Neutrinos

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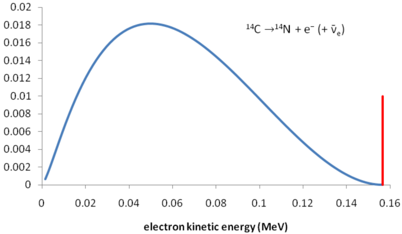
UNIVERSITY OF SHARJAH

Outline

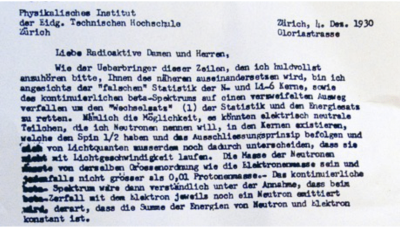
Some History about Neutrinos

- In 1896, the Franch physicist Henri Bequerel discovered the radiocativity.
- As a consequence, a neutron (1932) may give a proton (1911) and an electron (1897): $n \to p + e^-$. BUT ...



Is the **ENERGY CONCERVATION PRINCIPLE CORRECT**??

- The proposal of the "neutrino" was put forward by W. Pauli in 1930. [Pauli Letter Collection, CERN]



Physikalisches Institut
der Eidg. Technischen Hochschule
Zürich

Zürich, 4. Dez. 1930
Gloriastrasse

Liebe Radioaktive Damen und Herren,

Wie der Ueberbringer dieser Zeilen, den ich huldvollst anzuhören bitte, Ihnen des näheren auseinandersetzen wird, bin ich angesichts der "falschen" Statistik der N- und Li-6 Kerne, sowie des kontinuierlichen beta-Spektrums auf einen verzweifelten Ausweg verfallen um den "Wechselsatz" (1) der Statistik und den Energiesatz zu retten. Nämlich die Möglichkeit, es könnten elektrisch neutrale Teilchen, die ich Neutronen nennen will, in den Kernen existieren, welche den Spin 1/2 haben und das Ausschliessungsprinzip befolgen und sich von Lichtquanten ausserdem noch dadurch unterscheiden, dass sie nicht mit Lichtgeschwindigkeit laufen. Die Masse der Neutronen müsste von derselben Grössenordnung wie die Elektronenmasse sein und jedenfalls nicht grösser als 0,01 Protonenmasse.- Das kontinuierliche beta- Spektrum wäre dann verständlich unter der Annahme, dass beim beta-Zerfall mit dem Elektron jeweils noch ein Neutron emittiert wird, derart, dass die Summe der Energien von Neutron und Elektron konstant ist.

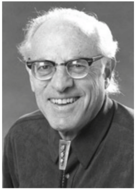
Dear radioactive ladies and gentlemen,

…I have hit upon a desperate remedy to save the … energy theorem. Namely the possibility that there could exist in the nuclei electrically neutral particles that I wish to call neutrons, which have spin 1/2 … The mass of the neutron must be … not larger than 0.01 proton mass. …in β decay a neutron is emitted together with the electron, in such a way that the sum of the energies of neutron and electron is constant.

- Since the neutron was discovered two years later by J. Chadwick, Fermi, following the proposal by E. Amaldi, used the name "neutrino" (little neutron) in 1932 and later proposed the Fermi theory of beta decay.

Some History about Neutrinos

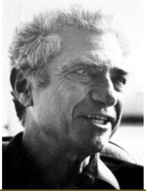
- Reines and Cowan discovered the neutrino in 1956 using inverse beta decay.



The Nobel Prize in Physics 1995

- Madame Wu in 1956 demonstrated that P is violated in weak interactions.

- Muon neutrinos were discovered in 1962 by L. Lederman, M. Schwartz and J. Steinberger.



The Nobel Prize in Physics 1988

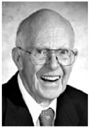
Some History about Neutrinos

- The first idea of neutrino oscillations was considered by B. Pontecorvo in 1957.
[B. Pontecorvo, J. Exp. Theor. Phys. 33 (1957)549.
B. Pontecorvo, J. Exp. Theor. Phys. 34 (1958) 247.]

- Mixing was introduced at the beginning of the '60 by Z. Maki, M. Nakagawa, S. Sakata,
Prog. Theor. Phys. 28 (1962) 870, Y. Katayama, K. Matumoto, S. Tanaka, E. Yamada, Prog. Theor. Phys. 28 (1962) 675 and M. Nakagawa, et. al., Prog. Theor. Phys. 30 (1963)727.

- First indications of ν oscillations came from solar ν.

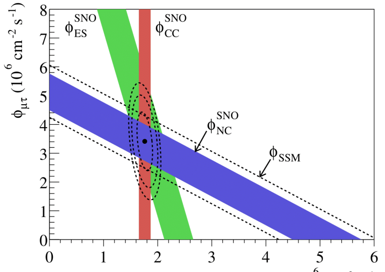
- R. Davis built the Homestake experiment to detect solar ν, based on an experimental technique by Pontecorvo.



Raymond Davis Jr.
1/4 of the prize
USA
University of Pennsylvania
Philadelphia, PA, USA

- Compared with the predicted solar neutrino fluxes (J. Bahcall et al.), a significant deficit was found. First results were announced [R. Davis, Phys. Rev. Lett. 12 (1964)302 and R. Davis et al., Phys. Rev. Lett. 20 (1968) 1205].

- This anomaly received further confirmation (SAGE, GALLEX, SuperKamiokande, SNO...) and was finally interpreted as neutrino oscillations.



SNO, PRL 89 2002

The Nobel Prize in Physics 2015

An anomaly was also found in atmospheric neutrinos.

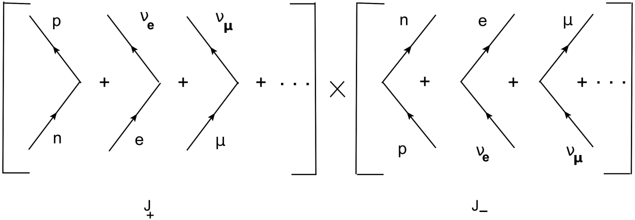
- Atmospheric neutrinos had been observed by various experiments but the first relevant indication of an anomaly was presented in 1988 [Kamiokande Coll., Phys. Lett. B205 (1988) 416], subsequently confirmed by MACRO.

- Strong evidence was presented in 1998 by SuperKamiokande (corroborated by Soudan2 and MACRO) [SuperKamiokande Coll., Phys. Rev. Lett. 81 (1998) 1562]. This is considered the start of "modern neutrino physics"!

The Nobel Prize in Physics 2015

Fermi Theory for Weak Interactions

- In 1933, E. Fermi put his theory to describe the beta decay $n \to p + e^- + \bar{\nu}$ but assuming a POINT interaction between two charged curents: hadronic (proton and neutron) and leptonic (el;ectron and neutrino) as $L_F = \frac{G_F}{\sqrt{2}}\ \bar{p}\gamma^\mu n\ \bar{e}\gamma_\mu \nu + h.c.$, with $G_F = 1.1663787 \times 10^{-5}\ GeV^{-2}$.
- When generalized

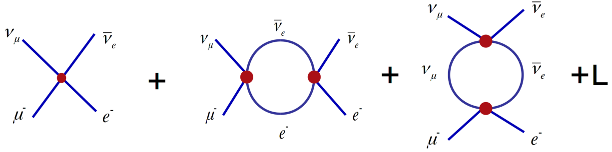


Later on, it had been realized that this theory is not valid at higher energies. It needed to be modified ...

Fermi Theory for Weak Interactions

There existed other reasons:

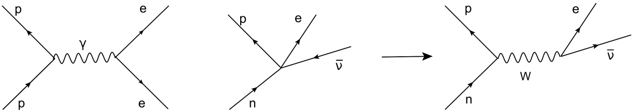
The Fermi theory has trouble with higher order weak contributions



Weak radiative corrections: infinite

- Can't be absorbed through suitable re-definition of G_F in H_{EFF}
- P violation : consider only left-handed Chirality ($P_L = \frac{1}{2}(1 - \gamma^5)$) for fermions in interaction with $W^\pm$.

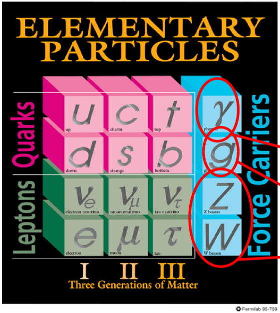
- Analogy with QED : in 1954, Yang & Mills introduced non-abelian gauge theory as a QED generalization ... the introduction to spin-1 particle like γ but charged $W^{\pm}$.



So the leptonic current should be $J_\ell = \bar{e}\gamma_\mu \frac{1}{2}(1-\gamma^5)\nu$ ... the so-called V-A theory (Sudarshan, Marshak, Feynman, Gell-Mann and Sakurai in 1957).

- Since only left-handed neutrino ($\nu_L = \frac{1}{2}(1-\gamma^5)\nu$) exists $\Longrightarrow$ **MUST BE MASSLESS**.

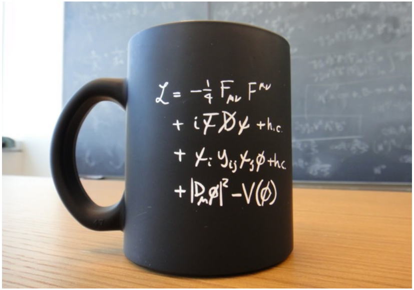
The Standard Model of particle physics is a triumph of late 20th century physics

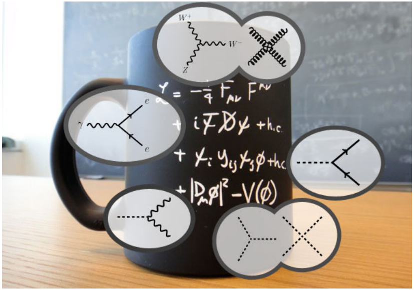


- Utilizes a simple & elegant symmetry principle to organize what we've observed

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

A Glimps on the Standard Model





SSB of Gauge symmetry

The effective field theory of superconductivity: a complex scalar field with charge q coupled to the U(1) gauge field

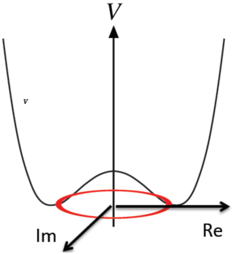
$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^\dagger D_\mu\phi - V(\phi) \qquad D_\mu = \partial_\mu - iqA_\mu$$

$$V(\phi) = -\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$$

$$\lambda > 0$$

Lowest energy configuration (vacuum):

$$\phi^\dagger\phi = \frac{\mu^2}{2\lambda} \equiv \frac{v^2}{2} \qquad \phi = \frac{v}{\sqrt{2}}e^{i\theta}$$



The ABEGH²KN Mechanism

Anderson-Brout-Englert-Guralnik-Hagen-Higgs-Kibble-Nambu

$$\mathcal{L}^{(2)} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{q^2v^2}{2}A_\mu A_\mu + \frac{1}{2}(\partial_\mu h\, \partial_\mu h - \mu^2 h^2) + \frac{v^2}{2}\partial_\mu\theta\, \partial_\mu\theta - qv^2\partial_\mu\theta A_\mu$$

Gauge transformation: $A'_\mu = A_\mu - \frac{1}{q}\partial_\mu\theta$

$$= -\frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + \frac{q^2v^2}{2}A'_\mu A'_\mu + \frac{1}{2}(\partial_\mu h\, \partial_\mu h - \mu^2 h^2)$$

Goldstone mode -> massive gauge field (ie. longitudinal polarization)

Goldstone mode "is eaten" by the gauge field to get massive: unitary gauge

Radial mode -> massive neutral scalar field

SM BEH mechanism

Of the full symmetry group: $\phi \to e^{iT^a \alpha^a} \phi \quad T^a = \left(\frac{\sigma^0}{2}, \frac{\vec{\sigma}}{2} \right)$

A U(1) subgroup remains unbroken

$$(T^0 + T^3)\langle\phi\rangle = 0$$

$$SU(2) \times U(1) \to U(1)_{em}$$

v

three broken generators: three massive gauge fields (W^{+-}, Z^0) and one massless photon

Gauge boson masses

$$\phi = e^{i\alpha^a T^a}\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

In unitary gauge:

$$D_\mu \phi^\dagger D_\mu \phi = \frac{1}{2}\partial_\mu h\ \partial_\mu h + \frac{1}{2}(0 \quad v+h)\left(gW_\mu^a \frac{\sigma^a}{2} + \frac{1}{2}g' B_\mu\right)^2 \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$= \frac{1}{2}\frac{v^2}{4}\{g^2\left((W_\mu^1)^2 + (W_\mu^2)^2\right) + (g' B_\mu - g W_\mu^3)^2\} = m_W^2 W_\mu^+ W_\mu^- + \frac{1}{2} m_Z^2 Z_\mu Z_\mu + O(h)$$

Charged weak: $W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \pm i W_\mu^2)$ $\qquad m_W \equiv g\frac{v}{2}$

Neutral weak $Z_\mu = \frac{1}{\sqrt{g^2 + g'^2}}(g W_\mu^3 - g' B_\mu)$ $\qquad m_Z \equiv \sqrt{g^2 + g'^2}\frac{v}{2}$

Electromagnetic $A_\mu = \frac{1}{\sqrt{g^2 + g'^2}}(g' W_\mu^3 + g B_\mu)$

Gauge Boson masses

Weak mixing angle

$$Z_\mu = \frac{1}{\sqrt{g^2 + g'^2}}\left(gW_\mu^3 - g'B_\mu\right)$$

$$A_\mu = \frac{1}{\sqrt{g^2 + g'^2}}\left(g'W_\mu^3 + gB_\mu\right)$$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}$$

$$\cos\theta_W \equiv \frac{g}{\sqrt{g^2 + g'^2}} \qquad \sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}$$

v

$$m_W = m_Z \cos\theta_W$$

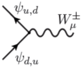
Neutral currents

All fermions are doublets $D_\mu\Psi = (\partial_\mu - igW^a_\mu\frac{\sigma^a}{2} - iY_\Psi g' B_\mu)\Psi$

$$D_\mu\Psi = \left(\partial_\mu - i\frac{g}{\sqrt{2}}(W^+_\mu T^+ + W^-_\mu T^-) - i\frac{g}{\cos\theta_W}(T^3 - \sin^2\theta_W Q_\Psi)Z_\mu - ieQ_\Psi A_\mu\right)\Psi$$

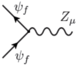
$$Q_\Psi \equiv T^3 + Y_\Psi \quad e \equiv \frac{gg'}{\sqrt{g^2+g'^2}} = g\sin\theta_W$$

Off-diagonal in isospin!

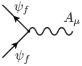


$i\frac{g}{\sqrt{2}}\gamma_\mu\frac{1-\gamma_5}{2}$

Diagonal in isospin!



$i\frac{g}{\cos\theta_W}\gamma_\mu\left(g^f_V - g^f_A\gamma_5\right)$



$-ieQ_f\gamma_\mu$

$$g^f_V = \frac{1}{2}T^3_f - Q_f\sin^2\theta_W, \quad g^f_A = \frac{1}{2}T^3_f\,.$$

Fermion Masses

Given the $SU(2)$ and $U(1)$ charges of the Higgs field and the fermions, Yukawa interactions are allowed:

$$-\mathcal{L} \supset Y_u Q u^c \phi + Y_d Q d^c \tilde{\phi} + Y_e L e^c \tilde{\phi} + h.c.,$$

where $\tilde{\phi} \equiv i\sigma_2 \phi^*$. Using

$$\langle\phi\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \langle\tilde{\phi}\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v \\ 0 \end{pmatrix},$$

$$-\mathcal{L} \supset (m_u)_{ij} u^i (u^c)^j + (m_d)_{ij} d^i (d^c)^j + (m_e)_{ij} e^i (e^c)^j + h.c.,$$

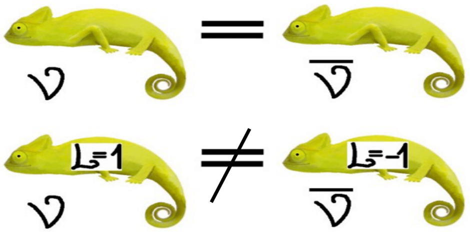
where $m_f \equiv Y_f v/\sqrt{2}$.

$u + u^c$, $d + d^c$, $e + e^c$ merge into Dirac fermions [note that all these fields have equal-but-opposite electric charges]. Neutrinos are left unpaired! $\rightarrow$ neutrinos are massless. Robust prediction, stable under quantum corrections.

Neutrinos can be Majorana or Dirac particles. In the SM only neutrinos can be Majorana because they are neutral.

Majorana particles are indistinguishable from antiparticles.

Dirac neutrinos are labelled by the lepton number.



The nature of neutrinos is linked to the conservation of the Lepton number (L). This information is crucial in understanding the Physics BSM: with or without L-conservation? and it can be linked to the existence of matter in the Universe.

Charge conjugation

This operation changes a field in its charge-conjugate (opposite quantum numbers):

$$\psi^c = C\bar{\psi}^T = i\gamma^2\psi^*$$

Properties: $C\gamma^{\alpha T}C^\dagger = -\gamma^\alpha \quad , \quad CC^\dagger = 1 \quad , \quad C^T = -C$
In Weyl representation: $C = i\gamma^2\gamma^0$
Let's apply it to a left-handed field

$$(\psi_L)^c = i\gamma^2\psi_L^* = i\begin{pmatrix} 0 & \sigma^2 \\ -\sigma^2 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ \eta^* \end{pmatrix} = \begin{pmatrix} i\sigma^2\eta^* \\ 0 \end{pmatrix}$$

We find that it behaves as a right-handed field!

$$(\psi_L)^c = (\psi^c)_R$$

Majorana fields

A Majorana field satisfies the Majorana condition

$$\psi = \psi^c$$

Majorana particles have 2 degrees of freedom:

$$\psi = \frac{1}{(2\pi)^{3/2}} \int \frac{1}{2E} \left(u_s(p) a_s(p) e^{ipx} + \xi v_s(p) a_s^\dagger(p) e^{-ipx} \right) d^3p$$

and, with respect to Dirac particles, the propagators

$$\overline{\psi(x_1)\psi^T}(x_2) = -S(x_1 - x_2)C \qquad \overline{\bar{\psi}^T(x_1)\bar{\psi}}(x_2) = C^\dagger S(x_1 - x_2)$$

Dirac $\quad \phi(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\mathbf{p}}}} \left(u_s^a(\mathbf{p}) \hat{a}^s_{\mathbf{p},a} e^{-ipx} + v_s^a(\mathbf{p}) \hat{b}^{s\dagger}_{\mathbf{p},a} e^{ipx} \right)$

Neutrino: Dirac or Majorana

A mass term for a fermion connects a left-handed field with a right-handed one. For example the "usual" Dirac mass

$$m_\psi(\bar{\psi}_R\psi_L + \mathrm{h.c.}) = m_\psi\bar{\psi}\psi$$

Dirac masses

This is the simplest case. We assume that we have two independent Weyl fields: ν_L , ν_R
and we can write down the term as above.

$$\mathcal{L}_{mD} = -m_\nu(\bar{\nu}_R\nu_L + \mathrm{h.c.})$$

This conserves lepton number!

$$\nu_L \to e^{i\alpha}\nu_L$$
$$\nu_R \to e^{i\alpha}\nu_R$$

$$\mathcal{L}_{mD} \to \mathcal{L}_{mD}$$

Diagonalize a Dirac mass term

If there are several fields, there will be a Dirac mass matrix.

$$\mathcal{L}_{mD} = -\bar{\nu}_{Ra}\ (m_D)_{ab}\ \nu_{Lb} + \text{h.c.}$$

This requires two unitary mixing matrices to diagonalise it

$$m_D = V m_{\text{diag}} U^\dagger$$

and the massive states are

$$n_L = U^\dagger \nu_L \quad n_R = V^\dagger \nu_R$$

This is the mixing matrix which enters in neutrino oscillations. So the form of the mass matrix determines the mixing pattern.

Majorana masses

If we have only the left-handed field, we can still write down a mass term, called Majorana mass term. We use the fact that

$$(\psi_L)^c = (\psi^c)_R$$

then the mass term is

$$\mathcal{L}_{mM} \propto -M_M \bar{\nu^c_L} \nu_L + \text{h.c.} = M_M \nu_L^T C^{-1} \nu_L$$

Hint:
$$\bar{\nu^c_L}\nu_L = (C\bar{\nu}_L^T)^\dagger \gamma^0 \nu_L = \bar{\nu}_L^* C^\dagger \gamma^0 \nu_L$$
$$= \nu_L^T \gamma^{0*} C^\dagger \gamma^0 \nu_L = -\nu_L^T C^{-1} \nu_L$$

This breaks lepton number!

$$\nu_L \to e^{i\alpha}\nu_L \qquad \mathcal{L}_{mM} \to e^{2i\alpha}\mathcal{L}_{mM}$$

Diagonalize a Majorana mass term

If there are several fields, there will be a Majorana mass matrix. We can show that it is symmetric.

$$M_M = M_M^T$$

In fact:

$$\nu_L^T M_M C^{-1} \nu_L = (\nu_L^T M_M C^{-1} \nu_L)^T$$
$$= -\nu_L^T M_M^T C^{-1,T} \nu_L = \nu_L^T M_M^T C^{-1} \nu_L$$

This implies that only one unitary mixing matrix is required to diagonalise it

$$M_M = (U^\dagger)^T m_{\text{diag}} U^\dagger$$

The massive fields are related to the flavour ones as

$$n_L = U^\dagger \nu_L$$

and the Lagrangian can be rewritten in terms of a Majorana field

$$\mathcal{L}_M = -\frac{1}{2}\bar{n_L^c} m_{\text{diag}} n_L - \frac{1}{2}\bar{n_L} m_{\text{diag}} n_L^c = -\frac{1}{2}\bar{\chi} m_{\text{diag}} \chi$$

with $$\chi \equiv n_L + n_L^c \Rightarrow \chi = \chi^c$$

A Majorana mass term (breaks L) leads to Majorana neutrinos (breaks L).

Dirac + Majorana masses

If we have both the left-handed and right-handed fields, we can write down three mass terms:

- a Dirac mass term
- a Majorana mass term for the left-handed field and
- a Majorana mass term for the right-handed field.

$$\mathcal{L}_{mD+M} = -m_\nu \bar{\nu}_R \nu_L - \frac{1}{2}\nu_L^T M_{M,L} C^{-1} \nu_L - \frac{1}{2}\nu_R^T M_{M,R} C^{-1} \nu_R + \text{h.c.}$$

This breaks lepton number, in both the Majorana mass terms.

The expectation is that, as lepton number is not conserved, neutrinos will be Majorana particles.

We start by rewriting $\mathcal{L}_{mD+M} = -\frac{1}{2}\overline{\psi_L^c}\mathcal{M}\psi_L + \text{h.c.}$

with $\psi_L \equiv \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}$ and $\mathcal{M} \equiv \begin{pmatrix} M_{M,L} & m_D^T \\ m_D & M_{M,R} \end{pmatrix}$

In fact

$$\mathcal{L}_{mD+M} = -\frac{1}{2}\overline{\nu_L^c}M_{M,L}\nu_L - \frac{1}{2}\bar{\nu_R}M_{M,R}\nu_R - \bar{\nu}_R m_D \nu_L + \text{h.c.}$$

and one can use $\overline{\nu_L^c} m_D^T \nu_R^c = \bar{\nu}_R m_D \nu_L$

Then, we need to diagonalise the full mass matrix, and we find the Majorana massive states, in analogy to what we have done for the Majorana mass case.

$$\chi \equiv n_L + n_L^c \Rightarrow \chi = \chi^c$$

The difference is that $n_L = U_j \nu_L + U_k \nu_R^c$

Not unitary

Mixing between mass states and sterile neutrinos

Summary of neutrino mass terms

Dirac masses

$$\mathcal{L}_{mD} = -m_\nu(\bar{\nu}_R \nu_L + \text{h.c.})$$

This term conserves lepton number.

Majorana masses

$$\mathcal{L}_{mM} \propto -M_M \bar{\nu_L^c} \nu_L + \text{h.c.} = M_M \nu_L^T C^{-1} \nu_L$$

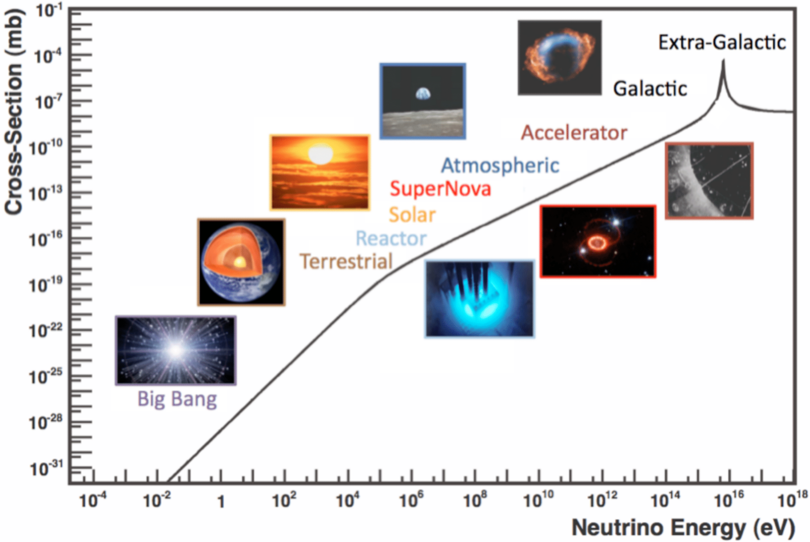
This term breaks lepton number.

Dirac + Majorana masses

$$\mathcal{L}_{mD+M} = -m_\nu \bar{\nu}_R \nu_L - \frac{1}{2}\nu_L^T M_{M,L} C^{-1} \nu_L - \frac{1}{2}\nu_R^T M_{M,R} C^{-1} \nu_R + \text{h.c.}$$

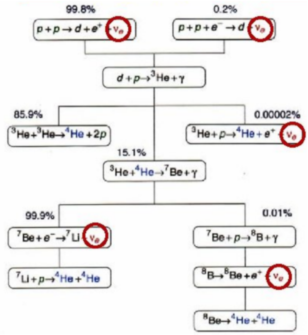
Lepton number is broken -> Majorana neutrinos.

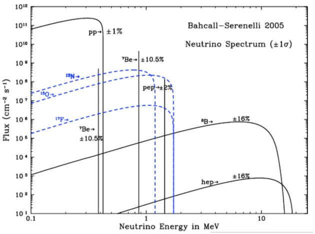
Neutrino sources

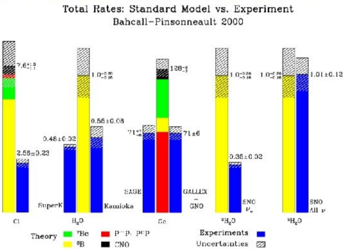


J. Formaggio and S. Zeller, 1305.7513

Neutrino Oscillations







The solution = neutrinos change their flavor during when traveling ... this is possible only if they are massive; and the mass eigenstates and the falvor eigenstates are DIFFERENT!!

Neutrino mixing

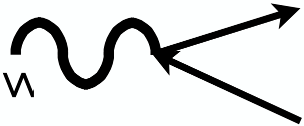
Mixing is described by the *Pontecorvo-Maki-Nakagawa-Sakata* matrix: $|\nu_\alpha\rangle = \sum_i U_{\alpha i}|\nu_i\rangle$

Flavour states (α) — Mass states (i)

which enters in the CC interactions

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}}\sum_{k\alpha}\left(U^*_{\alpha k}\bar{\nu}_{kL}\gamma^\rho l_{\alpha L}W_\rho + \text{h.c.}\right)$$

This implies that in an interaction with an electron, the corresponding (anti-)neutrino will be produced, as a superposition of different mass eigenstates.



Positron

electron neutrino $= \sum_i U_{ei}\nu_i$

- 2-neutrino mixing matrix depends on 1 angle only. The phases get absorbed in a redefinition of the leptonic fields (a part from 1 Majorana phase).

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

- 3-neutrino mixing matrix has 3 angles and 1(+2) CPV phases.

$$\begin{pmatrix} \bar{\nu}_1 & \bar{\nu}_2 & \bar{\nu}_3 \end{pmatrix} e^{i\psi} \begin{pmatrix} e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \text{CKM-} \\ \text{type} \end{pmatrix} \begin{pmatrix} e^{i\rho_e} & 0 & 0 \\ 0 & e^{i\rho_\mu} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}$$

Rephasing
$$\begin{aligned} e &\rightarrow e^{-i(\rho_e+\psi)} e \\ \mu &\rightarrow e^{-i(\rho_\mu+\psi)} \mu \\ \tau &\rightarrow e^{-i\psi} \tau \end{aligned}$$
the kinetic, NC and mass terms are not modified: these phases are unphysical.

For Dirac neutrinos, the same rephasing can be done.
For Majorana neutrinos, the Majorana condition forbids such rephasing: 2 physical CP-violating phases.

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{pmatrix}$$

For antineutrinos, $U \to U^*$

CP-conservation requires U is real $\Rightarrow \delta = 0, \pi$

Neutrinos oscillations in vacuum

Let's assume that at t=0 a muon neutrino is produced

$$|\nu, t=0\rangle = |\nu_\mu\rangle = \sum_i U_{\mu i}|\nu_i\rangle$$

The time-evolution is given by the solution of the Schroedinger equation with free Hamiltonian:

$$|\nu, t\rangle = \sum_i U_{\mu i} e^{-iE_i t}|\nu_i\rangle$$

In the same-momentum approximation:

$$E_1 = \sqrt{p^2 + m_1^2} \quad E_2 = \sqrt{p^2 + m_2^2} \quad E_3 = \sqrt{p^2 + m_3^2}$$

At detection one projects over the flavour state as these are the states which are involved in the interactions.
The probability of oscillation is

$$P(\nu_\mu \to \nu_\tau) = |\langle \nu_\tau | \nu, t \rangle|^2$$

$$= \left| \sum_{ij} U_{\mu i} U^*_{\tau j} e^{-iE_i t} \langle \nu_j | \nu_i \rangle \right|^2$$

$$= \left| \sum_{i} U_{\mu i} U^*_{\tau i} e^{-iE_i t} \right|^2$$

Typically, neutrinos are very relativistic: $E_i \simeq p + \frac{m_i^2}{2p}$

$$= \left| \sum_{i} U_{\mu i} U^*_{\tau i} e^{-i\frac{m_i^2}{2E} t} \right|^2$$

$$= \left| \sum_{i} U_{\mu i} U^*_{\tau i} e^{-i\frac{m_i^2 - m_1^2}{2E} t} \right|^2$$

Δm^2_{i1}

Implications of the existence of neutrino oscillations

The oscillation probability implies that

$$P(\nu_\alpha \to \nu_\beta) = \left| \sum_i U_{\alpha 1} U^*_{\beta 1} e^{-i \frac{\Delta m^2_{i1}}{2E} L} \right|^2$$

- **neutrinos have mass** (as the different components of the initial state need to propagate with different phases)

- **neutrinos mix** (as U needs not be the identity. If they do not mix the flavour eigenstates are also eigenstates of the propagation Hamiltonian and they do not evolve)

General properties of neutrino oscillations

- Neutrino oscillations conserve the total lepton number: a neutrino is produced and evolves with times
- They violate the flavour lepton number as expected due to mixing.
- Neutrino oscillations do not depend on the overall mass scale and on the Majorana phases.
- CPT invariance: $$P(\nu_\alpha \to \nu_\beta) = P(\bar{\nu}_\beta \to \bar{\nu}_\alpha)$$
- CP-violation:

$P(\nu_\alpha \to \nu_\beta) \neq P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$ requires $U \neq U^*(\delta \neq 0, \pi)$

2-neutrino case

Let's recall that the mixing is

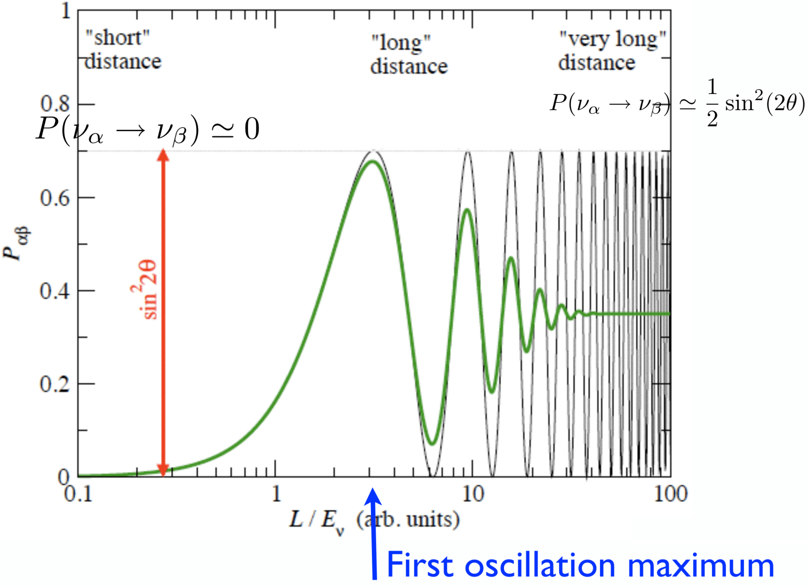
$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

We compute the probability of oscillation

$$\begin{aligned} P(\nu_\alpha \to \nu_\beta) &= \left| U_{\alpha 1} U^*_{\beta 1} + U_{\alpha 2} U^*_{\beta 2} e^{-i\frac{\Delta m^2_{21}}{2E}L} \right|^2 \\ &= \left| \cos\theta \sin\theta - \cos\theta \sin\theta e^{-i\frac{\Delta m^2_{21}}{2E}L} \right|^2 \\ &= \boxed{\sin^2(2\theta)\sin^2(\frac{\Delta m^2_{21}}{4E}L)} \end{aligned}$$

$$\boxed{\frac{\Delta m^2_{21}}{4E}L = 1.27\,\frac{\Delta m^2_{21}[\mathrm{eV}^2]}{4\ E[\mathrm{GeV}]}L[\mathrm{km}]}$$

Neutrino Oscillations



Properties of 2-neutrino oscillations

- Appearance probability:

$$P(\nu_\alpha \to \nu_\beta) = \sin^2(2\theta)\sin^2(\frac{\Delta m^2_{21}}{4E}L)$$

- Disappearance probability:

$$P(\nu_\alpha \to \nu_\alpha) = 1 - \sin^2(2\theta)\sin^2(\frac{\Delta m^2_{21}}{4E}L)$$

- No CP-violation as there is no Dirac phase in the mixing matrix

$$P(\nu_\alpha \to \nu_\beta) = P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$$

- Consequently, no T-violation (using CPT):

$$P(\nu_\alpha \to \nu_\beta) = P(\nu_\beta \to \nu_\alpha)$$

3-neutrino oscillations

They depend on two mass squared-differences

$$\Delta m^2_{21} \ll \Delta m^2_{31}$$

In general the formula is quite complex

$$P(\nu_\alpha \to \nu_\beta) = \left| U_{\alpha 1} U^*_{\beta 1} + U_{\alpha 2} U^*_{\beta 2} e^{-i \frac{\Delta m^2_{21}}{2E} L} + U_{\alpha 3} U^*_{\beta 3} e^{-i \frac{\Delta m^2_{31}}{2E} L} \right|^2$$

Interesting 2-neutrino limits

For a given L, the neutrino energy determines the impact of a mass squared difference. Various limits are of interest in concrete experimental situations.

- $\boxed{\frac{\Delta m^2_{21}}{4E} L \ll 1}$, applies to atmospheric, reactor (Daya Bay...), current accelerator neutrino experiments...

The oscillation probability reduces to a 2-neutrino limit:

$$P(\nu_\alpha \to \nu_\beta) = \left| \underline{U_{\alpha 1}U^*_{\beta 1} + U_{\alpha 2}U^*_{\beta 2}} + U_{\alpha 3}U^*_{\beta 3} e^{-i\frac{\Delta m^2_{31}}{2E}L} \right|^2$$

We use the fact that $U_{\alpha 1}U^*_{\beta 1} + U_{\alpha 2}U^*_{\beta 2} + U_{\alpha 3}U^*_{\beta 3} = \delta_{\alpha\beta}$

$$= \left| -U_{\alpha 3}U^*_{\beta 3} + U_{\alpha 3}U^*_{\beta 3} e^{-i\frac{\Delta m^2_{31}}{2E}L} \right|^2$$

$$= \left| U_{\alpha 3}U^*_{\beta 3} \right|^2 \underline{\left| -1 + e^{-i\frac{\Delta m^2_{31}}{2E}L} \right|^2}$$

The same we have encountered in the 2-neutrino case

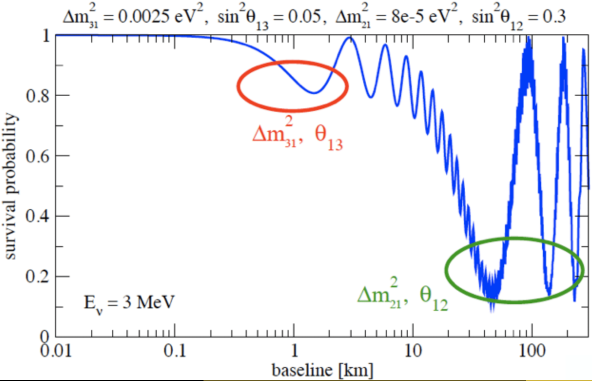
$$= \boxed{2 \left| U_{\alpha 3}U_{\beta 3} \right|^2 \sin^2\left(\frac{\Delta m^2_{31}}{4E}L\right)}$$

Neutrino Oscillations

- $\frac{\Delta m^2_{31}}{4E} L \gg 1$: for reactor neutrinos (KamLAND).

The oscillations due to the atmospheric mass squared differences get averaged out.

$$P(\bar{\nu}_e \to \bar{\nu}_e; t) \simeq c_{13}^4 \left(1 - \sin^2(2\theta_{12}) \sin^2 \frac{\Delta m^2_{21} L}{4E}\right) + s_{13}^4$$



CP-violation will manifest itself in neutrino oscillations, due to the delta phase. Let's consider the CP-asymmetry:

$$P(\nu_\alpha \to \nu_\beta; t) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta; t) =$$

$$= \left| U_{\alpha 1} U^*_{\beta 1} + U_{\alpha 2} U^*_{\beta 2} e^{-i\frac{\Delta m^2_{21} L}{2E}} + U_{\alpha 3} U^*_{\beta 3} e^{-i\frac{\Delta m^2_{31} L}{2E}} \right|^2 - (U \to U^*)$$

$$= U_{\alpha 1} U^*_{\beta 1} U^*_{\alpha 2} U_{\beta 2} e^{i\frac{\Delta m^2_{21} L}{2E}} + U^*_{\alpha 1} U_{\beta 1} U_{\alpha 2} U^*_{\beta 2} e^{-i\frac{\Delta m^2_{21} L}{2E}} - (U \to U^*) + \cdots$$

$$= 4 s_{12} c_{12} s_{13} c^2_{13} s_{23} c_{23} \sin\delta \left[\sin\left(\frac{\Delta m^2_{21} L}{2E}\right) + \left(\frac{\Delta m^2_{23} L}{2E}\right) + \left(\frac{\Delta m^2_{31} L}{2E}\right) \right].$$

- CP-violation requires all angles to be nonzero.
- It is proportional to the sine of the delta phase.
- If one can neglect Δm^2_{21}, the asymmetry goes to zero as we have seen that effective 2-neutrino probabilities are CP-symmetric.

Neutrinos oscillations in matter

- When neutrinos travel through a medium, they interact with the background of electron, proton and neutrons and acquire an effective mass.

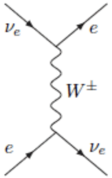
- This modifies the mixing between flavour states and propagation states and the eigenvalues of the Hamiltonian, leading to a different oscillation probability w.r.t. vacuum.

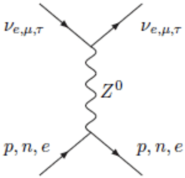
- Typically the background is CP and CPT violating, e.g. the Earth and the Sun contain only electrons, protons and neutrons, and the resulting oscillations are CP and CPT violating.

Effective potentials

Inelastic scattering and absorption processes go as G_F^2 and are typically negligible. Neutrinos undergo also forward elastic scattering, in which they do not change momentum. [L. Wolfenstein, Phys. Rev. D 17, 2369 (1978); ibid. D 20, 2634 (1979), S. P. Mikheyev, A. Yu Smirnov, Sov. J. Nucl. Phys. 42 (1986) 913.]

Electron neutrinos have CC and NC interactions, while muon and tau neutrinos only the latter.





We treat the electrons as a background, averaging over it and we take into account that neutrinos see only the left-handed component of the electrons.

$$\langle \bar{e}\gamma_0 e \rangle = N_e \quad \langle \bar{e}\vec{\gamma} e \rangle = \langle \vec{v}_e \rangle \quad \langle \bar{e}\gamma_0\gamma_5 e \rangle = \langle \frac{\vec{\sigma}_e \cdot \vec{p}_e}{E_e} \rangle \quad \langle \bar{e}\vec{\gamma}\gamma_5 e \rangle = \langle \vec{\sigma}_e \rangle$$

For an unpolarised at rest background, the only term is the first one. N_e is the electron density.

The neutrino dispersion relation can be found by solving the Dirac eq with plane waves, in the ultrarelativistic limit

$$E \simeq p \pm \sqrt{2} G_F N_e$$

medium	A_{CC} for $\nu_e, \bar{\nu}_e$ only	A_{NC} for $\nu_{e,\mu,\tau}, \bar{\nu}_{e,\mu,\tau}$
$e, \bar{e}$	$\pm\sqrt{2}G_F(N_e - N_{\bar{e}})$	$\mp\sqrt{2}G_F(N_e - N_{\bar{e}})(1-4s_W^2)/2$
$p, \bar{p}$	0	$\pm\sqrt{2}G_F(N_p - N_{\bar{p}})(1-4s_W^2)/2$
$n, \bar{n}$	0	$\mp\sqrt{2}G_F(N_n - N_{\bar{n}})/2$
ordinary matter	$\pm\sqrt{2}G_F N_e$	$\mp\sqrt{2}G_F N_n/2$

The Hamiltonian

Let's start with the vacuum Hamiltonian for 2-neutrinos

$$i\frac{d}{dt}\begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix} = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}\begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix}$$

Recalling that $|\nu_\alpha\rangle = \sum_i U_{\alpha i}|\nu_i\rangle$, one can go into the flavour basis

$$\begin{aligned} i\frac{d}{dt}\begin{pmatrix} |\nu_\alpha\rangle \\ |\nu_\beta\rangle \end{pmatrix} &= U\begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}U^\dagger\begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix} \\ &= \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix}\begin{pmatrix} |\nu_\alpha\rangle \\ |\nu_\beta\rangle \end{pmatrix} \end{aligned}$$

We have neglected common terms on the diagonal as they amount to an overall phase in the evolution.

The full Hamiltonian in matter can then be obtained by adding the potential terms, diagonal in the flavour basis. For electron and muon neutrinos

$$i\frac{d}{dt}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta + \sqrt{2}G_F N_e & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix}$$

For antineutrinos the potential has the opposite sign.

In general the evolution is a complex problem but there are few cases in which analytical or semi-analytical results can be obtained.

2-neutrino case in constant density

$$i\frac{d}{dt}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta + \sqrt{2}G_F N_e & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix} \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix}$$

If the electron density is constant (a good approximation for oscillations in the Earth crust), it is easy to solve. We need to diagonalise the Hamiltonian.

- Eigenvalues:

$$E_A - E_B = \sqrt{\left(\frac{\Delta m^2}{2E}\cos(2\theta) - \sqrt{2}G_F N_e\right)^2 + \left(\frac{\Delta m^2}{2E}\sin(2\theta)\right)^2}$$

- The diagonal basis and the flavour basis are related by a unitary matrix with angle in matter

$$\tan(2\theta_m) = \frac{\frac{\Delta m^2}{2E}\sin(2\theta)}{\frac{\Delta m^2}{2E}\cos(2\theta) - \sqrt{2}G_F N_e}$$

Neutrino Oscillations

- If $\sqrt{2}G_F N_e \ll \frac{\Delta m^2}{2E}\cos 2\theta$, we recover the vacuum case and $\theta_m \simeq \theta$
- If $\sqrt{2}G_F N_e \gg \frac{\Delta m^2}{2E}\cos(2\theta)$, matter effects dominate and oscillations are suppressed.
- If $\sqrt{2}G_F N_e = \frac{\Delta m^2}{2E}\cos 2\theta$: resonance and maximal mixing $\theta_m = \pi/4$

- The resonance condition can be satisfied for
  - neutrinos if $\Delta m^2 > 0$
  - antineutrinos if $\Delta m^2 < 0$

$$P(\nu_e \to \nu_\mu; t) = \sin^2(2\theta_m)\sin^2\frac{(E_A - E_B)L}{2}$$

Neutrino Oscillations

We have

$$|\nu_\alpha\rangle = U(t)|\nu_I\rangle, \qquad U^\dagger(t)H_{m,fl}U(t) = \mathrm{diag}(E_A(t), E_B(t))$$

Starting from the Schroedinger equation, we can express it in the instantaneous basis

$$i\frac{d}{dt}U_m(t)\begin{pmatrix} |\nu_A\rangle \\ |\nu_B\rangle \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta + \sqrt{2}G_F N_e(t) & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix} U_m(t)\begin{pmatrix} |\nu_A\rangle \\ |\nu_B\rangle \end{pmatrix}$$

$$i\frac{d}{dt}\begin{pmatrix} |\nu_A\rangle \\ |\nu_B\rangle \end{pmatrix} = \begin{pmatrix} E_A(t) & -i\dot{\theta}(t) \\ i\dot{\theta}(t) & E_B(t) \end{pmatrix}\begin{pmatrix} |\nu_A\rangle \\ |\nu_B\rangle \end{pmatrix}$$

The evolution of ν_A and ν_B are not decoupled. In general, it is very difficult to find an analytical solution to this problem.

Adiabatic case

In the adiabatic case, each component evolves independently. In the non adiabatic one, the state can "jump" from one to the other.

If the evolution is sufficiently slow (adiabatic case):

$$|\dot{\theta}(t)| \ll | E_A - E_B|$$

we can follow the evolution of each component independently.

Adiabaticity condition

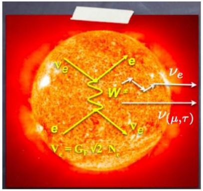
$$\gamma^{-1} \equiv \frac{2|\dot{\theta}|}{|E_A - E_B|} = \frac{\sin(2\theta)\frac{\Delta m^2}{2E}}{|E_A - E_B|^3}|\dot{V}_{CC}| \ll 1$$

In the Sun, typically we have $\gamma \sim \frac{\Delta m^2}{10^{-9}\mathrm{eV}^2}\frac{\mathrm{MeV}}{E_\nu}$

Neutrino Oscillations

Matter oscillations

Linc Wolfenstein
(1978)



MSW effect:
Electron neutrinos feel a "drag" due to extra contribution from W exchange

Effective θ_M and Δm^2

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - x)^2}$$

$$\Delta m_M^2 = \Delta m^2 \sqrt{\sin^2 2\theta + (\cos 2\theta - x)^2}$$

$$x = \frac{2\sqrt{2} G_F N_e E}{\Delta m^2}$$

N_e = electron density

1. Low electron density (the Earth):

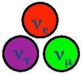
   $$P_{\nu_\alpha \to \nu_\beta} = \sin^2 2\theta_M \sin^2\left(\frac{\Delta m_M^2 L}{4E}\right)$$

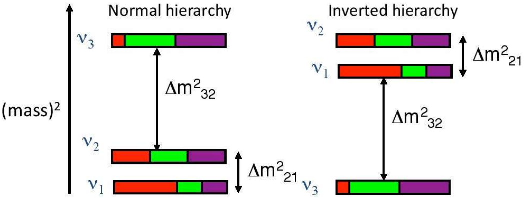
2. Resonant MSW:
   $\theta_M = \pi/4$
   Total transition between two flavours

3. Varying N_e (the Sun):
   $d\theta_M/dx \neq 0$
   Adiabatic transition between effective mass eigenstates

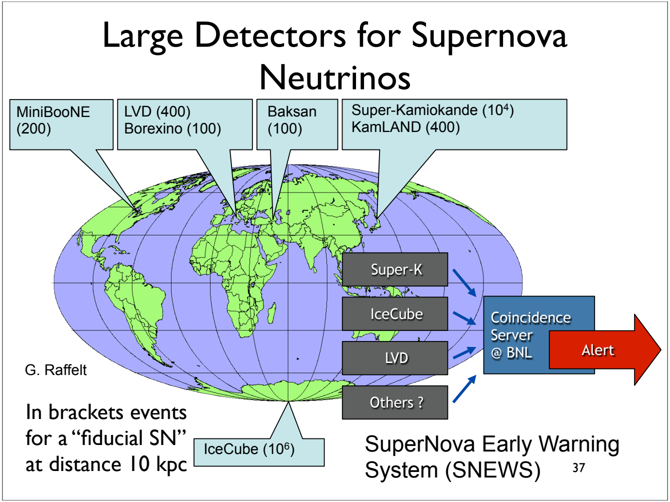
Mass hierarchy

We don't know the ordering the mass splittings Δm_2 –
but we do know that $\nu_2 >> \nu_1$





Neutrinos experiments



Radiochemical Experiments

This technique uses the production of radioactive isotopes.

Davis-Pontecorvo experiment was the first attempt to use this to look at solar neutrinos

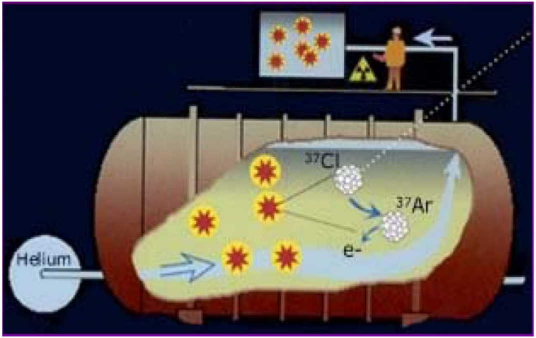
$$\nu_e + Cl^{37} \rightarrow Ar^{37} + e^- \qquad \nu_e + Ga^{71} \rightarrow \mathrm{Ge}^{37} + e^-$$

The isotopes Ar or Ge are radioactive. In this type of experiment the isotopes are chemically extracted and counted using their decay

Disadvantage is that there is no information on interaction time, neutrino direction or flavours other than ν_e

The Davis Experiment

The very first solar neutrino experiment in the Homestake mine in South Dakota



615 tonnes of CCl4
Ran from 1968 to 1994

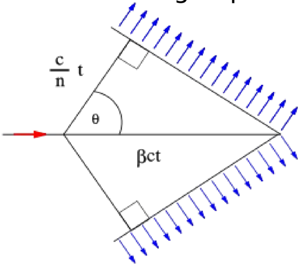
Individual argon atoms are captured and counted.

1 atom per 2 days.

Threshold : 814 keV

Water Experiments

Water is a very cheap target material – these experiments detect charged particles using Cerenkov radiation.

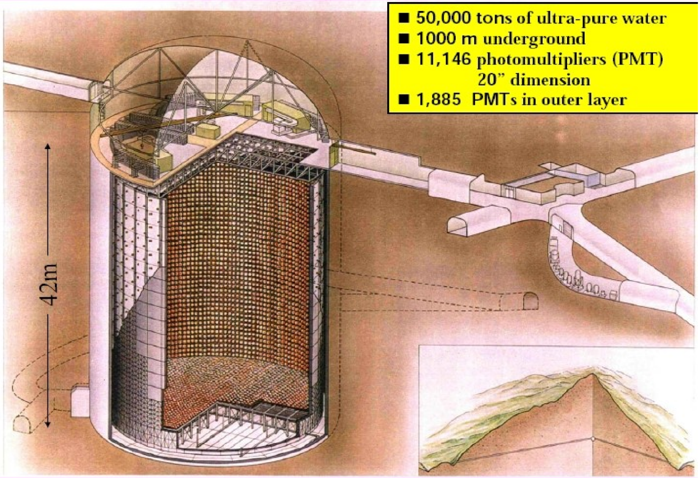


If a charged particle moves through a material with β > 1/n it produces an EM shockwave at a particular angle.

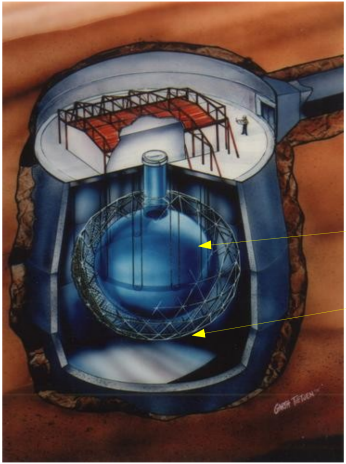
$$\cos\theta = 1/\beta n$$

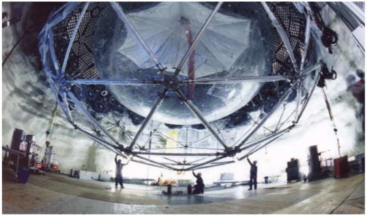
The shockwave can be detected and used to measure the particle direction and vertex.
Particles below threshold and neutral particles are not detected

Super-Kamiokande



SNO





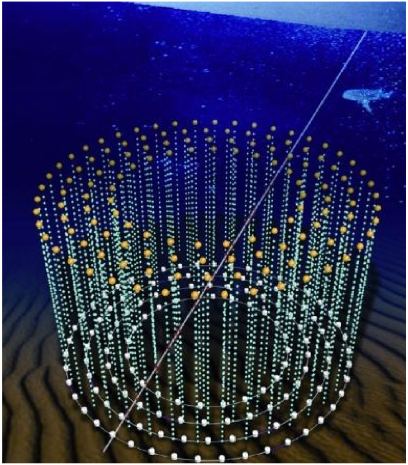
1000 tonnes of $D_2 0$

6500 tons of $H_2 0$

Viewed by 10,000 PMTS

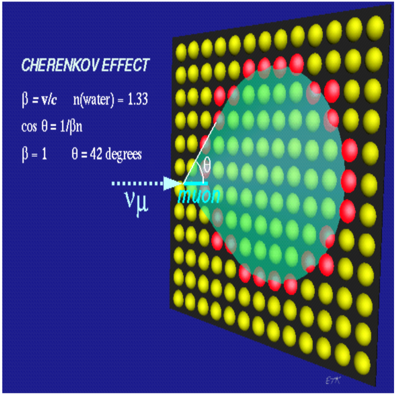
In a salt mine 2km underground in Sudbury, Canada

Deep Water Detectors -KM3Net



Sited off Toulon in the Mediterranean @2400m depth

Principle of operation



- Cerenkov light detected as a ring or circle by PMTs
- Vertex from timing
- Direction from cone
- Energy from summed light
- No neutrals or charged particles under Cerenkov threshold
- Low multiplicity events

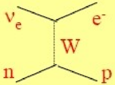
ν Reactions in SNO

Charged Current Reaction:

$\nu_e + d \rightarrow p + p + e^-$ $E_{thres} = 1.4$ MeV

CC

- 6-9 events per day
- n_e flux and energy spectrum
- Some directional sensitivity $(1 - 1/3 COS\theta_e)$



Neutral Current Reaction:

$\nu_x + d \rightarrow \nu_x + p + n$ $E_{thres} = 2.2$ MeV

NC

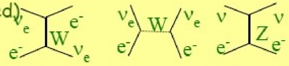
- 1-2 or 6-8 events per day (different detection mechanisms)
- Total solar ^8B active neutrino flux

Elastic Scattering Reaction:

$\nu_x + e^- \rightarrow \nu_x + e^-$ $E_{thres} = 0$ MeV

ES

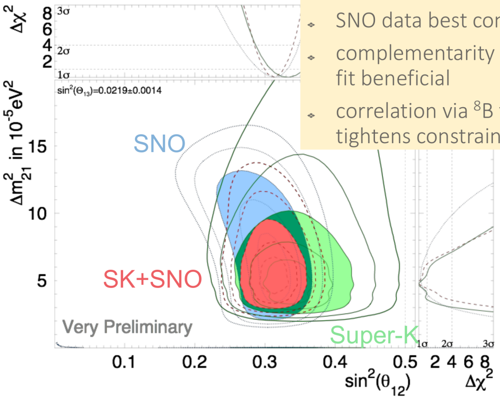
- 1-2.5 events per day
- Directional sensitivity (very forward peaked)



Neutrinos experiments

Solar ν Angle θ_{12} & Mass² Difference

- Super-K data best constrains Δm^2_{21}
- SNO data best constrains $\sin^2\theta_{12}$
- complementarity makes combined fit beneficial
- correlation via ^{8}B flux further tightens constraints



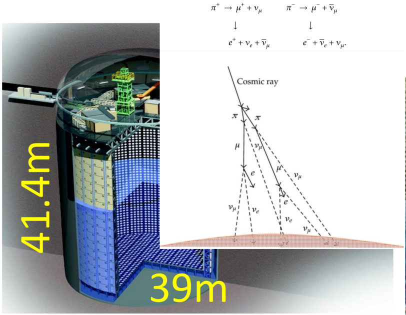
M. Ikeda, for Super-Kamiokande, Neutrino 2018

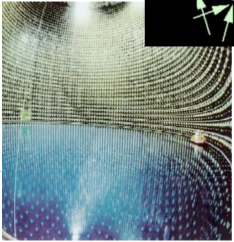
14

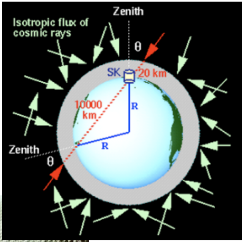
Solar experiments best constrain the "solar mixing" angle of theta$_{12}$ to be large (but non-maximal). The mass squared difference is around 7×10^{-5} eV^2.

Cosmic rays hit the atmosphere and produce pions (and kaons) which decay producing lots of muon and electron (anti-) neutrinos.

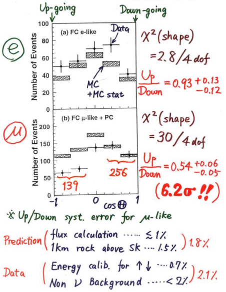
- Typical energies: 100 MeV - 100 GeV
- Typical distances: 100-10000 km.

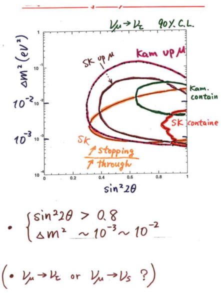






SuperKamiokande Coll.





http://www-sk.icrr.u-tokyo.ac.jp/nu98/

first evidence of neutrino oscillations in 1998 with atmospheric neutrinos (nu_{mu}->nu_{tau}).

T. Kajita's talk at Neutrino 1998

SK and MINOS went on to measure the atmospheric mixing angle to be large (mainly maximal) and the atm mass squared different at ~2.5x 10^{-3} eV^2.

# of tau events

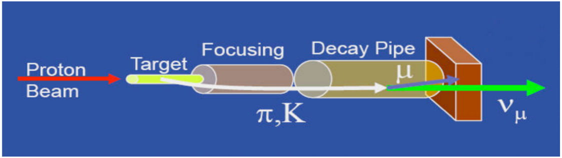
338.1 ± 72.7 (stat.+ sys.) events

Reject no-tau-appearance @ 4.6σ.

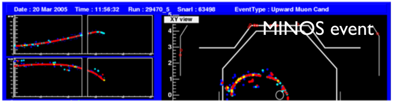
(Exp. significance is 3.3σ)

Accelerator neutrinos

Conventional beams: muon neutrinos from pion decays



Neutrino production.
Credit: Fermilab



- Typical energies:

MINOS: E~4 GeV; T2K: E~700 MeV; NOvA: E~2 GeV.
OPERA and ICARUS: E~20 GeV.

- Typical distances: 100 km - 2000 km.

MINOS: L=735 km; T2K: L=295 km; NOvA: L=810 km.
OPERA and ICARUS: L=700 km.

How to get naturally small neutrino mass?

NEUTRINO MASS GENERATION:
SEE-SAW MECHANISM

Neutrinos in BSM Theories (ν-mass)

Assume Dirac and Majorana Masses, with right handed v with LARGE Mass

$$\mathcal{L}^{\mathrm{D+M}} = -\frac{1}{2}\,\bar{\nu}_L\, M_L^{\mathrm{M}} (\nu_L)^c - \bar{\nu}_L\, M^{\mathrm{D}}\, \nu_R - \frac{1}{2}\,\overline{(\nu_R)^c}\, M_R^{\mathrm{M}} \nu_R + \mathrm{h.c.}$$

Three mass parameters:
m_L (Left-handed Majorana), m_D (Dirac), m_R (Right-handed Majorana)

As seen above: Particles with ***definite mass*** are Majorana

$$m'_{1,2} = \frac{1}{2}(m_R + m_L) \mp \frac{1}{2}\sqrt{(m_R - m_L)^2 + 4\,m_D^2}$$

Mixing angle $\tan 2\theta = \frac{2m_D}{m_R - m_L}, \quad \cos 2\theta = \frac{m_R - m_L}{\sqrt{(m_R - m_L)^2 + 4\,m_D^2}}$

BASIC ASSUMPTIONS FOR SEE-SAW

(I) No left-handed Majorana Mass $m_L = 0$

(II) Dirac mass term generated by standard Higgs-mechanism (Yukawa couplings), so m_D = O(quark or lepton mass)

(III) Right-handed Majorana mass, which ***breaks Lepton number*** conservation, is much heavier than the electroweak scale

$$m_R \equiv M_R \gg m_D$$

$$\tan 2\theta = \frac{2m_D}{m_R - m_L}$$

$$\eta_1 = -1 \qquad \eta_2 = 1.$$

$$m_1 \simeq \frac{m_D^2}{M_R} \ll m_D, \quad m_2 \simeq M_R \gg m_D.$$

$$\theta \simeq \frac{m_D}{M_R} \ll 1.$$

Neutrinos in BSM Theories (ν-mass)

FLAVOUR FIELDS IN THE SEE-SAW ONE GENERATION EXAMPLE

$$\nu_L = i\,\nu_{1L} + \frac{m_D}{M_R}\,\nu_{2L}$$

$$(\nu_R)^c = -i\,\frac{m_D}{M_R}\,\nu_{1L} + \nu_{2L}.$$

SMALLNESS OF ACTIVE NEUTRINO MASSES IN SEE-SAW DUE TO: ***SUPPRESSION*** BY THE SCALE AT WHICH ***LEPTON NUMBER*** IS ***VIOLATED,*** WHICH IS MUCH LARGER THAN THE ELECTROWEAK SCALE (DIRAC MASS).

Example:

$$\begin{aligned} m_D &= \mathcal{O}\Big(m_t = 175\ \text{GeV}\Big) \\ m_1 &= \text{heaviest of neutrino mass in neutrino mass hierarchy} \\ &= \mathcal{O}\Big(5\times 10^{-2}\ \text{eV}\Big) \end{aligned}$$

$\Rightarrow$ $M_R \simeq \frac{m_D^2}{m_1} \simeq 10^{15}$ GeV.

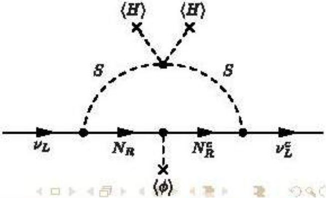
i.e. Lepton number violation @ Grand Unification Scale in this case...

This model is a SI-generalization of the SI-Soctogenic model, Ma, PRD73 (2006) 077301.
SM + $\phi \sim (1,1,0)$ + $S \sim (1,2,1)$ + 3 $N_{iR} \sim (1,1,0)$ with the symmetry Z_2: $\{N_{iR}, S\} \rightarrow \{-N_{iR}, -S\}$.
The Lagrangian is given by

$$\mathcal{L} \supset i\bar{N}_R\gamma^\mu\partial_\mu N_R + \frac{1}{2}(\partial^\mu\phi)^2 + |D^\mu S|^2 - \frac{y_i}{2}\,\phi\,\overline{N^c_{iR}}\,N_{iR} - g_{i\alpha}\overline{N_{iR}}\,L_\alpha S - V(\phi, S, H),$$

$$V_0(H,\phi,S_{1,2}) = \lambda|H|^4 + \frac{\lambda_\phi}{4}\phi^4 + \frac{\lambda}{2}|S|^4 + \frac{\lambda_\phi}{2}\phi^2|H|^2 + \frac{\lambda_\phi}{2}\phi^2|S|^2 + \lambda_3|H|^2|S|^2$$
$$+\lambda_4\,|H^\dagger S|^2 + \frac{\lambda_5}{2}(S^\dagger H)^2 + \mathrm{H.c.}.$$

$$(M_\nu)_{\alpha\beta} = \sum_i \frac{g_{i\alpha}g_{i\beta}M_i}{16\pi^2}\left\{\frac{M^2_{S^0}}{M^2_{S^0}-M_i^2}\ln\frac{M^2_{S^0}}{M_i^2} - \frac{M_A^2}{M_A^2-M_i^2}\ln\frac{M_A^2}{M_i^2}\right\}..$$



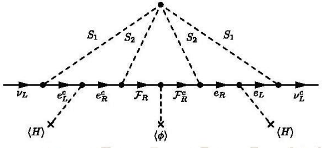
This model is a SI-generalization of the Krauss-Nasri-Trodden model, PRD67(2003) 085002.
SM + $\phi \sim (1,1,0)$ + $S_{1,2} \sim (1,1,2)$ + 3 $N_i \sim (1,1,0)$ with the symmetry Z_2: $\{N_i, S_2\} \rightarrow \{-N_i, -S_2\}$.
The Lagrangian is given by

$$\mathcal{L} \supset -\{f_{\alpha\beta}\, \overline{L^c_\alpha}\, L_\beta S_1^+ + g_{i\alpha}\overline{N_i^c} S_2^+ e_{\alpha R} + \mathrm{H.c}\} - \frac{1}{2}\tilde{y}_i \phi \overline{N_i^c} N_i - V(H, S_{1,2}, \phi),$$

$$V_0(H,\phi,S_{1,2}) \supset \lambda_H\, |H|^4 + \frac{\lambda_{\phi H}}{2}|H|^2\phi^2 + \frac{\lambda_\phi}{4}\phi^4 + \frac{\lambda_S}{4}(S_1^-)^2(S_2^+)^2 + \sum_{a=1,2}\frac{1}{2}(\lambda_{Ha}\,|H|^2 + \lambda_{\phi a}\,\phi^2)|S_a|^2.(1)$$

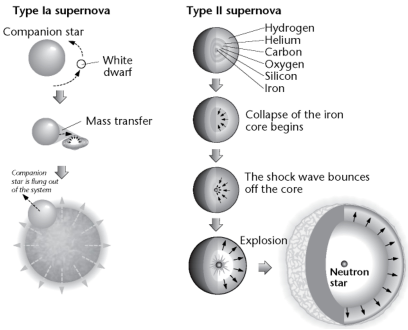
$$(M_\nu)_{\alpha\beta} = \frac{\lambda_S}{(4\pi^2)^3}\frac{m_\sigma m_\rho}{M_{S_2}} g^*_{\sigma i}\, g^*_{\rho i}\, f_{\alpha\sigma}\, f_{\beta\rho} \times F_{loop}\left(\frac{M^2_{N_i}}{M^2_{S_2}}, \frac{M^2_{S_1}}{M^2_{S_2}}\right),$$

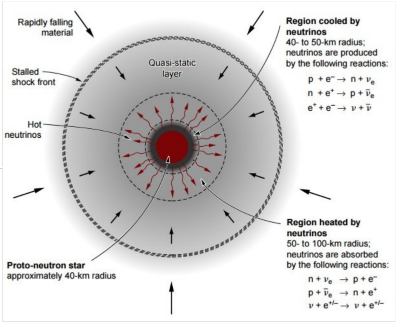
$$F_{loop}(\alpha,\beta) = \frac{\sqrt{\alpha}}{8\beta^2}\int_0^\infty dr \frac{r}{r+\alpha}\left(\int_0^1 dx \ln\frac{x(1-x)r+(1-x)\beta+x}{x(1-x)r+x}\right)^2.$$



Neutrinos in Astrophysics & Cosmology

Supernova type-II and neutrinos





Matter-Antimatter Asymmetry

Observation of acoustic peaks in cosmic microwave background radiation (CMB) has led to precision measurement of the baryon asymmetry $\eta_B \simeq (\eta_B - \eta_{\overline{B}}) = n_B/n_\gamma$ by WMAP collaboration,

$$\eta_B^{CMB} = (6.1^{+0.3}_{-0.2}) \times 10^{-10} \, ;$$

'measurement' of η_B at temperature $T_{CMB} \sim 1$ eV, i.e. time $t_{CMB} \sim 3 \times 10^5 y \simeq 10^{13} s$, assumes Friedmann universe.

Second determination of η_B from nucleosynthesis, i.e. abundances of the light elements, D, ^{3}He, ^{4}He, ^{7}Li, yields

$$\eta_B^{BBN} = \frac{n_B}{n_\gamma} = (2.6 - 6.2) \times 10^{-10} \, ;$$

'measurement' of η_B at temperature $T_{BBN} \sim 10$ MeV , i.e. time $t_{BBN} \sim 10s$; consistency of η_B^{CMB} and η_B^{BBN} remarkable test of standard cosmological model.

A matter-antimatter asymmetry can be dynamically generated in an expanding universe if the particle interactions and the cosmological evolution satisfy Sakharov's conditions,

- baryon number violation ,
- C and CP violation ,
- deviation from thermal equilibrium .

Baryon asymmetry provides important relationship between the standard model of cosmology and the standard model of particle physics as well as its extensions.

Scenarios for baryogenesis: classical GUT baryogenesis, leptogenesis, electroweak baryogenesis, Affleck-Dine baryogenesis (scalar field dynamics).

Theory of baryogenesis depends crucially on nonperturbative properties of standard model,

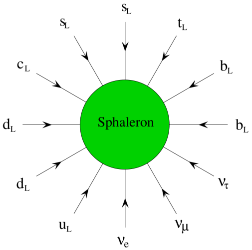
- electroweak phase transition: 'symmetry restoration' at high temperatures, $T > T_{EW} \sim 100$ GeV, smooth transition for large Higgs masses, $m_H > m_H^c \simeq 72$ GeV

- sphaleron processes: relate baryon and lepton number at high temperatures, in thermal equilibrium in temperature range,

$$T_{EW} \sim 100\mathrm{GeV} < T < T_{SPH} \sim 10^{12}\mathrm{GeV}\,.$$

Baryon and lepton number violating sphaleron processes

Kuzmin, Rubakov, Shaposhnikov '85



$O_{B+L} = \prod_i (q_{Li} q_{Li} q_{Li} l_{Li})$,

$\Delta B = 3$, $\Delta L = 3$,

$B - L$ conserved

Processes are in thermal equilibrium above electroweak phase transition, for temperatures

$$T_{EW} \sim 100\text{GeV} < T < T_{SPH} \sim 10^{12}\text{GeV} .$$

Sphaleron processes have a profound effect on the generation of cosmological baryon asymmetry. Analysis of chemical potentials of all particle species in the high-temperature phase yields relation between the baryon asymmetry (B) and L and $B-L$ asymmetries,

$$\langle B\rangle_T = c_S \langle B-L\rangle_T = \frac{c_S}{c_S-1}\langle L\rangle_T \ ,$$

with c_S number $\mathcal{O}(1)$; in standard model $c_s = 28/79$.

This relation suggests that lepton number violation is needed to explain the cosmological baryon asymmetry. However, it can only be weak, since otherwise any baryon asymmetry would be washed out. The interplay of these conflicting conditions leads to important contraints on neutrino properties and on extensions of the standard model in general.

Thermal leptogenesis

Fukugita, Yanagida '86

Lightest (heavy) Majorana neutrino, N_1, is ideal candidate for baryogenesis: no SM gauge interactions, hence out-of-equilibrium condition o.k.; N_1 decays to lepton-Higgs pairs yield lepton asymmetry $\langle L\rangle_T \neq 0$, partially converted to baryon asymmetry $\langle B\rangle_T \neq 0$.

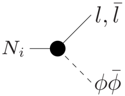
The generated baryon asymmetry is proportional to the CP asymmetry in N_1-decays ($H_1 = H_2^* = \phi$, seesaw relation, . . . Flanz et al. '95, Covi et al. '96,. . .),

$$\begin{aligned}\varepsilon_1 &= \frac{\Gamma(N_1 \to l\phi) - \Gamma(N_1 \to \bar{l}\bar{\phi})}{\Gamma(N_1 \to l\phi) + \Gamma(N_1 \to \bar{l}\bar{\phi})} \\ &\simeq -\frac{3}{16\pi}\frac{M_1}{(hh^\dagger)_{11}v^2}\mathrm{Im}\left(h^* m_\nu h^\dagger\right)_{11} .\end{aligned}$$

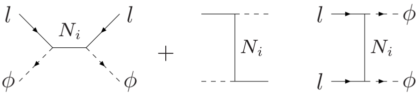
Rough estimate for ε_1 in terms of neutrino masses (dominance of the decays (D), inverse decays (ID)

PROCESSES in PLASMA

$$N_i \leftrightarrow l\,\phi\,,\ \bar{l}\,\bar{\phi}$$



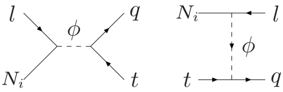
$\Delta L = 2$ processes (N_i virtual)



$l\,\bar{\phi} \leftrightarrow \bar{l}\,\phi$ (N)

$l\,l \leftrightarrow \phi\,\phi$
$\bar{l}\,\bar{l} \leftrightarrow \bar{\phi}\,\bar{\phi}$ (N, t)

$\Delta L = 1$ processes (N_i real)



$N_i\,l \leftrightarrow \bar{t}\,q$ (ϕ, s)

$N_i\,t \leftrightarrow \bar{l}\,q$ (ϕ, t)

largest eigenvalue of m_ν, phases $\mathcal{O}(1)$),

$$\varepsilon_1 \sim \frac{3}{16\pi}\frac{M_1 m_3}{v^2} \sim 0.1\ \frac{M_1}{M_3}\ ;$$

order of magnitude of CP asymmetry is given by the mass hierarchy of the heavy Majorana neutrinos, e.g. $\varepsilon_1 \sim 10^{-6}$ for $M_1/M_3 \sim m_u/m_t \sim 10^{-5}$.

Baryon asymmetry for given CP asymmetry ε_1,

$$\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} = \frac{\kappa}{f} c_S \varepsilon_1 \sim 10^{-10} \ldots 10^{-9}\ ,$$

with $f \sim 10^2$ dilution factor which accounts for the increase of the number of photons in a comoving volume element between baryogenesis and today; determination of the washout factor κ requires Boltzmann equations (for estimate, $\kappa \sim 0.01 \ldots 0.1$).

Thank you for your attension.

One has to mention that when preparing this lecture, I relied on many excellent lectures and talks by many scientists like Pascoli, Buchmuller, de-Gouvea and others.