Transient property analysis of the APS cavity in E-Driven capture linac

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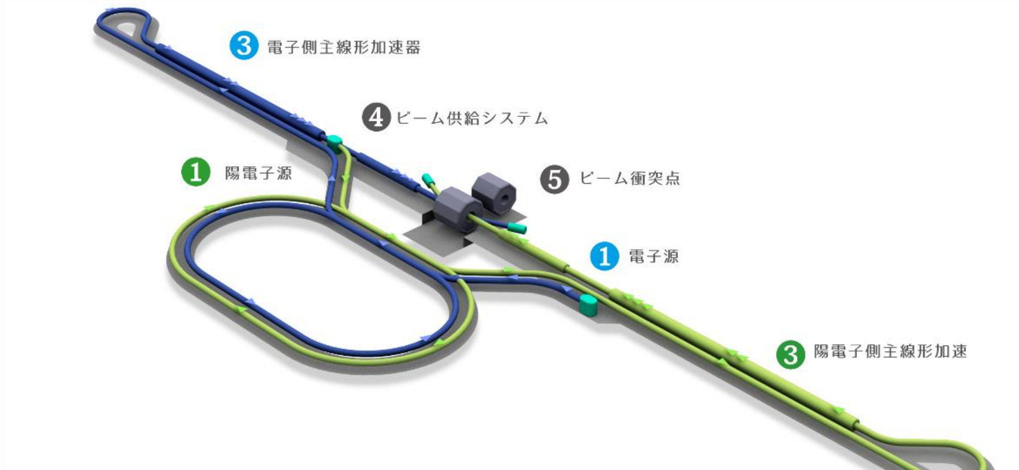
ILC(International Linear Collider)

- Electron-positron linear collider.

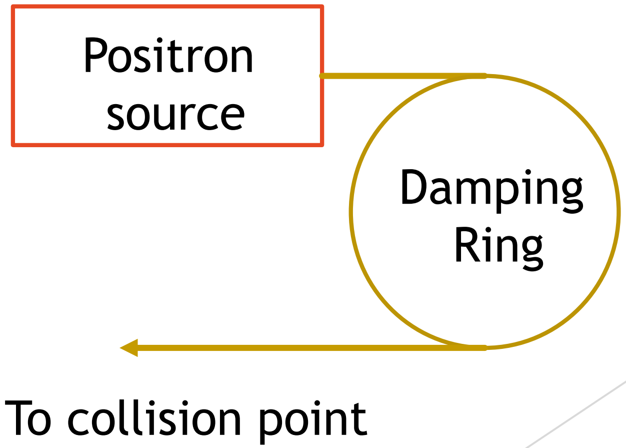
- Center of mass energy is 250GeV~1TeV.

- Expected to find new physics (ex. Higgs particle supersymmetric particle).

- Kitakami mountain(Iwate, Japan) is one candidate for ILC.



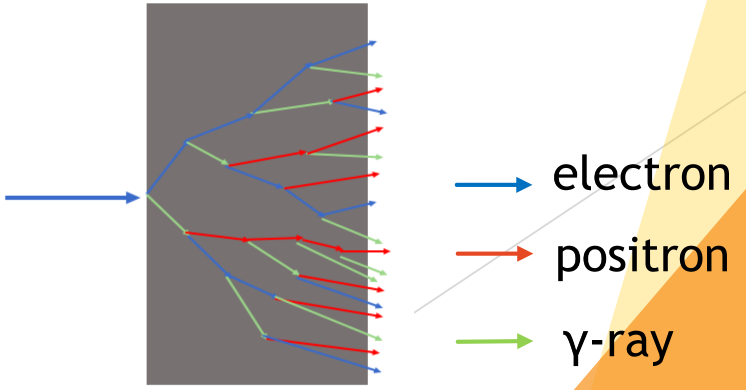
Schematic view of ILC



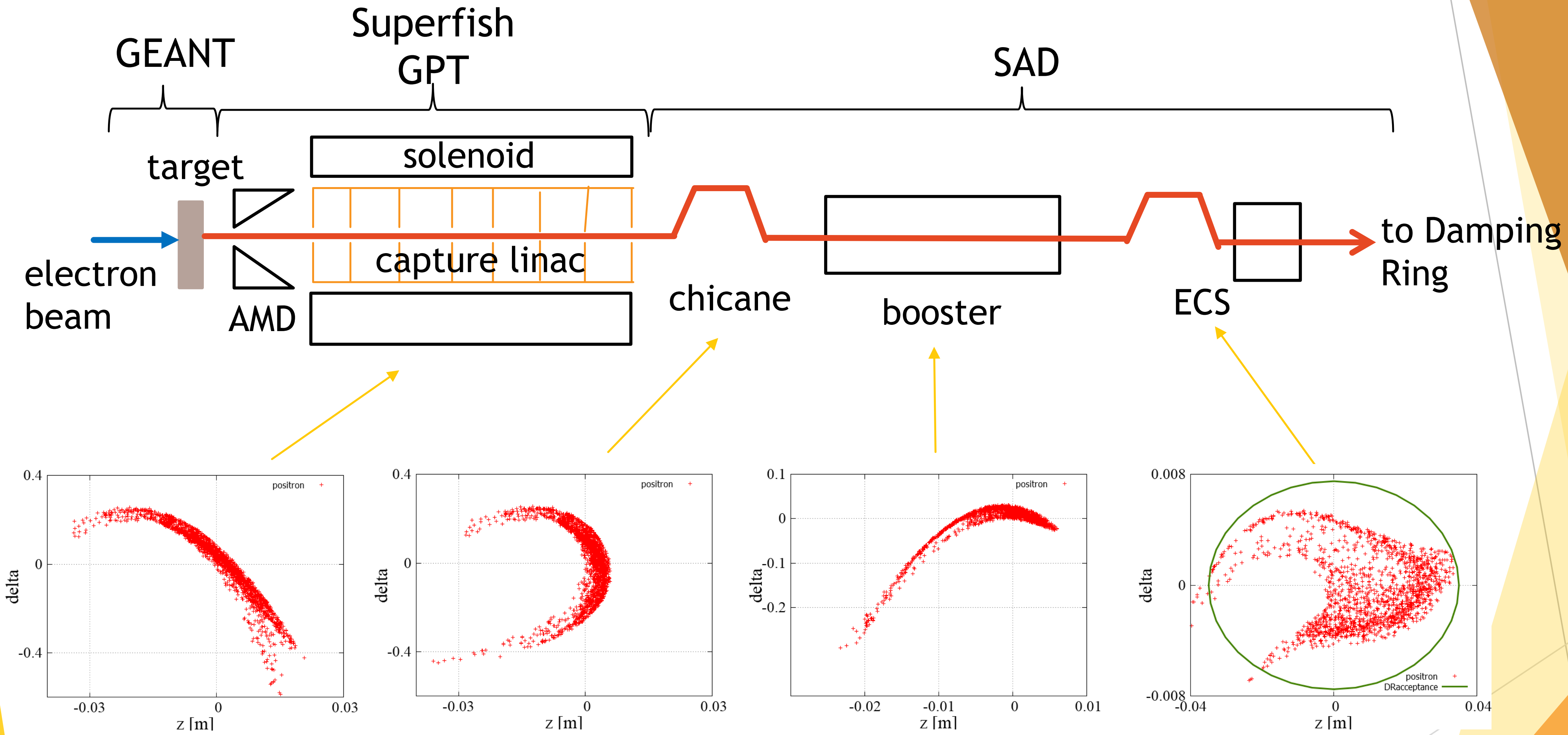
ILC (positron source)

e-driven method

- Positron rarely exists, so we should generate artificially.
- To generate positron, we use e-driven method.
- Not only positron but electron and γ-ray are emitted.
- We should consider beam loading current coursed by electron.



Constitution of e-driven positron source



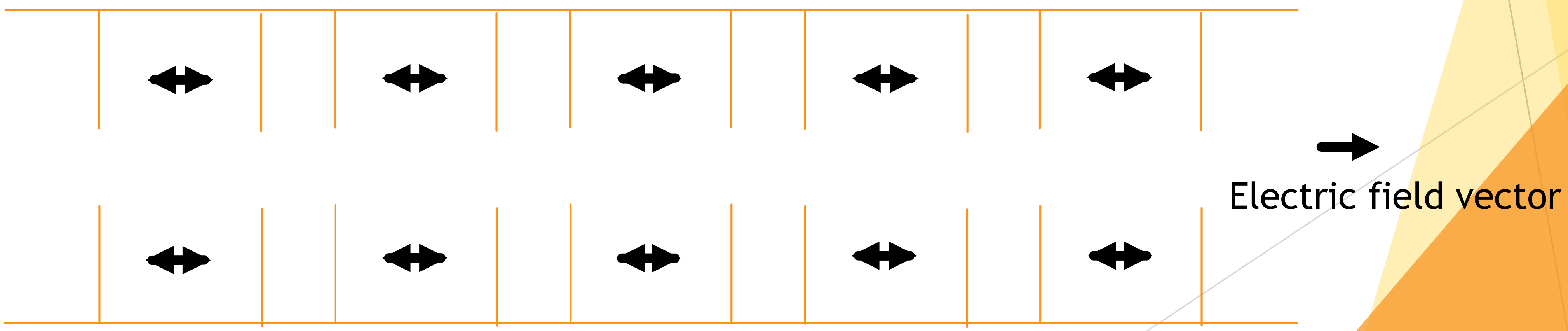
Phase space at each section

$$z = s - s_{ave}$$

$$\delta = \frac{\gamma - \gamma_{ave}}{\gamma_{ave}}$$

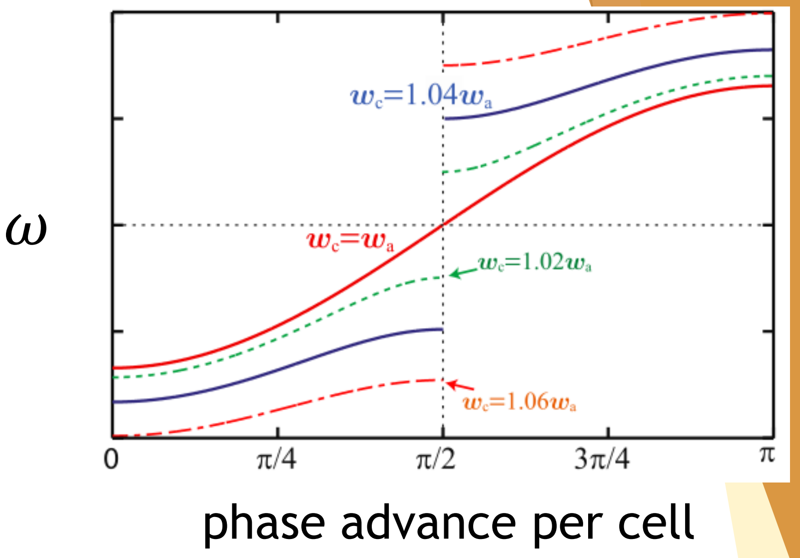
APS(Alternate Periodic Structure)cavity

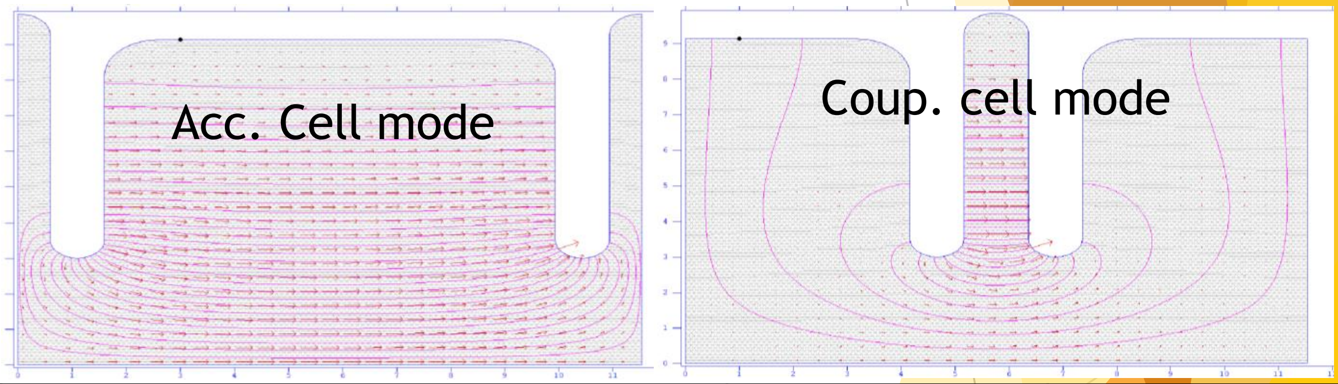
- Standing wave cavity and phase advance per cell is π/2 (π/2-mode).
- There are cells generated electric field(accelerating cell) and cells doesn’t generated electric field(coupling cell).
- In APS cavity, accelerating cell is longer than coupling cell. From this, the particle can be accelerated efficiently



APS cavity designed by superfish

- Aperture radius is a=30mm.
- If the frequencies of accelerating cell and coupling cell are adjusted, the group velocity v_g of π/2-mode is non-zero.
- The non-zero v_g improves the uniformity of the field across the structure, especially for high coupling beta.

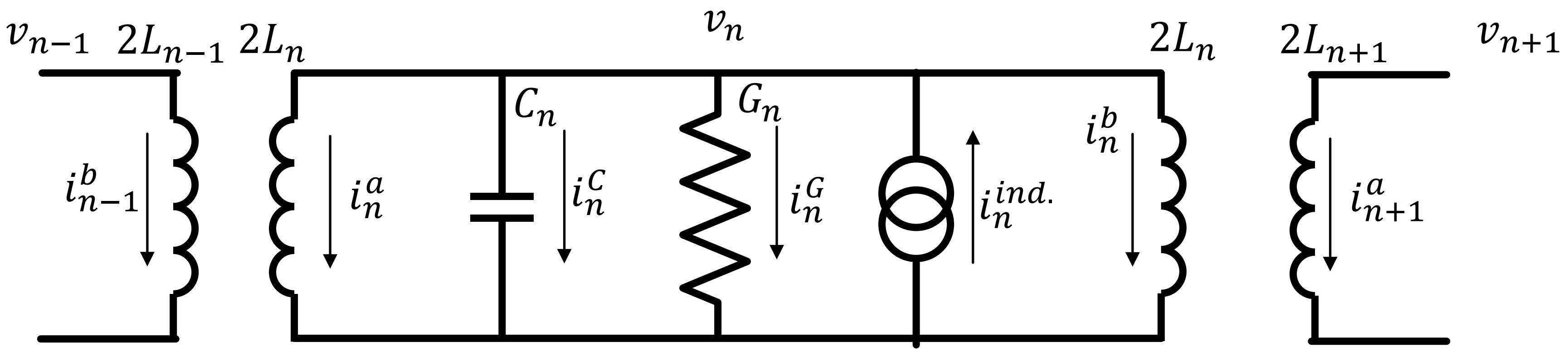




	Accelerating cell	Coupling cell
Resonance frequency[MHz]	1300.02	1299.94
Shunt impedance[Ohm/m]	52.65	2.09
Q value	24972	5915
R/Q[Ohm]	145.5	30.5

Equivalent circuit model

- Transient property of APS cavity is simulated with an equivalent circuit model as follows;



$$\frac{1}{\omega_n^2}\frac{d^2\hat{v}_n}{dt^2}+\frac{1}{\omega_n Q_0}\frac{d\hat{v}_n}{dt}+\hat{v}_n=\frac{1}{2}k(\hat{v}_{n-1}+\hat{v}_{n+1})+\frac{1}{\omega_n}\frac{d\hat{i}_n^{ind.}}{dt}$$

$$\hat{v}_n=\sqrt{C_n}v_n=\frac{v_n}{\sqrt{\omega_n\left({}^R/_Q\right)_n}} \qquad \hat{i}_n=\sqrt{L_n}i_n=\sqrt{\frac{\left({}^R/_Q\right)_n}{\omega_n}}i_n$$

All field variables oscillate coherently with an angular frequency ω as

$$\hat{v}(t) = \hat{V}(t)e^{i\omega t}$$

$$\hat{V}_n{}'' + \left(2i + \frac{1}{Q_0}\right)\hat{V}_n{}' + \left(i\frac{1}{Q_0} - 2\delta_n\right)\hat{V}_n = \frac{1}{2}k\left(\hat{V}_{n-1} + \hat{V}_{n+1}\right) + \hat{I}_n^{ind.\prime} + i\hat{I}_n^{ind.}$$

The equation is discretized with a step in phase $\Delta\theta = \omega\Delta t$ by index m.

$$\hat{V}_n = \hat{V}_n^m$$

$$\hat{V}_n{}' = \frac{\hat{V}_n^{m+1} - \hat{V}_n^{m-1}}{2\Delta\theta}$$

$$\hat{V}_n{}'' = \frac{\hat{V}_n^{m+1} + \hat{V}_n^{m-1} - 2\hat{V}_n^m}{(\Delta\theta)^2}$$

$$\hat{I}_n^{ind.} = \hat{I}_n^{m\ ind.}$$

$$\hat{I}_n^{ind.\prime} = \frac{\hat{I}_n^{m+1\ ind.} - \hat{I}_n^{m-1\ ind.}}{2\Delta\theta}$$

The temporal evolution of the field is given by a recursion formula as,

$$\hat{V}_n^{m+1} = \begin{pmatrix} a_1 & a_2 & a_3 & a_4 \end{pmatrix} \begin{pmatrix} \hat{V}_{n-1}^{m} \\ \hat{V}_{n}^{m} \\ \hat{V}_{n+1}^{m} \\ \hat{V}_{n}^{m-1} \end{pmatrix}$$

$$+\begin{pmatrix} b_1 & b_2 & b_3 \end{pmatrix} \begin{pmatrix} \hat{I}_n^{ind.\,m-1} \\ \hat{I}_n^{ind.\,m} \\ \hat{I}_n^{ind.\,m+1} \end{pmatrix}$$

$$a_1 = \frac{\frac{1}{2}k}{\frac{1}{(\Delta\theta)^2} + \frac{1}{2\Delta\theta}\left(2i + \frac{1}{Q_0}\right)}$$

$$a_2 = \frac{\frac{2}{(\Delta\theta)^2} - i\frac{1}{Q_0} + 2\delta_n}{\frac{1}{(\Delta\theta)^2} + \frac{1}{2\Delta\theta}\left(2i + \frac{1}{Q_0}\right)}$$

$$a_3 = \frac{\frac{1}{2}k}{\frac{1}{(\Delta\theta)^2} + \frac{1}{2\Delta\theta}\left(2i + \frac{1}{Q_0}\right)}$$

$$a_4 = \frac{\frac{1}{2\Delta\theta}\left(2i + \frac{1}{Q_0}\right) - \frac{1}{(\Delta\theta)^2}}{\frac{1}{(\Delta\theta)^2} + \frac{1}{2\Delta\theta}\left(2i + \frac{1}{Q_0}\right)}$$

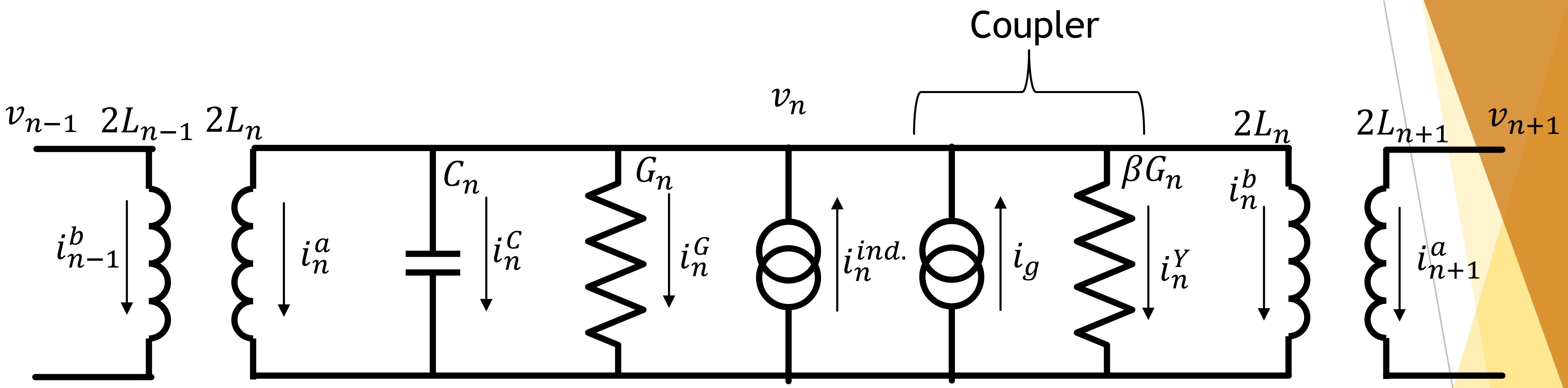
$$b_1 = \frac{-\frac{1}{2\Delta\theta}}{\frac{1}{(\Delta\theta)^2} + \frac{1}{2\Delta\theta}\left(2i + \frac{1}{Q_0}\right)}$$

$$b_2 = \frac{i}{\frac{1}{(\Delta\theta)^2} + \frac{1}{2\Delta\theta}\left(2i + \frac{1}{Q_0}\right)}$$

$$b_3 = \frac{\frac{1}{2\Delta\theta}}{\frac{1}{(\Delta\theta)^2} + \frac{1}{2\Delta\theta}\left(2i + \frac{1}{Q_0}\right)}$$

$$\delta_n = \frac{\omega - \omega_n}{\omega_n}$$

For the RF coupler cell, we add a RF generator



Coupling beta

Generator current

$$\frac{1}{\omega_n^2}\frac{d^2\hat{v}_n}{dt^2} + \frac{1}{\omega_n}\frac{1+\beta_e}{Q_0}\frac{d\hat{v}_n}{dt} + \hat{v}_n = \frac{1}{2}k(\hat{v}_{n-1} + \hat{v}_{n+1}) + \frac{1}{\omega_n}\frac{d}{dt}\left(\hat{i}_n^{ind.} + \hat{i}_g\right)$$

$$\hat{V}_n^{m+1} = \begin{pmatrix} a_1 & a_2 & a_3 & a_4 \end{pmatrix} \begin{pmatrix} \hat{V}_{n-1}^{m} \\ \hat{V}_n^{m} \\ \hat{V}_{n+1}^{m} \\ \hat{V}_n^{m-1} \end{pmatrix} + \begin{pmatrix} b_1 & b_2 & b_3 \end{pmatrix} \begin{pmatrix} \hat{I}_n^{ind.\,m-1} + \hat{I}_g^{m-1} \\ \hat{I}_n^{ind.\,m} + \hat{I}_g^{m} \\ \hat{I}_n^{ind.\,m+1} + \hat{I}_g^{m+1} \end{pmatrix}$$

$$a_1 = \frac{\frac{1}{2}k}{\frac{1}{(\Delta\theta)^2} + \frac{1}{2\Delta\theta}\left(2i + \frac{1+\beta_e}{Q_0}\right)}$$

$$a_2 = \frac{\frac{2}{(\Delta\theta)^2} - i\frac{1+\beta_e}{Q_0} + 2\delta_n}{\frac{1}{(\Delta\theta)^2} + \frac{1}{2\Delta\theta}\left(2i + \frac{1+\beta_e}{Q_0}\right)}$$

$$a_3 = \frac{\frac{1}{2}k}{\frac{1}{(\Delta\theta)^2} + \frac{1}{2\Delta\theta}\left(2i + \frac{1+\beta_e}{Q_0}\right)}$$

$$a_4 = \frac{\frac{1}{2\Delta\theta}\left(2i + \frac{1+\beta_e}{Q_0}\right) - \frac{1}{(\Delta\theta)^2}}{\frac{1}{(\Delta\theta)^2} + \frac{1}{2\Delta\theta}\left(2i + \frac{1+\beta_e}{Q_0}\right)}$$

$$b_1 = \frac{-\frac{1}{2\Delta\theta}}{\frac{1}{(\Delta\theta)^2} + \frac{1}{2\Delta\theta}\left(2i + \frac{1+\beta_e}{Q_0}\right)}$$

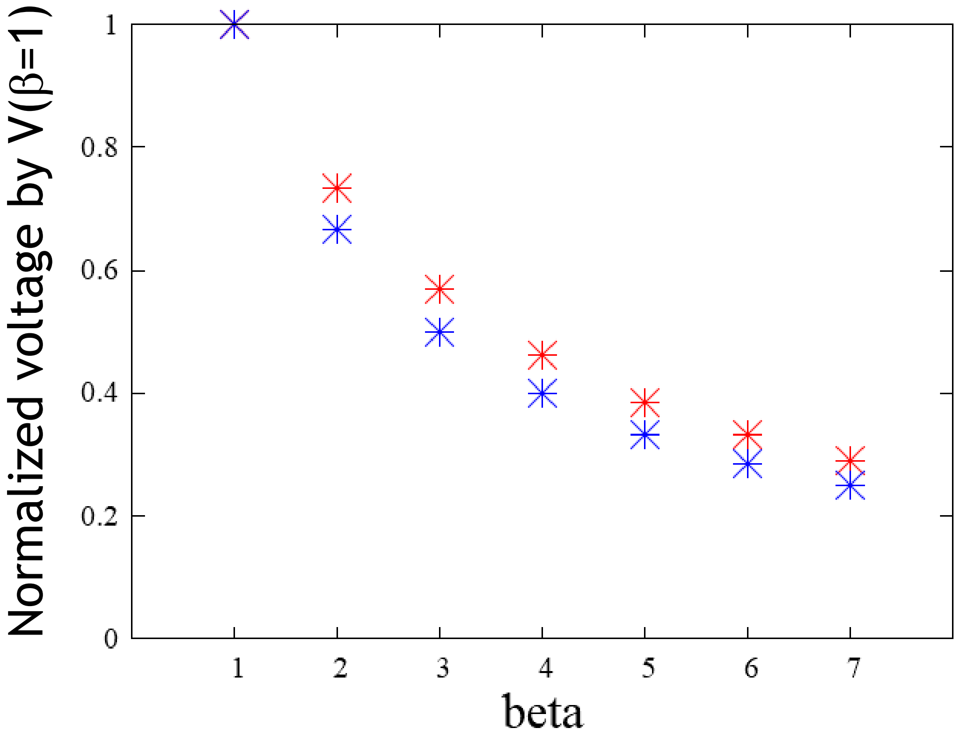
$$b_2 = \frac{i}{\frac{1}{(\Delta\theta)^2} + \frac{1}{2\Delta\theta}\left(2i + \frac{1+\beta_e}{Q_0}\right)}$$

$$b_3 = \frac{\frac{1}{2\Delta\theta}}{\frac{1}{(\Delta\theta)^2} + \frac{1}{2\Delta\theta}\left(2i + \frac{1+\beta_e}{Q_0}\right)}$$

Beam induced voltage (Beam loading voltage)

By a single cell model, the voltage (asymptotic value at the steady state)

$$V \propto \frac{1}{1+\beta}$$



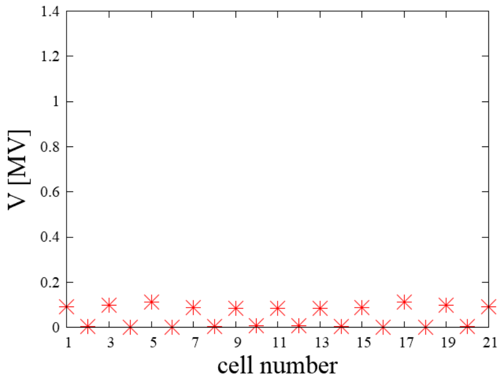
✳ :Circuit model (multi cells model)

✳: Single cell model

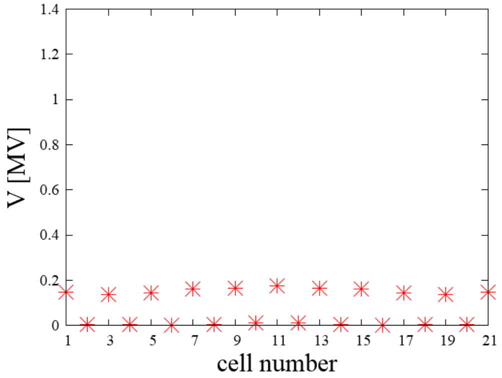
The relation is still valid for the multi cell model.

$$V \propto \frac{1}{1+\beta}$$

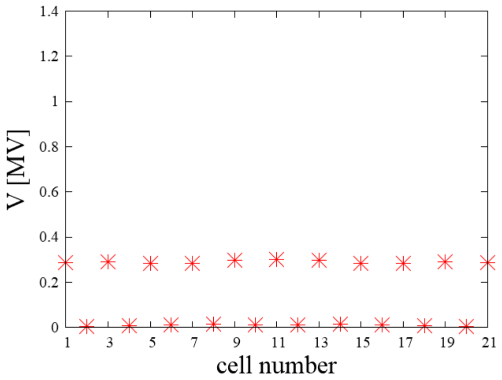
Voltage evolution by RF input and no beam.



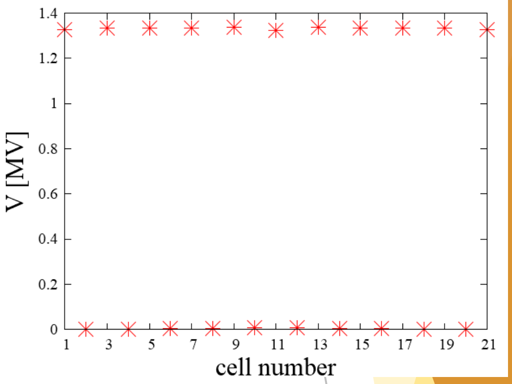
0.3μsec



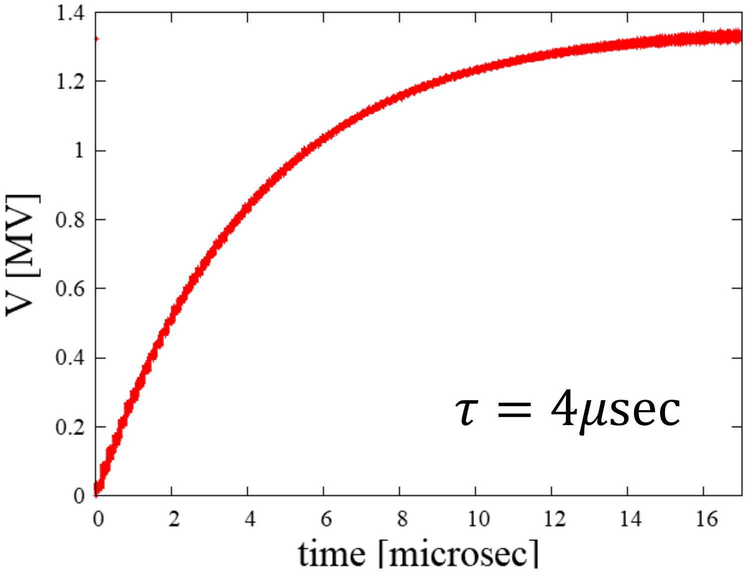
0.5μsec



1μsec



16μsec



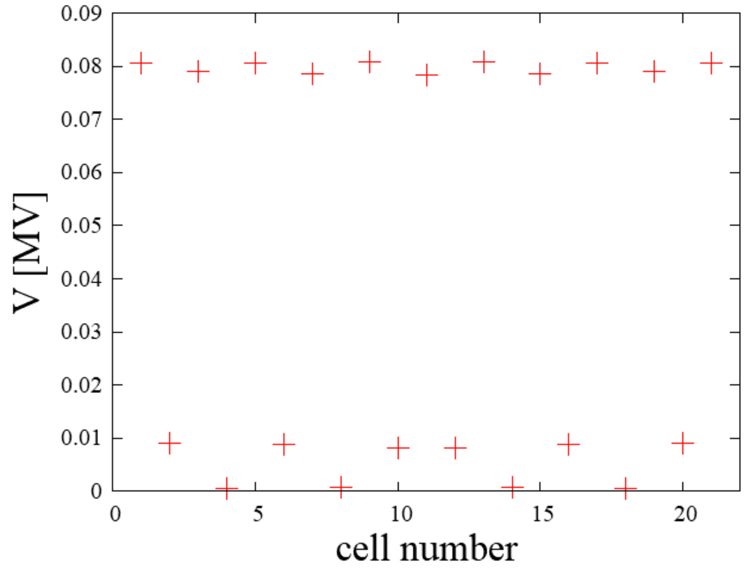
Voltage of 11th cell (coupler cell)

Time constant τ

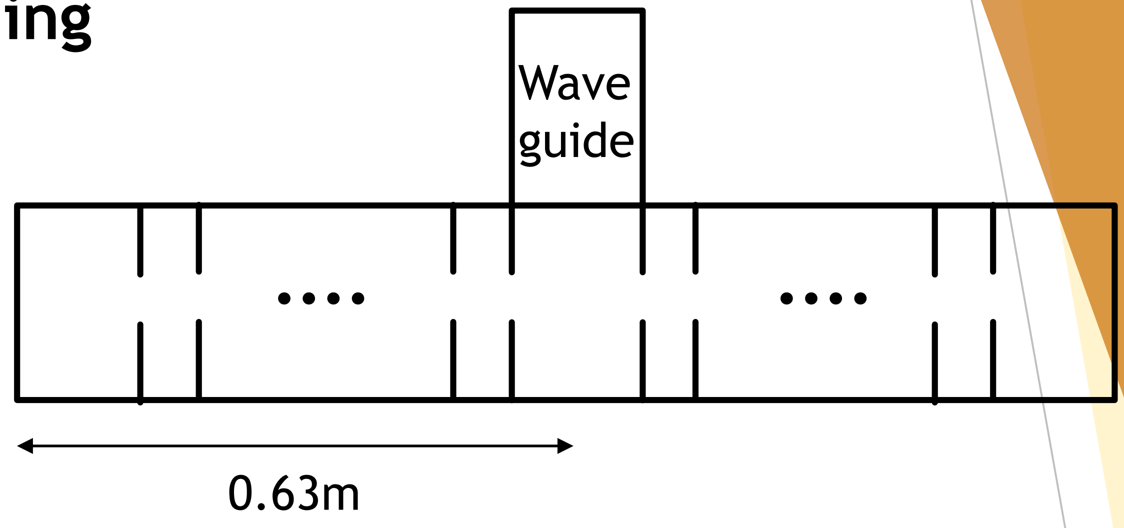
$$\tau = \frac{2Q_0}{(1+\beta_e)\omega} = 1\mu sec$$

Field uniformity by beam loading

$I_n^{ind.} = 1\text{A}$



16μsec



The propagation time of power from the end cell to the coupler cell with $v_g = 0.028c$ is

$$\frac{0.63}{0.028c} = 75\text{nsec}$$

is much less τ resulting a good field uniformity.

We can treat the APS cavity as a big single cell.

Summary

- We designed an APS cavity with the large aperture and the high gradient for the capture linac of ILC e-Driven positron source.

- We analyzed the transient response of the APS cavity with an equivalent circuit model.

- We can treat it as a single cell because of the high v_g, the field uniformity.

- The beam loading compensation based on the single cell model works well.