

# **Higgs Precision calculations for $e^+e^-$ colliders**

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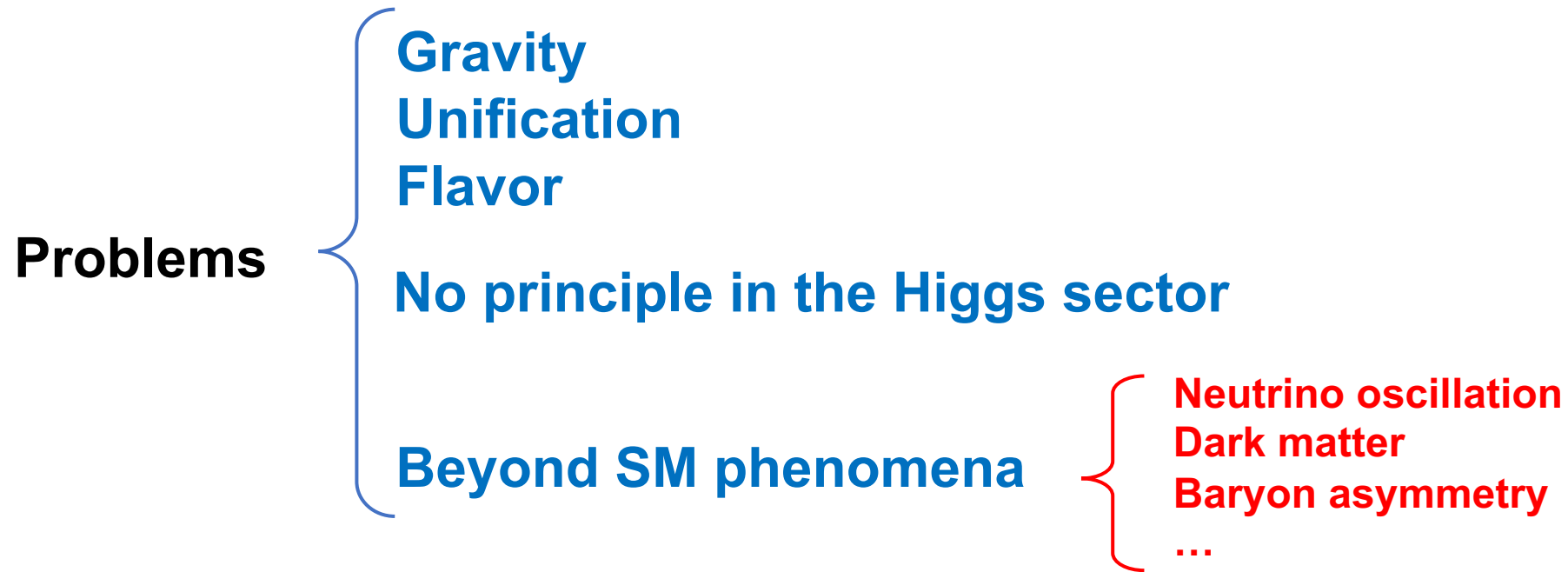
# Contents

- **Introduction**
- **Extended Higgs sectors**
- **Study on Radiative Corrections**
- **Higgs to Higgs decays**
- **Triple Higgs boson coupling**
- **Summary**

# Introduction

# Beyond the SM

SM is a good description of the nature around the EW scale, however ....



**SM must be replaced by a more fundamental theory**

# Higgs sector is the key

Although the Higgs boson was found, the Higgs sector remains unknown

Elementary or composite?

Multiplet Structure?

Higgs potential (Dynamics of EWSB, EWPT, ...)

Yukawa structure (Flavor Physics, CPV, ...)

Higgs sector: **no principle**

The SM Higgs sector can be modified

⇒ BSM phenomena may be explained

Tiny neutrino mass

Phase Transition (1<sup>st</sup> Order)

CPV sources for baryogenesis

DM candidates

...

Higgs sector is testable at future collider experiments

**Higgs sector is a probe of new physics**

# EFT and BSM models

- **Current situation**

- Higgs was found to be 125 GeV
- No new particle found

⇒ **Nature seems nearly SM around EW scale.**

- **Description**

- **EFT approaches**                      General, but each coefficient is independent

- **Precision analysis of concrete (renormalizable) BSM models**  
MSSM, NMSSMs, ...  
Minimal Composite models, ...  
General extended Higgs models,  
...

Model dependent, but there are correlations among couplings  
predictive power at loop levels ⇒ interesting

**Complementary!**<sub>6</sub>

# Extended Higgs sectors

# Extended Higgs sectors?

## Multiplet Structure (with additional scalars)

$\Phi_{\text{SM}} +$  Isospin **Singlet**,

$\Phi_{\text{SM}} +$  **Doublet** (2HDM),

$\Phi_{\text{SM}} +$  **Triplet**,

...

## Additional Symmetry

Discrete or Continuous?

Exact or Approximate or Softly broken?

## Interaction

Weakly coupled or Strongly Coupled?



Hint for  
BSM models

# Simple example: 2HDM (softly-broken $Z_2$ )

$$V_{\text{THDM}} = +m_1^2 |\Phi_1|^2 + m_2^2 |\Phi_2|^2 - \underline{m_3^2 (\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1)} \\ + \frac{\lambda_1}{2} |\Phi_1|^4 + \frac{\lambda_2}{2} |\Phi_2|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2 \\ + \lambda_4 |\Phi_1^\dagger \Phi_2|^2 + \frac{\lambda_5}{2} \left[ (\Phi_1^\dagger \Phi_2)^2 + (\text{h.c.}) \right]$$

$$\Phi_1 \text{ and } \Phi_2 \Rightarrow \underbrace{h, H, A^0, H^\pm}_{\substack{\uparrow \text{CPEven} \\ \uparrow \text{CPodd}}} \oplus \text{Goldstone bosons}$$

$\uparrow$  charged

$$\Phi_i = \begin{bmatrix} w_i^+ \\ \frac{1}{\sqrt{2}}(h_i + v_i + ia_i) \end{bmatrix} \quad (i = 1, 2)$$

## Diagonalization

$$\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} H \\ h \end{bmatrix} \quad \begin{bmatrix} z_1^0 \\ z_2^0 \end{bmatrix} = \begin{bmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{bmatrix} \begin{bmatrix} z^0 \\ A^0 \end{bmatrix} \\ \begin{bmatrix} w_1^\pm \\ w_2^\pm \end{bmatrix} = \begin{bmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{bmatrix} \begin{bmatrix} w^\pm \\ H^\pm \end{bmatrix}$$

masses

$$\left\{ \begin{aligned} m_h^2 &= v^2 \left( \lambda_1 \cos^4 \beta + \lambda_2 \sin^4 \beta + \frac{\lambda}{2} \sin^2 2\beta \right) + \mathcal{O}\left(\frac{v^2}{M_{\text{soft}}^2}\right), \\ m_H^2 &= M_{\text{soft}}^2 + v^2 (\lambda_1 + \lambda_2 - 2\lambda) \sin^2 \beta \cos^2 \beta + \mathcal{O}\left(\frac{v^2}{M_{\text{soft}}^2}\right), \\ m_{H^\pm}^2 &= M_{\text{soft}}^2 - \frac{\lambda_4 + \lambda_5}{2} v^2, \\ m_A^2 &= M_{\text{soft}}^2 - \lambda_5 v^2. \end{aligned} \right.$$

$M_{\text{soft}}$ : soft breaking scale

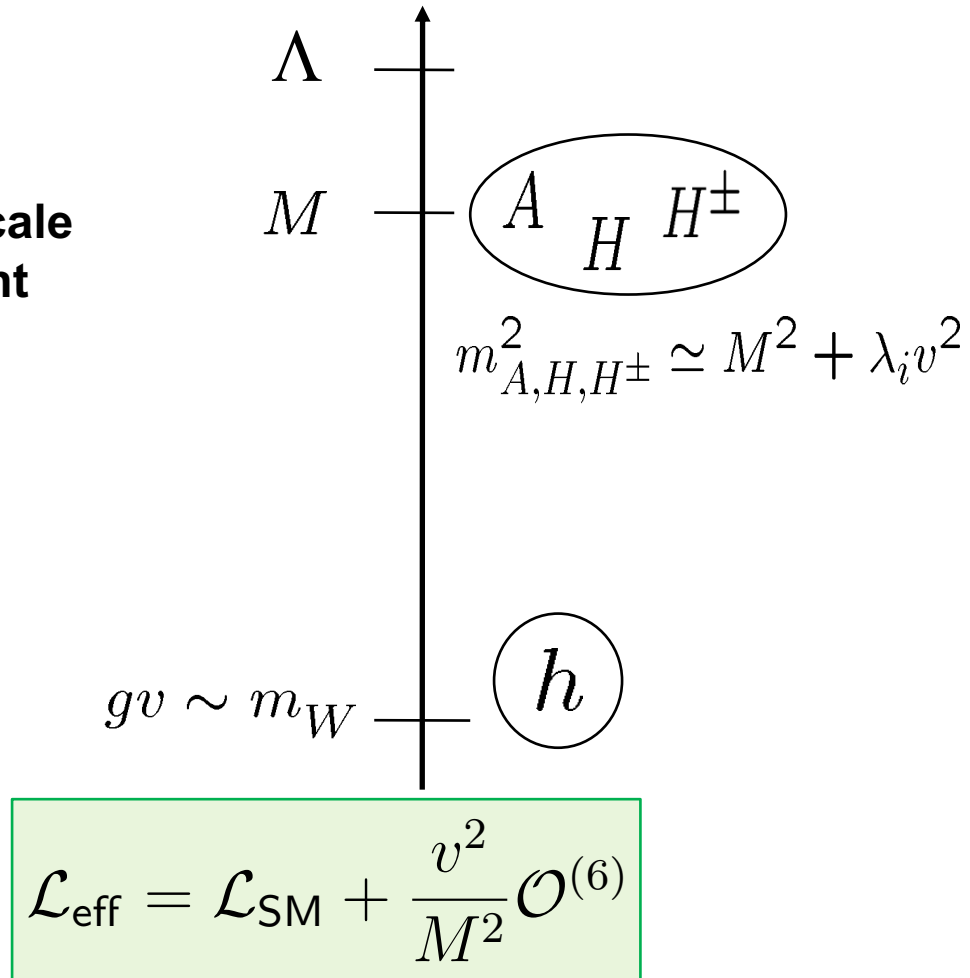
$$\frac{v_2}{v_1} \equiv \tan \beta$$

$$M_{\text{soft}} \left( = \frac{m_3}{\sqrt{\cos \beta \sin \beta}} \right):$$

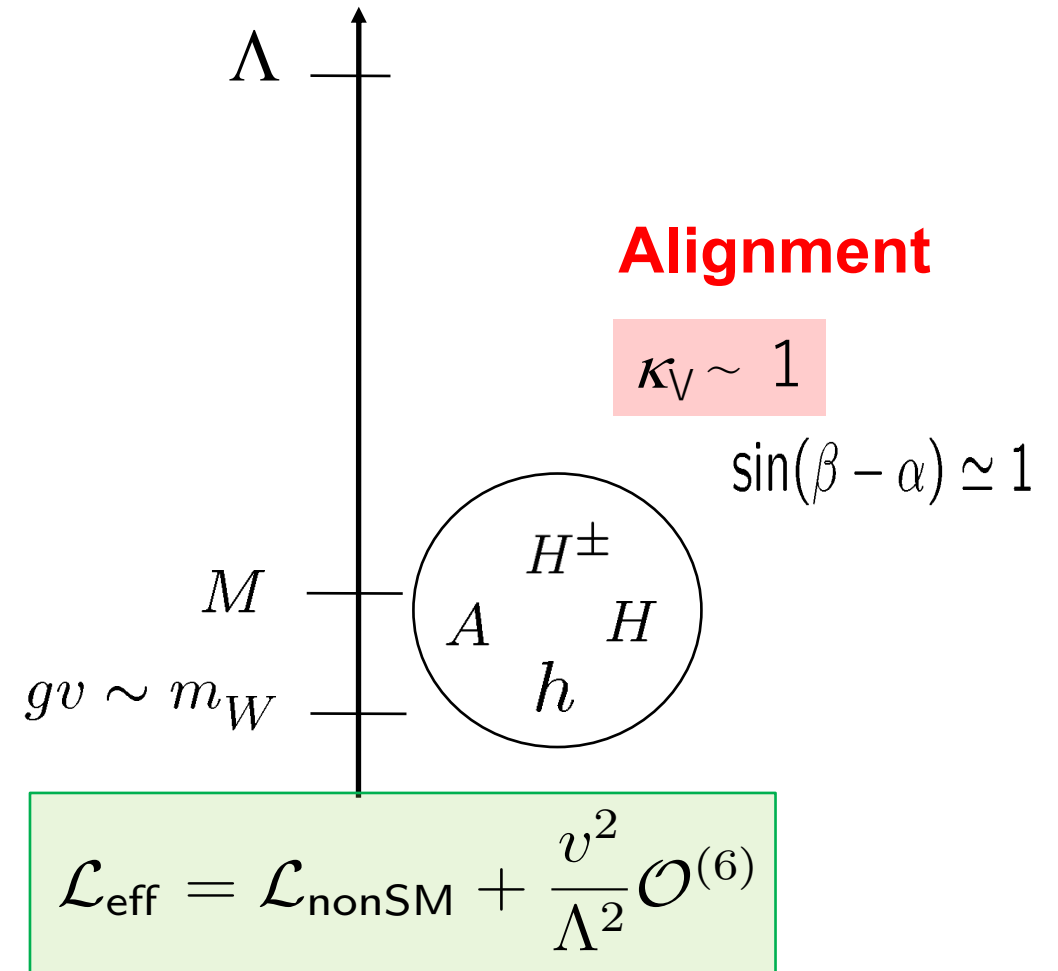
soft-breaking scale  
of the discrete symm.

# Two Possibilities satisfying current data

$\Lambda$  : Cutoff  
 $M$  : Mass scale irrelevant to VEV



Effective Theory is the SM  
**Decoupling**



Effective Theory is an extended Higgs sector  
**Alignment without decoupling**

# Small deviation ( $\kappa_V \neq 1$ ) = New Phys. Scale

No-Lose Theorem Lee, Quigg, Thacker 1977

Unitarity: Higgs boson must appear below 1 TeV,  
Otherwise, new physics must appear below 1 TeV

Motivation to build SSC, LHC

Lead to Higgs discovery at LHC!

$m_h = 125$  GeV

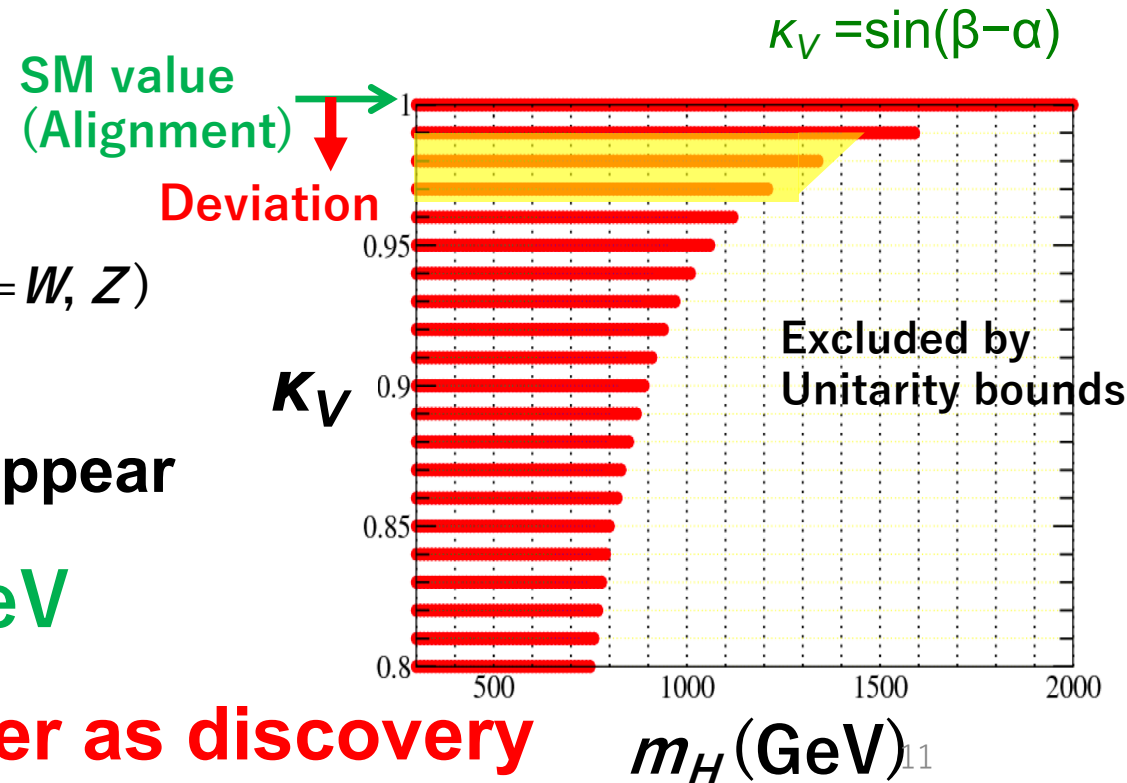
**New No-Lose Theorem**

$hVV$  coupling ( $V=W, Z$ )

Unitarity: If deviations in Higgs couplings,  
the second Higgs  $H$  or new physics must appear

$$\Delta \kappa_V = 2\% \Leftrightarrow m_H < 1.4 \text{ TeV}$$

Finding deviations has the same power as discovery



# Pattern of deviations

Gauge couplings      Yukawa couplings

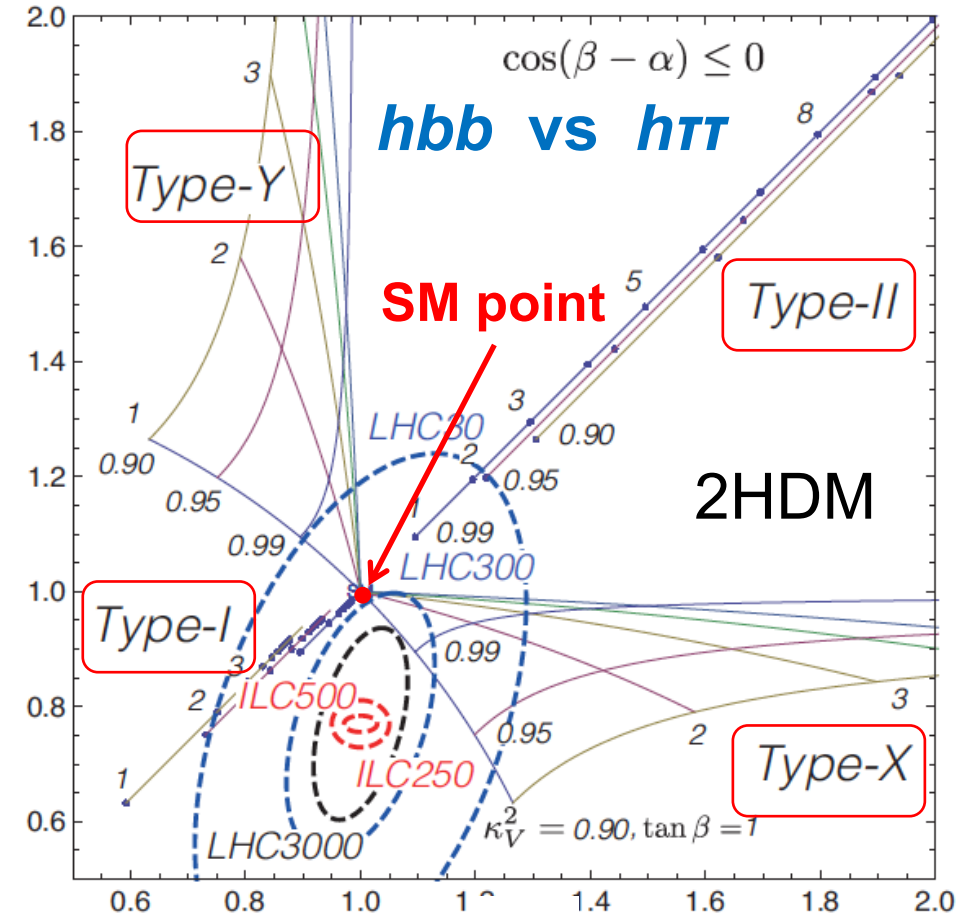
$hVV$        $h\tau\tau$        $hbb$        $hcc$

	$K_V$	$K_T$	$K_b$	$K_c$
Type-I	↓	↓	↓	↓
Type-II	↓	↑	↑	↓
Type-X	↓	↑	↓	↓
Type-Y	↓	↓	↑	↓

$\cos(\beta - \alpha) < 0$

Direction of deviation in each coupling

$K_b$



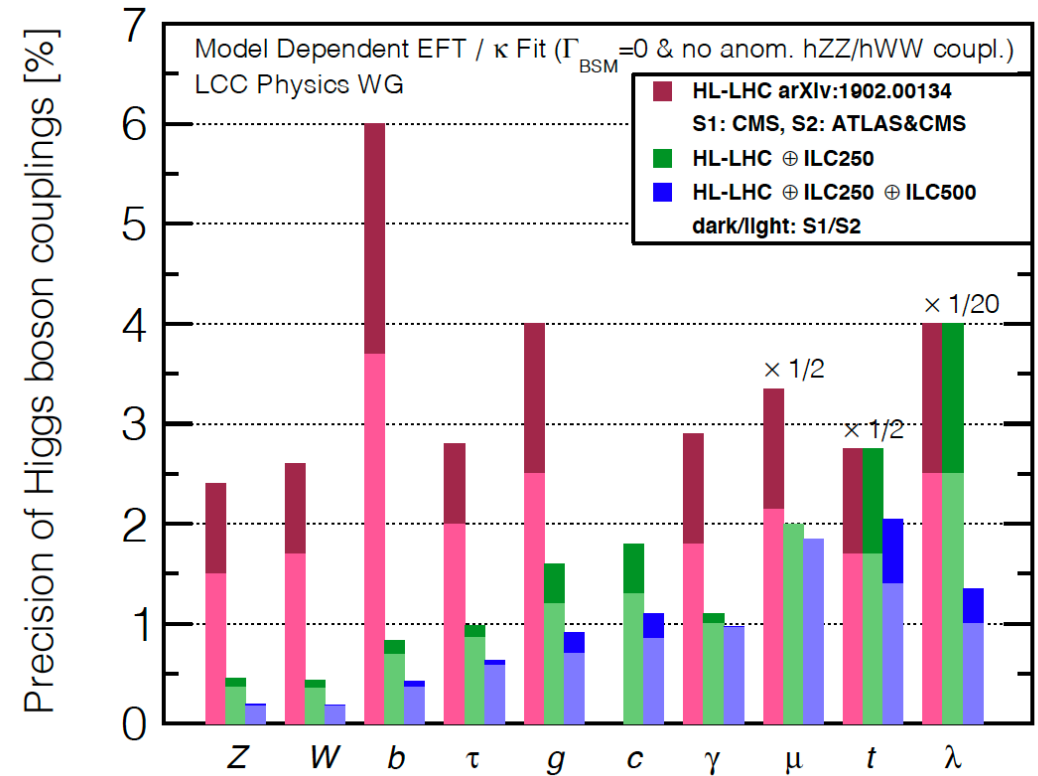
Ellipse = 68.27% CL       $K_T$

We can fingerprint extended Higgs models from the pattern of deviations in Higgs couplings

2HDM

# Radiative Corrections

1903.01629

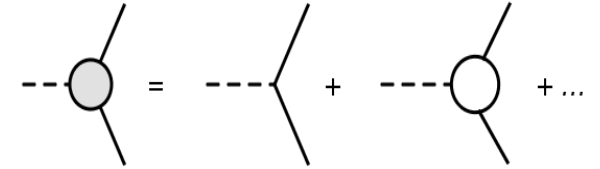


**Permille to percent level precision measurements at future colliders!**

# Radiative Corrections

Higgs couplings  $h\gamma\gamma$ ,  $hgg$ ,  $hWW$ ,  $hZZ$ ,  $h\tau\tau$ ,  $hbb$ ,  $htt$ , ... will be measured thoroughly in the future

Analyses with radiative corrections are necessary



Accurate Theory Predictions

×

Precision Measurements

New Physics!

**H-COUP Project** *SK, Kikuchi, Mawatari, Sakurai, Yagyu*

Full set of Fortran codes to systematically calculate quantum corrections to Higgs decays in various extended Higgs models [arXiv:1710.04603](https://arxiv.org/abs/1710.04603)

Program H-COUP ver. 2 [Manual [arXiv: 1910.12769](https://arxiv.org/abs/1910.12769)]

See also 2HDECAY [Klaue, Muhlleitner, Spira 1810.00768](https://arxiv.org/abs/1810.00768)

Models

Additional Singlet

2HDM (I)

2HDM (II)

2HDM (X)

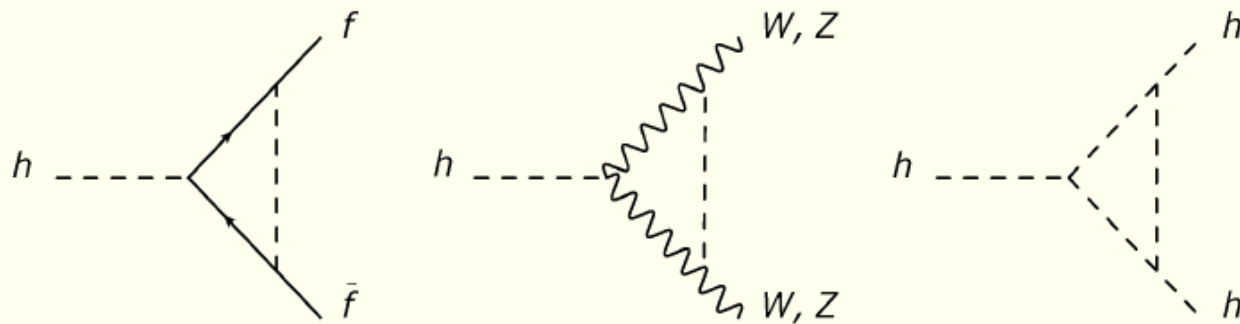
2HDM (Y)

Inert doublet/singlet

Triplet model

# *H-COUP*

Heavy Higgs decays  
will be included in ver. 3



**NEW!!** H-COUP version 2.3 was released (30 Apr. 2020)

H-COUP version 2 (1 Sep. 2019) is a calculation tool composed of a set of Fortran codes to compute the Higgs boson decay rates and the branching ratios with radiative corrections (NNLO for QCD and NLO for EW) in various non-minimal Higgs models, such as the Higgs singlet model, four types of two Higgs doublet models and the inert doublet model. H-COUP ver. 2 contains all the functions in H-COUP ver. 1.

Authors:

Shinya Kanemura, Mariko Kikuchi, Kentarou Mawatari, Kodai Sakurai and Kei Yagyu

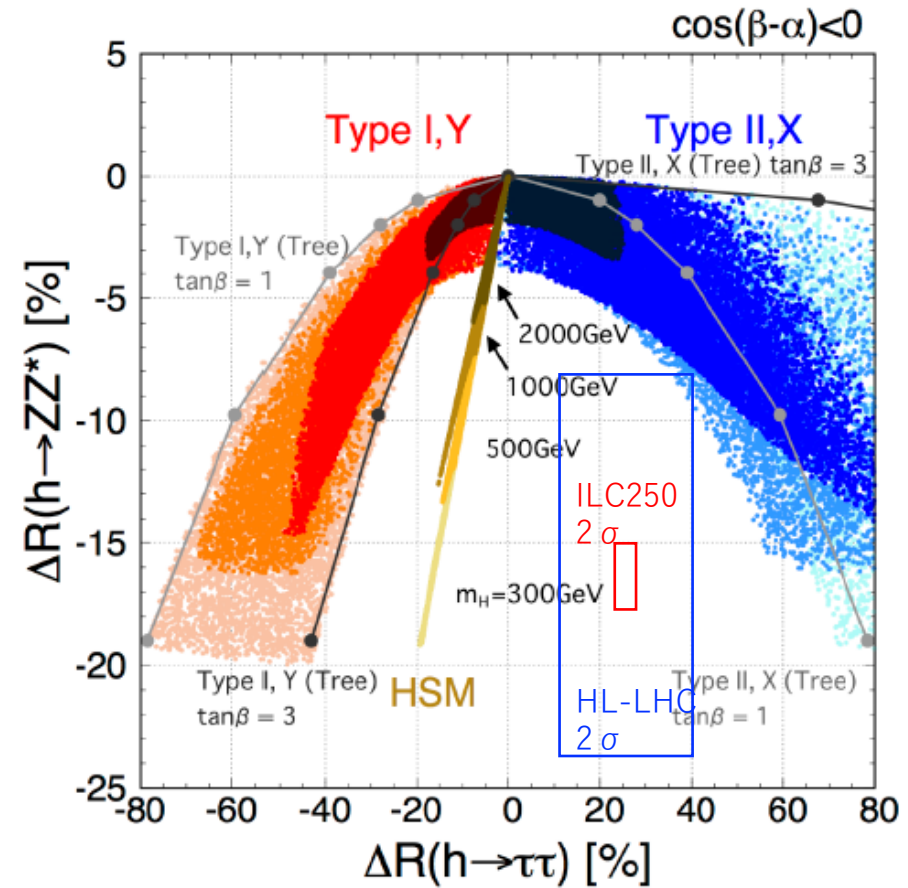
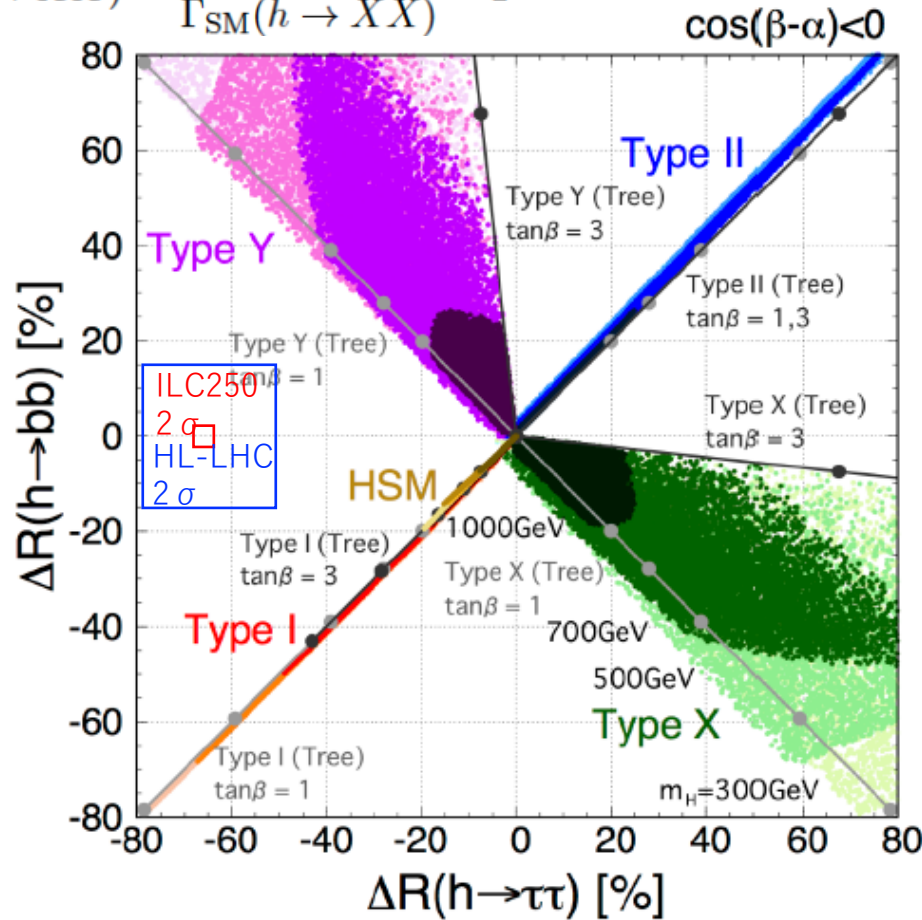
The manual for H-COUP version 2 can be taken on [arXiv:1910.12769 \[hep-ph\]](https://arxiv.org/abs/1910.12769).

# Fingerprinting the new physics

*H-COUP Project*

*SK, Kikuchi, Mawatari, Sakurai, Yagyu 2018*

$$\Delta R(h \rightarrow XX) = \frac{\Gamma_{\text{NP}}(h \rightarrow XX)}{\Gamma_{\text{SM}}(h \rightarrow XX)} - 1$$

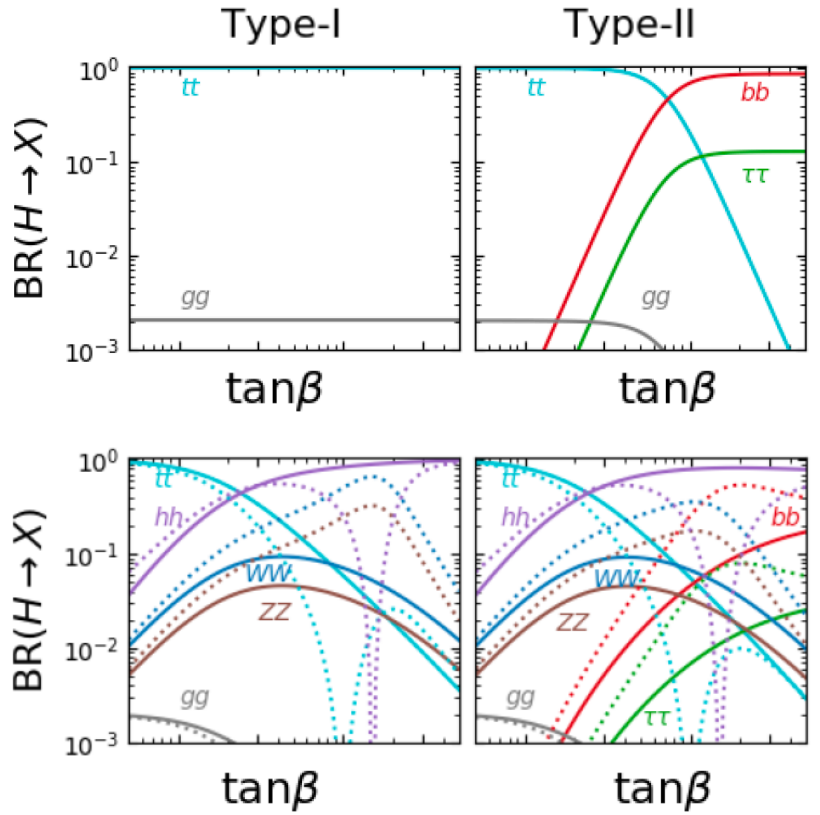


**Full set of 1-loop corrections (EW + QCD + Higgs) to the decay rates in various Higgs sectors and future precision measurements at ILC250 make us possible to fingerprint models and also to get information of inner parameters such as mass of the second Higgs boson**

# **Higgs to Higgs decays (Synergy with LHC)**

# HL-LHC and ILC250 can explore cases of $\kappa \neq 1$ but not the case of $\kappa=1$

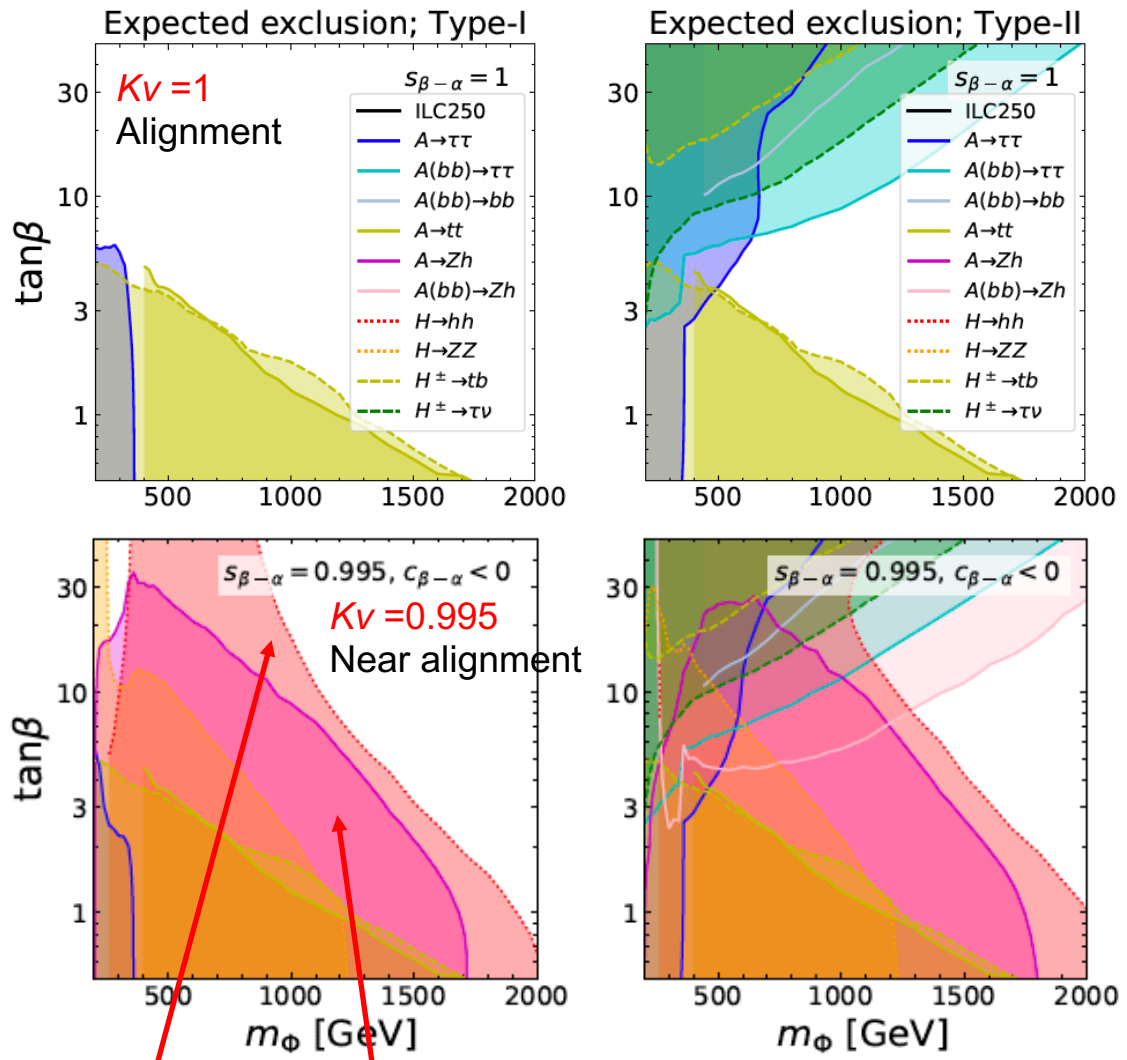
Near and exact are drastically different



$K_V = 1$   
Alignment

$K_V = 0.995$   
Near alignment  
H to H decays become important

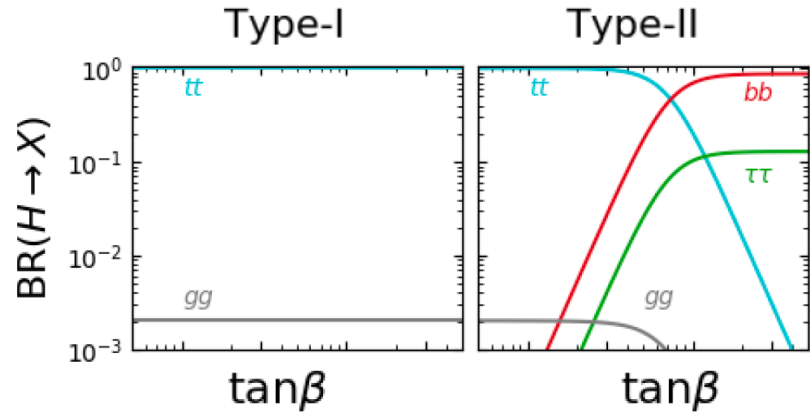
Region of alignment without decoupling can be explored by higher energy e+e- collisions, and also by the  $hhh$  measurement



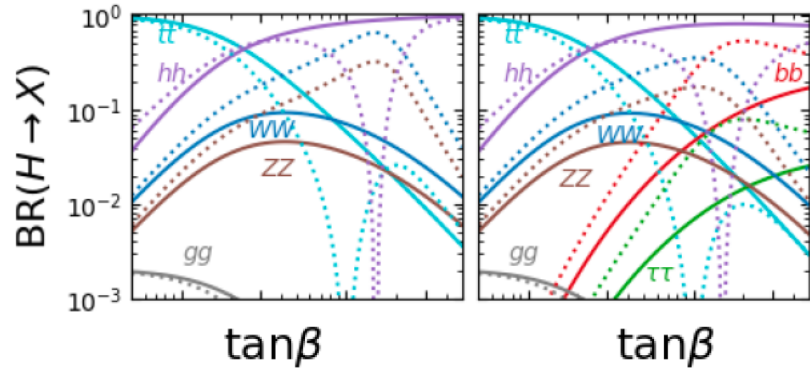
$H \rightarrow hh, A \rightarrow hZ$  very sensitive for  $\kappa \neq 1$

HL-LHC and the Higgs Factory (CEPC) can explore cases of  $\kappa \neq 1$  but not the case of  $\kappa = 1$

Near and exact are drastically different



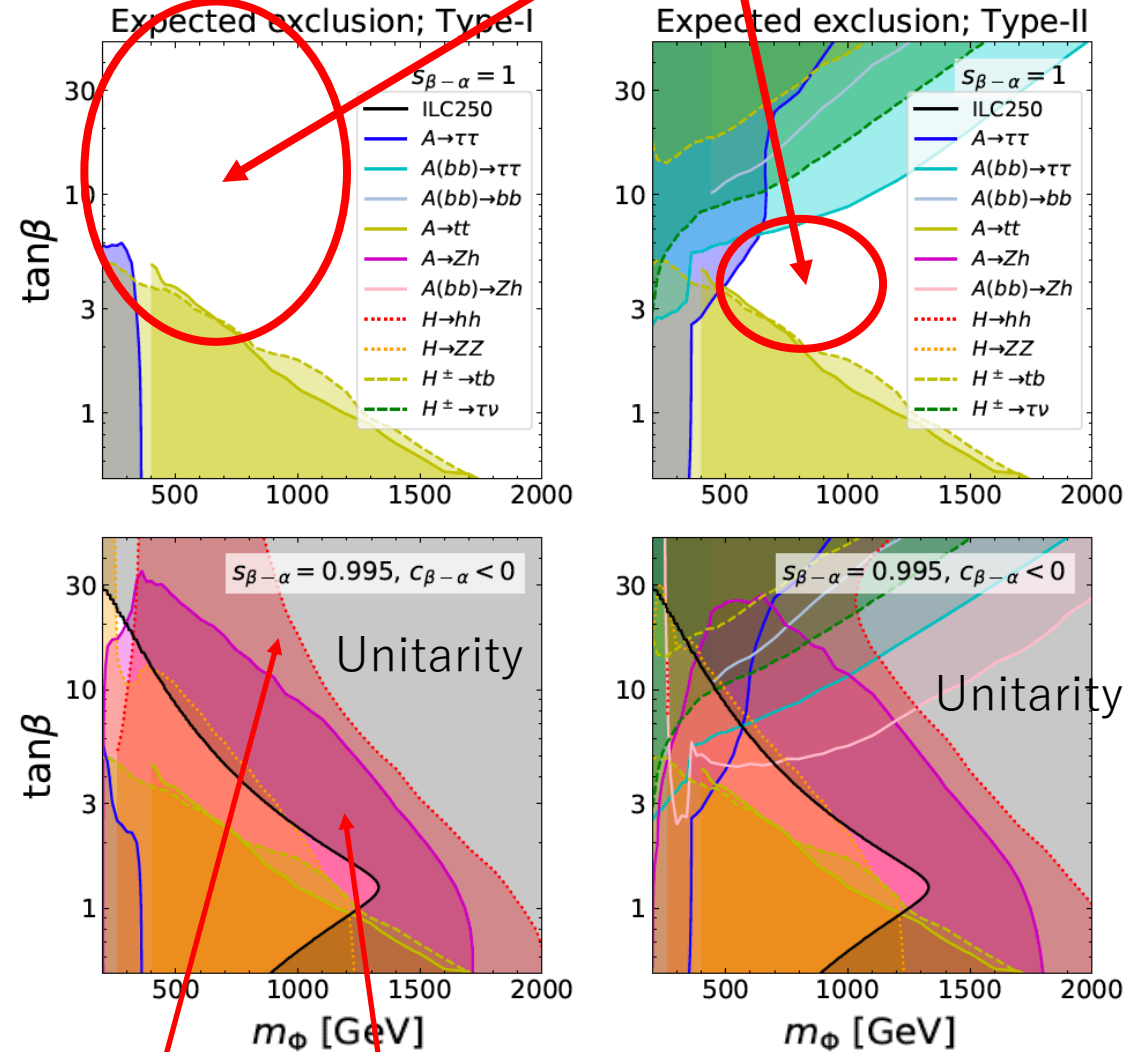
$Kv = 1$   
Alignment



$Kv = 0.995$   
Near alignment  
H to H decays become important

Region of alignment without decoupling can be explored by higher energy e+e- collisions, and also by the  $hhh$  measurement

Regions of alignment without decoupling



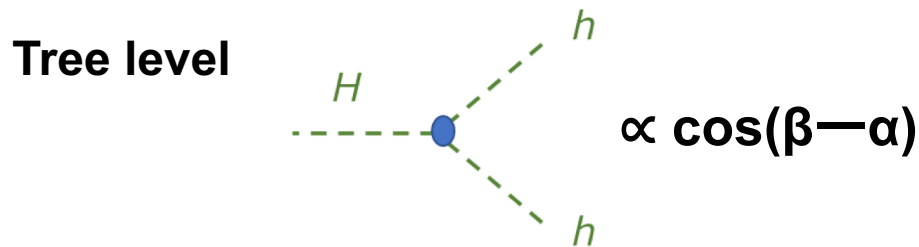
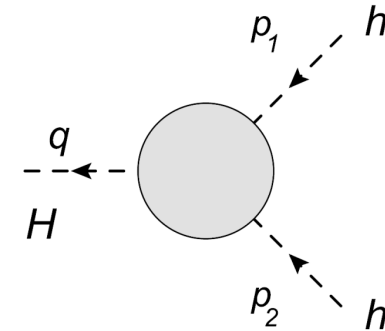
$H \rightarrow hh, A \rightarrow hZ$  very sensitive for  $\kappa \neq 1$

# Radiative Corrections to $H \rightarrow hh$

SK, Kikuchi, Yagyu

$$\Gamma[H \rightarrow hh] \propto \left| H \begin{array}{c} \diagup h \\ \diagdown h \end{array} + \text{1PI} + \text{crossed} \right|^2$$

Renormalization in the improved on-shell scheme  
(scheme in H-COUP projects)



Alignment limit  
 $\Gamma_{tree} = 0$

One-loop corrected amplitude

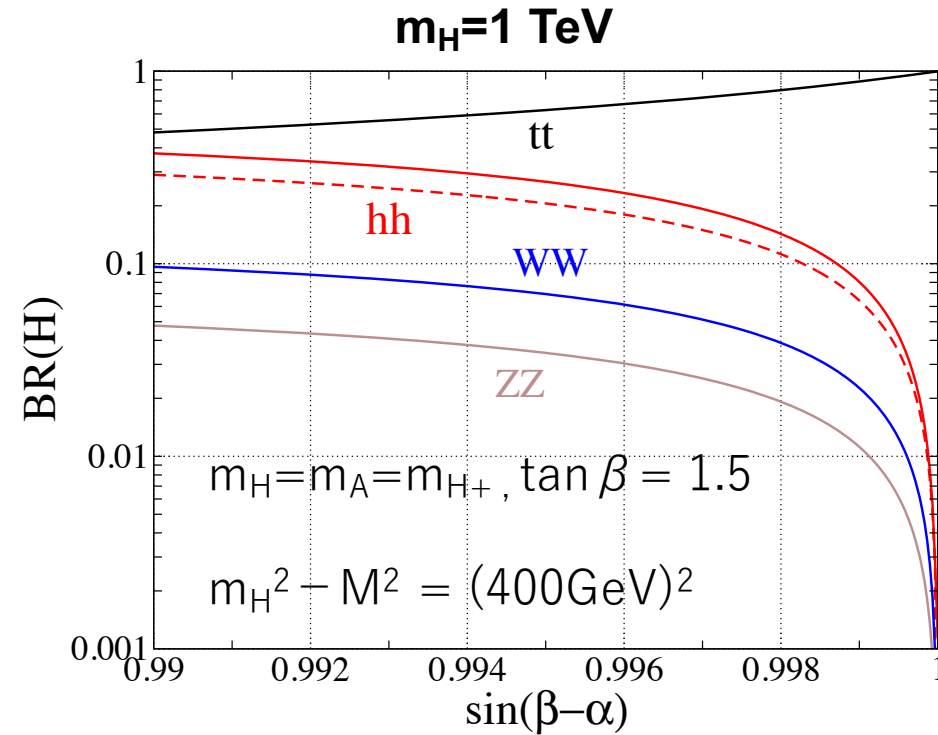
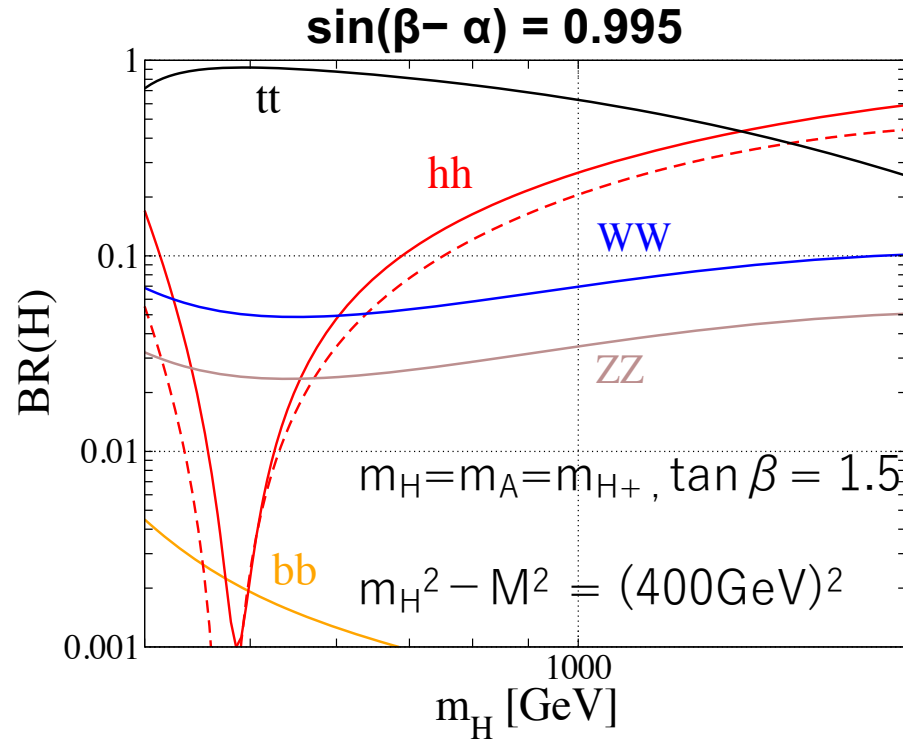
$$\Gamma_{tree} = \cos^2(\beta - \alpha) \tilde{\Gamma}_{tree}$$

$$\Gamma = \underbrace{\tilde{\Gamma}_{tree}}_{\text{tree}} \left( \underbrace{c_{\beta-\alpha}^2}_{\text{tree}} + \underbrace{c_{\beta-\alpha} \xi \text{Re}[\Delta_{1-loop}] + \zeta |\Delta_{1-loop}|^2}_{\text{1-loop}} \right)$$

Near alignment ( $c_{\beta-\alpha} \ll 1$ ), one-loop contributions become important!

Preliminary

# $B(H \rightarrow hh)$ at one loop

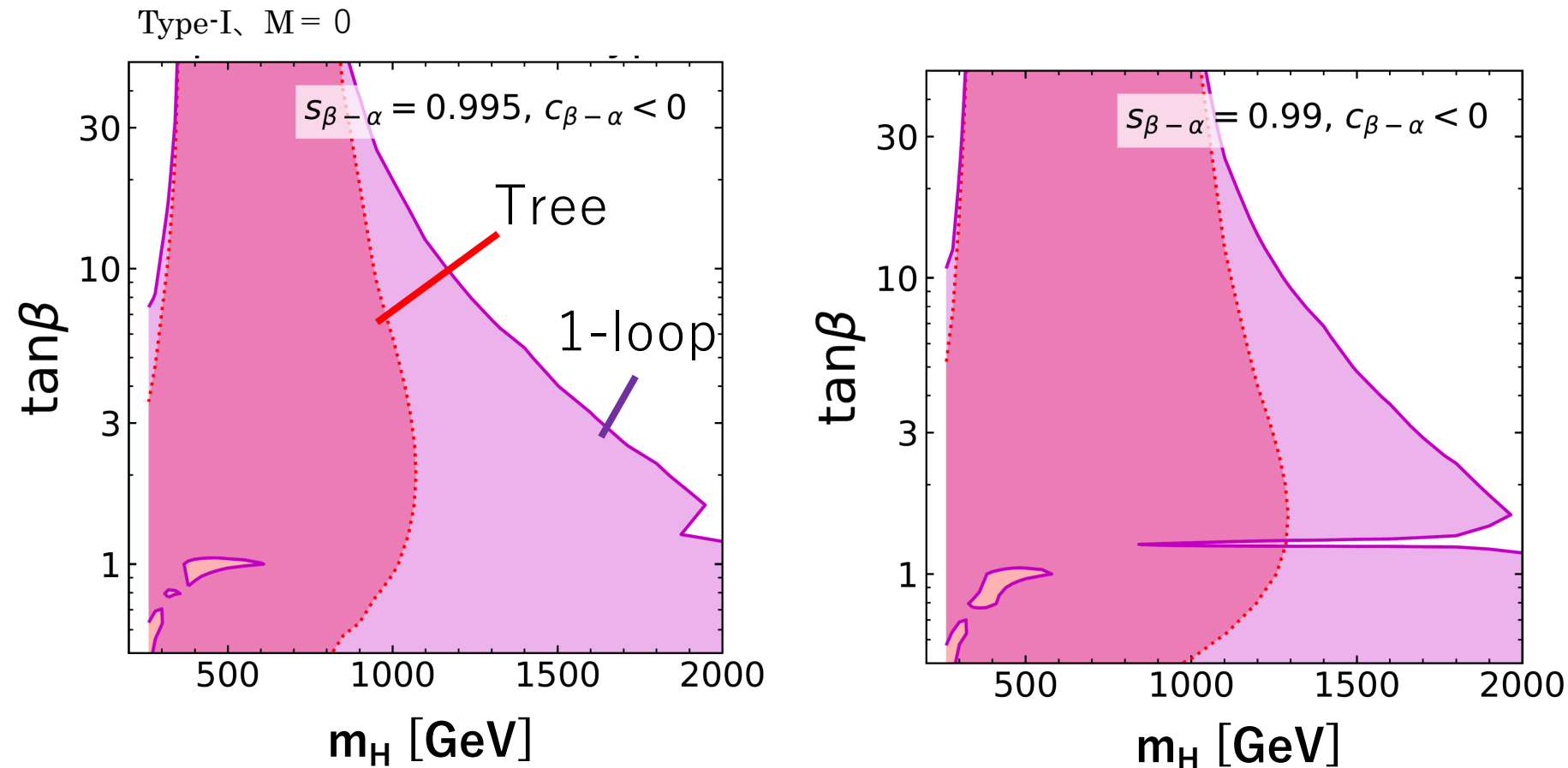


The one-loop correction can be 30% near alignment

Excluded region with radiative corrections should be necessary (under go)

# Excluded region by $H \rightarrow hh$ at HL-LHC

Preliminary



One loop results can be drastically different from the tree level results.  
Analyses including radiative correction are very important for Higgs to Higgs decays too!!

# Alignment without decoupling

If  $\kappa_V \neq 1$  is not detected, two possibilities

(1) Alignment with decoupling

New physics out of reach?

(2) Alignment without decoupling

Even  $\kappa_V=1$ , the Higgs self-coupling can still exhibit large deviations from the SM predictions because of large non-decoupling effects in BSM loops

$O(10) - O(100)$  % deviation possible in extended Higgs sectors

SK, Okada, Senaha, Yuan, 2004, etc

Expected measurement of the hhh coupling

HL-LHC:	~ 50-100%
0.5-1TeV lepton linear colliders:	~ 10-30%
100TeV hadron colliders	~ 5-7%

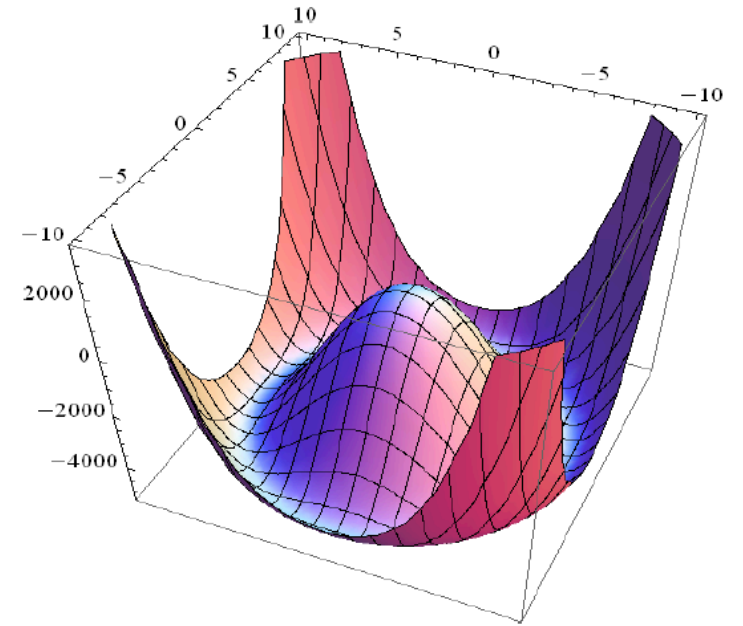
de Blas et al. 1905.03764,  
Cepeda et al. 1902.00134  
Di Vita et al. 1711.03978  
Fujii et al. 1506.05992  
Roloff et al. 1901.05897  
Chang et al. 1908.00753  
Etc ...

# Triple Higgs boson coupling

# Higgs Potential

Dynamics of EWSB  $SU(2)_I \times U(1)_Y \rightarrow U(1)_{em}$

It is important to know the  $hhh$  coupling to reconstruct the Higgs potential



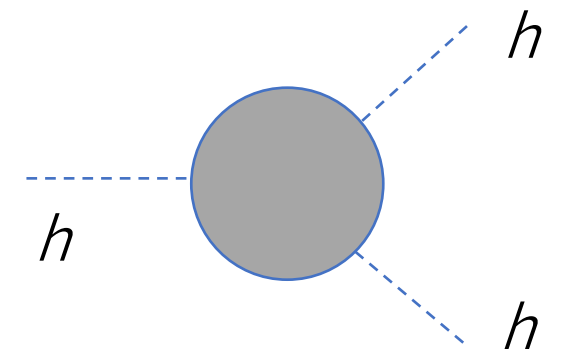
$$V_{\text{Higgs}} = \frac{1}{2} \underline{m_h^2} h^2 + \frac{1}{3!} \underline{\lambda_{hhh}} h^3 + \frac{1}{4!} \lambda_{hhhh} h^4 + \dots$$

Top loop effect on the  $hhh$  coupling

$$\lambda_{hhh}^{\text{SMloop}} \sim \frac{3m_h^2}{v} \left( 1 - \frac{N_c m_t^4}{3\pi^2 v^2 m_h^2} + \dots \right)$$

Top loop effect in the SM

**Non-decoupling effect**



How about new physics effect?

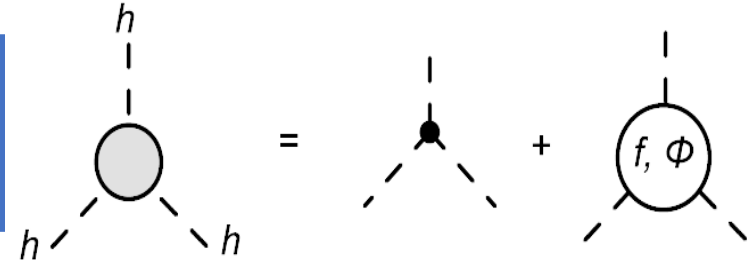
# The $hhh$ coupling in extended Higgs sectors

Alignment  
2HDM

$$\lambda_{hhh}^{2\text{HDM}} \simeq \frac{3m_h^2}{v} \left[ 1 + \frac{m_\Phi^4}{12\pi^2 m_h^2} \left( 1 - \frac{M^2}{m_\Phi^2} \right)^3 - \frac{m_t^4}{\pi^2 v^2 m_h^2} \right]$$

Extra scalar  
loop

Top loop



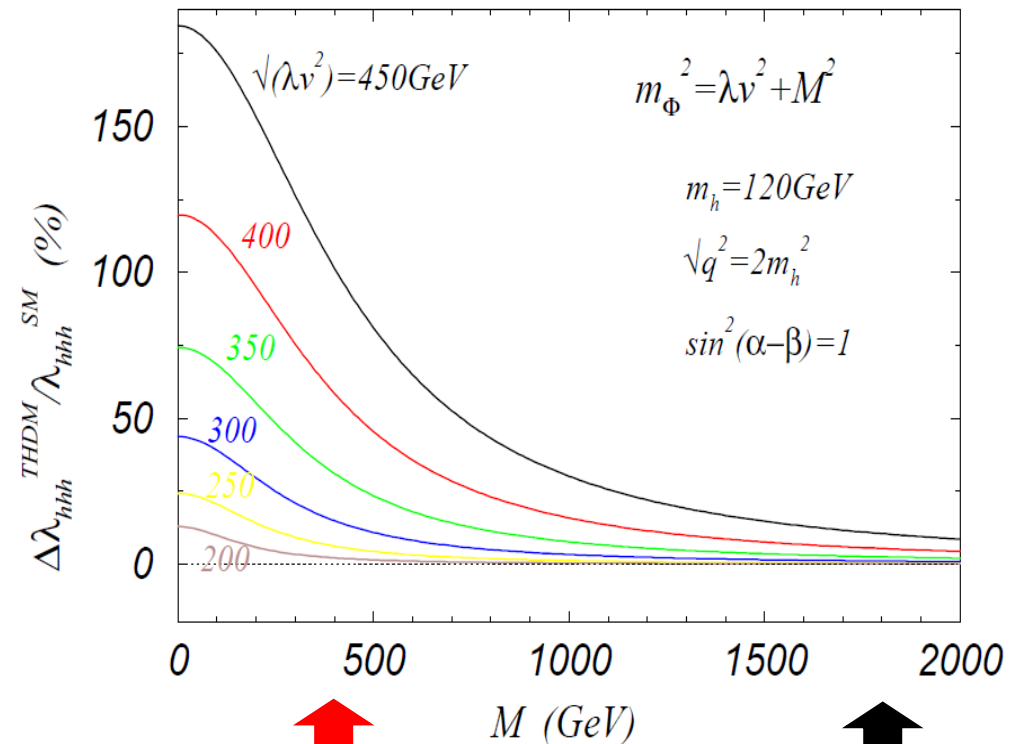
$\Phi = H, A, H^\pm$

Mass of extra Higgs bosons

$$m_\Phi^2 = M^2 + \lambda_i v^2 \quad (\Phi = H, A, H^\pm)$$

- Alignment Case
- At 1-loop, powerlike effect  $m_\Phi^4$

Deviation can be very large  
 $\sim 100\%$  (NLO)  
under constraint from  
unitarity bound



Non-decoupling effect

Decoupling

# 2 loop calculation for the $hhh$ coupling

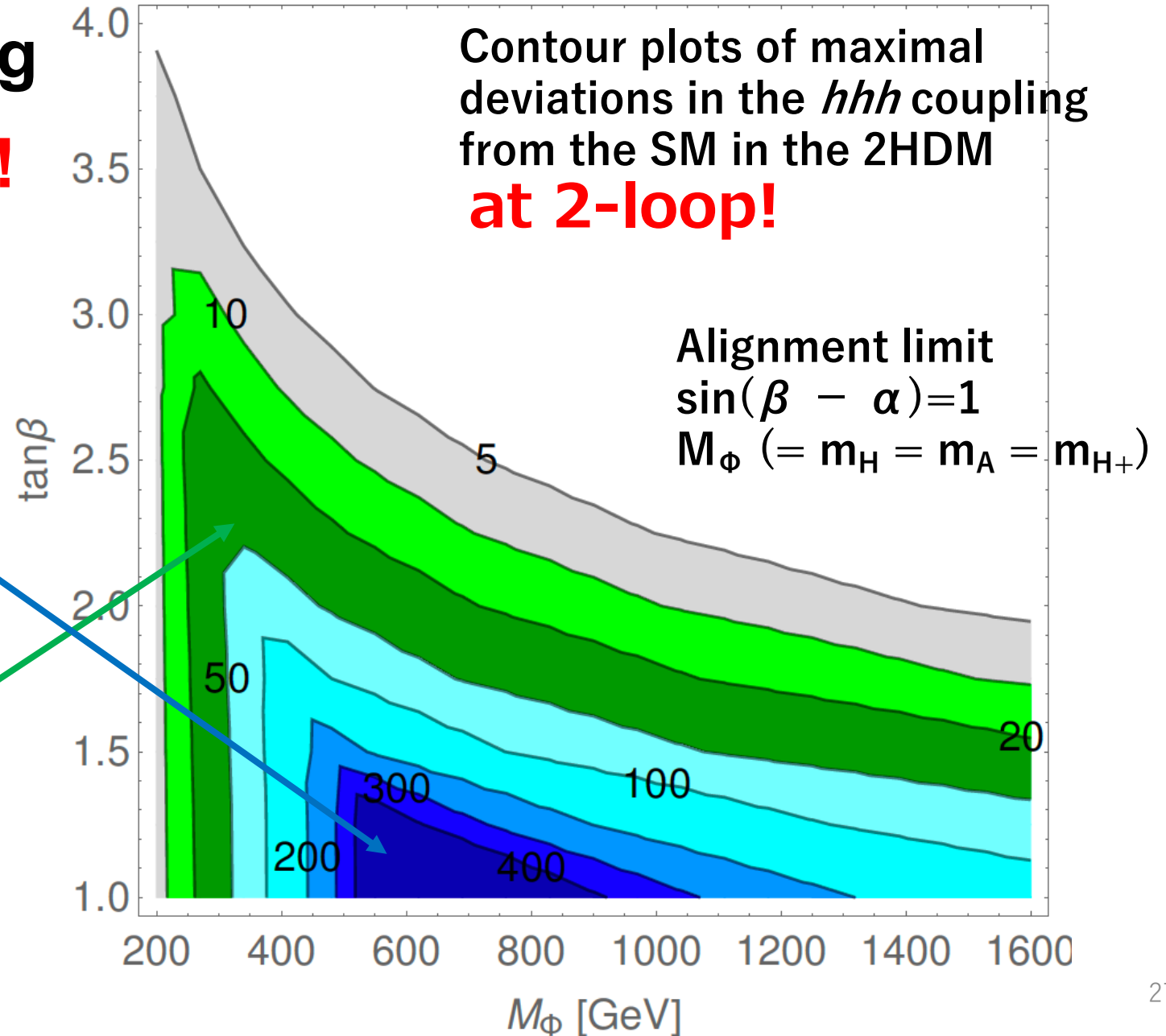
See Braathen's Talk!

HL-LHC  
can test?

ILC1TeV  
CLIC,  
HE-LHC, ...  
can test?

$\delta R$  [%]

J. Braathen, SK (2019)



# 1<sup>st</sup> Order Phase Transition

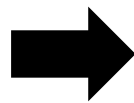
## Electroweak Baryogenesis

Sakharov 3<sup>rd</sup> condition

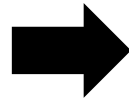
Departure from Thermal Equilibrium

Sphaleron Decoupling condition

$$\Gamma_{\text{sph}} < H(T_c)$$

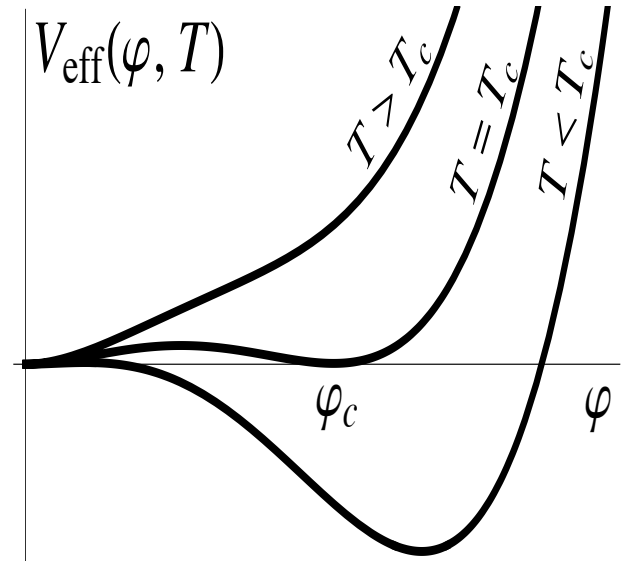


$$\frac{\varphi_c}{T_c} \gtrsim 1$$



Strong 1st Order Phase Transition

$$\Gamma_{\text{sph}} \sim e^{-E_{\text{sph}}/T_c} \sim e^{-\alpha' \varphi_c/T_c}$$



High Temp Exp.

$$V_{\text{eff}}(\varphi, T) \simeq D(T^2 - T_0^2)\varphi^2 - ET\varphi^3 + \frac{\lambda_T}{4}\varphi^4 + \dots$$

$$\frac{\varphi_C}{T_C} = \frac{2E}{\lambda_T} = \frac{6m_W^3 + 3m_Z^3 + \dots}{3\pi v m_h^2} \ll 1$$

SM with  $m_h=125\text{GeV}$ ,  
1<sup>st</sup> OPT not realized

# 1<sup>st</sup> Order Phase Transition

Extended Higgs: Strongly 1<sup>st</sup> OPT possible

Quantum effect of additional Scalar field  $\Phi$  ( $= H, A, H^+, \dots$ )

Realized by Quantum Effect by  $\Phi$  ( $= H, A, H^+, \dots$ )

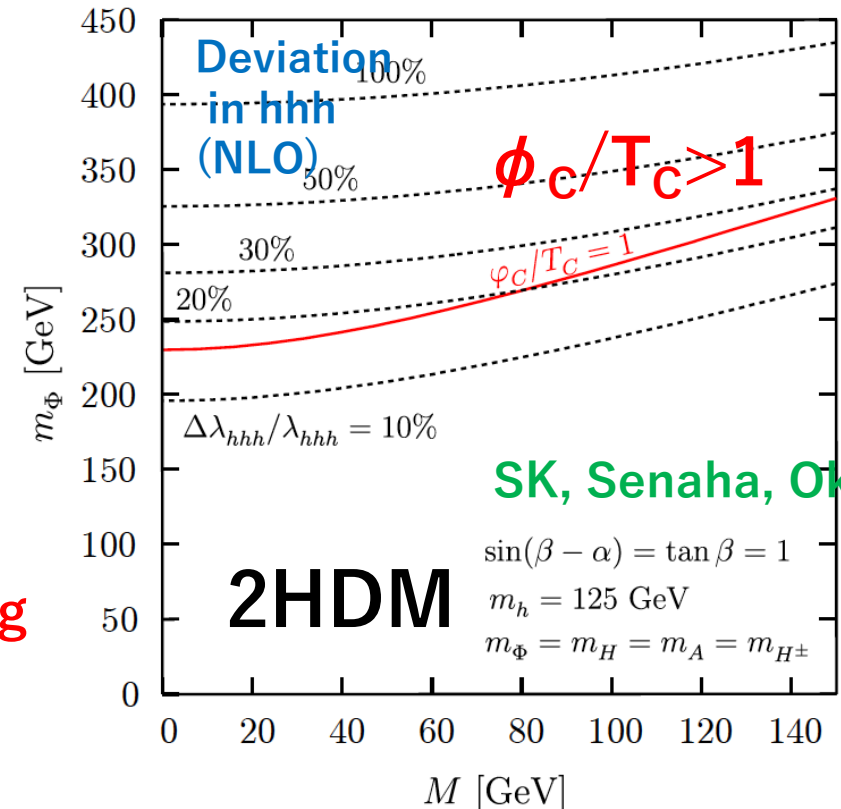
$$\frac{\varphi_C}{T_C} \simeq \frac{1}{3\pi v m_h^2} \left\{ 6m_W^3 + 3m_Z^3 + \sum_{\Phi} m_{\Phi}^3 \left( 1 - \frac{M^2}{m_{\Phi}^2} \right)^3 \left( 1 + \frac{3M^2}{2m_{\Phi}^2} \right) \right\} > 1$$



Large deviation in the  $hhh$  coupling

$$\lambda_{hhh} \simeq \frac{3m_h^2}{v^2} \left\{ 1 - \frac{m_t^4}{\pi^2 v^2 m_h^2} + \sum_{\Phi} \frac{m_{\Phi}^4}{12\pi^2 v^2 m_h^2} \left( 1 - \frac{M^2}{m_{\Phi}^2} \right)^3 \right\} > \lambda_{hhh}^{SM}$$

1<sup>st</sup> OPT can be tested by measuring the  $hhh$  coupling



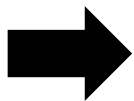
# Improvement

SK, M. Tanaka  
Preliminary

Sphaleron Decoupling Condition

$$\Gamma_{\text{sph}} < H(T_c)$$

$$\frac{\varphi_c}{T_c} \gtrsim 1$$



$$\frac{\phi_B(r=0)}{T_n} > \zeta_{\text{sph}}(T_n)$$

More Exact Form

$$\zeta_{\text{sph}}(T) \equiv \frac{g_2}{4\pi\mathcal{E}(T)} \left[ 42.8 + 7 \ln \frac{v(T)}{T} - \ln \left( \frac{T}{100\text{GeV}} \right) + \dots \right]$$

Gan, Long, Wang, 2017

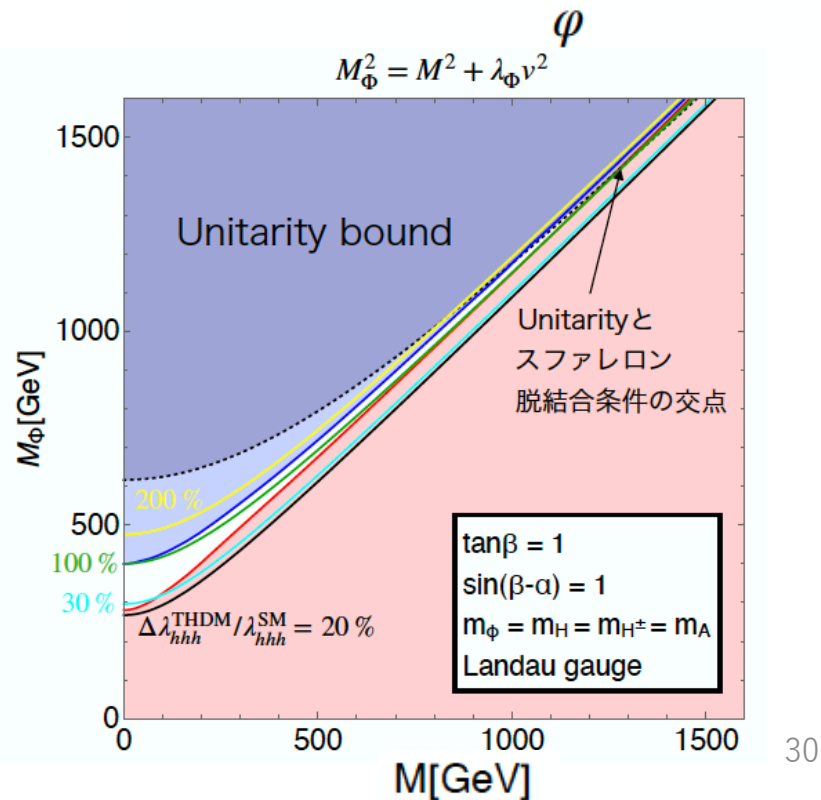
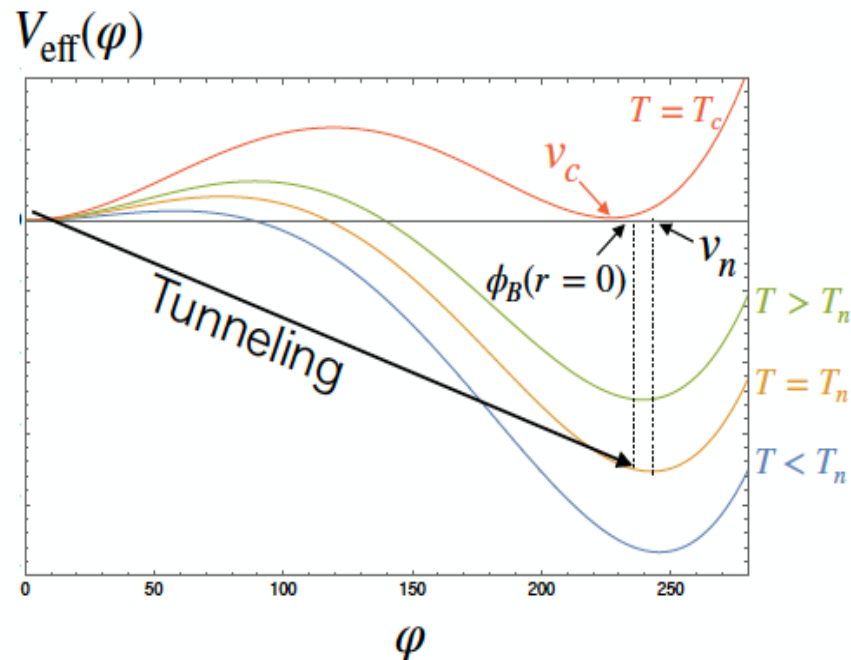
Improvement

Sphaleron Energy  
Temp.  $T_n$  dependences  
Use  $\phi_B$  instead of  $\varphi_c$

$$\frac{\Delta\lambda_{hhh}^{\text{THDM}}}{\lambda_{hhh}^{\text{SM}}} = 25 - 100 \%$$

$$m_\phi < 1.4 \text{ TeV}$$

with  
 $\Gamma/H > 1$   
unitarity bound



# Future measurement of the hhh coupling

HE-LHC, FCC<sub>hh</sub>, ...

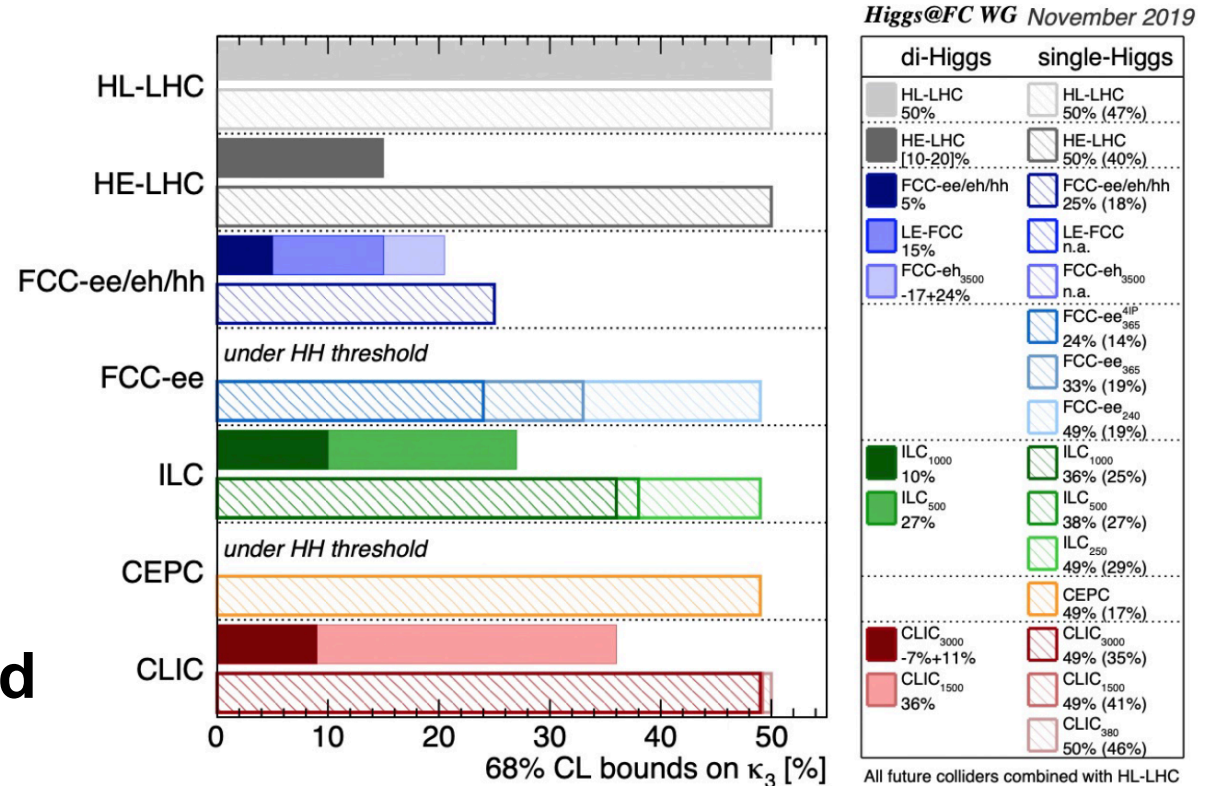
$$pp \rightarrow hhX$$

ILC500, ILC1000, CLIC, ...

$$e^+e^- \rightarrow Zhh$$

$$e^+e^- \rightarrow \nu\bar{\nu}hh$$

The triple Higgs coupling is expected to be measured with **10%** at ILC1000, CLIC3000, FCC<sub>hh</sub>, ...



arXiv: 1905.03764

We do not miss to test the 1<sup>st</sup> order phase transition for EW baryogenesis

# Synergy with GW experiments

## Singlet-Doublet mixing model

$\Phi$ : isospin doublet scalar (SM Higgs)

$S$ : singlet scalar

$$V_0 = -\mu_\Phi^2 |\Phi|^2 + \lambda_\Phi |\Phi|^4 + \mu_{\Phi S} |\Phi|^2 S + \frac{\lambda_{\Phi S}}{5} |\Phi|^2 S^2 + \mu_S^3 S + \frac{m_S^2}{2} S^2 + \frac{\mu'_S}{3} S^3 + \frac{\lambda_S}{4} S^4$$

Scaling factor of  $h$ -couplings:  $\kappa = \cos\theta$

$$h = \varphi \cos\theta \varphi + S \sin\theta$$

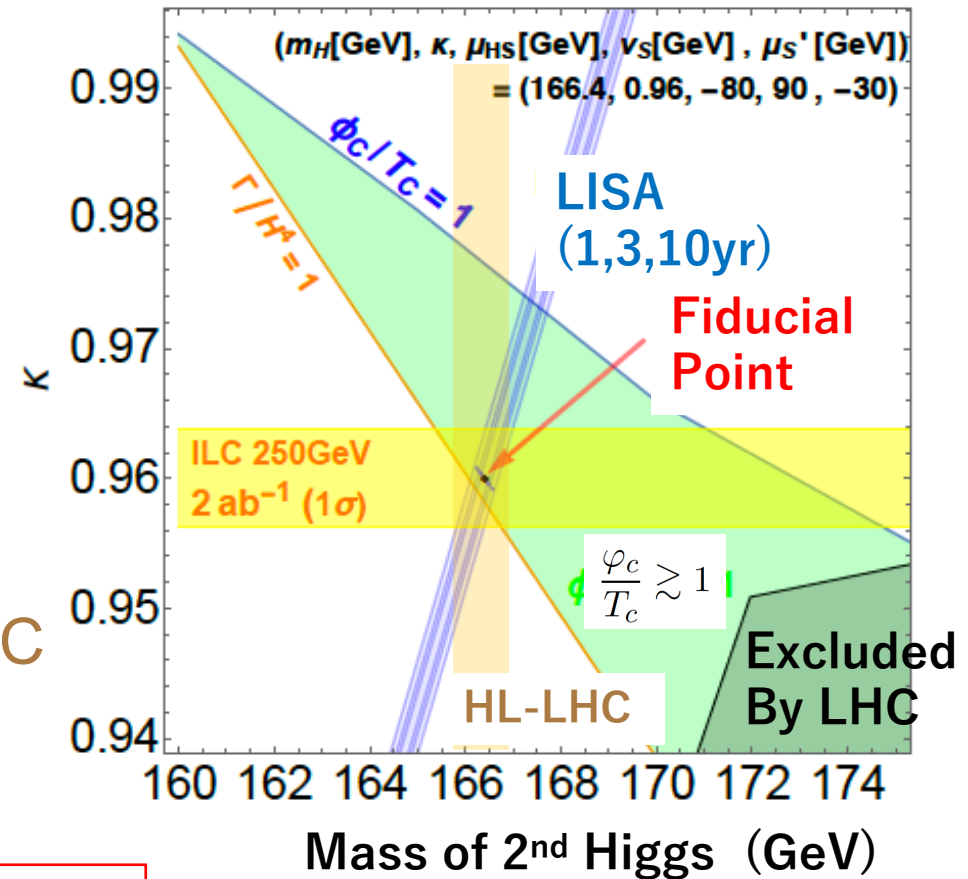
Condition for 1<sup>st</sup> OPT ( $\varphi_c/T_c > 1$ )

Searches of 2<sup>nd</sup> Higgs ( $H$ ) at HL-LHC

Coupling measurements at ILC250

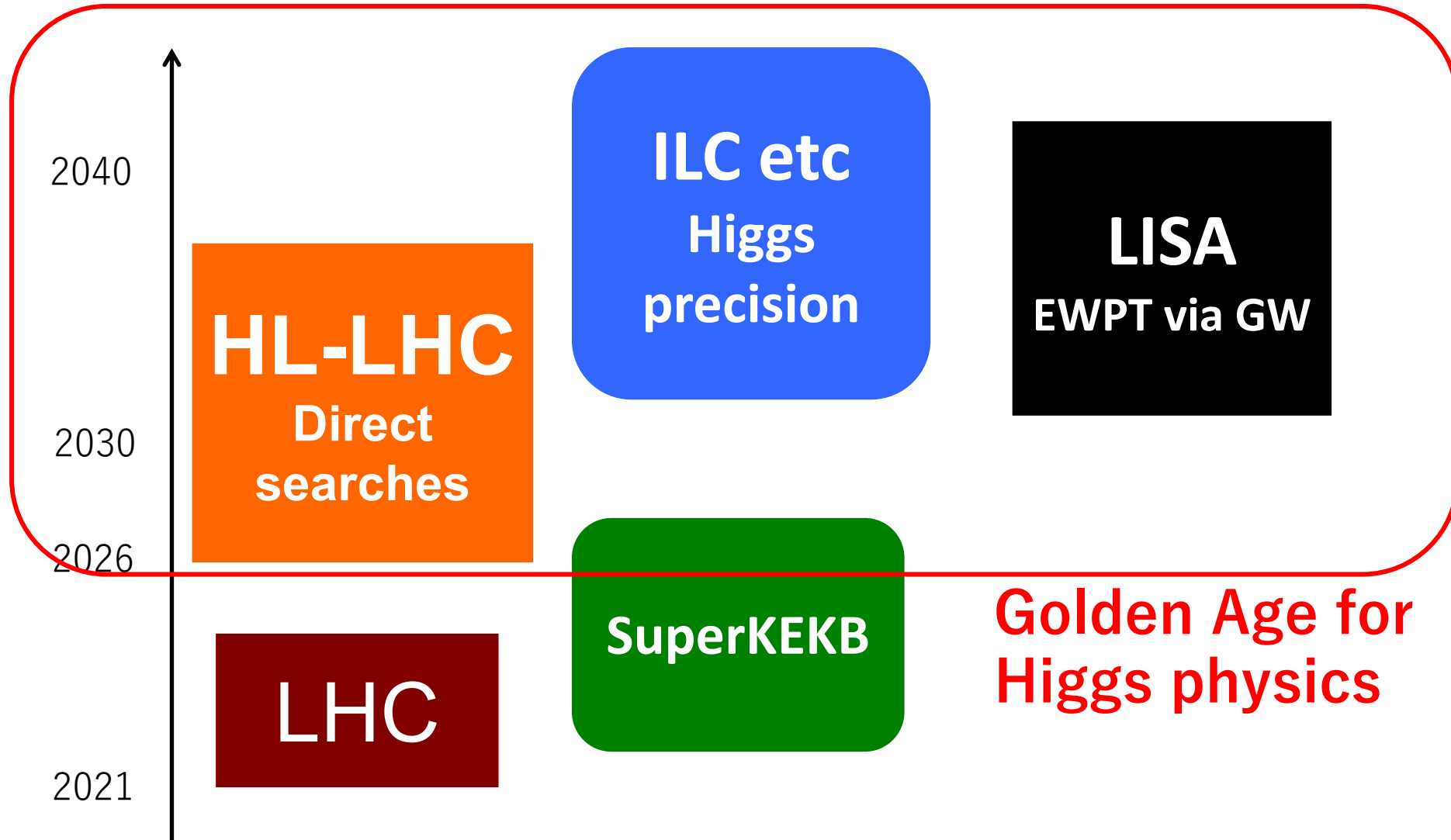
GWs measurements at LISA

The synergy between colliders and GW can narrow down the BSM model



Hashino, Kakizaki, Jinno, SK, Takahashi, Takimoto (2019)

# Future experiments



# Summary

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- Precision study of  $h(125)$  with radiative corrections in extended Higgs sectors are important to explore BSM at  $e^+e^-$  colliders.
- At HL-LHC direct searches for additional Higgs bosons, Higgs to Higgs decays ( $H \rightarrow h h, A \rightarrow h Z, \dots$ ) are crucial in the non-alignment ( $\kappa_V \neq 1$ ) case. Study with radiative corrections necessary.
- Synergy of HL-LHC (direct searches of  $H, A, H^+, \dots$ ) and ILC250 can explore most of the cases with  $\kappa_V \neq 1$ .
- If there is nothing (no deviation, no new particles) found,
  - (1) Just decoupling
  - (2) Alignment w/o decoupling ( $\kappa=1$ )
- Case (2) (where 1<sup>st</sup> Order PT occurs) can be tested by measuring in the  $hhh$  coupling at future ILC500, ILC1000, CLIC, FCC<sub>hh</sub>. Synergy with future GW experiments is also interesting.

**Higgs precision is important as a probe of new physics!**

**Thank you for your attention!**