

$\mathcal{B}(B_c^+ \rightarrow \tau^+ \nu_\tau)$ prospects at FCC-ee

Based on work detailed in [arXiv:2105.13330](https://arxiv.org/abs/2105.13330)

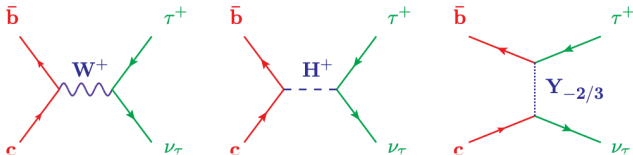
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FCC Week 2021

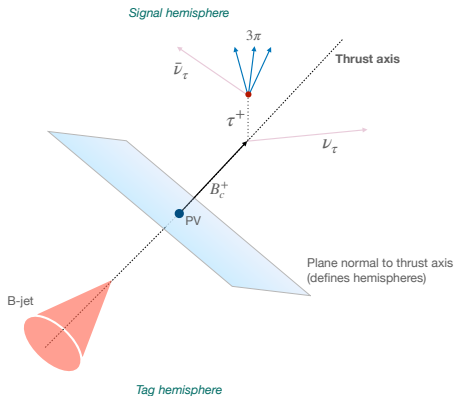
Introduction

- $B_c^+ \rightarrow \tau^+ \nu_\tau$ is a unique flavour opportunity at FCC-ee
 - Not possible at LHCb due to missing energy - lack of constraints and reconstructed information
 - No B_c^+ mesons produced at Belle II
- Involves the same Feynman vertex factors as $b \rightarrow c \tau \nu_\tau$ decays
 - Measurements of $R(D)$ and $R(D^*)$ show a $\sim 3 - 4\sigma$ tension with SM
 - Measuring $\mathcal{B}(B_c^+ \rightarrow \tau^+ \nu_\tau)$ would provide an independent test of lepton universality violation, and yield strong constraints on possible NP explanations
 - $\mathcal{B}(B_c^+ \rightarrow \tau^+ \nu_\tau)^{\text{SM}} = (1.94 \pm 0.09)\%$



Event topology for $B_c^+ \rightarrow \tau^+ \nu_\tau$

- Reconstruct the thrust axis and use this to define which hemisphere the particles fall in
- Due to high missing energy in signal decay, the two hemispheres have rather different energy distributions



Using $\tau^+ \rightarrow \pi^+\pi^+\pi^-\bar{\nu}_\tau$

- Existing feasibility study for CEPC used electron and muon τ decays
- Use tree-prong **hadronic mode** $\tau^+ \rightarrow \pi^+\pi^+\pi^-\bar{\nu}_\tau$ for this work
 - Provides τ decay vertex, and thus a measure of combined B_c^+ and τ^+ flight (B_c^+ lifetime 3x lower than $B^{0/+}$)
 - Branching fraction of 9% provides sufficient decay rate
- Reject background from $Z \rightarrow b\bar{b}/c\bar{c}/q\bar{q}$ using event-level missing energy properties and properties of the reconstructed 3π candidate
 - Two-stage MVA selection

Samples and software

- Using official **spring2021** production of events (see [here](#))
 - 10 million signal MC with $\tau^+ \rightarrow \pi^+ \pi^+ \pi^- \bar{\nu}_\tau$ generated using EvtGen
 - 1 billion each of $Z \rightarrow b\bar{b}, c\bar{c}, q\bar{q}$ in Pythia ($b\bar{b}$ crosschecked with EvtGen and found to be very similar)
 - Samples of exclusive B hadron background decays also generated to study selection efficiencies more accurately
- Dedicated production of 1 billion events with orthogonal seeds for MVA training (see [here](#))
- Common software used throughout from EDM4hep to [FCCAnalyses](#) and common [Python analysis tools](#)

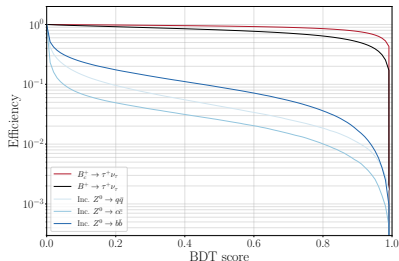
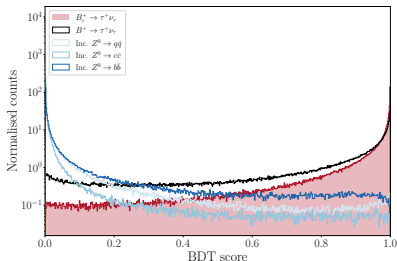
- The following cuts are applied prior to first-stage MVA training:
 - A primary vertex is reconstructed
 - At least one reconstructed 3π candidate in the event
 - One of the 3π must be in the lower energy hemisphere
- These pre-selection cuts are applied to signal and inclusive $Z \rightarrow b\bar{b}/c\bar{c}/q\bar{q}$ background samples

First-stage MVA

- Train on general event-level properties, harnessing difference in hemisphere energies between $B_c^+ \rightarrow \tau^+ \nu_\tau$ and $Z \rightarrow b\bar{b}/c\bar{c}/q\bar{q}$
- Also use information on PV multiplicity, number of vertices reconstructed in event, number of 3π candidates, and distance between decay vertices and PV
- Background training sample is a mix of inclusive $Z \rightarrow b\bar{b}/c\bar{c}/q\bar{q}$, combined according to Z branching ratios and efficiency of pre-selection requirements

First-stage MVA performance

- Strong rejection of all three backgrounds, with ROC = **0.984**
 - $Z \rightarrow b\bar{b}$ rejected least since it produces most missing energy
- $B^+ \rightarrow \tau^+ \nu_\tau$ also rejected to an extent (different event-level properties compared to B_c^+)
 - Important background, which is enhanced 1000x due to B^+ vs. B_c^+ production rate but lower due to CKM suppression ($|V_{ub}|^2$ vs. $|V_{cb}|^2$)
 - Expect 5 times more $B^+ \rightarrow \tau^+ \nu_\tau$ than signal before any cuts



Selection cuts before second-stage MVA

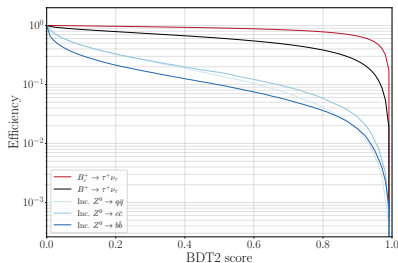
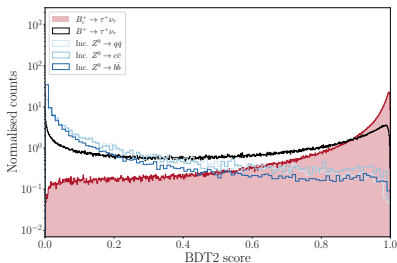
- Apply $MVA1 > 0.6$ cut to remove “easy” stuff
- Select the reconstructed 3π candidate with the smallest displaced vertex χ^2
- Selected vertex must be in the hemisphere with less energy
- Apply $m(\pi\pi)$ cuts to 3π candidates to select $\rho \rightarrow \pi^+\pi^-$ region $[0.6, 1.0]$ GeV
 - Signal decays purely via $\tau \rightarrow a_1 \rightarrow \rho\pi$ resonance
- Require $m(3\pi) < m_\tau$, and that the difference in hemisphere energies exceeds 10 GeV

Second-stage MVA

- Trained on signal and inclusive background passing pre-selection (previous slide)
- Use properties of the 3π candidate (mass, momentum, IP, distance from PV, angle of momentum to thrust axis)
- Use IP to the PV for other decay vertices in the event
 - Associated charm hadron produced with $B_c^+ \rightarrow \tau^+ \nu_\tau$ originates from PV, unlike the charm hadron in $B^0 \rightarrow D^{(*)-} \tau^+ \nu_\tau$ for example
- Mass of PV - this is larger for background because the associated charm hadron in $B_c^+ \rightarrow \tau^+ \nu_\tau$ carries energy away from the PV
- Nominal B energy = $m(Z)$ - sum of all reco. event energy apart from $E(3\pi)$

Second-stage MVA performance

- All backgrounds rejected very well, with ROC = **0.966**
- Excellent rejection of $B^+ \rightarrow \tau^+ \nu_\tau$ mode as well, even though it is not used in training
 - This reduces the B^+ mode to a level where it must be constrained from independent measurements
 - Described more in toy fit setup later



2D MVA cut optimisation

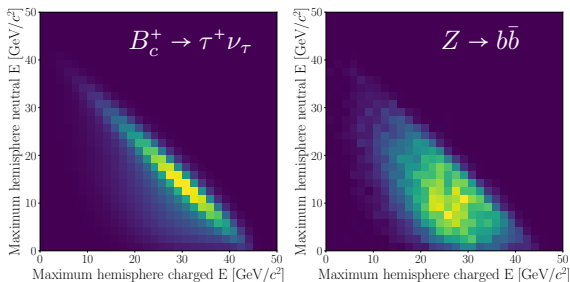
- Tune the two MVA cuts to maximise purity of the signal $S/(S + B)$, to enable a precise $\mathcal{B}(B_c^+ \rightarrow \tau^+ \nu_\tau)$ measurement
 - Estimate S and B expected from 5×10^{12} Z at FCC-ee
- Inclusive $Z \rightarrow c\bar{c}$ and $Z \rightarrow q\bar{q}$ samples are rejected at 10^9 level with sufficiently tight MVA cuts
 - All generated events are rejected
 - Do not consider these in subsequent studies
- Inclusive $Z \rightarrow b\bar{b}$ statistics are insufficient to assess background rejection to a high enough level or create fit templates
 - Generate a set of exclusive B^0 , B^+ , B_s^0 , and Λ_b^0 modes where all B -hadron decay products are decayed inclusively
 - Focus on modes where a τ or charm hadron produces a displaced 3π , along with another charm hadron

Cut optimisation - results

- Estimate yields with $N_Z = 5 \times 10^{12}$, $\mathcal{B}(Z \rightarrow b\bar{b}) = 0.1512$, and B hadron production fractions from Pythia
- Estimate signal yield using $\mathcal{B}(B_c^+ \rightarrow \tau^+ \nu_\tau) = 1.94\%$ (SM prediction), $f_c = 0.0004$ from Pythia, and selection efficiency from the signal MC
- Estimate $B^+ \rightarrow \tau^+ \nu_\tau$ and exclusive B -hadron background yields using PDG \mathcal{B} 's, production fractions, and MC ϵ 's
- **Best signal yield of 4295 events with 85% purity ($\epsilon = 0.39\%$)**
- **285 events expected for $B^+ \rightarrow \tau^+ \nu_\tau$ mode ($\epsilon = 4.3 \times 10^{-5}$)**
- **Background level of 448 events** estimated from a sum of all exclusive modes, scaled by 2.5 (inclusive vs. exclusive ratio)

Signal vs. background discrimination for fit

- In an inclusive $Z \rightarrow b\bar{b}$ event, either side could have max. E
 - However, in a signal decay, the max. E hemisphere will most likely be the **non-signal side**
- **Result:** in signal events, max. E hemisphere looks a lot like an inclusive decay (peak near $m(Z)/2$ with tail)
 - In background, our selection requirements bias this hemisphere **down in energy**, giving us discrimination

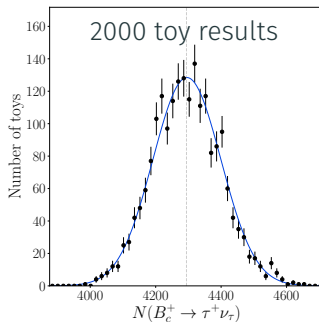
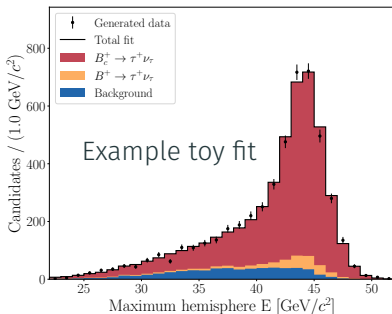


Toy fit to measure signal yield

- Energy of the maximum energy hemisphere used as the fit variable
- Create histogram templates in $B_c^+ \rightarrow \tau^+ \nu_\tau$, $B^+ \rightarrow \tau^+ \nu_\tau$, and exclusive (B^0 , B^+ , B_s^0 , Λ_b^0) background
 - Sum exclusive backgrounds according to their expected yields from the MVA cut optimisation
- $B^+ \rightarrow \tau^+ \nu_\tau$ yield constrained in fit to the value found in cut optimisation, with **5% relative uncertainty**
 - Expected uncertainty on $\mathcal{B}(B^+ \rightarrow \tau^+ \nu_\tau)$ from Belle II physics book
 - This mode contributes a **yield of 5% relative to signal** after all cuts, so the uncertainty from this is negligible

Toy fit to measure signal yield

- Toy datasets generated from a summed histogram PDF of signal, $B^+ \rightarrow \tau^+ \nu_\tau$, and total background, with yields as mentioned
 - Each bin is Poisson varied independently to create toys
- **Signal** and **background** yields vary freely in the toy fit - signal is measured with **2.4% relative precision**
 - Fit with 10× higher background level measures 2.9% uncertainty



Converting from signal yield to $\mathcal{B}(B_c^+ \rightarrow \tau^+ \nu_\tau)$

$$\mathcal{B}(B_c^+ \rightarrow \tau^+ \nu_\tau) = \frac{N(B_c^+ \rightarrow \tau^+ \nu_\tau)}{N(B_c^+ \rightarrow J/\psi \mu^+ \nu_\mu)} \times \frac{\epsilon(B_c^+ \rightarrow J/\psi \mu^+ \nu_\mu)}{\epsilon(B_c^+ \rightarrow \tau^+ \nu_\tau)}$$
$$\times \frac{\mathcal{B}(J/\psi \rightarrow \mu\mu)}{\mathcal{B}(\tau^+ \rightarrow 3\pi \nu_\tau)} \times \mathcal{B}(B_c^+ \rightarrow J/\psi \mu^+ \nu_\mu)$$

- Signal yield measured in toy fit is based on an assumed $\mathcal{B}(B_c^+ \rightarrow \tau^+ \nu_\tau) = 1.94\%$
- Convert toy fit yield back to $\mathcal{B}(B_c^+ \rightarrow \tau^+ \nu_\tau)$ via correction factors, and assess the precision on branching ratio
- Use some external branching fractions as well as a theory prediction for $\mathcal{B}(B_c^+ \rightarrow J/\psi \mu^+ \nu_\mu) = 0.0135 \pm 0.0011$

$\mathcal{B}(B_c^+ \rightarrow \tau^+ \nu_\tau)$ calculation inputs

- Expected signal yield from cut optimisation procedure
- Yield statistical uncertainty from toy fits, **scaled up by $\sqrt{2}$ to account for potential systematics** (template statistics, background modelling)
- Normalisation yield calculated in similar manner to signal, assuming fairly high efficiency for three-muon mode
 - Main loss due to $m(J/\psi\mu) > 5.3$ GeV cut needed to reject the B^{+0} background (see [arXiv:1407.2126](https://arxiv.org/abs/1407.2126))

Term	Value	Explanation
$\epsilon(B_c^+ \rightarrow \tau^+ \nu_\tau)$	$(3.93 \pm 0.04) \times 10^{-3}$	From our MC (assume 1% precision)
$\epsilon(B_c^+ \rightarrow J/\psi\mu^+\nu_\mu)$	0.100 ± 0.001	Assume with 1% precision
$\mathcal{B}(J/\psi \rightarrow \mu\mu)$	$(5.96 \pm 0.03) \times 10^{-2}$	From PDG
$\mathcal{B}(\tau^+ \rightarrow 3\pi\nu_\tau)$	$(9.31 \pm 0.05) \times 10^{-2}$	From PDG
$N(B_c^+ \rightarrow \tau^+ \nu_\tau)$	4295 ± 146	Uses $N_{b\bar{b}}$, f_c , \mathcal{B} 's, and ϵ
$N(B_c^+ \rightarrow J/\psi\mu^+\nu_\mu)$	48670 ± 220	Uses $N_{b\bar{b}}$, f_c , \mathcal{B} 's, and ϵ
$\mathcal{B}(B_c^+ \rightarrow J/\psi\mu^+\nu_\mu)$	0.0135 ± 0.0011	Calculation from O. Sumensari

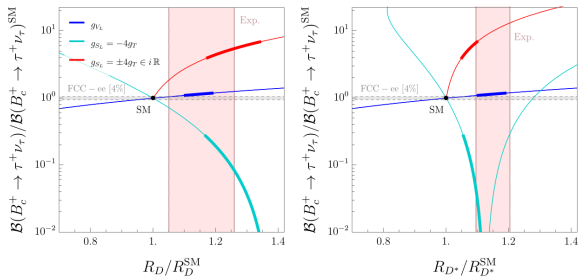
$\mathcal{B}(B_c^+ \rightarrow \tau^+ \nu_\tau)$ anticipated precision

$$\mathcal{B}(B_c^+ \rightarrow \tau^+ \nu_\tau) = (1.941 \pm 0.175) \times 10^{-2}$$

- Recover the input branching ratio central value as expected
- Relative uncertainty is **9.0%**, including uncertainties on all of the input terms
- Uncertainty dominated by the current theory uncertainty on $\mathcal{B}(B_c^+ \rightarrow J/\psi \mu^+ \nu_\mu)$, which is 8.1%
 - Using [this](#) and [this](#) lattice paper for current estimate along with exclusive $|V_{cb}|$
 - **Uncertainty is reducible if decay form factors are measured** e.g. in an angular analysis (LHCb & FCC-ee can both do this)
- Signal yield uncertainty from fit is 2.4%, which increases to 3.4% assuming the same level of systematics
 - **Excellent precision using only the $\tau^+ \rightarrow 3\pi \nu_\tau$ mode!**

NP sensitivity

- Can consider the ratio $R_c = \mathcal{B}(B_c^+ \rightarrow \tau^+ \nu_\tau) / \mathcal{B}(B_c^+ \rightarrow J/\psi \mu^+ \nu_\mu)$, which has $\sim 4\%$ expected experimental precision
 - $|V_{cb}|$ independent
 - Can assume no NP in the muonic mode for theory calculation
- R_c measurement at FCC-ee can strongly constrain both 2HDM and leptoquark parameter space in a complementary manner to other key observables
 - Leptoquark couplings can introduce $\mathcal{O}(10) - \mathcal{O}(100)$ variations!



Summary

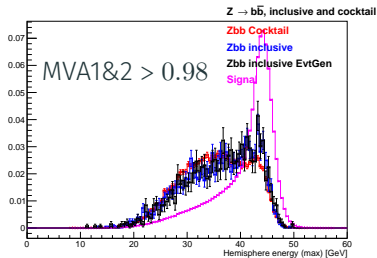
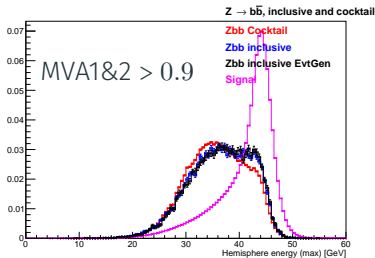
- Full framework in place using FCC-ee software tools for generation, processing, and analysis of $B_c^+ \rightarrow \tau^+ \nu_\tau$ mode and backgrounds
 - Many other exciting flavour modes need to be studied, and the tools are now in place to help you get involved!
- Selection exploiting missing energy signature of signal and 3π properties can achieve a high purity final dataset
 - Analysis still feasible with higher background level (fit stable with 10 times larger background and gives similar precision)
- Signal yield measurement $\sim 3\%$ precision, but requires accurate $\mathcal{B}(B_c^+ \rightarrow J/\psi \mu^+ \nu_\mu)$ prediction to convert to $\mathcal{B}(B_c^+ \rightarrow \tau^+ \nu_\tau)$
 - R_c ratio also highly valuable, where 4% precision is achievable
- Pre-print [arXiv:2105.13330](https://arxiv.org/abs/2105.13330) to be submitted to JHEP soon

Backup

Exclusive background

- Focus on modes where a τ or charm hadron produces a displaced 3π , along with another charm hadron (this topology is most like the signal)
 - $B \rightarrow D^{(*)} D_s^{(*)}$
 - $B \rightarrow D^{(*)} 3\pi$
 - $B \rightarrow D^{(*)} \tau^+ \nu_\tau$
- The sum of all these exclusive modes matches the distributions in inclusive $Z \rightarrow b\bar{b}$ rather well
 - An overall rate difference of **2.5** is observed, since we are only generating about 10% of possible B hadron decay widths
 - **Multiply the exclusive sample yields by 2.5 in the cut optimisation to account for this**

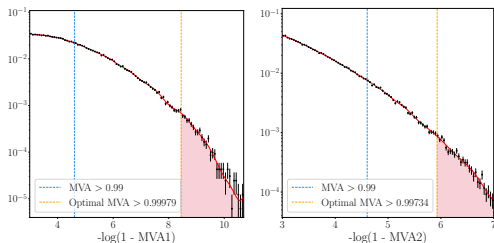
Comparing inclusive, exclusive, and signal (normalised)



- Comparing shape of **inclusive Pythia8 $Z \rightarrow b\bar{b}$** , **inclusive EvtGen $Z \rightarrow b\bar{b}$** , **exclusive sum of B hadron modes**, and **signal**
- Maximum hemisphere energy considered as fit variable
- All plots can be found [here](#)

Exclusive background - efficiency determination

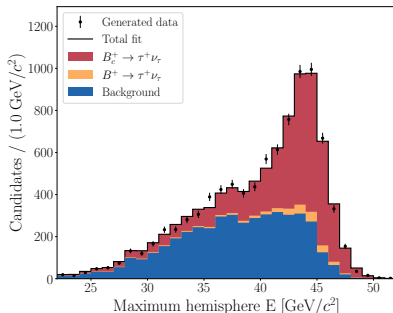
- Full selection leaves too few events in exclusive samples to determine an accurate background efficiency
- Measure efficiency of $[MVA1 > 0.95 \text{ \& } MVA2 > 0.95]$ cut for each exclusive background mode
- Parameterise shape of remaining MVA distributions using summed sample of exclusive modes
 - Fit $-\log(1 - MVA)$ with **cubic spline**
 - Integrate splines above certain cuts to determine efficiencies (and multiply by $\epsilon[MVA1 > 0.95 \text{ \& } MVA2 > 0.95]$)



Decay mode	N(expected)	N(generated)	Expected / Generated	Final ϵ
$B^+ \rightarrow \bar{D}^0 \tau^+ \nu_\tau$	5.01×10^9	2×10^8	25.0	1.46×10^{-9}
$B^+ \rightarrow \bar{D}^{*0} \tau^+ \nu_\tau$	1.22×10^{10}	2×10^8	61.1	1.1×10^{-9}
$B^+ \rightarrow \bar{D}^0 3\pi$	3.64×10^9	1.9×10^8	19.2	1.56×10^{-9}
$B^+ \rightarrow \bar{D}^{*0} 3\pi$	6.7×10^9	2×10^8	33.5	1.04×10^{-9}
$B^+ \rightarrow \bar{D}^0 D_s^+$	5.85×10^9	2×10^8	29.3	2.52×10^{-10}
$B^+ \rightarrow \bar{D}^{*0} D_s^+$	4.94×10^9	1.75×10^8	28.2	2.72×10^{-10}
$B^+ \rightarrow \bar{D}^{*0} D_s^{*+}$	1.11×10^{10}	2×10^8	55.6	2.42×10^{-10}
$B^0 \rightarrow D^- \tau^+ \nu_\tau$	7.02×10^9	2×10^8	35.1	2.69×10^{-9}
$B^0 \rightarrow D^{*-} \tau^+ \nu_\tau$	1.02×10^{10}	2×10^8	51.0	1.25×10^{-9}
$B^0 \rightarrow D^- 3\pi$	3.9×10^9	2×10^8	19.5	3.4×10^{-9}
$B^0 \rightarrow D^{*-} 3\pi$	4.69×10^9	2×10^8	23.4	9.84×10^{-10}
$B^0 \rightarrow D^- D_s^+$	4.68×10^9	2×10^8	23.4	3.23×10^{-10}
$B^0 \rightarrow D^{*-} D_s^+$	5.2×10^9	2×10^8	26.0	2.32×10^{-10}
$B^0 \rightarrow D^{*-} D_s^{*+}$	1.15×10^{10}	2×10^8	57.5	2.35×10^{-10}
$B_s^0 \rightarrow D_s^- \tau^+ \nu_\tau$	3.53×10^9	2×10^8	17.6	3.71×10^{-9}
$B_s^0 \rightarrow D_s^{*-} \tau^+ \nu_\tau$	2.35×10^9	2×10^8	11.8	2.27×10^{-9}
$B_s^0 \rightarrow D_s^- 3\pi$	8.85×10^8	2×10^8	4.4	5.53×10^{-9}
$B_s^0 \rightarrow D_s^{*-} 3\pi$	1.05×10^9	2×10^8	5.2	3.38×10^{-9}
$B_s^0 \rightarrow D_s^- D_s^+$	6.39×10^8	2×10^8	3.2	4.09×10^{-10}
$B_s^0 \rightarrow D_s^{*-} D_s^+$	2.02×10^9	2×10^8	10.1	3.17×10^{-10}
$B_s^0 \rightarrow D_s^{*-} D_s^{*+}$	2.09×10^9	2×10^8	10.5	2.56×10^{-10}
$\Lambda_b^0 \rightarrow \Lambda_c^- \tau^+ \nu_\tau$	1.83×10^9	2×10^8	9.1	1.36×10^{-9}
$\Lambda_b^0 \rightarrow \Lambda_c^{*-} \tau^+ \nu_\tau$	1.83×10^9	2×10^8	9.1	9.44×10^{-10}
$\Lambda_b^0 \rightarrow \Lambda_c^- 3\pi$	4.31×10^8	2×10^8	2.2	5.58×10^{-9}
$\Lambda_b^0 \rightarrow \Lambda_c^{*-} 3\pi$	4.31×10^8	2×10^8	2.2	9.21×10^{-10}
$\Lambda_b^0 \rightarrow \Lambda_c^- D_s^+$	6.15×10^8	2×10^8	3.1	3.46×10^{-10}
$\Lambda_b^0 \rightarrow \Lambda_c^{*-} D_s^+$	6.15×10^8	2×10^8	3.1	2.72×10^{-10}
$\Lambda_b^0 \rightarrow \Lambda_c^{*-} D_s^{*+}$	6.15×10^8	2×10^8	3.1	2.5×10^{-10}

Toy fit with 10x larger background level

- Expected background yield from optimisation increased by 10x (purity drops from 85% to 47%)
- Signal yield still measured without bias, but uncertainty increases from 2.4% to 2.9%
- Analysis should thus not be highly sensitive to absolute rate of the background



2HDM constraints

- Strong constraints expected on 2HDM parameter space from Belle II $B^+ \rightarrow \tau^+ \nu_\tau$ result (5% precision)
- A similarly precise $B_c^+ \rightarrow \tau^+ \nu_\tau$ result from FCC-ee can rule out even more parameter space

