HITRI Meeting on evaluation of power and energy consumption of Hadron th. SC facility vs resitive one
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# Basic scaling for resistive magnets

*Davide Tommasini, CERN*

# Beam rigidity

$E_{0(proton)} = 938\ MeV\ ;\ E_{0(nucleon)} = 931\ MeV\ ;\ q = ze$

$Centripetal\ force\ F = qvB = \frac{pv}{\rho} \rightarrow \boldsymbol{B r} = \frac{p}{q}$ $is\ defined\ as\ \boldsymbol{magnetic\ rigidity}.$

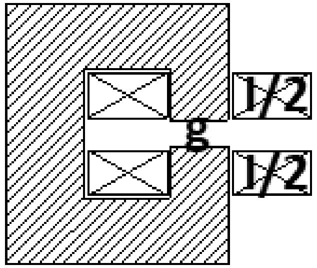
$By\ expressing\ the\ momentum\ in\ \frac{GeV}{c},\ we\ obtain:\ \boldsymbol{Br}[\boldsymbol{Tm}] = \frac{10^9 p}{cz} = \boldsymbol{3.3356\frac{p}{z}} = 3.3356\frac{\sqrt{E^2 - E_0^2}}{z}$

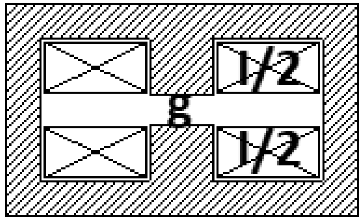
| Particle | Beam rigidity B·r [Tm] |
|---|---|
| 250 MeV electrons | 0.84 |
| 150 MeV protons | 1.84 |
| 250 MeV/u $^{12}C^{+6}$ | 4.85 |
| 250 MeV protons | 2.43 |
| 430 MeV/u $^{12}C^{+6}$ | 6.62 |

# Estimate of power consumption : bending magnets

We consider a reference bending magnet where:

$$B \sim \mu_0 I / g$$




The dissipated power in the conductor is :

$$P = \rho V J_{rms}^2 = 2\rho l p I J \text{ where } p = \frac{J_{rms}^2}{J^2}$$ *(ex for a sinusoid p=1/2)*

We remark that if $l$ is the magnet length we are neglecting the losses in the coil ends: for a short magnet these can be as large (or even larger)  than the losses of the “active field part”.

We obtain:

$$P/Bl = \frac{2\rho g}{\mu_0} \cdot Jp$$

# What does this mean in terms of power per deflected angle

The required power to produce a deflection angle α as a function of the beam rigidity Br is:

$$P/\alpha Br = \frac{2\rho g}{\mu_0} \cdot Jp$$

If you neglect the contribution of the coil ends, for the same bending angle making a long magnet (large radius) at low field does not change the power consumption with respect to a shorter magnet (small radius) at a higher field.

In reality in a short magnet the ends will contribute more and the magnet will consume more.

Example for carbon ions, every $\frac{\pi}{2}$ angle :

$$P = 0.33 gJp$$

The power depends linearly from the magnet gap and the current density

For a gap of 70 mm and a current density of 5 $A/mm^2$ with p=1 (DC) we obtain:

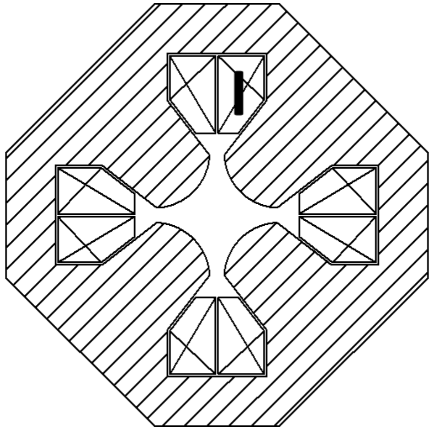
$$P = 115\, kW\ for\ a\ 90°\ bend$$

To this, we shall also add the contribution of the coil ends, which depend on the horizontal magnet aperture

Warning: there are a number of “hidden” considerations behind this result

# Estimate of power consumption : quadrupole magnets

We consider a reference quadrupole magnet where:




$$G \sim 2\mu_0 I / R^2$$

The dissipated power in the conductor is :

$$P = \rho V J_{rms}^2 = 4 \cdot 2\rho l I J p$$

We remark that if $l$ is the magnet length we are neglecting the losses in the coil ends: for a short magnet these can be as large (or even larger) than the losses of the "active field part".

We obtain:

$$P/Gl = \frac{4\rho R^2}{\mu_0} \cdot Jp$$

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