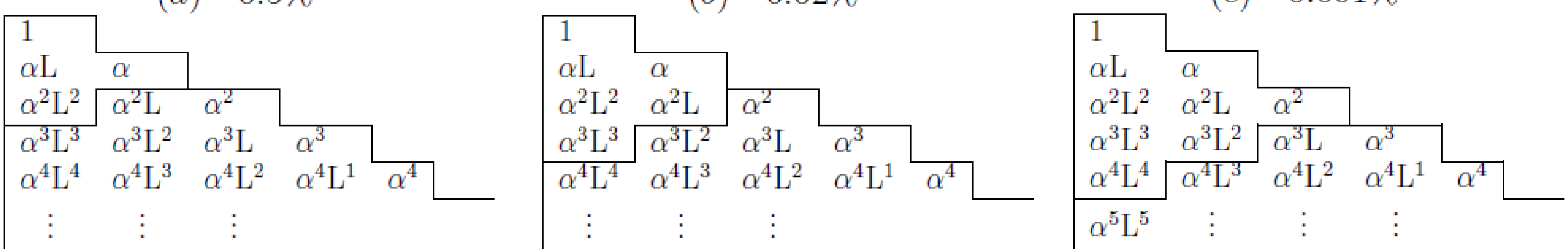
Monte-Carlo methods for QED effects

- Implementation in MC program to evaluate exp. efficiency and particle ID
- Current state of art: e.g. `KORALZ, KKMC` — Jadach, Ward, ...
  $\rightarrow \mathcal{O}(\alpha^2 L)$ accuracy $[L = \ln(s/m_e^2)]$
- One to two orders improvement needed:

Observable	Where from	Present (LEP)	FCC stat.	FCC syst	$\frac{\text{Now}}{\text{FCC}}$
M_Z [MeV]	Z linesh. [28]	$91187.5 \pm 2.1\{0.3\}$	0.005	0.1	3
Γ_Z [MeV]	Z linesh. [28]	$2495.2 \pm 2.1\{0.2\}$	0.008	0.1	2
$R_l^Z = \Gamma_h/\Gamma_l$	$\sigma(M_Z)$ [33]	$20.767 \pm 0.025\{0.012\}$	$6 \cdot 10^{-5}$	$1 \cdot 10^{-3}$	12
σ^0_{had}[nb]	σ^0_{had} [28]	$41.541 \pm 0.037\{0.25\}$	$0.1 \cdot 10^{-3}$	$4 \cdot 10^{-3}$	6
N_ν	$\sigma(M_Z)$ [28]	$2.984 \pm 0.008\{0.006\}$	$5 \cdot 10^{-6}$	$1 \cdot 10^{-3}$	6
$\sin^2\theta_W^{eff} \times 10^5$	$A_{FB}^{lept.}$ [33]	$23099 \pm 53\{28\}$	0.3	0.5	55
$A_{FB,\mu}^{M_Z \pm 3.5\text{GeV}}$	$\frac{d\sigma}{d\cos\theta}$ [28]	$\pm 0.020\{0.001\}$	$1.0 \cdot 10^{-5}$	$0.3 \cdot 10^{-5}$	100

Jadach, Skrzypek ’19

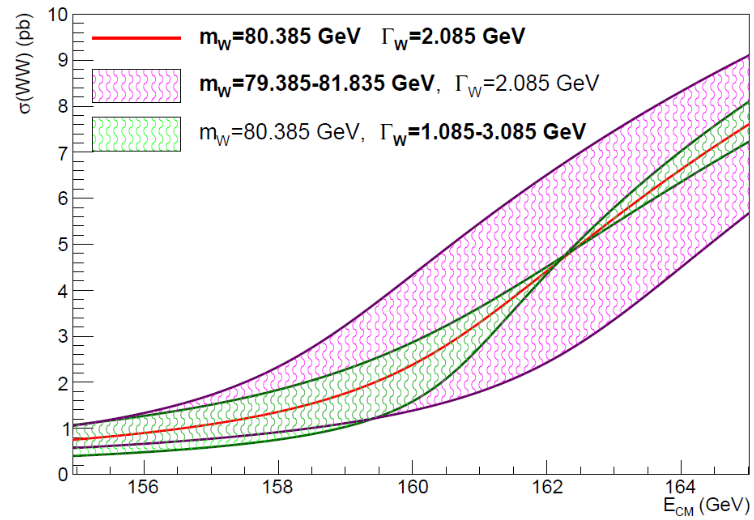
(a) 0.5% (b) 0.02% (c) 0.001%



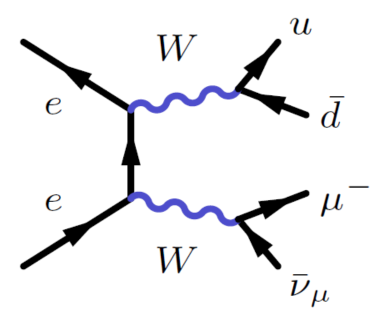
$\rightarrow$ Need matching of h.o. matrix elements with QED parton shower (exclusive in all fs particles)

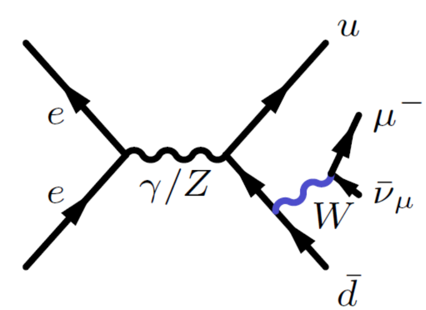
WW threshold

- High-precision measurement of M_W from $e^+e^- \to W^+W^-$ at threshold
- a) Corrections near threshold enhanced by
  $1/\beta$ and $\ln\beta$
  $$\beta \sim \sqrt{1 - 4\frac{M_W^2 - iM_W\Gamma_W}{s}} \sim \sqrt{\Gamma_W/M_W}$$
  b) Non-resonant contributions are important



- Full $\mathcal{O}(\alpha)$ calculation of $e^+e^- \to 4f$
  Denner, Dittmaier, Roth, Wieders '05
- EFT expansion in $\alpha \sim \Gamma_W/M_W \sim \beta^2$
  Beneke, Falgari, Schwinn, Signer, Zanderighi '07
  - NLO corrections with NNLO Coulomb correction ($\propto 1/\beta^n$): $\delta_{\text{th}} M_W \sim 3$ MeV
    Actis, Beneke, Falgari, Schwinn '08
  - Adding NNLO corrections to $ee \to WW$ and $W \to f\bar{f}$ and NNLO ISR: $\delta_{\text{th}} M_W \lesssim 0.6$ MeV





WW threshold

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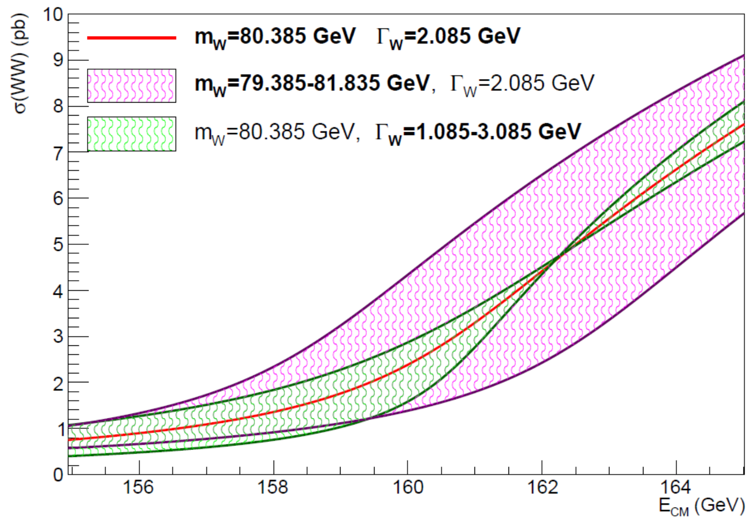
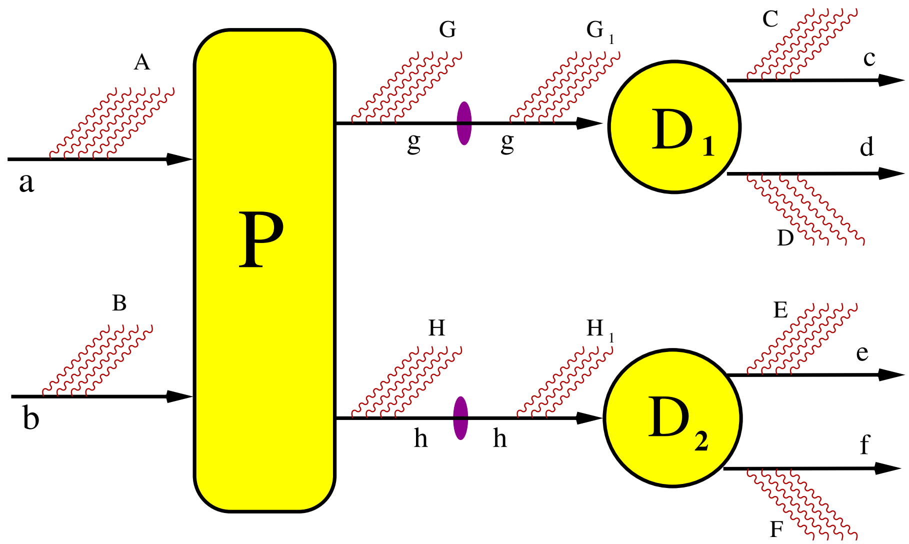


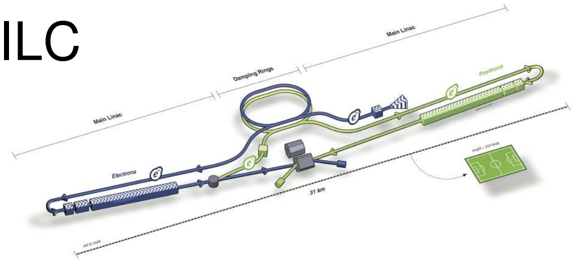
- Resummation of soft photon radiation

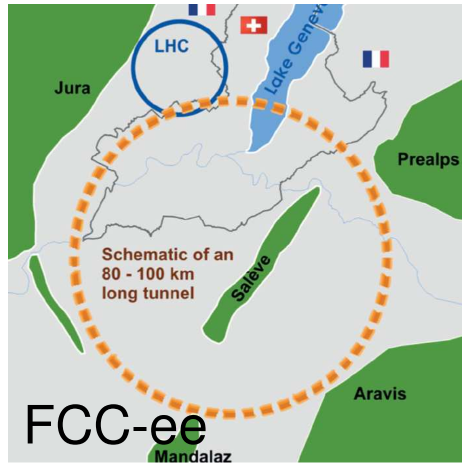
Jadach, Płaczek, Skrzypek ’19

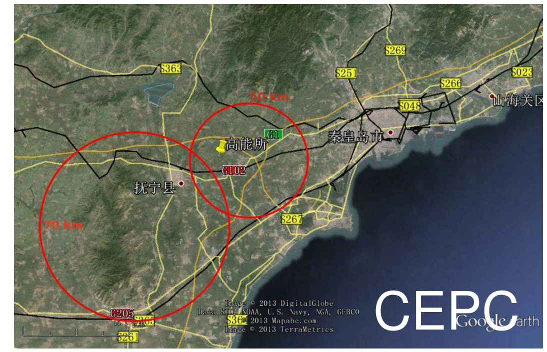


Electroweak precision tests at future colliders

ILC







$\sqrt{s}$	M_Z	$2M_W$	
ILC/GigaZ	100 fb^{-1}	500 fb^{-1} (6 pts.)	beam pol. (P_{e^-}=0.8, P_{e^+}=0.3)
FCC-ee	230 ab^{-1}	10 ab^{-1} (2 pts.)	2 detectors
CEPC	45 ab^{-1}	2.6 ab^{-1} (3 pts.)	2 detectors

Anticipated precision for EWPOs:

	Current exp.	ILC/GigaZ	CEPC	FCC-ee
M_{W} [MeV]	15	1–2 ^{a,e}	1 ^{e}	1 ^{e}
M_{Z} [MeV]	2.1	–	0.5 ^{e}	0.1 ^{e}
Γ_Z [MeV]	2.3	1 ^{a}	0.5 ^{e}	0.1 ^{e}
$R_\ell = \Gamma_{\mathrm{Z}}^{\mathrm{had}}/\Gamma_{\mathrm{Z}}^{\ell}$ $[10^{-3}]$	25	6 ^{b}	2 ^{b}	1 ^{b}
$R_b = \Gamma_{\mathrm{Z}}^{b}/\Gamma_{\mathrm{Z}}^{\mathrm{had}}$ $[10^{-5}]$	66	15 ^{c}	4.3 ^{c}	6 ^{c}
$\sin^2\theta_{\mathrm{eff}}^{\ell}$ $[10^{-5}]$	16	1 ^{d}	<1 ^{e}	0.5 ^{e}

Systematics:

^{a} energy scale

^{b} acceptance

^{c} flavor tagging

^{d} polarization

^{e} beam energy calibration / beam-beam interactions

Comparison of EWPOs with theory

- To probe new physics, compare EWPOs with SM theory predictions
- Need to take theory error into account:

	Current exp.	Current th.†	CEPC	FCC-ee
M_{W} [MeV]	15	4*	1	1
Γ_Z [MeV]	2.3	0.4	0.5	0.1
$R_\ell = \Gamma_Z^{\mathrm{had}}/\Gamma_Z^\ell$ [10^{-3}]	25	5	2	1
$R_b = \Gamma_Z^b/\Gamma_Z^{\mathrm{had}}$ [10^{-5}]	66	10	4.3	6
$\sin^2\theta_{\mathrm{eff}}^\ell$ [10^{-5}]	16	4.5	<1	0.5

* computed from G_μ † full NNLO and leading NNNLO

- Theory error estimate is not well defined, ideally $\Delta_{\mathrm{th}} \ll \Delta_{\mathrm{exp}}$
- Common methods:
  - Count prefactors (α, N_c, N_f, ...)
  - Extrapolation of perturbative series
  - Renormalization scale dependence
  - Renormalization scheme dependence

Comparison of EWPOs with theory

- **Electroweak precision tests** at future e^+e^- colliders require 1–2 orders improvement in SM theory calculations and tools
  - Z-pole: 3-loop & leading 4-loop EW + multi-loop/leg merging for QED MC
  - off Z-pole / backgrounds: ($\geq$2)-loop EW
  - WW 2-loop EW for 2$\to$2 processes (+ 4-loop QCD)
    ($\geq$1)-loop for backgr. and non-resonant terms

BSM reach: effective operator analysis

SMEFT: Gauge-invariant operators with SU(2) Higgs doublet

$$\mathcal{L} = \sum_i \frac{c_i}{\Lambda^2}\mathcal{O}_i + \mathcal{O}(\Lambda^{-3}) \qquad (\Lambda \gg M_Z)$$

$$\mathcal{O}_{\phi 1} = (D_\mu\Phi)^\dagger\Phi\,\Phi^\dagger(D^\mu\Phi) \qquad \alpha\Delta T = -\frac{v^2}{2}\frac{c_{\phi 1}}{\Lambda^2}$$

$$\mathcal{O}_{BW} = \Phi^\dagger B_{\mu\nu}W^{\mu\nu}\Phi \qquad \alpha\Delta S = -e^2v^2\frac{c_{BW}}{\Lambda^2}$$

$$O_{LL}^{(3)e} = (\bar{L}_L^e\sigma^a\gamma_\mu L_L^e)(\bar{L}_L^e\sigma^a\gamma^\mu L_L^e) \qquad \Delta G_F = -\sqrt{2}\frac{c_{LL}^{(3)e}}{\Lambda^2}$$

$$O_R^f = i(\Phi^\dagger \overleftrightarrow{D}_\mu \Phi)(\bar{f}_R\gamma^\mu f_R) \qquad f = e, \mu\,\tau, b, lq$$

$$O_L^F = i(\Phi^\dagger \overleftrightarrow{D}_\mu \Phi)(\bar{F}_L\gamma^\mu F_L) \qquad F = \binom{\nu_e}{e}, \binom{\nu_\mu}{\mu}, \binom{\nu_\tau}{\tau}, \binom{u, c}{d, s}, \binom{t}{b}$$

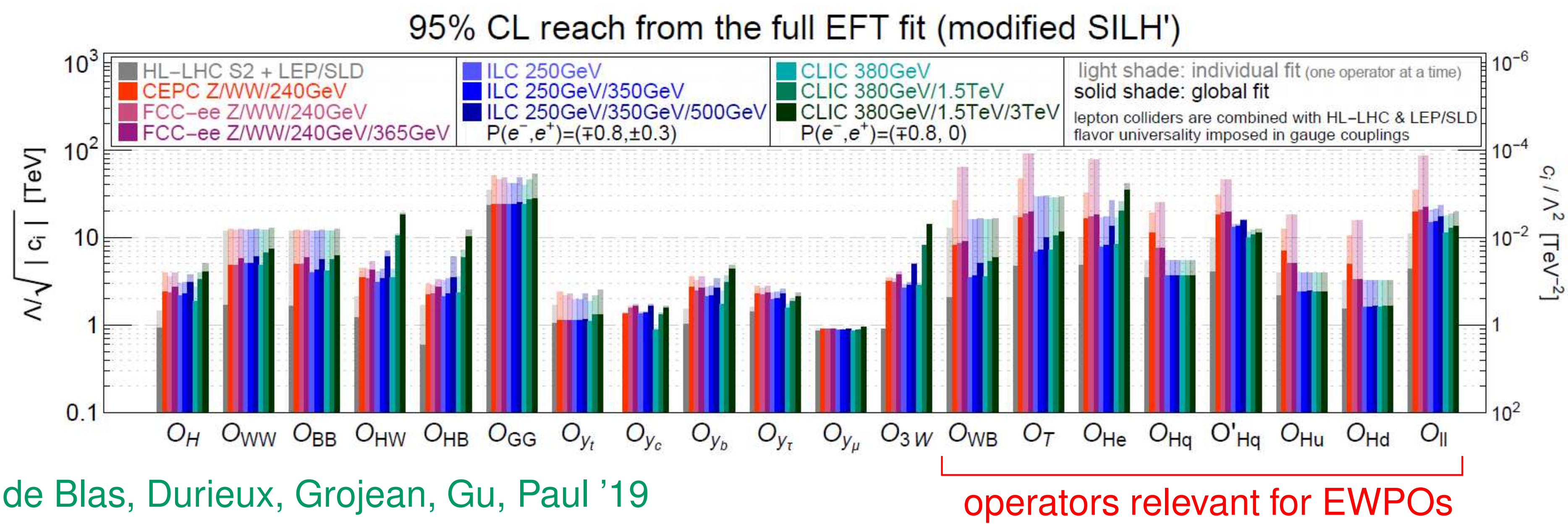
$$O_L^{(3)F} = i(\Phi^\dagger \overleftrightarrow{D}^a_\mu \Phi)(\bar{F}_L\sigma_a\gamma^\mu F_L)$$

More operators than EWPOs

→ Need to make flavor assumptions and/or
use other obs. (e.g. W production and decay)

BSM reach: effective operator analysis

Projected reach assuming Minimal Flavor Violation:

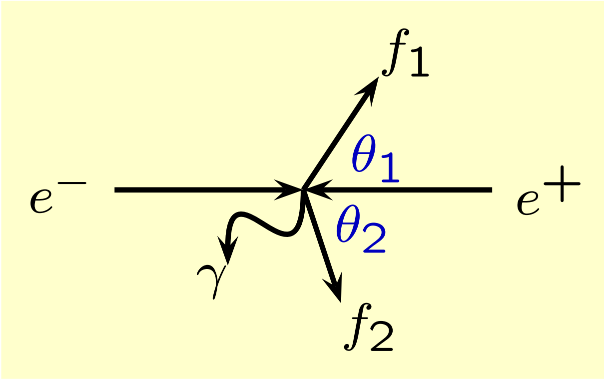


de Blas, Durieux, Grojean, Gu, Paul '19

Electroweak precision at $\sqrt{s} = 250$ GeV

EWPOs accessible through **radiative return** $e^+e^- \to \gamma Z$

- γ mostly collinear with beam
- Reduction in cross-section by $\sim \frac{\alpha}{\pi} \ln \frac{s}{m_e^2} \sim 0.06$
- Precise det. of m_{ff} from measured angles:



$$m_{ff}^2 = s\,\frac{1-\beta}{1+\beta}, \quad \beta = \frac{|\sin(\theta_1+\theta_2)|}{\sin\theta_1 + \sin\theta_2}$$

- Additional backgrounds from $e^+e^- \to WW, ZZ$ that are not flat in m_{ff}

Ueno '19

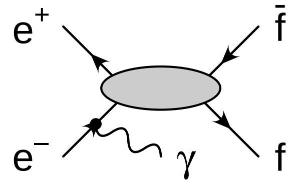
$Z\gamma$ electroweak precision: observables

Fujii et al. '19

- $A_{\mathsf{LR}} \to \sin^2\theta^\ell_{\mathsf{eff}}$ (limited by sys. err. on beam polarization)
- $A^{\mu,\tau,b}_{\mathsf{FB}}$ (statistics limited)
- R_ℓ, R_c, R_b (limited by sys. err. on flavor tag)
- No competitive measurements on M_Z, Γ_Z, σ^0 (need to use LEP values)

$Z\gamma$ electroweak precision: theory input

Leading effect: Soft+collinear multi-γ ISR



$$\sigma[e^+e^- \to f\bar{f}] = \mathcal{R}_{\text{ini}}(s,s') \otimes \sigma_{\text{hard}}(s')$$

Kureav, Fadin '85
Berends, Burgers, v. Neerven '88
Kniehl, Krawczyk, Kühn, Stuart '88
Beenakker, Berends, v. Neerven '89
Bardin et al. '91; Skrzypek '92
Montagna, Nicrosini, Piccinini '97

$$\mathcal{R}_{\text{ini}} = \sum_n \left(\frac{\alpha}{\pi}\right)^n \sum_{m=0}^{n} h_{nm} \ln^m\left(\frac{s}{m_e^2}\right)$$

Universal ($m{=}n$) logs known to $n = 6$, also some sub-leading terms

Ablinger, Blümlein, De Freitas, Schönwald '20

Exclusive description: MC tools

Subleading effects:

Radiative corrections to $e^+e^- \to f\bar{f}\gamma\,(+n\gamma)$

- Some corrections cancel for A_{LR}, A_{FB}, BRs
- NLO for $ee \to f\bar{f}\gamma$ + NNLO for $ee \to Z\gamma$, $Z \to f\bar{f}$ could be sufficient

W mass

W mass measurement from $e^+e^- \to WW$: Baak et al. '13

- $\ell\nu_\ell\ell'\nu_{\ell'}$: Endpoints of E_ℓ or other distributions
- $\ell\nu_\ell jj$: Kinematic reconstruction
- $jjjj$: Systematic uncertainty from color reconnection

Expected precision with $\mathcal{L}_{\text{int}} = 2\ \text{ab}^{-1}$ at $\sqrt{s} = 250$ GeV: $\Delta M_W \approx 2.5$ MeV

Theory needs: Small impact of loop corrections, but accurate decription of FSR QED effects needed

Electroweak precision tests at future colliders

Anticipated precision for EWPOs:

Fujii et al. '19

	ILC-250*	ILC/GigaZ	CEPC	FCC-ee
M_{W} [MeV]	2.5 ^{a}	1–2 ^{a,e}	1 ^{e}	1 ^{e}
M_{Z} [MeV]	–	–	0.5 ^{e}	0.1 ^{e}
Γ_{Z} [MeV]	–	1 ^{a}	0.5 ^{e}	0.1 ^{e}
$R_\ell = \Gamma_{\mathrm{Z}}^{\mathrm{had}}/\Gamma_{\mathrm{Z}}^{\ell}$ [10^{-3}]	16 ^{c}	6 ^{b}	2 ^{b}	1 ^{b}
$R_b = \Gamma_{\mathrm{Z}}^{b}/\Gamma_{\mathrm{Z}}^{\mathrm{had}}$ [10^{-5}]	23 ^{c}	15 ^{c}	4.3 ^{c}	6 ^{c}
$\sin^2\theta_{\mathrm{eff}}^{\ell}$ [10^{-5}]	2 ^{d}	1 ^{d}	$<$1 ^{e}	0.5 ^{e}

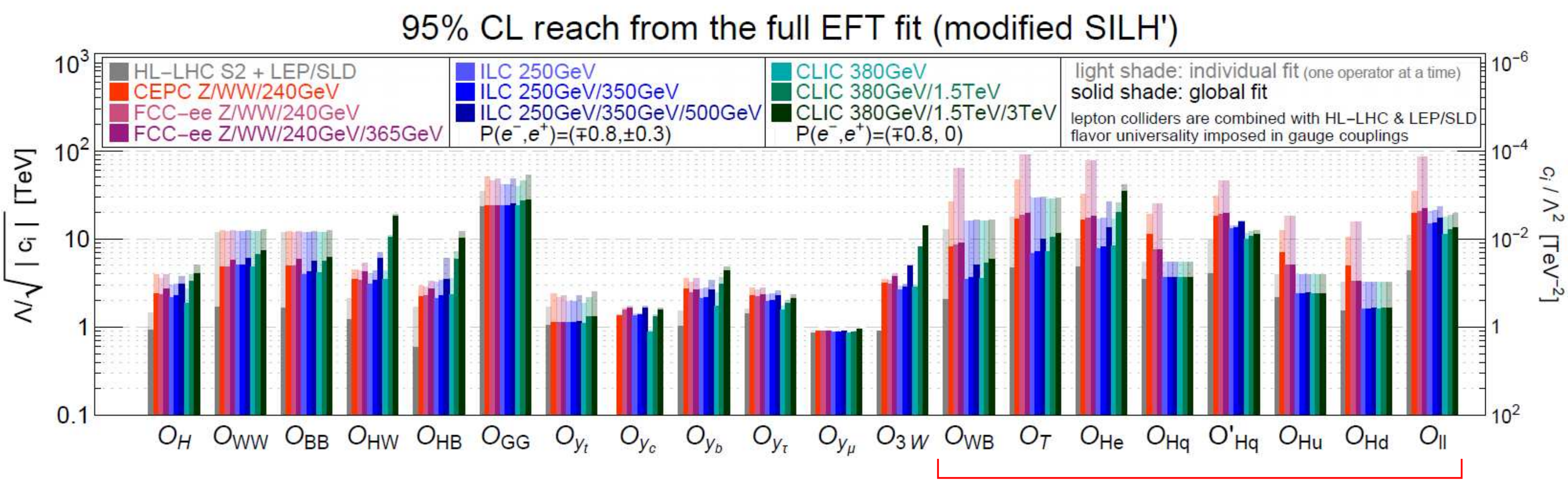
Systematics:

^{a} energy scale
^{b} acceptance
^{c} flavor tagging
^{d} polarization
^{e} beam energy calibration / beam-beam interactions

* $\sqrt{s} = 250$ GeV, $\mathcal{L}_{\mathrm{int}} = 2$ ab^{-1}

BSM reach: effective operator analysis

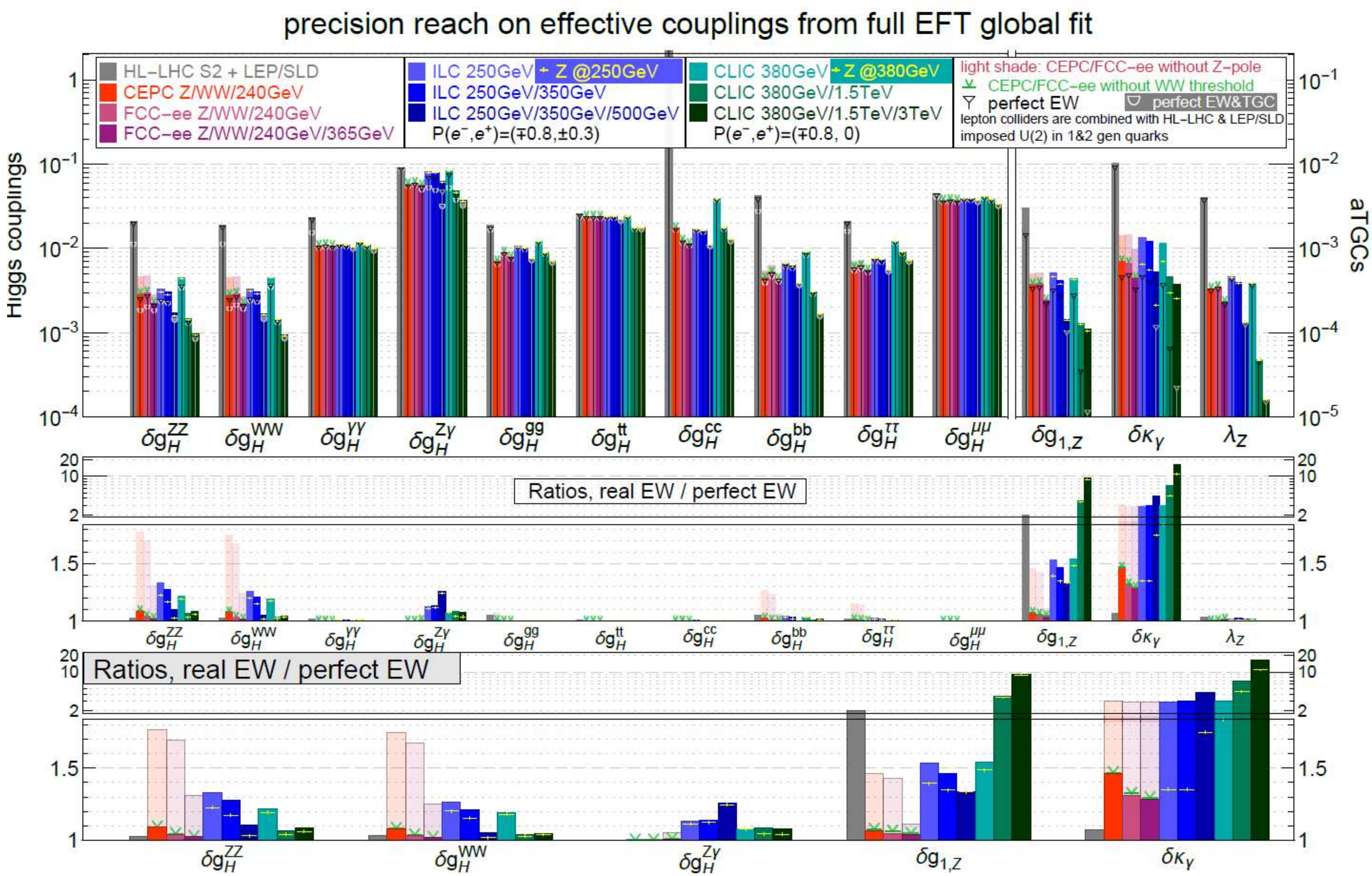
Projected reach assuming Minimal Flavor Violation:



de Blas, Durieux, Grojean, Gu, Paul '19

Correlation between EW and Higgs physics

- Improvement of Z-pole data important for Higgs physics and aGC:



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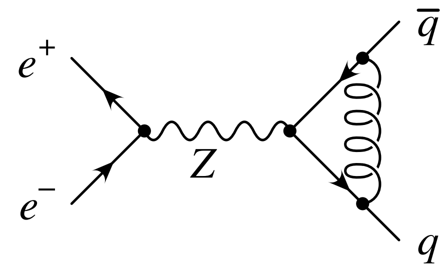
Strong coupling

- Electroweak precision ($R_\ell = \Gamma_Z^{\text{had}}/\Gamma_Z^\ell$):
  $\alpha_s = 0.120 \pm 0.003$ PDG '18



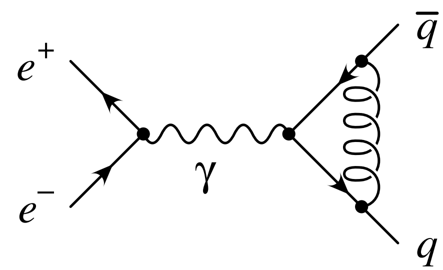
  $\longrightarrow$ No (negligible) non-perturbative QCD effects

  FCC-ee: $\delta R_\ell \sim 0.001$

  $\Rightarrow$ $\delta\alpha_s < 0.0002$ (subj. to theory error)

  Caviat: R_ℓ could be affected by new physics

- $R = \frac{\sigma[ee \to \text{had.}]}{\sigma[ee \to \mu\mu]}$ at lower $\sqrt{s}$



  e.g. CLEO ($\sqrt{s} \sim 9$ GeV): $\alpha_s = 0.110 \pm 0.015$
  Kühn, Steinhauser, Teubner '07

  $\longrightarrow$ dominated by s-channel photon, less room for new physics
  $\longrightarrow$ QCD still perturbative

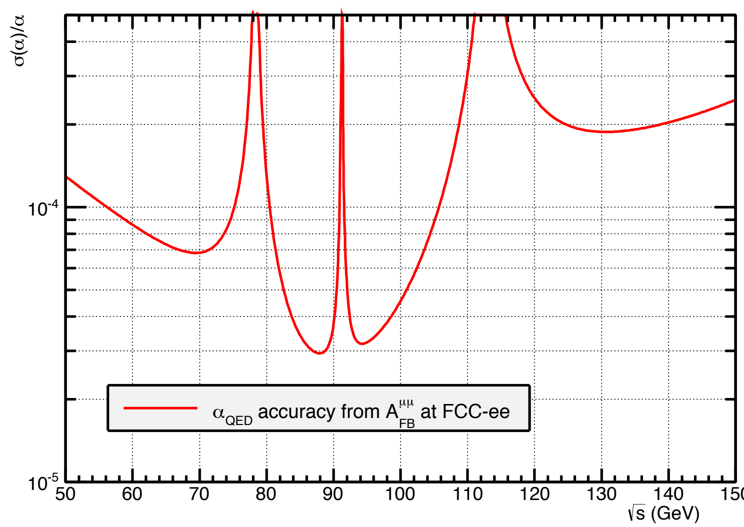
  naive scaling to 50 ab^{-1} (BELLE-II): $\delta\alpha_s \sim 0.0001$

Shift of finestructure constant

- $\Delta\alpha_{\text{had}}$: Could be limiting factor
  - a) From $e^+e^- \to$ had. using dispersion relation
    Current: $\delta(\Delta\alpha_{\text{had}}) \sim 10^{-4}$
    Improvement to $\delta(\Delta\alpha_{\text{had}}) \sim 5 \times 10^{-5}$ likely
  - b) Direct determination at FCC-ee from $e^+e^- \to \mu^+\mu^-$ off the Z peak
    (i.e. $A_{\text{FB}}^{\mu\mu}$ at $\sqrt{s} \sim 88$ GeV and $\sqrt{s} \sim 95$ GeV)

    $\to \delta(\Delta\alpha_{\text{had}}) \sim 3 \times 10^{-5}$ with $\mathcal{L}_{\text{tot}} = 85$ ab^{-1} Janot '15

    Requires high-precision theory prediction for $e^+e^- \to \mu^+\mu^-$ including 2/3-loop corrections for γ-exchange and box contributions



Summary

- **Electroweak precision tests** at future e^+e^- colliders require allow to probe multi-TeV BSM physics (or feebly coupled lighter physics)
- Improved measurements in **several sectors** important:
  EW masses and couplings, Higgs couplings, top mass, α_s and $\alpha(s)$
- Theory progress needed both for **fixed-order loop corrections** as well as **MC tools**
- **ILC-250** can deliver similar physics goals as GigaZ, but with reduced precision
  $\rightarrow$ Open question: Direct determination of α_s and $\alpha(s)$
- Unique theory challenges for description of $ee \to Z\gamma$ vs. $ee \to Z$

Backup slides

Z lineshape

- Deconvolution of initial-state QED radiation:

$$\sigma[e^+e^- \to f\bar{f}] = \mathcal{R}_{\text{ini}}(s,s') \otimes \sigma_{\text{hard}}(s')$$

- Subtraction of γ-exchange, γ–Z interference, box contributions:

$$\sigma_{\text{hard}} = \sigma_Z + \sigma_\gamma + \sigma_{\gamma Z} + \sigma_{\text{box}}$$

- Z-pole contribution:

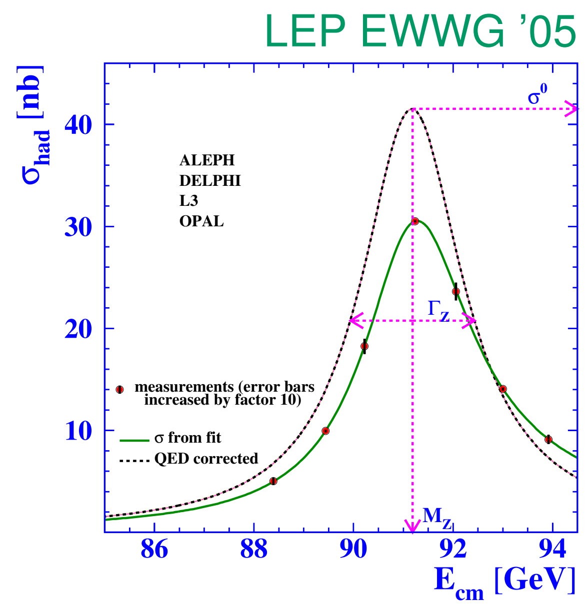
$$\sigma_Z = \frac{R}{(s-\overline{M}_Z^2)^2 + \overline{M}_Z^2\overline{\Gamma}_Z^2} + \sigma_{\text{non-res}}$$

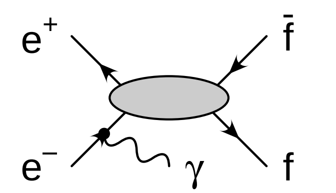
- In experimental analyses:

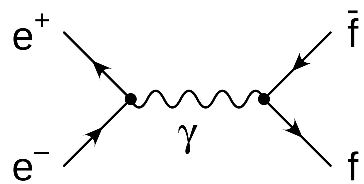
$$\sigma \sim \frac{1}{(s-M_Z^2)^2 + s^2\Gamma_Z^2/M_Z^2}$$

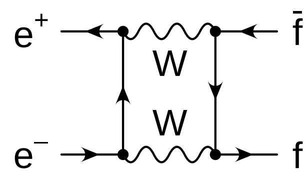
$$\overline{M}_Z = M_Z\Big/\sqrt{1+\Gamma_Z^2/M_Z^2} \approx M_Z - 34\text{ MeV}$$

$$\overline{\Gamma}_Z = \Gamma_Z\Big/\sqrt{1+\Gamma_Z^2/M_Z^2} \approx \Gamma_Z - 0.9\text{ MeV}$$









“Hard” matrix element

Consistent (gauge-invariant) theory setup:

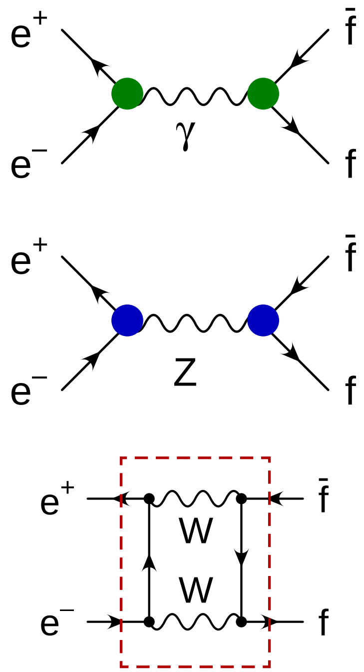
Expansion of $\mathcal{A}[e^+e^- \to \mu^+\mu^-]$ about $s_0 = M_Z^2 - iM_Z\Gamma_Z$:

$$\mathcal{A}[e^+e^- \to f\bar{f}] = \frac{R}{s - s_0} + S + (s - s_0)T + \ldots$$

$$R = g_Z^e(s_0)g_Z^f(s_0)$$

$$S = \left[\frac{1}{M_Z^2}g_\gamma^e g_\gamma^f + g_Z^e g_Z^{f\,\prime} + g_Z^{e\,\prime} g_Z^f + S_{\text{box}}\right]_{s=s_0}$$

$g_V^f(s)$: effective $Vf\bar{f}$ couplings



At NNLO: Need R at $\mathcal{O}(\alpha^2)$, S at $\mathcal{O}(\alpha)$, etc.

Z-pole asymmetries

Blondel scheme: (if e^- and e^+ polarization available) Blondel '88

Four independent measurements for $P_{e^+}/P_{e^-} = ++, +-, -+, --$

$$A_{\mathrm{LR}} = \sqrt{\frac{(\sigma_{++}+\sigma_{-+}-\sigma_{+-}-\sigma_{--})(-\sigma_{++}+\sigma_{-+}-\sigma_{+-}+\sigma_{--})}{(\sigma_{++}+\sigma_{-+}+\sigma_{+-}+\sigma_{--})(-\sigma_{++}+\sigma_{-+}+\sigma_{+-}-\sigma_{--})}}$$

Note: No need to know $|P_{e^\pm}|$!

Main systematic uncertainties:

- Difference of $|P|$ for $P > 0$ and $P < 0$
- Difference of $\mathcal{L}$ for $P > 0$ and $P < 0$

$\delta A_{\mathrm{LR}} \approx 10^{-4}$ $\Rightarrow$ $\delta \sin^2\theta^{\ell}_{\mathrm{eff}} \approx 1.3 \times 10^{-5}$ Mönig, Hawkings '99

Theory calculations: Status

- Many seminal works on 1-loop and leading 2-loop corrections

  Veltman, Passarino, Sirlin, Marciano, Bardin, Hollik, Riemann, Degrassi, Kniehl, ...

- Full 2-loop results for M_W, Z-pole observables

  Freitas, Hollik, Walter, Weiglein '00
  Awramik, Czakon '02
  Onishchenko, Veretin '02
  Awramik, Czakon, Freitas, Weiglein '04
  Awramik, Czakon, Freitas '06
  Hollik, Meier, Uccirati '05,07
  Awramik, Czakon, Freitas, Kniehl '08
  Freitas '14
  Dubovyk, Freitas, Gluza, Riemann, Usovitsch '16,18

- Approximate 3- and 4-loop results (enhance by Y_t and/or N_f)

  Chetyrkin, Kühn, Steinhauser '95
  Faisst, Kühn, Seidensticker, Veretin '03
  Boughezal, Tausk, v. d. Bij '05
  Schröder, Steinhauser '05
  Chetyrkin et al. '06
  Boughezal, Czakon '06
  Chen, Freitas '20

Theory and parametric uncertainties

	CEPC	perturb. error with 3-loop†	Param. error CEPC*	main source
M_W [MeV]	1	1	2.1	m_t, $\Delta\alpha$
Γ_Z [MeV]	0.5	0.15	0.15	m_t, α_s
R_b [10^{-5}]	4.3	5	< 1	
$\sin^2\theta^\ell_{eff}$ [10^{-5}]	$<$1	1.5	2	m_t, $\Delta\alpha$

† **Theory scenario:** $\mathcal{O}(\alpha\alpha_s^2)$, $\mathcal{O}(N_f\alpha^2\alpha_s)$, $\mathcal{O}(N_f^2\alpha^2\alpha_s)$, leading 4-loop

(N_f^n = at least n closed fermion loops)

Parametric inputs:

***CEPC:** $\delta m_t = 600$ MeV, $\delta\alpha_s = 0.0002$, $\delta M_Z = 0.5$ MeV,

$\delta(\Delta\alpha) = 5 \times 10^{-5}$

Theory and parametric uncertainties

	CEPC	perturb. error with 3-loop†	Param. error CEPC*	main source
M_W [MeV]	1	1	0.6	$\Delta\alpha$
Γ_Z [MeV]	0.5	0.15	0.1	α_s
R_b $[10^{-5}]$	4.3	5	< 1	
$\sin^2\theta^\ell_\mathrm{eff}$ $[10^{-5}]$	<1	1.5	1	$\Delta\alpha$

† **Theory scenario:** $\mathcal{O}(\alpha\alpha_\mathrm{s}^2)$, $\mathcal{O}(N_f\alpha^2\alpha_\mathrm{s})$, $\mathcal{O}(N_f^2\alpha^2\alpha_\mathrm{s})$, leading 4-loop

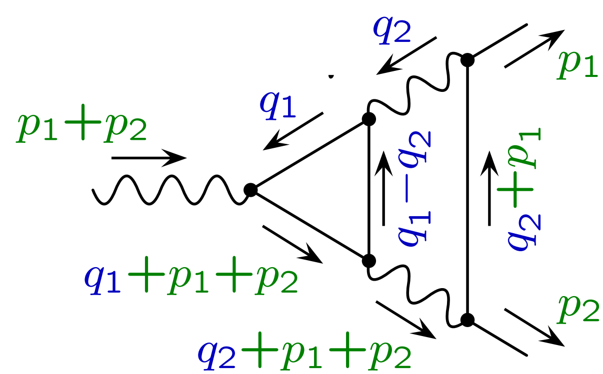
(N_f^n = at least n closed fermion loops)

Parametric inputs:

***FCC-ee:** $\delta m_t = 50$ MeV, $\delta\alpha_\mathrm{s} = 0.0002$, $\delta M_\mathrm{Z} = 0.5$ MeV, $\delta(\Delta\alpha) = 3\times 10^{-5}$

Calculational techniques

Experimental precision requires inclusion of **radiative corrections** in theory (1-loop, 2-loop, and partial 3-loop)



Integrals over loop momenta:

$$\int d^4q_1 d^4q_2 \; f(q_1, q_2, p_1, p_2, \ldots, m_1, m_2, \ldots)$$

Computer algebra tools:

- Generation of diagrams, $\mathcal{O}(100) - \mathcal{O}(10000)$
- Lorentz and Dirac algebra
- Integral simplification (and expansion)

Evaluation of loop integrals:

- In general not possible analytically
- Numerical methods are more general, but computing intensive
- Special numerical techniques can balance precision and evaluation time

Analytic calculations

- Mostly used for diagrams with few mass scales
- Reduce to **master integrals** with integration-by-parts and other identities

  Chetyrkin, Tkachov '81; Gehrmann, Remiddi '00; Laporta '00; ...

  Public programs:

Public programs:	`Reduze`	von Manteuffel, Studerus '12
	`FIRE`	Smirnov '13,14
	`LiteRed`	Lee '13
	`KIRA`	Maierhoefer, Usovitsch, Uwer '17

  → Large need for computing time and memory

- Evaluate master integrals with differential equations or Mellin-Barnes rep.

  Kotikov '91; Remiddi '97; Smirnov '00,01; Henn '13; ...

  → Result in terms of Goncharov polylogs / multiple polylogs

  → Some problems need iterated elliptic integrals / elliptic multiple polylogs

  Broedel, Duhr, Dulat, Trancredi '17,18

  Ablinger er al. '17

  → Even more classes of functions needed in future?

Asymptotic expansions

- Exploit large mass ratios, $e.\,g.\; M_Z^2/m_t^2 \approx 1/4$
- Evaluate coeff. integrals analytically
- Fast numerical evaluation

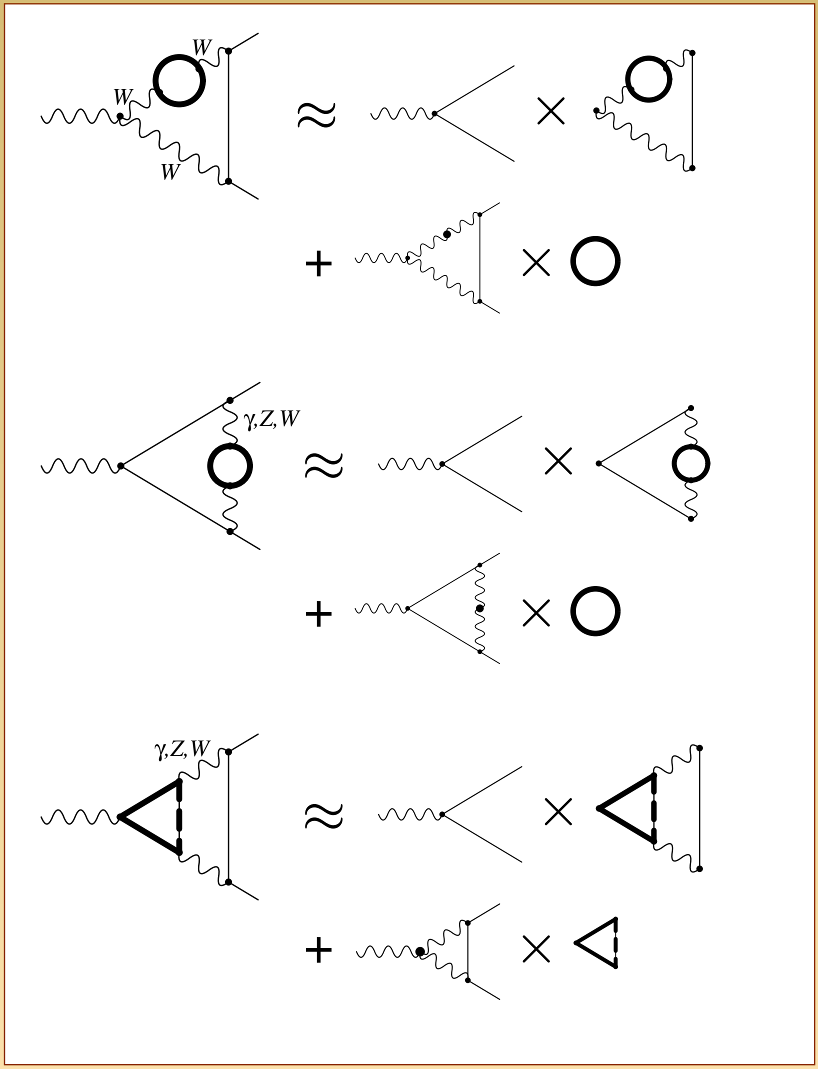
→ Used in some 2/3-scale problems

→ Public programs:

`exp` Harlander, Seidensticker, Steinhauser '97
`asy` Pak, Smirnov '10

→ Possible limitations:

- Difficult coefficient integrals
- bad convergence



Numerical integration

Two general approches:

→ Automated treatment of UV/IR divergencies

→ No restriction on number of loops or legs

- **Sector decomposition:**

Public programs:	SecDec	Carter, Heinrich '10; Borowka et al. '12,15,17
	FIESTA	Smirnov, Tentyukov '08; Smirnov '13,15

- **Mellin-Barnes representations:**

Public programs:	MB/MBresolve	Czakon '06; Smirnov, Smirnov '09
	AMBRE/MBnumerics	Gluza, Kajda, Riemann '07 Dubovyk, Gluza, Riemann '15 Usovitsch, Dubovyk, Riemann '18

- Diagrams with internal thresholds can cause numerical instabilities
- Specialized techniques (for some type of diagrams) often improve computing time, robustness, precision (but not automated)

Specialized numerical techniques

Example: HZ double boxes

Song, Freitas '21

- Introduce Feynman parameters and disp. rel.
- Expressions for second loop from, e.g., `LoopTools`
  Hahn, Perez-Victoria '98
- 3-dim. numerical integral with adaptive Gaussian integration
- $\mathcal{O}(0.1\%)$ precision in $\mathcal{O}$(min.) on laptop