FCC

ROBUST MODELLING OF FCC-EE WITH ANALYTICAL EQUATIONS AND SIMULATIONS

Leon van Riesen-Haupt, Rogelio Tomas, Tobias Persson, Tessa Charles, Helmut Burkhardt, Katsunobu Oide, Frank Zimmermann

Overview of Areas Covered

- **Extensive comparisons between MADX and SAD**
  - Linear optics, amplitude and momentum detuning (presentation)
  - Emittance (presentation) and radiation integrals (presentation)
  - On axis and tilted solenoid (presentation)
- **Tapering Implementation in MADX**
  - Presented during FCC November Week 2020 (presentation)
  - Optics in good agreement with SAD results
  - Able to get correct emittance from tracking
- **Analytical equations for emittance estimate**
  - Quadrupole roll errors and sextupole misalignments
- **Dynamic Aperture Studies**
  - Computation of dynamic aperture without radiation in PTC (presentation)
- **Input for future code development at CERN**
  - Input for MAD-NG development (presentation)

VALIDATION OF SIMULATION CODES AND TAPERING

Code Comparisons

Introduction

- **Comparison of SAD, MADX and MADX-PTC**
  - Multipurpose codes but development and benchmarked with emphasis on different purposes
- **Strategic Accelerator Design (SAD)**
  - Developed and maintained in Japanese High Energy Accelerator Research Organization (KEK)
  - Features and development driven by KEK-B and circular lepton colliders
- **Methodical Accelerator Design (MADX)**
  - And its Polymorphic Tracking Code (PTC) implementation
  - Maintained in the European Organization for Nuclear Research (CERN).
  - Recent developments stimulated by CERN’s hadron infrastructure and more recently also FCC-ee

Code Comparisons

Linear Optics

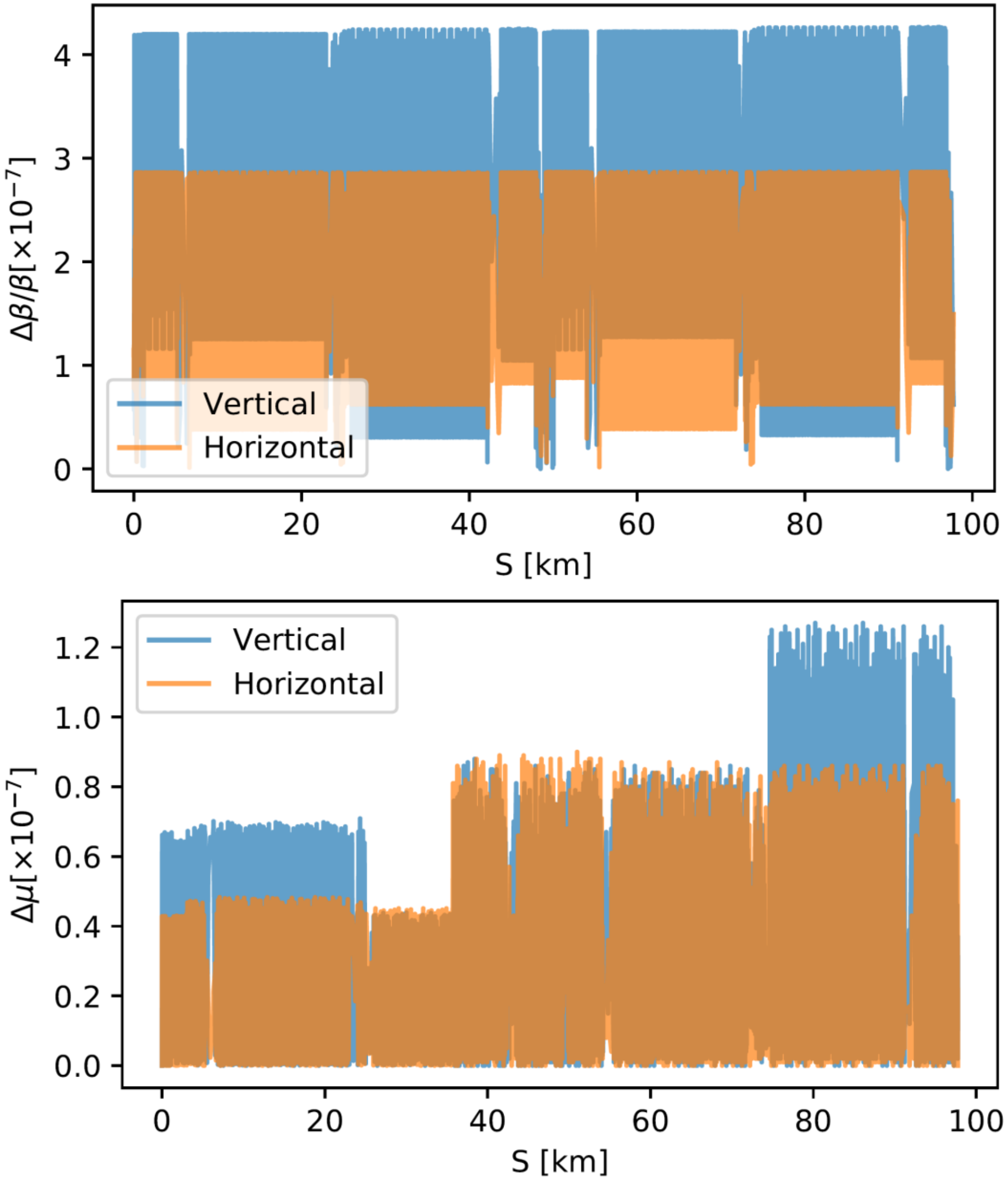
FCC-ee ZZ version 213 (No radiation)

Comparison between MADX and SAD

- MADX and MADX-PTC known to be good

Error in agreement oscillates with a magnitude of about 10^{-7}

- Likely numerical tolerances of closed orbit search



Difference in Linear Optics Between MADX and SAD

Code Comparisons

Momentum Detuning

FCC-ee ZZ version 213 (No Radiation)

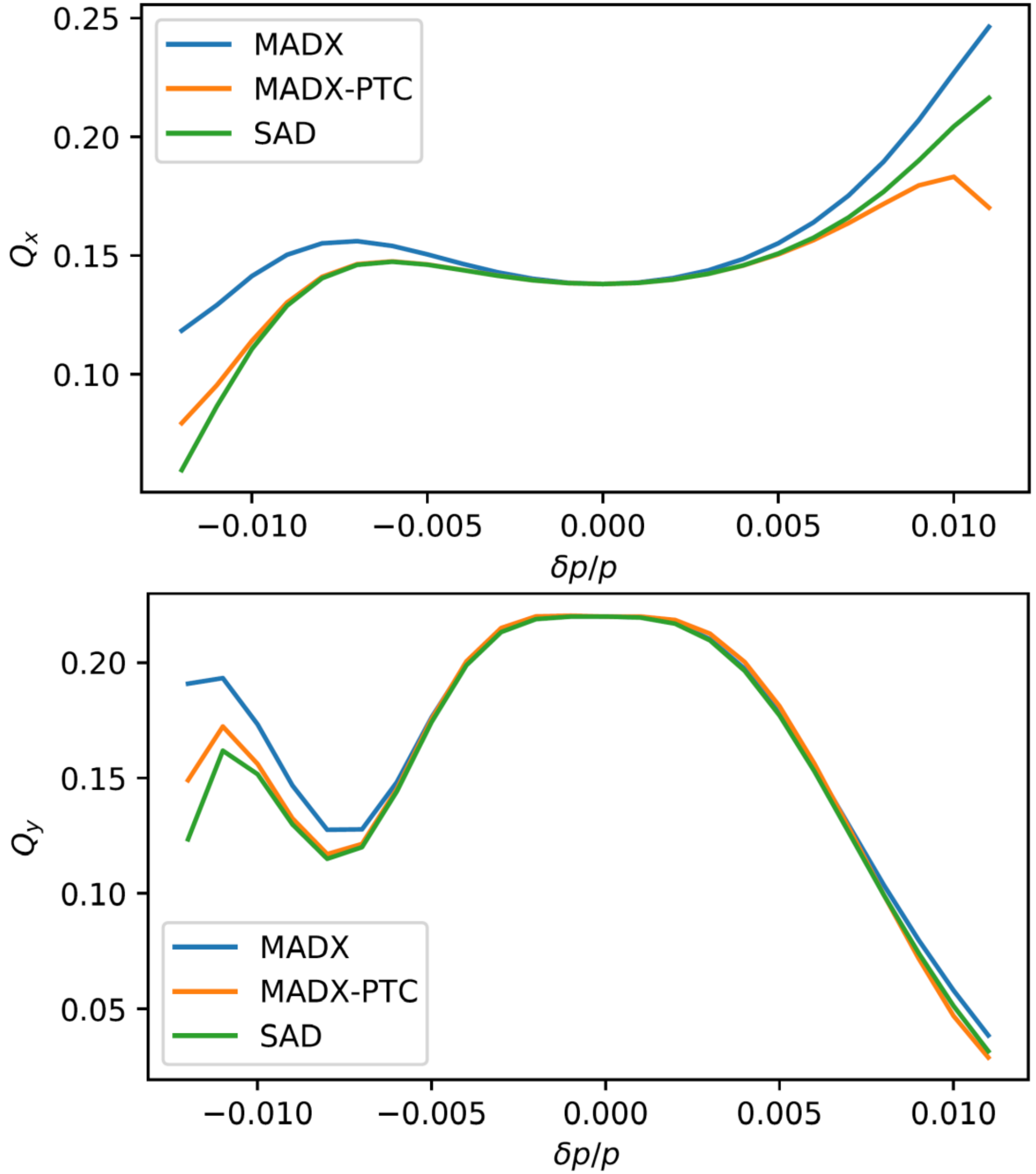
Comparison between all three codes

Compute closed twiss with momentum offset

- Plot tune vs momentum offset
- Range over which closed twiss is found
  - Before hitting resonances

Good agreement between all three codes

- Best agreement between MADX-PTC and SAD



Horizontal and Vertical Momentum Detuning

Code Comparisons

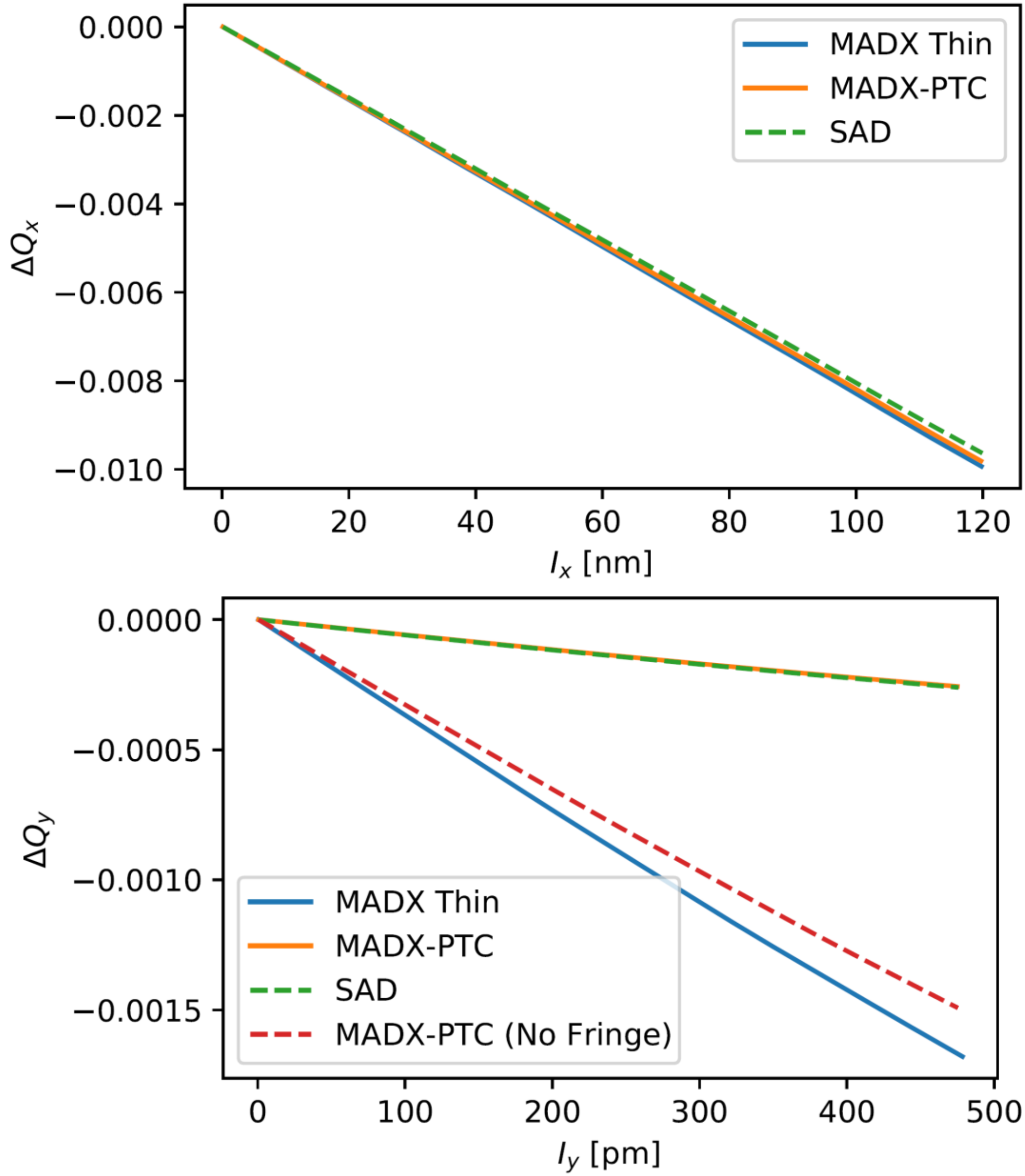
Amplitude Detuning

FCC-ee ZZ version 213 (no radiation)

Tune change computed from tracking
- Tune and amplitude computed using harpy
- MADX tracking requires slicing of magnets

Detuning heavily dependant on number of slices/integration steps of IR sextupoles and doublet
- Convergence found at:
  - 20 thin slices in MADX
  - 15 steps in MADX-PTC
  - EPS = 0.01 in SAD

MADX results resemble PTC results with fringe flag turned off (no fringe fields from dipoles)



Horizontal and Vertical Amplitude Detuning

Code Comparisons

Radiation Integrals

FCC-ee ZZ version 213

Computed in MADX and SAD

- < 0.0001% difference

I_4 integral momentum dependant in SAD but not MADX

- Momentum dependence now captured by I_8 integral implemented in MADX
- MADX also now includes I_6 integral
  - Quantify energy loss in quadrupoles

	SAD	MADX
I_1	1.441818	1.441817674
I_2	5.8860828×10^{-4}	$5.886083969\times 10^{-4}$
I_3	5.4659284×10^{-8}	$5.465930449\times 10^{-8}$
I_4	$-2.2581083\times 10^{-10}$	$-2.258108575\times 10^{-10}$
I_5	5.2274385×10^{-11}	$5.226180922\times 10^{-11}$

Radiation Integrals Determined by SAD and MADX

Code Comparisons

Tapering

Adjust magnetic strengths to compensate for local beam energy

- Variation in energy due to strong radiation
- Required for FCC-ee

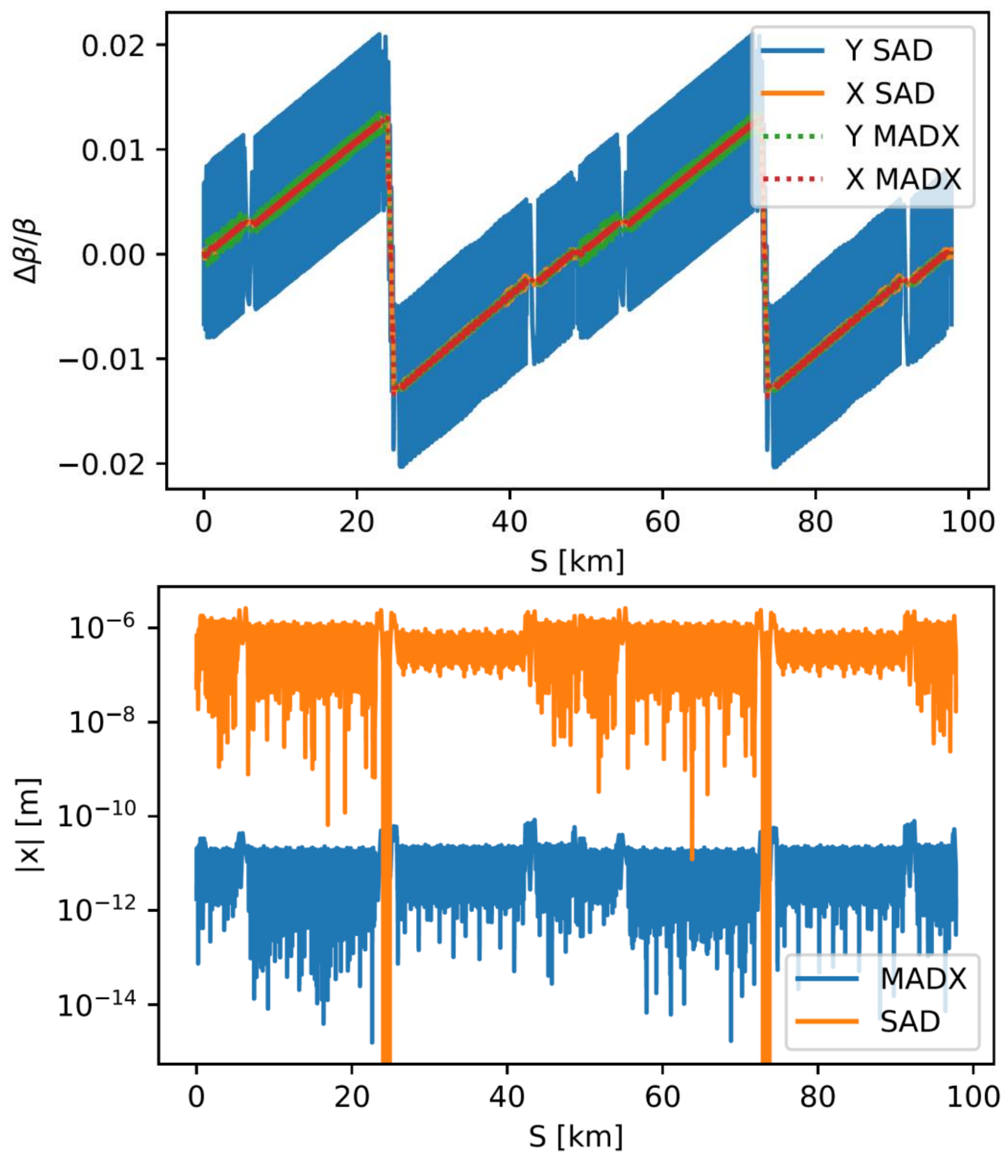
Existing feature in SAD, recently implemented in MADX

- MADX version 5.6.00 or later

FCC-ee tt version 213

MADX tapering manages to reduce residual orbit sufficiently

- Also minimal residual β-beating
- Higher residual orbit in SAD could again be due to closed orbit tolerances



β-beating and Residual Orbit of tapered lattices

Code Comparisons

Emittance in Tapered Lattice

Emittance in new tapered lattices can be computed using EMIT module

- Compared to SAD
- Can also be compared to value from EMIT at 1 GeV scaled by [E GeV]^2
  - Previously shown that at GeV there is good agreement between MADX, SAD and MADX-PTC (Envelope Module)

Also good agreement with longitudinal emittance between MADX and SAD

- Scaling more complicated due to dependence on cavity voltage

Lattice	Energy	ϵ_x @ 1 GeV	Scaled ϵ_x	Tapered ϵ_x	SAD ϵ_x
Z	45.6 GeV	1.30×10^{-5} nm	0.27 nm	0.27 nm	0.27 nm
WW	80 GeV	1.30×10^{-4} nm	0.83 nm	0.83 nm	0.84 nm
ZH	120 GeV	4.35×10^{-5} nm	0.63 nm	0.63 nm	0.63 nm
tt	182.5 GeV	4.35×10^{-5} nm	1.45 nm	1.45 nm	1.46 nm

Code Comparisons

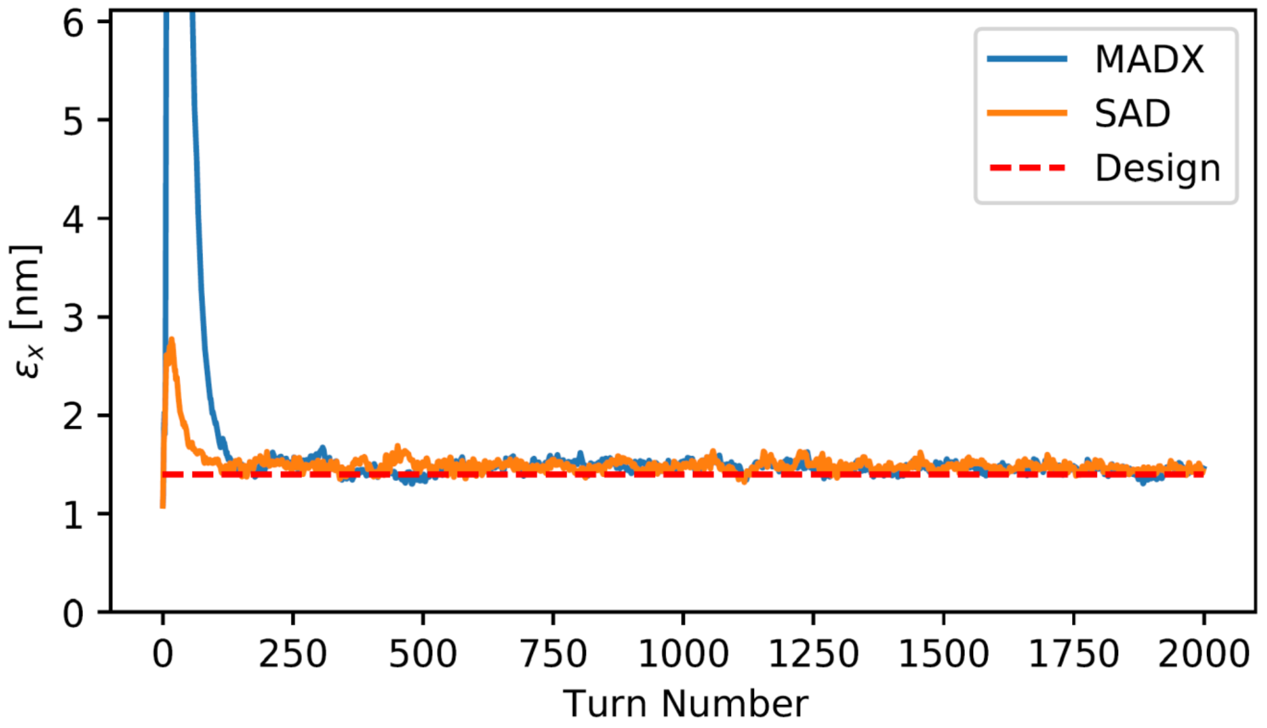
Emittance from Tracking

FCC-ee tt version 213

Tracking with radiation
- Requires tapered lattice

Tracking of 5000 particles
- Identical distribution in both codes
- Include damping and quantum excitations
- Compute emittance every turn

Motion in MADX and SAD damps to similar levels
- Good agreement with design emittance



Emittance from Tracking over 2000 turns

Code Comparisons

On-Axis Solenoid

FCC-ee ZZ version 213 (no radiation)

- 2T solenoid and anti-solenoid added manually

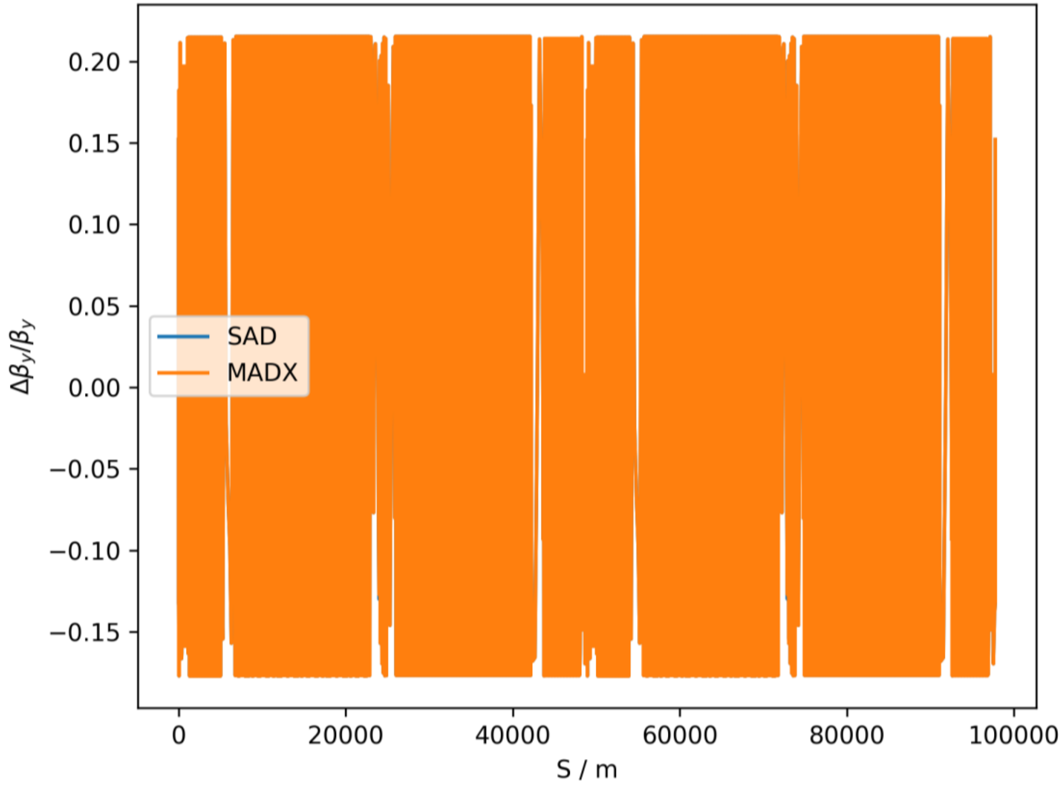
Slight differences in the MADX and SAD implementation

- Solenoid treated as element in MADX
- Solenoid markers in SAD indicate change in longitudinal magnetic field
  - Allows for overlapping elements

β-beating due to solenoid virtually identical for both codes

- Significantly larger beating in vertical plane
- Smaller beating but similar in horizontal plane

Good agreement for emittance in both codes



Beta beating between lattice with and without solenoid

Code Comparison

Tilted Solenoid and Other Future Work

Currently discrepancies between tilted solenoid in MADX and SAD

- Significant differences in optics, emittance and tracking
  - Including synchrotron radiation effects
  - Ongoing efforts also from MADX team

Further comparison and validation test, especially with radiation

Aid in implementation of key features needed from MADX e.g. solenoid fringe fields

Making comparison scripts available to the FCC-ee community

- Currently shared with some colleagues on AFS, git repository by end of July

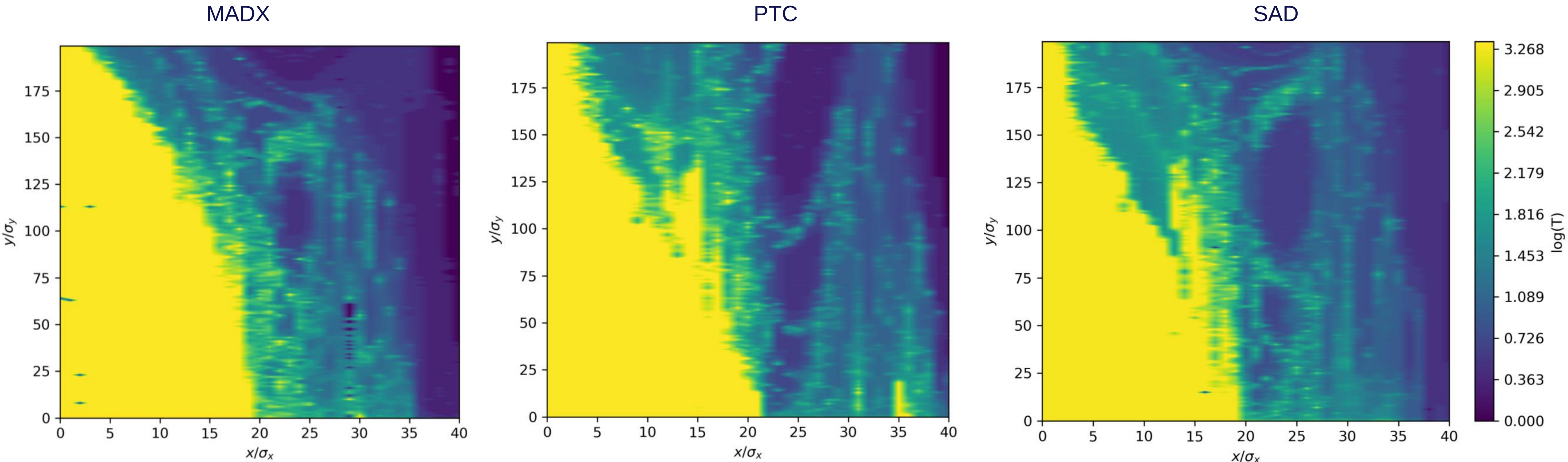
DYNAMIC APERTURE

Dynamic Aperture

Comparison Study

Comparison study

- FCC-ee ZZ version 213, 2000 turns, no radiation



Dynamic Aperture

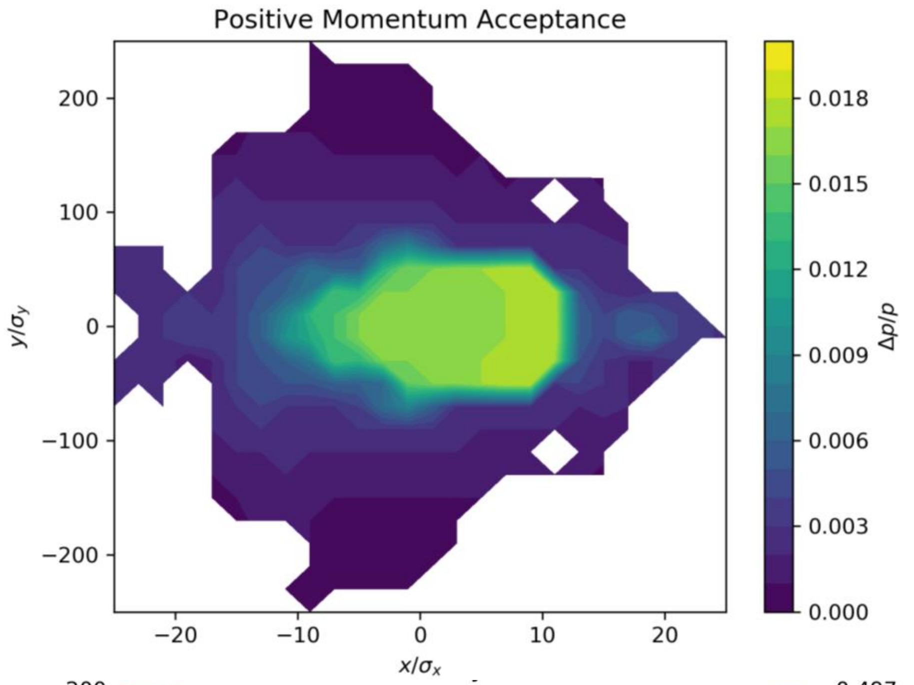
PTC Studies Without Radiation

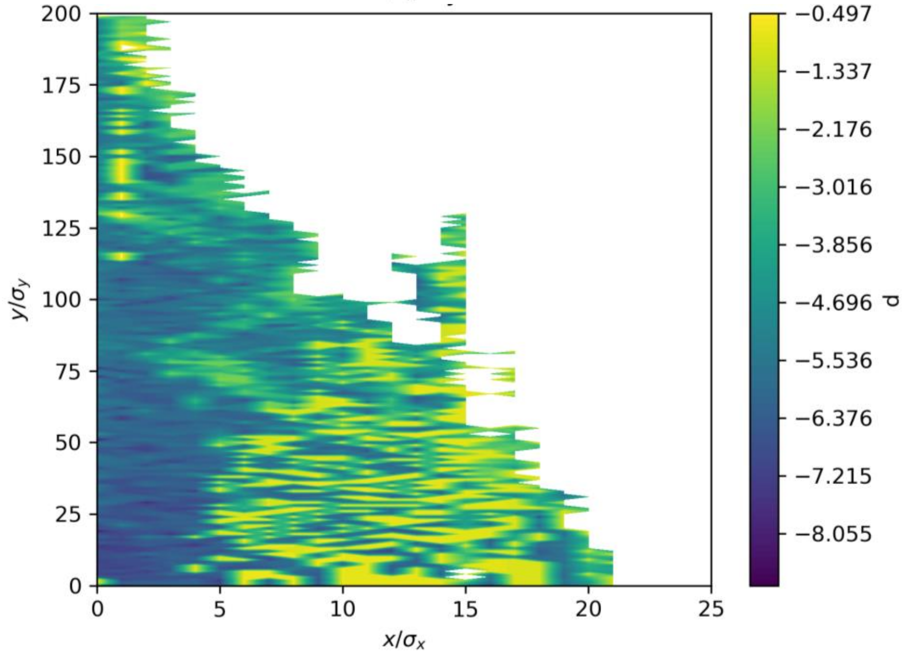
FCC-ee ZZ version 213 (No radiation)

Extensive dynamic apertures studies performed using PTC

- PTC settings based on findings of comparison studies
- Optimised for parallelisation and HTCondor using OMCPython libraries

Complete results in optics talk (link)





Dynamic Aperture

Corrected Lattices

First studies with lattices corrected by T. Charles

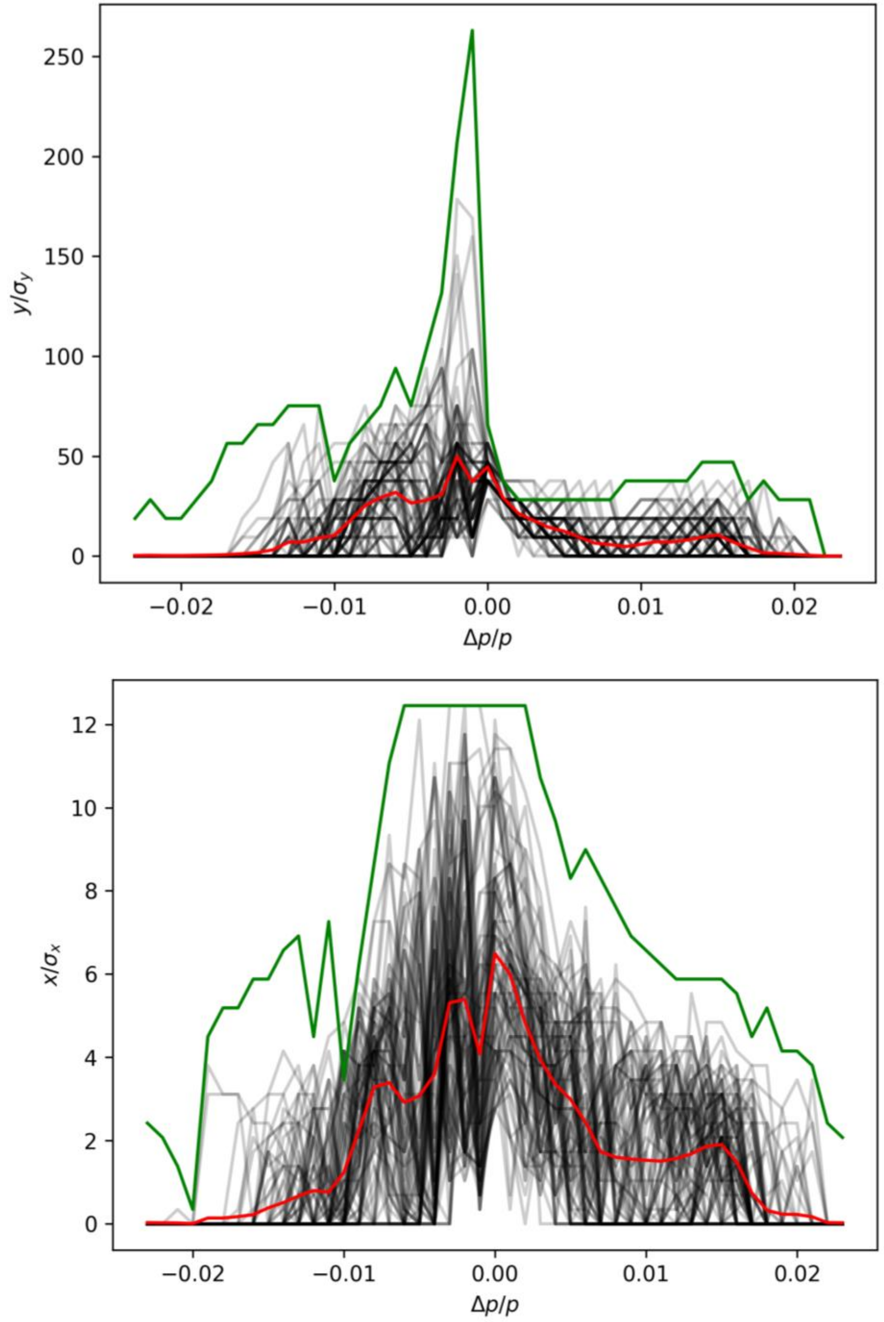
- 4IP tt Lattice version
- No radiation

Plots showing maximum stable amplitude

- Survives 2000 turns
- 100 seeds
- Green = no errors
- Red = Average

Ongoing effort and future requirements

- Including radiation, especially for tt lattice
  - Using MADX tapering
- Dedicated corrections to increase dynamic aperture (currently optimised for emittance)
- Inclusion of other effects



ANALYTICAL EMITTANCE ESTIMATES

Analytical Emittance Studies

Motivation and Background

Tool for alignment tolerance budget for FCC-ee

- Additional tool to help identify critical magnets and assign different tolerances to different magnets
- Compliment more precise emittance studies by T. Charles et.al.

Estimate using analytical formulas

- Formulas readily available in the literature, e.g. SLAC-PUB-4937

$$\frac{\epsilon_y}{< y_{sext}^2 >} \approx \frac{J_x\left(1-\cos(2\pi\nu_x)\cos(2\pi\nu_y)\right)\epsilon_x}{J_y\left(\cos(2\pi\nu_x)-\cos(2\pi\nu_y)\right)^2}\sum_{sext}\beta_x\beta_y\left(\frac{k_2L}{2}\right)^2+\frac{J_z\sigma_\delta^2}{\sin^2(\pi\nu_y)}\sum_{sext}\beta_y\eta_x^2\left(\frac{k_2L}{2}\right)^2$$

$\frac{k_2L}{2} \to k_1L$ and $< y_{sext}^2 > \to < \theta_{quad}^2 >$ for quads

Analytical Emittance Studies

Quadrupole Rolls

Analytical formula compared to MADX simulations with 100 error seeds

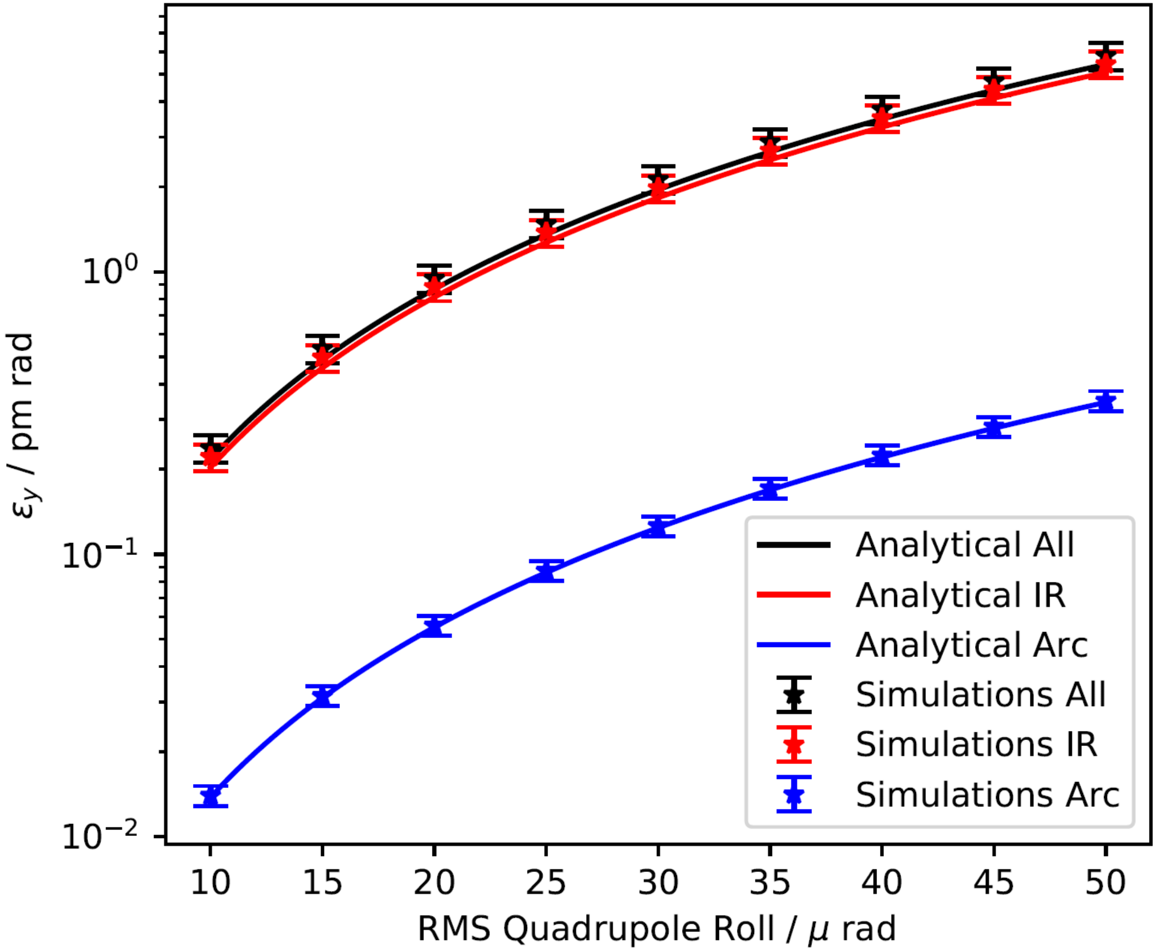
- Various RMS error sizes

Simulations with errors applied to all quadrupoles

Simulations with errors only applied to arc or IR quadrupoles

Very good agreement between simulations and estimates

Most emittance due to IR quadrupoles



Emittance due to Quadrupole Roll Errors

Analytical Emittance Studies

Single Quadrupoles

Compare how well formula describes contribution from single quadrupoles

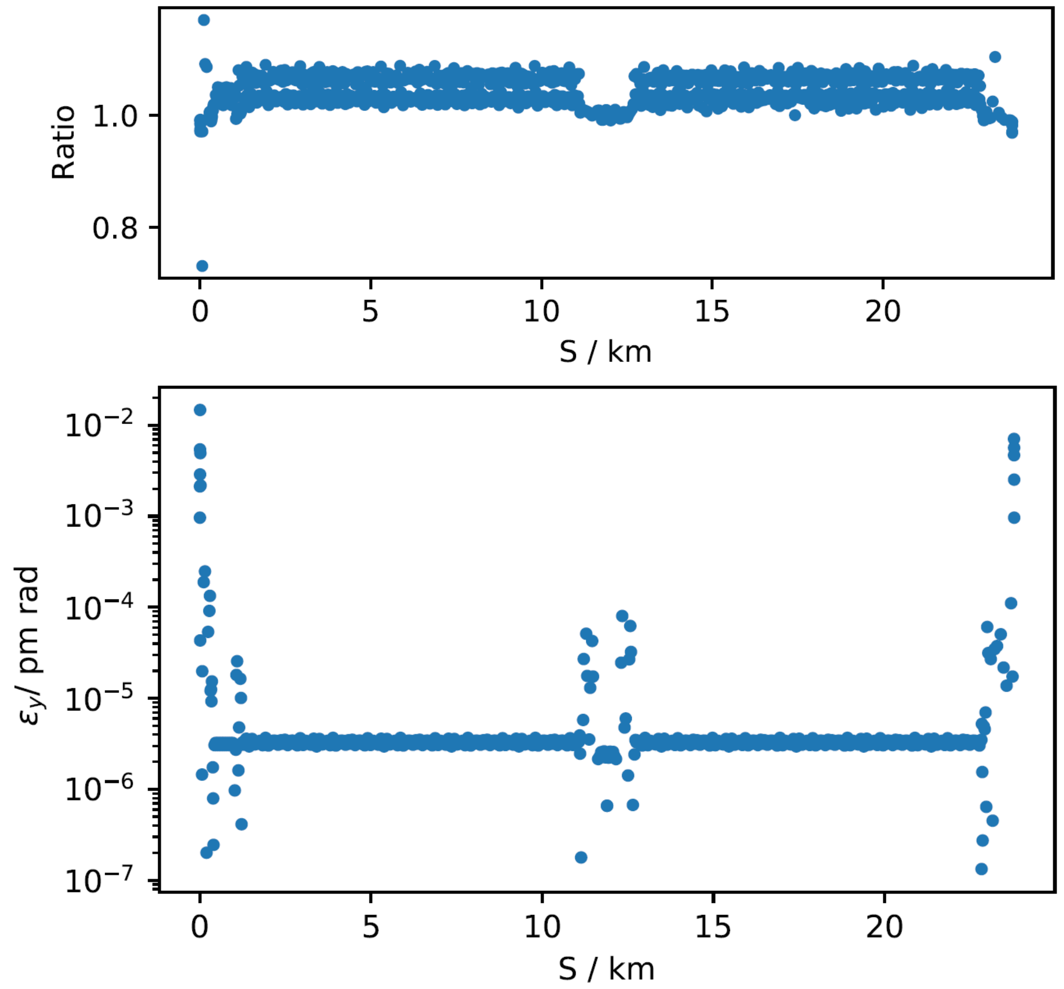
- In MADX, cycle through the machine, turn each quadrupole by 10 μrad
- Divide analytical solution by simulation value

Very good agreement

- Slight overestimate in many arc quadrupoles
- Slight underestimate in some IR quadrupoles

Plot emittance contribution by single quadrupoles

- Clearly see how some IR quadrupoles dominate total emittance



Emittance due to Single Quadrupole Roll and Ratio between MADX and analytical

Analytical Emittance Studies

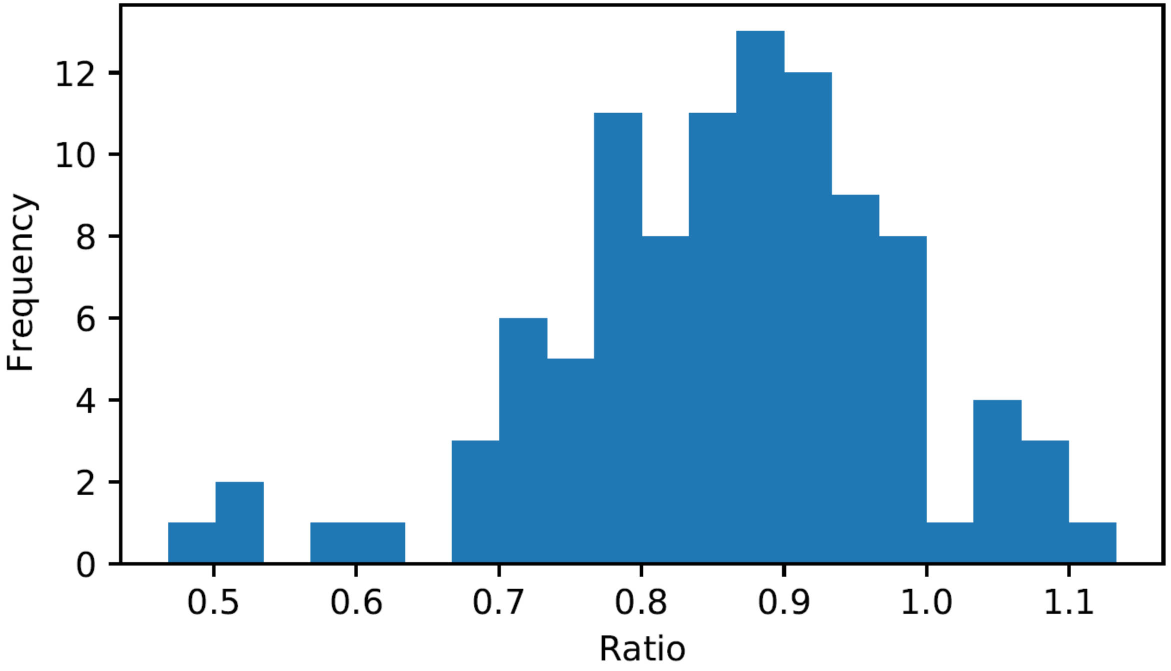
Groups of Quadrupoles

Explore if contributions of single quadrupoles add linearly

- Select 100 groups of 300 random quadrupoles
- Perform simulations with 100 error seeds for each group
- For each group divide analytical result by average from simulations

Good agreement:

- Mean ratio: 0.86
- Standard deviation 0.12
- Slight underestimate



Ratio Between Analytical and MADX Emittance for Different Magnet Groups

Analytical Emittance Studies

Sextupole Misalignments

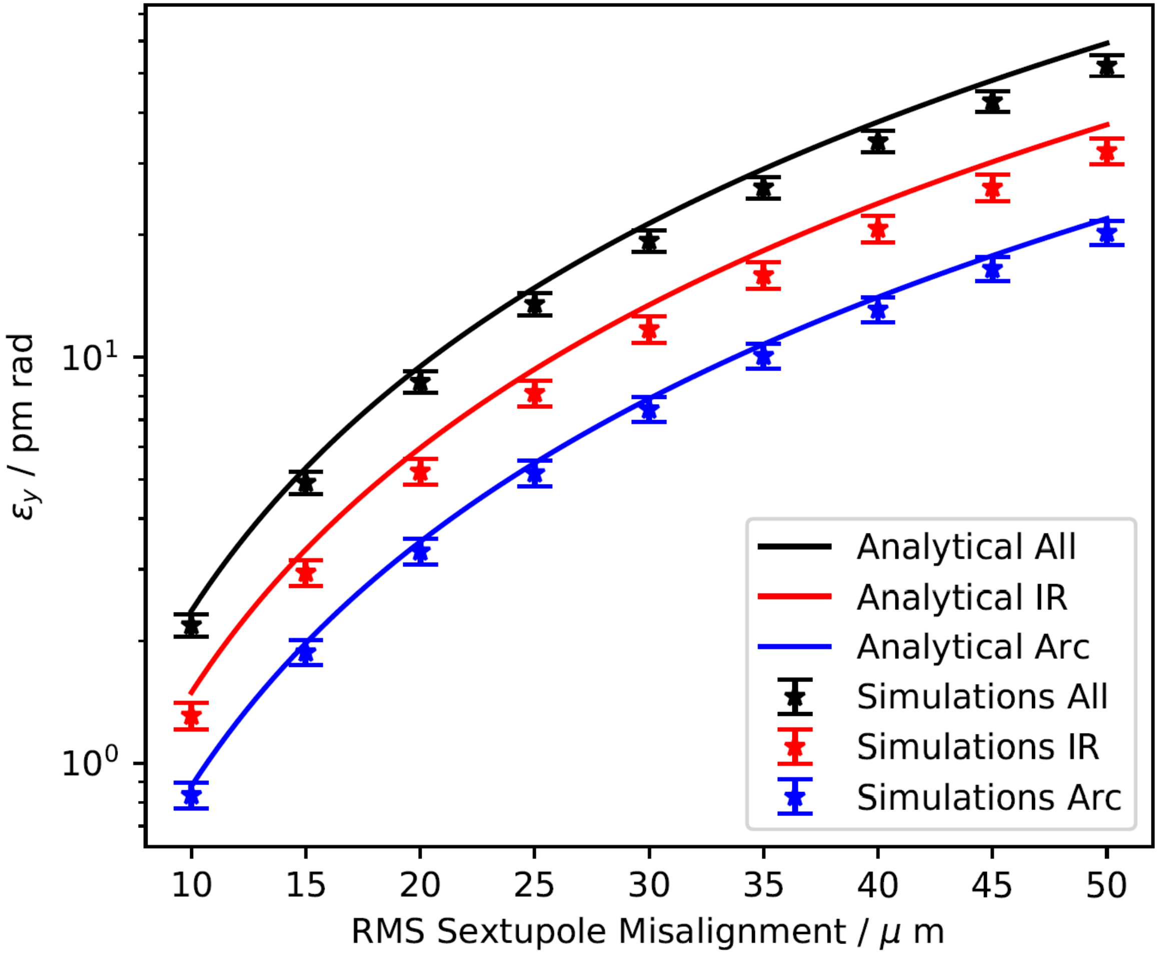
Similar setup as for quadrupoles

Analytical formulas tend to be a slight overestimate

- In particular in simulations involving IR sextupoles

IR sextupoles contribute more than arc sextupoles

- Difference less significant than for quadrupoles



Emittance due to Sextupole Alignment Errors

Analytical Emittance Studies

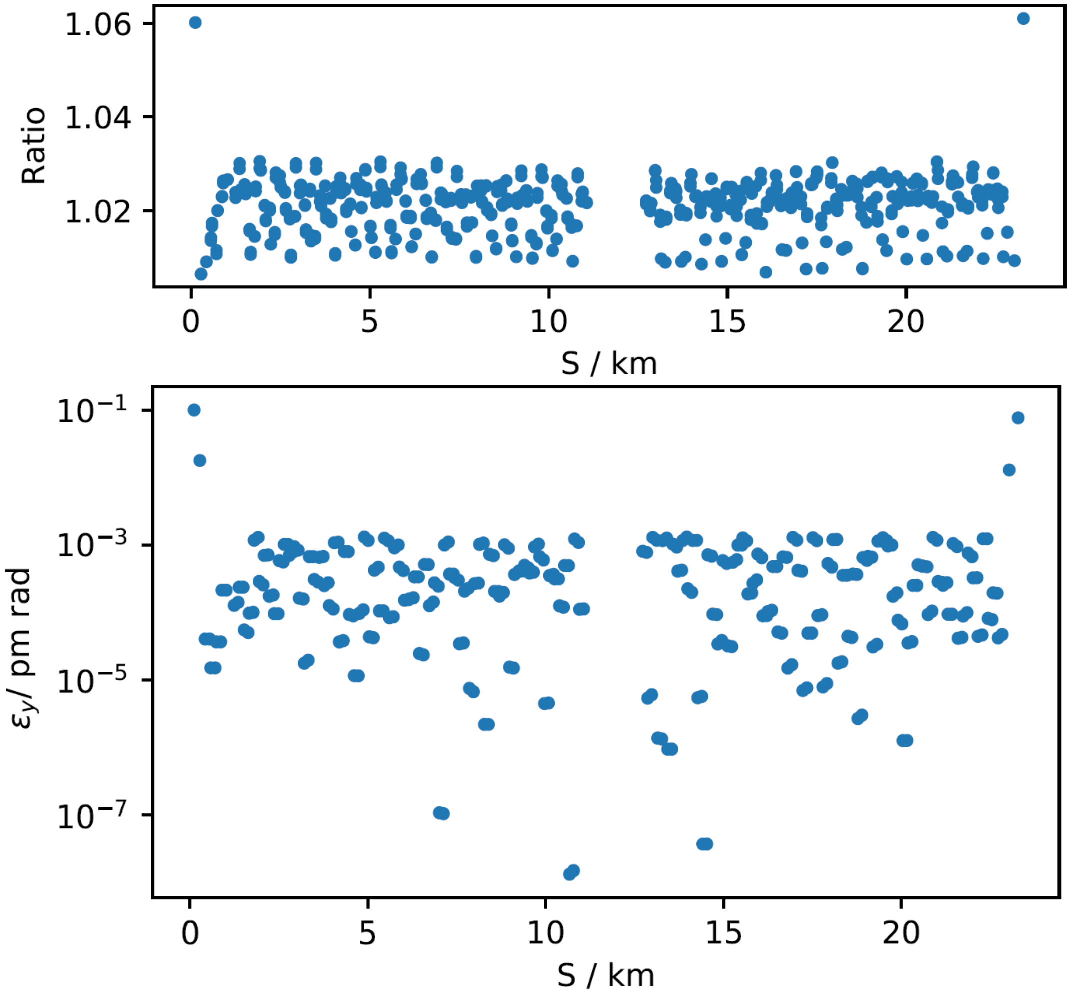
Single Sextupoles

Similar study to case with quadrupoles

Smaller distribution in of ratio between analytical and simulations than for quadrupoles

Lower difference in emittance contribution between arcs and IR than for quadrupoles

Larger variation in emittance contributions within arcs than for quadruples



Emittance due to Single Sextupole Misalignment and Ratio between MADX and analytical

Analytical Emittance Studies

Next Steps

Extend to further error types

- Analytically – using methods in the literature or own derivations
- Empirically – determine effect of single magnet by simulation and exploit good behaviour in groups and with scaling

Explore combinations of error types

- Start by looking into similar errors e.g. sextupole misalignments and quadrupole rolls

Ultimately aim to have an emittance budget for every magnet type