QCD and Event Generators

Lecture 2 of 2

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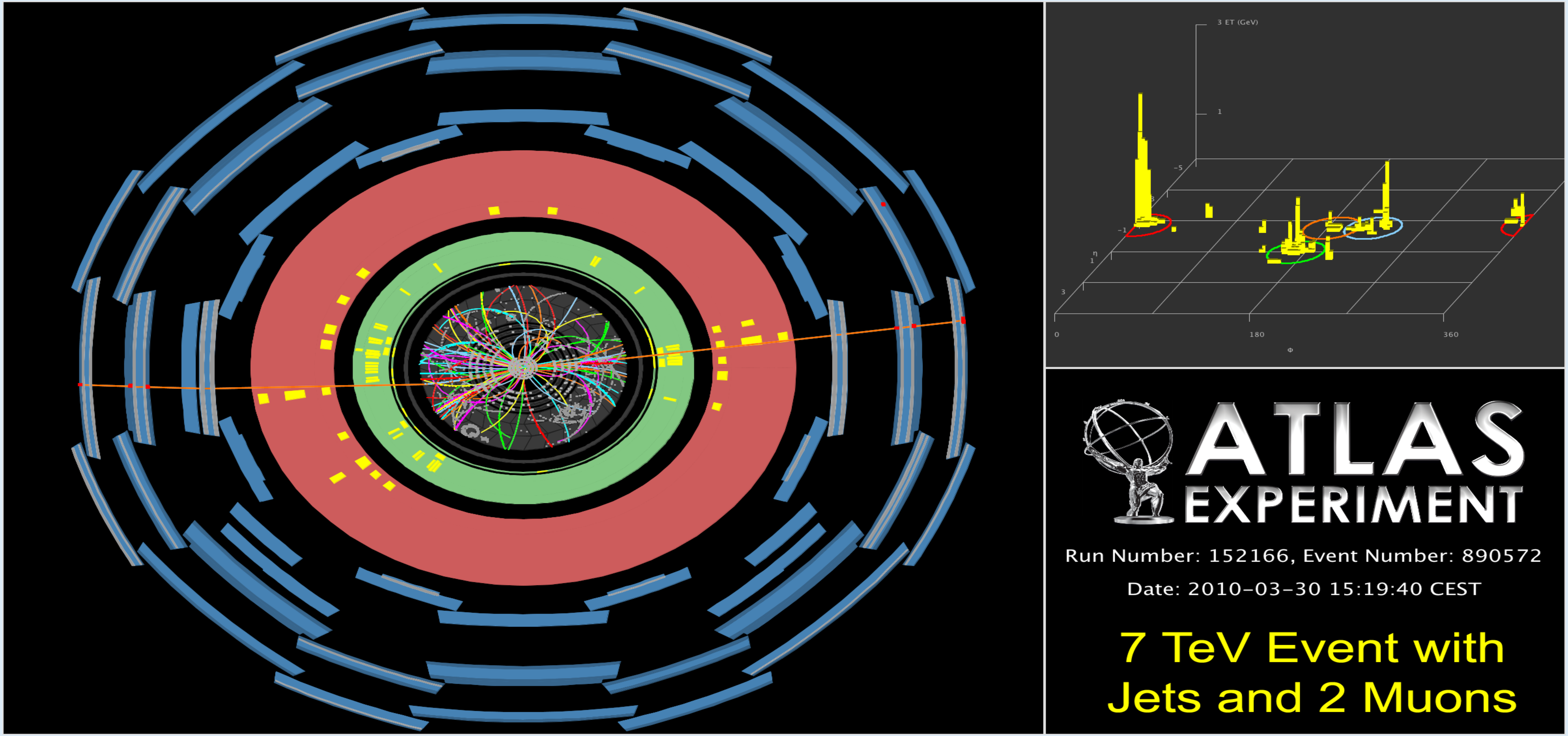
Some slides of these Lectures are based on:

- talk by P. Skands at HCPSS, 2020
- Lectures by T. Sjöstrand at Lund, 2018

...But Fixed Order QCD is not enough



Accelerated charges

Accelerated charged particle with charge $\pm Ze$ will continuously emit radiation (when changing its velocity from β_1 to β_2)

$$\lim_{\omega \to 0} \frac{\mathrm{d}^2 N}{\mathrm{d}\omega \mathrm{d}\Omega_\gamma} = \frac{Z^2 \alpha}{4\pi^2 \omega} \left| \epsilon^* \cdot \left(\frac{\beta_2}{1 - \mathbf{n}\cdot\beta_2} - \frac{\beta_1}{1 - \mathbf{n}\cdot\beta_1} \right) \right|^2$$

(see J. D. Jackson, Classical Electrodynamics)

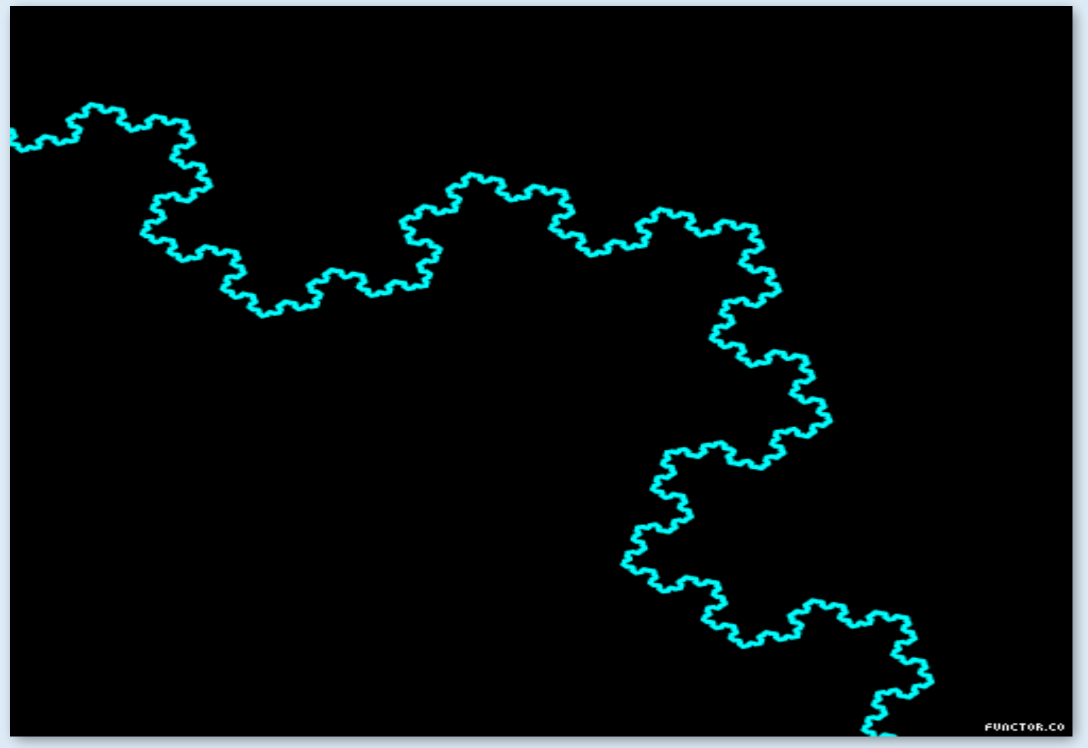
Two important consequences:

- The particle is very fast; $\beta_i \cdot \mathbf{n} = 0$ (collinear singularity)
- $N \propto \int \mathrm{d}\omega/\omega \implies$ infinitely many infinitely soft emitted photons

  but the net energy taken is finite.

$$E_{\mathrm{Total}} \propto \hbar N \omega \approx \hbar \int \mathrm{d}\omega \to \text{finite}$$

(soft photons continuously emitted & reabsorbed)



In QCD, the situation is quite similar with two main differences:

- QCD is a non-Abelian gauge theory; we have also gluon emission off a gluon.
- The strong coupling diverges for $Q^2 \to \Lambda^2_{\mathrm{QCD}}$.

Small couplings are not sufficient?

Naively, QCD radiation suppressed by $\alpha_s \approx 0.1$

→ Truncate at fixed order = LO, NLO, …

But beware the jet-within-a-jet-within-a-jet …

$\Longrightarrow$ 100 GeV can be "soft" at the LHC

Example: SUSY pair production at LHC$_{14}$, with M$_{SUSY}$ ≈ 600 GeV

LHC - sps1a - m~600 GeV — Plehn, Rainwater, PS PLB645(2007)217

FIXED ORDER pQCD	σ_{tot}[pb]	$\tilde{g}\tilde{g}$	$\tilde{u}_L\tilde{g}$	$\tilde{u}_L\tilde{u}_L^*$	$\tilde{u}_L\tilde{u}_L$	TT
$p_{T,j} > 100$ GeV	σ_{0j}	4.83	5.65	0.286	0.502	1.30
inclusive **X + 1 "jet"**	σ_{1j}	2.89	2.74	0.136	0.145	0.73
inclusive **X + 2 "jets"**	σ_{2j}	1.09	0.85	0.049	0.039	0.26
$p_{T,j} > 50$ GeV	σ_{0j}	4.83	5.65	0.286	0.502	1.30
	σ_{1j}	5.90	5.37	0.283	0.285	1.50
	σ_{2j}	4.17	3.18	0.179	0.117	1.21

(Computed with SUSY-MadGraph)

σ for X + jets much larger than naive factor-α_s estimate

σ for 50 GeV jets ≈ larger than total cross section
→ what is going on?

All the scales are high, $Q \gg 1$ GeV, so perturbation theory **should** be OK

Peter Skands (HCPSS 2020)

Why is fixed-order QCD not enough?

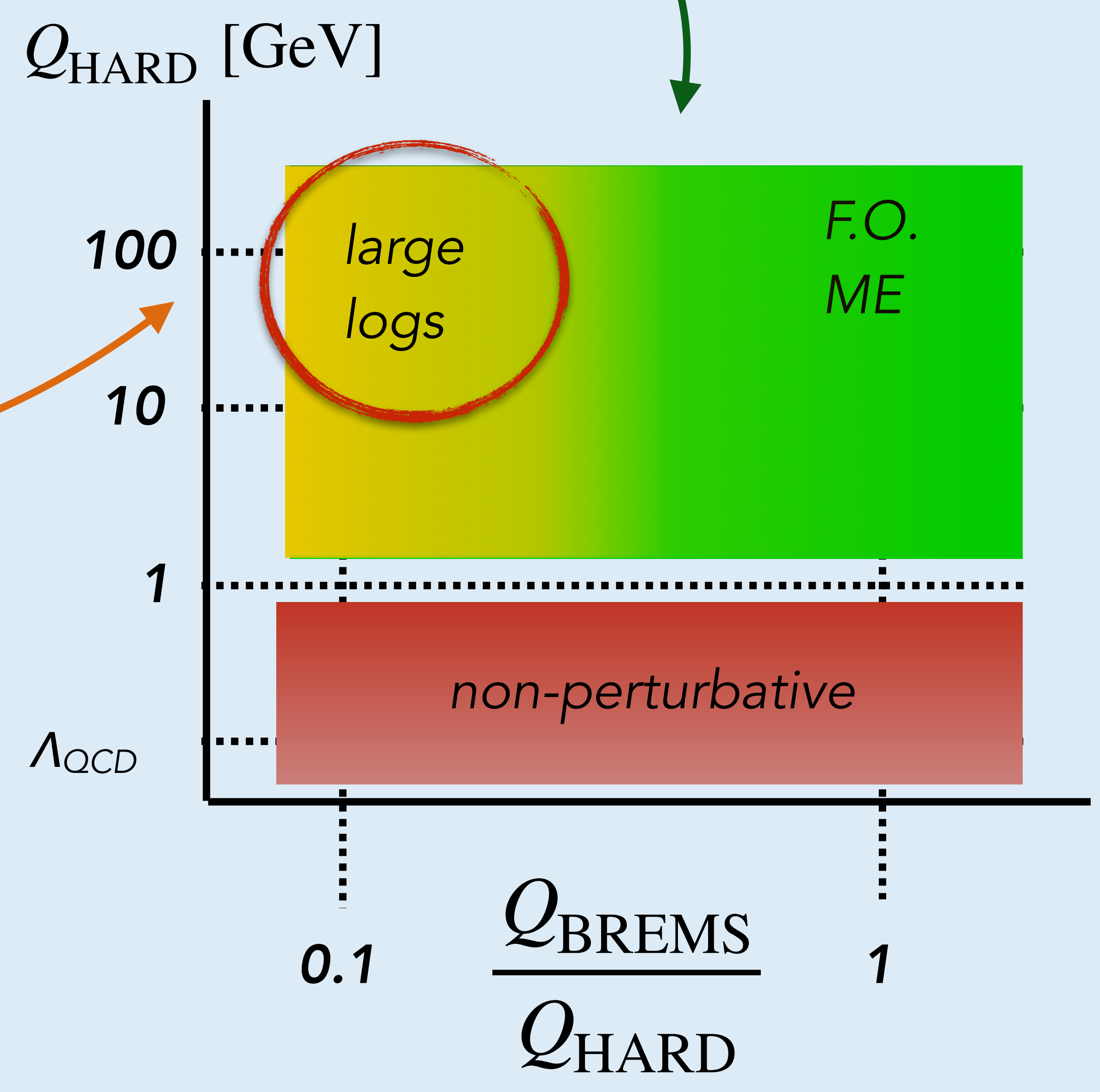
Fixed Order (**F.O.**) **QCD** requires **Large scales** (α_s small enough to be perturbative → high-scale processes)

F.O. QCD also requires **No hierarchies**

Bremsstrahlung propagators $\propto 1/Q^2$ integrated over phase space $\propto dQ^2$ → **logarithms**

$$\alpha_s^n \ln^m \left(Q^2_{\mathrm{Hard}}/Q^2_{\mathrm{Brems}}\right) \quad ; \; m \leq 2n$$

→ cannot truncate at any fixed order n if upper and lower integration limits are **hierarchically different**



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Harder Processes are accompanied by Harder Jets

The hard process "kicks off" **a shower** of successively softer **radiation**

Fractal structure: if you look at $Q_{JET}/Q_{HARD} \ll 1$, you **will** resolve substructure.

So it's **not** like you can put a cut at X (e.g., 50, or even 100) GeV and say: "Ok, now fixed-order matrix elements will be OK"

Extra radiation:

Will generate **corrections to your kinematics**

Extra jets from bremsstrahlung can be important **combinatorial background** especially if you are looking for decay jets of similar p_T scales (often, $\Delta M \ll M$)

Is an unavoidable aspect of the **quantum description of quarks and gluons**
(no such thing as a "bare" quark or gluon; they depend on how you look at them)

This is what parton showers are for

Peter Skands (HCPSS 2020)

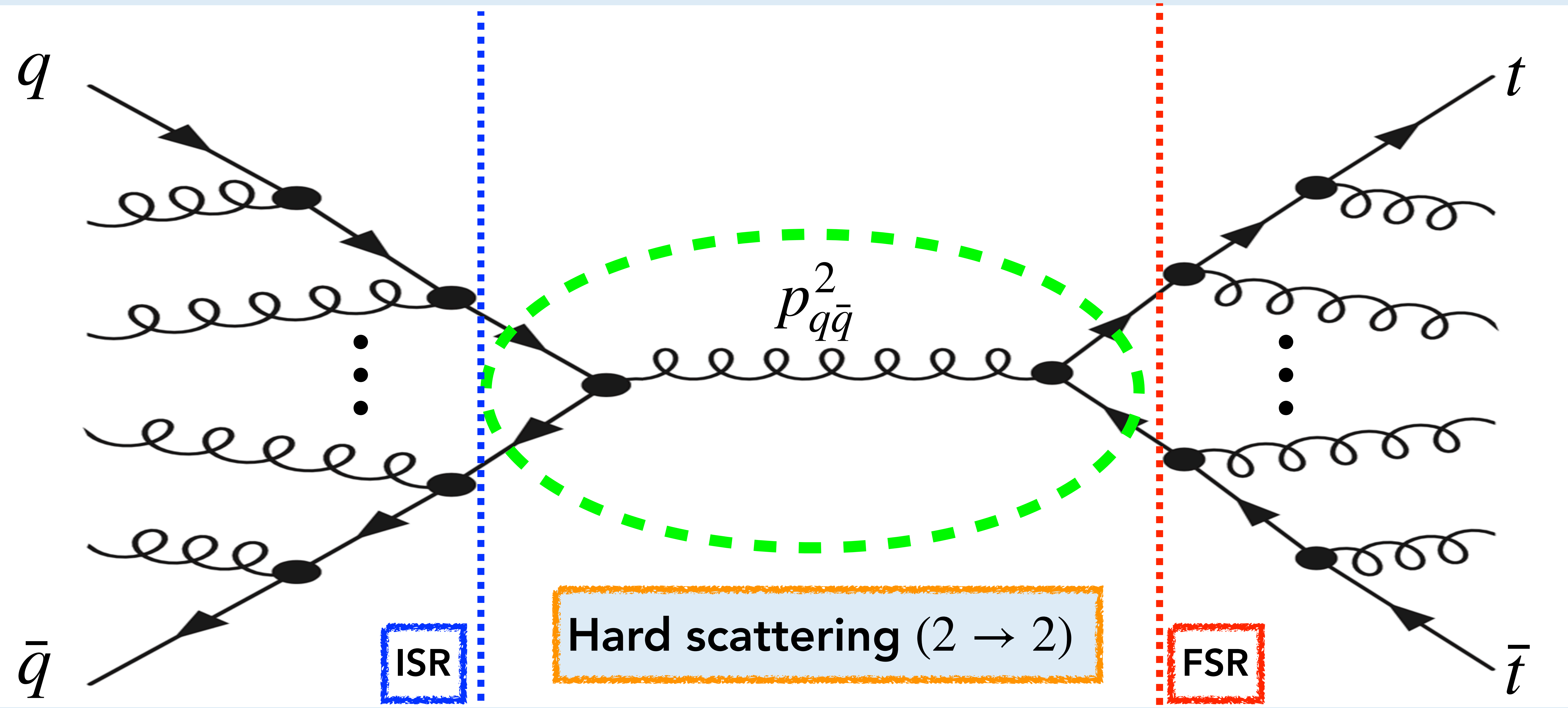
Parton showers: General formalism

Consider $q\bar{q} \to t\bar{t}$

Extra radiation implies a $2 \to N$ (not a $2 \to 2$) process.

Factorise and conquer $\Longrightarrow \; 2 \to N \equiv 2 \to 2 \oplus \text{ISR} \oplus \text{FSR}$

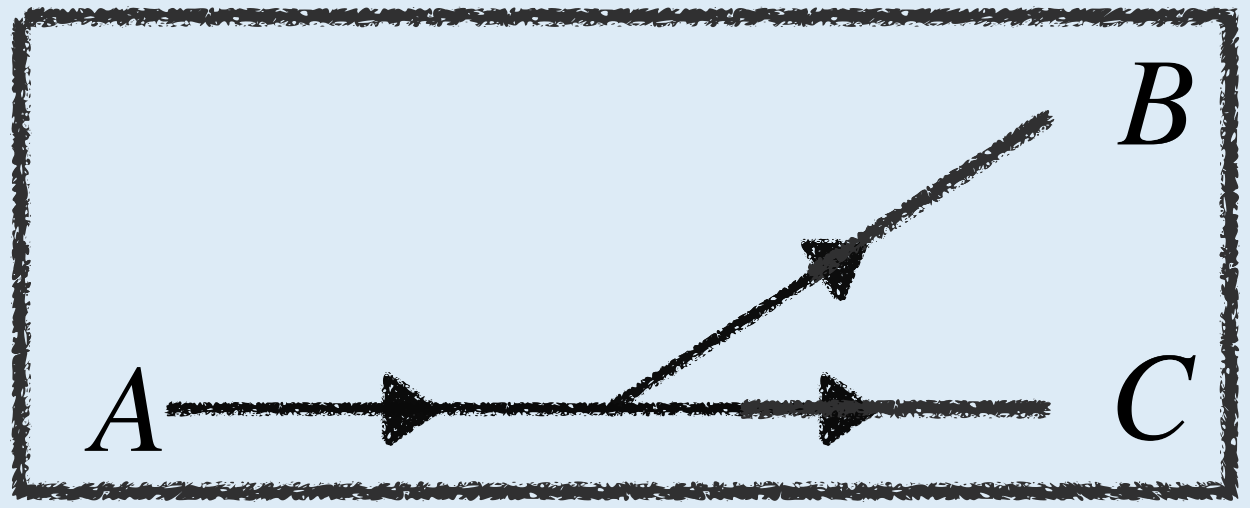
Initial-State radiation (ISR):
Space-like showers, i.e. $Q_{ij}^2 < 0$



Final-State Radiation (FSR):
Time-like showers
$Q_{ij}^2 > 0$

Time-like vs space-like showers

Consider the following branching $A \to BC$



$$p_i = (E_i, p_{\perp,i}, p_{L,i}); i \in \{A, B, C\}$$

(Choose a frame where $p_{\perp,A} = 0$)

The emitting particle is off-shell while the branching products are massless; i.e. if A branches; then $M_A^2 \neq 0$; $M_B^2 = M_C^2 = 0$

Using energy-momentum conservation ($p_{i,\pm} = E_i \pm p_{L,i}$):

$$\frac{p_{+,A}p_{-,A}}{p_{+,A}} = \frac{p_{+,A}p_{-,B}}{p_{+,A}} + \frac{p_{+,A}p_{-,C}}{p_{+,A}} \implies \frac{p_{+,A}p_{-,A}}{p_{+,A}} = \frac{p_{+,B}p_{-,B}}{z\, p_{+,A}} + \frac{p_{+,C}p_{-,C}}{(1-z)\, p_{+,A}}$$

we used $p_B = zp_A$, $p_C = (1-z); z \in [0,1]$

$$\implies \frac{m_A^2 + p_{\perp,A}}{p_{+,A}} = \frac{m_B^2 + p_{\perp,B}}{z\, p_{+,A}} + \frac{m_A^2 + p_{\perp,C}}{(1-z)\, p_{+,A}} \implies m_A^2 = \frac{m_B^2}{z} + \frac{m_C^2}{1-z} + \frac{p_\perp^2}{z\,(1-z)}$$

ISR: B branches; $m_A = m_C = 0 \implies m_B^2 = -\frac{p_\perp^2}{(1-z)} < 0$ **(space-like showers)**

FSR: A branches; $m_B = m_C = 0 \implies m_A^2 = \frac{p_\perp^2}{z(1-z)} > 0$ **(time-like showers)**

Parton showers & production rates

Parton showering implies a probabilistic function which does not change the total cross section (see next slides).

The situation is more complicated that just a multiplicative factor with total probability of unity:

- ISR: parton showers affect the evolution of the PDFs. Therefore, it's impact enter in the expression of the inclusive cross section:

$$\sigma_{pp\to X} \equiv \sum_{i,j} \int\int \mathrm{d}x_1 \mathrm{d}x_2 f_i(x_1, Q_F^2) f_j(x_2, Q_F^2) \hat{\sigma}_{ij\to X}(\alpha_s(Q_R^2), x_1, x_2)$$

$f_i(x_1, Q_F^2)$ can be determined through Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations.

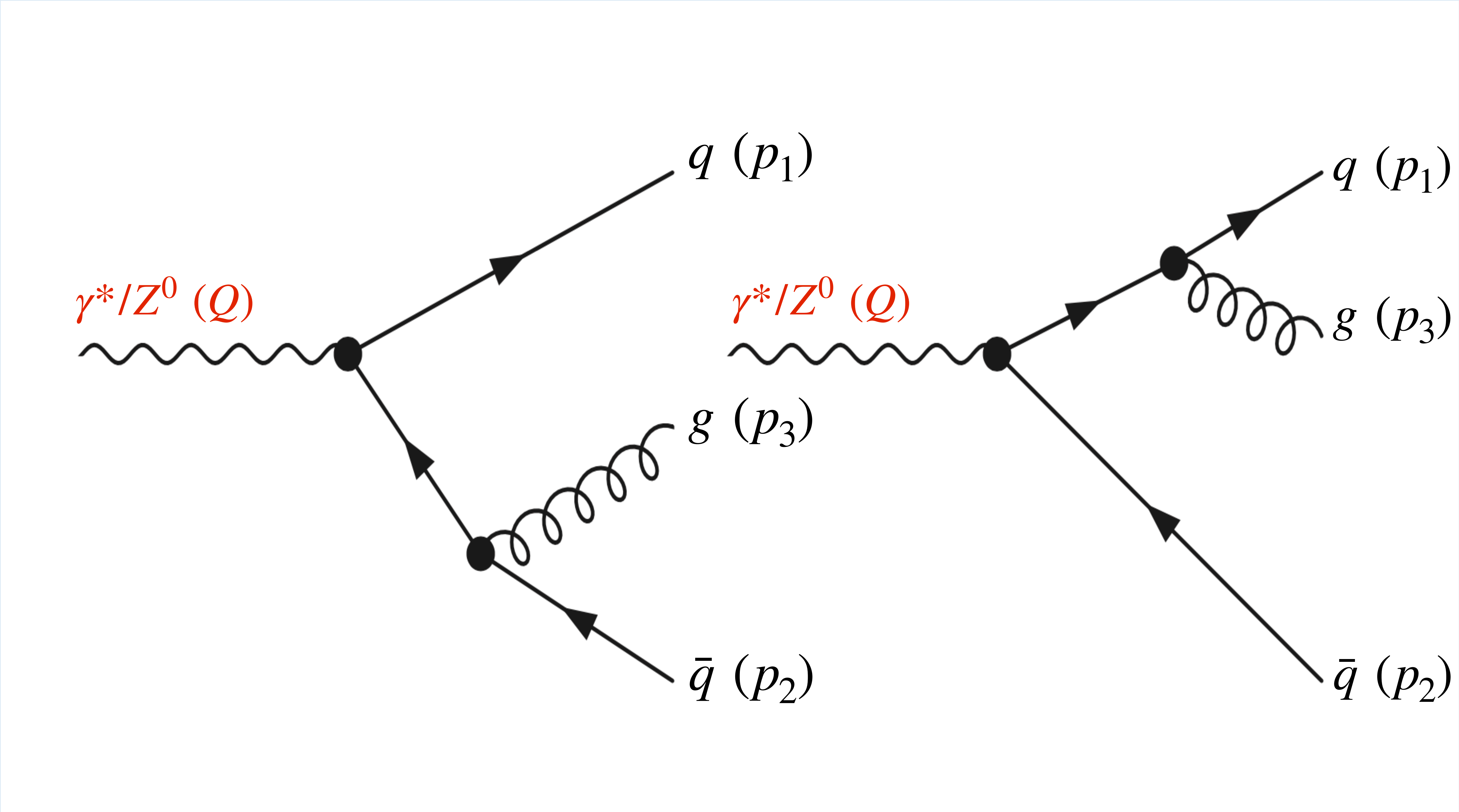
- **G. Altarelli, G. Parisi; Nuclear Physics B, 126 (2): 298-318 (1977).**
- **Yu. L. Dokshitzer; Sov. Phys. JETP 46:461 (1977).**
- **N. Gribov, L. N. Lipatov; Sov. J. Nucl. Phys. 15:438 (1972).**

- FSR: parton showers has important effects on the kinematics (event-shapes).

→ Assume we have the production of two jets with $p_{\perp 1} = p_{\perp 2} = 150$ GeV. Parton showers may produce a third jet plus recoiling the existing two; $p_{\perp 1} = 180$ GeV, $p_{\perp 2} = 140$ GeV and $p_{\perp 3} = 40$ GeV

Parton showers: formalism

We consider the following process: $e^+e^- \to q\bar{q}g$



Define

$$x_i = \frac{2Q \cdot p_i}{Q^2} = \frac{2E_i}{\sqrt{s}} = 1 - \frac{m_{jk}^2}{s}$$

$$\implies x_1 + x_2 + x_3 = 2 \quad (0 \le x_i \le 1)$$

Notable limits

$$x_1, x_2 \to 1 \Leftrightarrow m_{qg}^2, m_{\bar{q}g}^2 \to 0$$

$\Longrightarrow$Propagator for $\bar{q} \to \bar{q}g$ and $q \to qg$ goes on shell

Parton showers: formalism

After some algebra

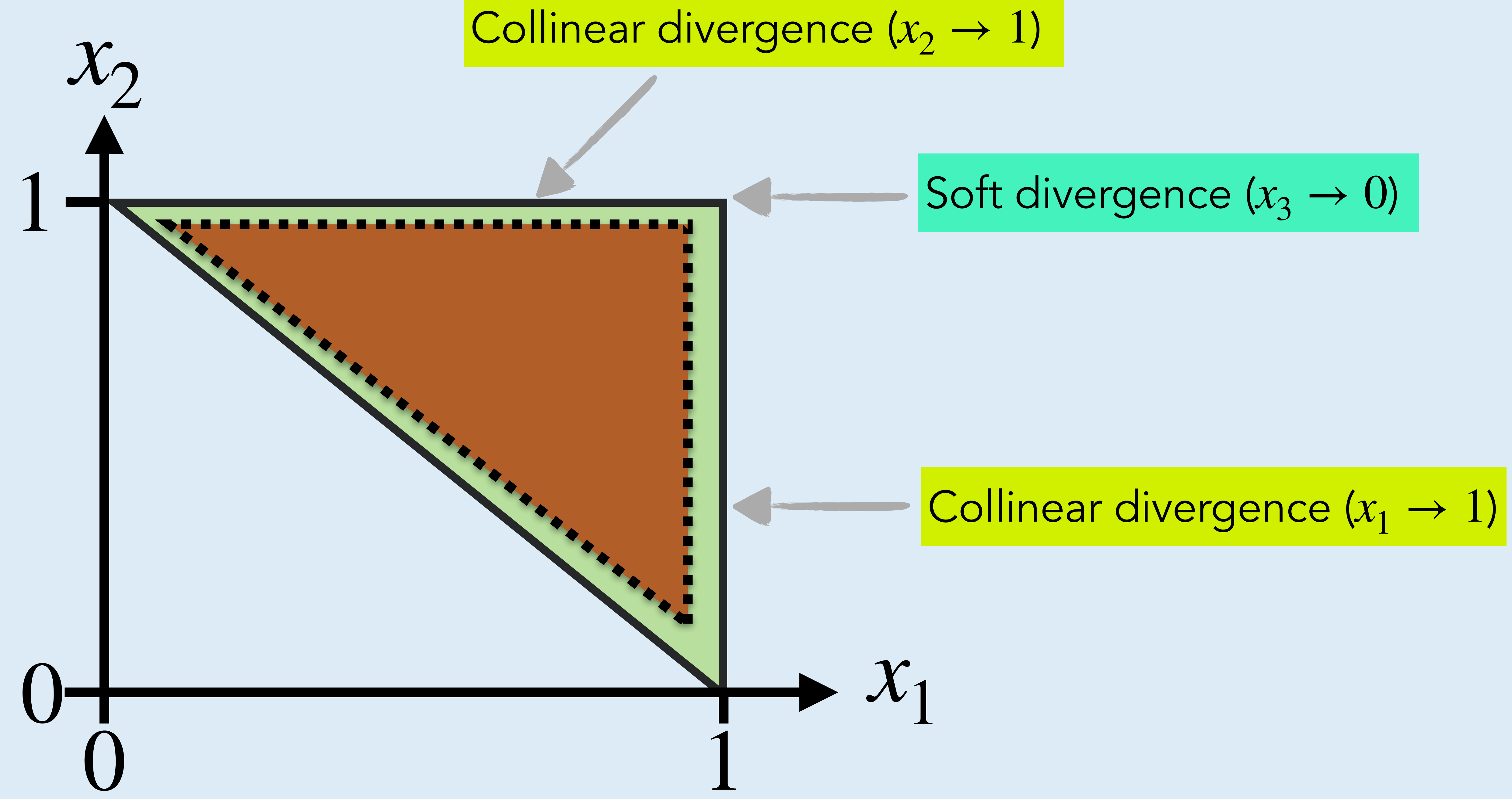
$$\frac{d^2\sigma}{\sigma_0} = \frac{\alpha_s}{2\pi} C_F \frac{x_1^2 + x_2^2}{(1 - x_1)\,(1 - x_2)} dx_1 dx_2 \approx \frac{\alpha_s}{2\pi} C_F \frac{2\; dx_1 dx_2}{(1 - x_1)\,(1 - x_2)}$$

For example, we can choose

$$\max\{x_1, x_2, x_3\} < 1 - \min\{y_{23}, y_{13}, y_{12}\}$$

$$\int_\delta \frac{d^2\sigma}{\sigma_0} \approx \frac{\alpha_s}{\pi} C_F \int_\delta \frac{dy_{23} dy_{13}}{y_{23} y_{13}} \approx C_F \frac{\alpha_s}{\pi} \log^2(\delta)$$

→ Double-logarithmic enhancement!!



DGLAP splitting kernels

Take, for example, the collinear limit ($x_1 \to 1$)

$$1 - x_1 = \frac{m_{23}^2}{Q^2} \implies \mathrm{d}x_2 = \frac{\mathrm{d}m_{23}^2}{Q^2}$$

Define z as the fraction the anti-quark takes in the branching $q \to qg$:

$$x_1 \approx z \implies \mathrm{d}x_1 \approx \mathrm{d}z \quad \text{and} \quad x_3 \approx (1 - z)$$

$$\implies \mathrm{d}\mathscr{P} = \frac{\mathrm{d}\sigma}{\sigma_0} = \frac{\alpha_s}{2\pi} C_F \frac{x_2^2 + x_1^2}{(1 - x_2)} \mathrm{d}x_1 \approx \frac{\alpha_s}{2\pi} \frac{\mathrm{d}m_{23}^2}{m_{23}^2} C_F \frac{1 + z^2}{1 - z} \mathrm{d}z$$

Collinear

Soft divergence

Universal and holds in the limit $x_2 \to 1$ as well

DGLAP splitting kernels

$$\mathrm{d}\mathscr{P}_{i\to jk} = \frac{\alpha_s}{2\pi}\frac{\mathrm{d}Q^2}{Q^2}P_{i\to jk}(z)\mathrm{d}z$$

In general, we can obtain the universal branching kernels (DGLAP)

$$P_{q\to qg} = P_{\bar{q}\to\bar{q}g} = C_F\frac{1+z^2}{1-z}$$

$$P_{g\to gg} = C_A\frac{(1-z(1-z))^2}{z(1-z)}$$

$$P_{g\to q\bar{q}} = n_f T_F(z^2+(1-z)^2)$$

These are the limits of any-matrix element in the collinear region (holds for any process).

The ordering variable

Now, we can generalize to multiple emissions.

$\Longleftrightarrow$ probabilities are large for one & should large for multiple (resummations)

$\Longleftrightarrow$ need to impose cuts on the soft/collinear to get rid of non-perturbative QCD

The choice of the ordering variable is not unique $\Longrightarrow$ If $Q^2 \equiv m_{jk}^2$ is a variable then $P^2 = f(z)Q^2$ can also be a variable

$$\left|\frac{\mathrm{d}(P^2, z)}{\mathrm{d}(Q^2, z)}\right| = \begin{vmatrix} \frac{\partial P^2}{\partial Q^2} & \frac{\partial P^2}{\partial z} \\ \frac{\partial z}{\partial Q^2} & \frac{\partial z}{\partial z} \end{vmatrix} = \begin{vmatrix} f(z) & \frac{\mathrm{d}f(z)}{\mathrm{d}z} \\ 0 & 1 \end{vmatrix} = f(z)$$

$$\Longrightarrow \quad \mathrm{d}\mathscr{P}_{i\to jk} = \frac{\alpha_s}{2\pi}\frac{f(z)\ \mathrm{d}Q^2}{f(z)\ Q^2}P_{i\to jk}(z)\mathrm{d}z = \mathrm{d}\mathscr{P}_{i\to jk} = \frac{\alpha_s}{2\pi}\frac{\mathrm{d}P^2}{P^2}P_{i\to jk}(z)\mathrm{d}z$$

Examples

- $P^2 = E^2\theta^2 \approx \dfrac{Q^2}{z(1-z)}$ (angular-ordered shower); used by HERWIG.
- $P^2 = p_\perp^2 \approx Q^2 z(1-z)$ (transverse-momentum shower); used by PYTHIA8.
- $P^2 = Q^2$ (virtuality ordered shower); used by PYTHIA6.

Sudakov factors

Using conservation of total probability (unitarity)

$$\mathscr{P}(\text{no emission}) = 1 - \mathscr{P}(\text{emission}); \quad \mathscr{P}_{0<t\leq T} = \mathscr{P}_{0<t\leq T_1} \times \mathscr{P}_{T_1<t\leq T}$$

Split the interval $[0, T]$ into infinitesimally small and equal intervals

$$\begin{aligned}
\mathscr{P}_{\text{no emission}}(0 < t \leq T) &= \lim_{N\to\infty} \prod_{i=0}^{N-1} \mathscr{P}_{\text{no emission}}(T_i < t \leq T_{i+1}) \\
&= \lim_{N\to\infty} \prod_{i=0}^{N-1} (1 - \mathscr{P}_{\text{emission}}(T_i < t \leq T_{i+1})) \\
&= \exp\left(- \lim_{N\to\infty} \sum_{i=0}^{N-1} \mathscr{P}_{\text{emission}}(T_i < t \leq T_{i+1}) \right) \\
&= \exp\left(- \int_0^T \frac{\mathrm{d}\mathscr{P}_{\text{emission}}(t)}{\mathrm{d}t} \mathrm{d}t \right)
\end{aligned}$$

Sudakov factors

We find the probability for the first emission

$$\mathrm{d}\mathscr{P}_{\text{first}}(T) = \mathrm{d}\mathscr{P}_{\text{emission}}(T) \times \exp\left(-\int_0^T \frac{\mathrm{d}\mathscr{P}_{\text{emission}}(t)}{\mathrm{d}t}\mathrm{d}t \right)$$

$$\underset{Q \simeq 1/t}{\Longrightarrow} \mathrm{d}\mathscr{P}_{i\to jk} = \frac{\alpha_s}{2\pi}\frac{\mathrm{d}Q^2}{Q^2}P_{i\to jk}(z)\mathrm{d}z \times \underbrace{\exp\left(-\sum_{j,k}\int_{Q_1^2}^{Q_2^2}\frac{\mathrm{d}Q^2}{Q^2}\int_{z_{\min}}^{z_{\max}}\frac{\alpha_s}{2\pi}P_{i\to jk}(z')\mathrm{d}z' \right)}$$

Sudakov form factors; $\Delta(Q_1^2, Q_2^2)$

Note that the total probability is one; i.e.

$$\sum_{j,k}\int_{Q^2}\int_z \mathrm{d}\mathscr{P}_{i\to jk}(z, Q^2) = 1$$

Connection to matrix elements

Take our favorite process as an example: $e^+e^- \to q\bar{q}g$

$\Longleftrightarrow$ use thrust cuts on the phase space (see slide 11)

$$\frac{\sigma_{\text{real}}}{\sigma_0} \approx C_F \frac{\alpha_s}{\pi} \log^2 \delta \qquad \text{and} \qquad \frac{\sigma_{\text{virtual}}}{\sigma_0} \approx \frac{\alpha_s}{\pi} - C_F \frac{\alpha_s}{\pi} \log^2 \delta$$

$$\Longrightarrow \quad \sigma_{\text{NLO}} = \sigma_0 + \sigma_{\text{real}}(\delta) + \sigma_{\text{virtual}}(\delta) = \left(1 + \frac{\alpha_s}{\pi}\right)\sigma_0$$

$\sigma_0 \equiv \sigma(e^+e^- \to q\bar{q})$

Neglect the small correction $(\alpha_s/\pi) \Longrightarrow \sigma_{\text{virtual}}(\delta) = -\sigma_{\text{real}}(\delta)$

$$\frac{\mathrm{d}\mathscr{P}}{\mathrm{d}y} = \frac{1}{\sigma_0}\frac{\mathrm{d}\sigma_{\text{real}}}{\mathrm{d}y} \exp\left(-\int_y^1 \frac{1}{\sigma_0}\frac{\mathrm{d}\sigma_{\text{real}}}{\mathrm{d}y'}\mathrm{d}y'\right) = \frac{1}{\sigma_0}\frac{\mathrm{d}\sigma_{\text{real}}}{\mathrm{d}y}\left(1 + \frac{\sigma_{\text{virtual}}(y)}{\sigma_0} + \left(\frac{\sigma_{\text{virtual}}(y)}{\sigma_0}\right)^2 + \cdots\right)$$

Parton showers: diagrammatic

Starting from an arbitrary Born ME, we can now:

Obtain tree-level ME with **any number of legs** (in soft/collinear approximation)

Universality (scaling)

⟶

Jet-within-a-jet-within-a-jet-…

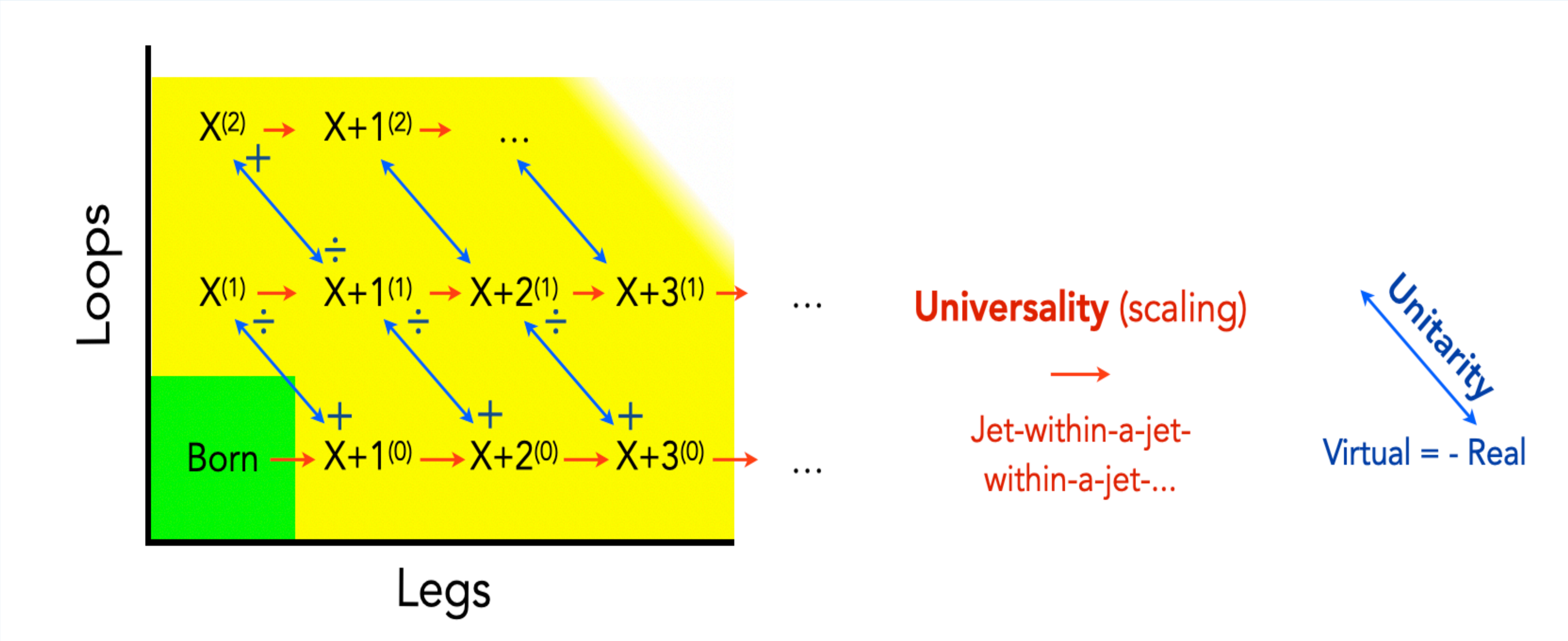
Doesn't look very "all-orders" though, does it? **What about the loops?**

Peter Skands (HCPSS 2020)

Parton showers: An all-order QCD

Showers impose **Detailed Balance** (a.k.a. Probability Conservation ↔ **Unitarity**)

When X branches to X+1 **: Gain** one X+1, **Lose** one X ➜ **Virtual Corrections**



➜ Showers do **"Bootstrapped Perturbation Theory"**

Imposed via differential **event evolution**

Peter Skands (HCPSS 2020)

Evolution ~ Fine-Graining the Description of the Event

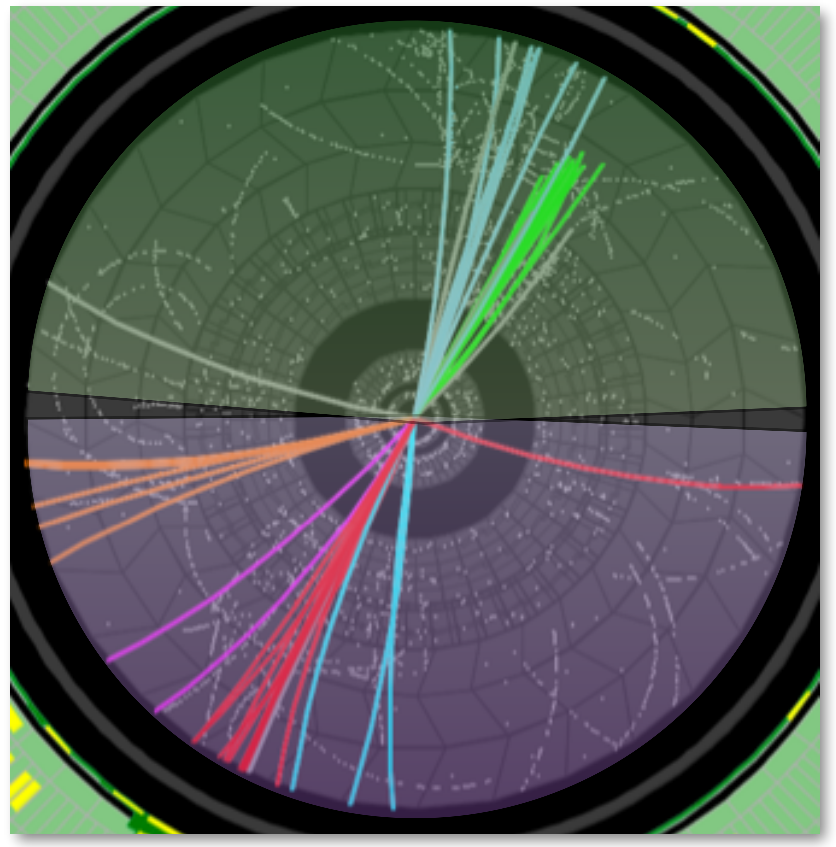
Resolution Scale

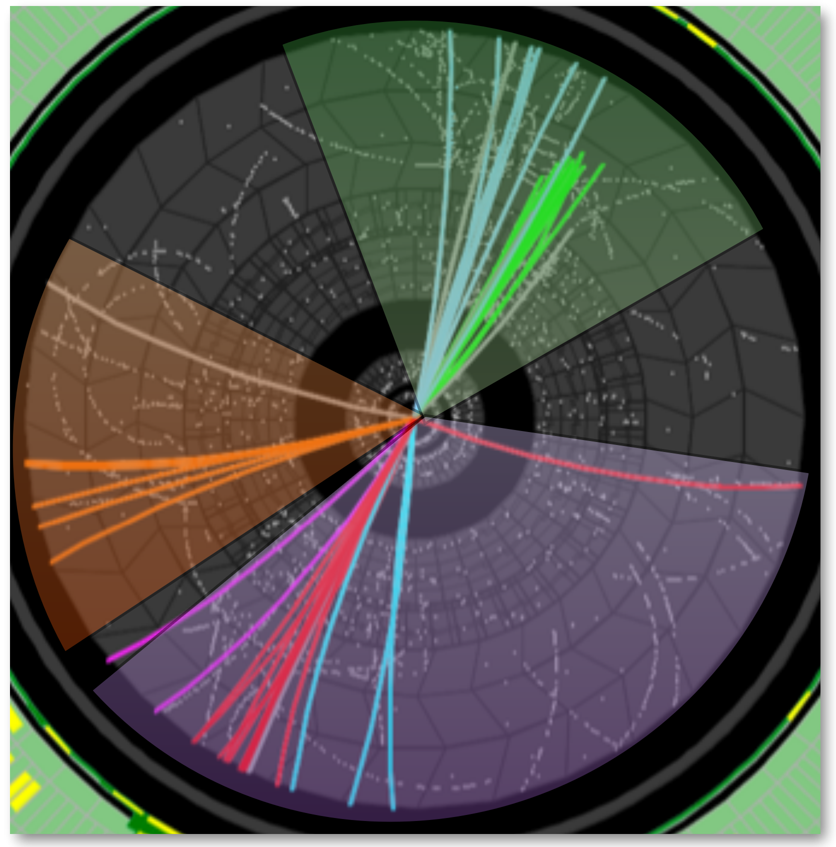
$Q \sim Q_{\mathrm{HARD}}$

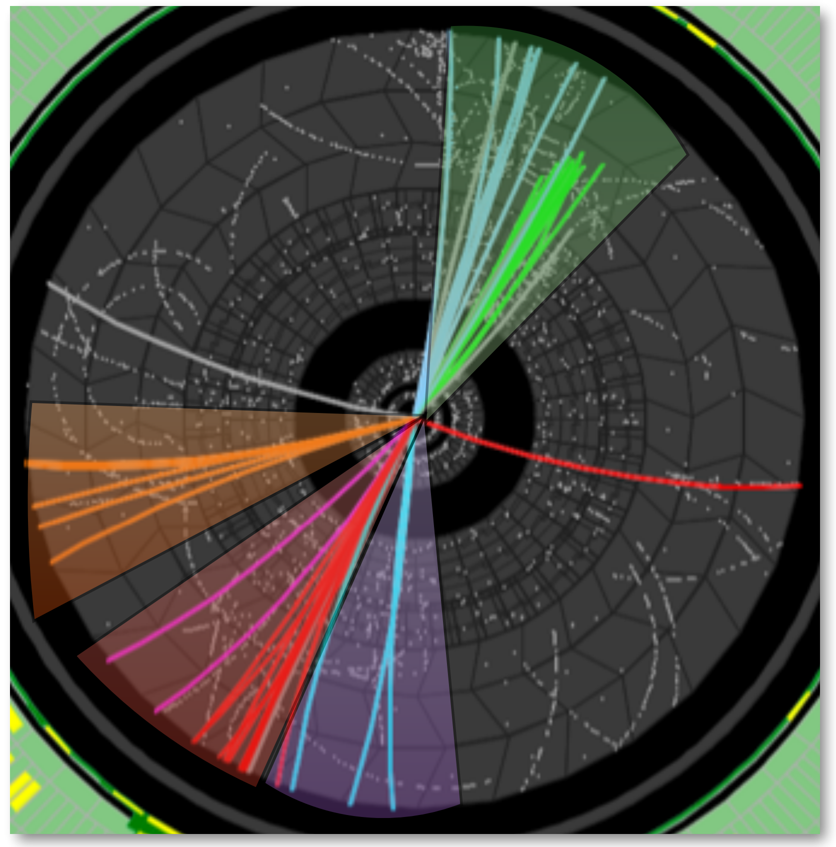
$Q_{\mathrm{HARD}}/Q <$ "A few"

$Q \ll Q_{\mathrm{HARD}}$

Scale Hierarchy!







At most inclusive level
"Everything is 2 jets"

At (slightly) finer resolutions,
some events have 3, or 4 jets

At high resolution, **most** events
have >2 jets

Cross sections

Fixed order:
$\sigma_{\mathrm{inclusive}}$

Fixed order:
$\sigma_{X+n} \sim \alpha_s^n \, \sigma_X$

Fixed order diverges:
$\sigma_{X+n} \sim \alpha_s^n \ln^{2n}(Q/Q_{\mathrm{HARD}})\sigma_X$

Unitarity ➜ ***number of splittings diverges***
while cross section remains $\sigma_{\mathrm{inclusive}}$

Peter Skands (HCPSS 2020)

Parton showers: some ambiguities

Final-state particles generated by any shower algorithm depends on many factors:

1. The choice of perturbative evolution variable(s) $t^{[i]}$. ← Ordering & Evolution-scale choices
2. The choice of phase-space mapping $\mathrm{d}\Phi^{[i]}_{n+1}/\mathrm{d}\Phi_n$. ← Recoils, kinematics
3. The choice of radiation functions a_i, as a function of the phase-space variables. ← Non-singular terms, Coherence, Subleading Colour
4. The choice of renormalization scale function μ_R.
5. Choices of starting and ending scales. ← Phase-space limits / suppressions for hard radiation and choice of hadronization scale

→ gives us additional handles for **uncertainty estimates**, beyond just μ_R
(+ ambiguities can be reduced by including more pQCD → **merging**!)

Peter Skands (HCPSS 2020)

Combining showers and fixed-order QCD

Fixed Order QCD

Provide solutions for a single process (fully automated at LO):

Most of the SM processes can be computed up to NLO; some can be computed at NNLO or even NNNLO.

Beyond the SM processes only known at LO (or NLO for some).

Accurate for hard process, to a given perturbative order. Good accuracy in the full phase space regions.

Limited generality:

Problem of multi scales (see slide 4) which needs resummations (analytical or semi-classical a.k.a. parton-showers)

➜ loss of accuracy.

All-orders QCD

Universal solutions to all the processes (SM or BSM).

Accurate in strongly ordered (soft/collinear) limits (=regions with enhanced probabilities)

Maximum generality:

Problem of process-dependence = sub-leading corrections, large for hard resolved jets.

$\Longrightarrow$ Jet Merging

Jet Merging: General Idea

Idea: combine fixed-order QCD with parton showers to get the best of the two!

Naive prescription:

Run generator for X + **shower**
Run generator for X+1 + **shower**
…
Run generator for X+m + **shower**

} Add all these generated samples together!

Problem:

If you do that, you get a "double counting" of terms present in both expansions:

e.g. the X + **shower** sample covers some of the phase-space in the (X+1) + **shower** sample.

Solution:

Develop algorithms to remove the "double counting":

- Based on Matrix-Element Corrections (MECs): Pythia8 and Powheg.
- Based on Phase-space slicing: MLM and CKKW-L.

Jet Merging: MECs

Idea: Modify parton shower to use radiation functions ∝ full matrix element for 1st emission:

$$\frac{P(z)}{Q^2} \to \frac{P'(z)}{Q^2} = \frac{P(z)}{Q^2} \frac{|M_{n+1}|^2}{\sum_i P_i(z)/Q^2 |M_n|^2}$$

Implemented in PYTHIA8 for:

all the SM processes and many BSM processes

Difficult to generalise beyond one emission

MECs (POWHEG with Loops)

Positive **W**eight **H**ardest **E**mission **G**enerator

Start at the Born level $|M_X|^2$

Generate shower emission $|M_{X+1}|^2 \approx \sum_i a_i |M_X|^2$

Correct to Matrix elements $a_i \to \frac{|M_{X+1}|^2}{\sum_i a_i |M_X|^2} a_i$

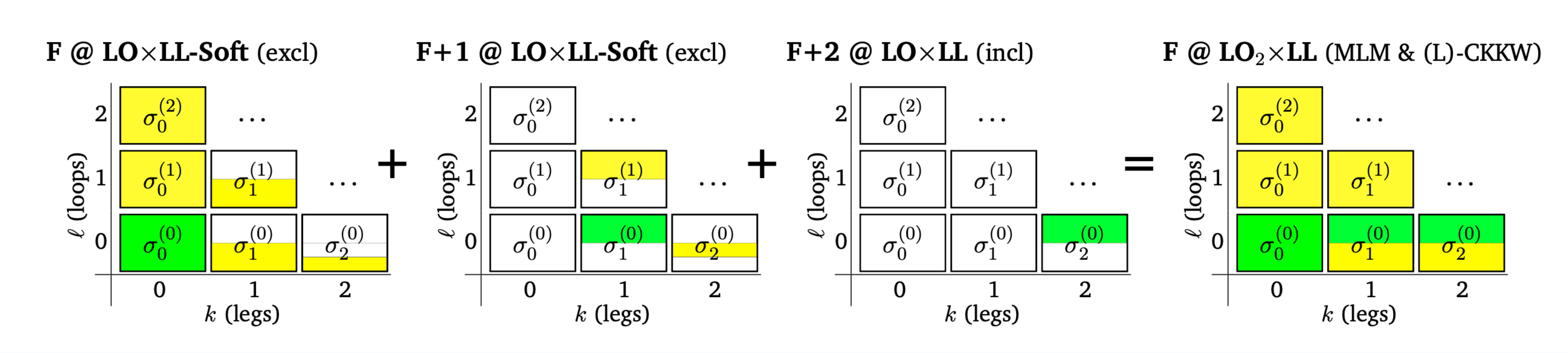
Unitarity of the shower $\text{Virtual} = -\int \text{Real}$

Correction: $|M_X|^2 \to |M_X|^2 + 2\text{Re}\{M_F^{(1)} M_F^{(0)}\} + \int \text{Real}$

Repeat parton showers

Nason, JHEP 0411 (2004) 040
Frixione, Nason, Oleari JHEP 0711 (2007) 070
+ POWHEG Box JHEP 1006 (2010) 043

Jet Merging: Slicing Algorithms



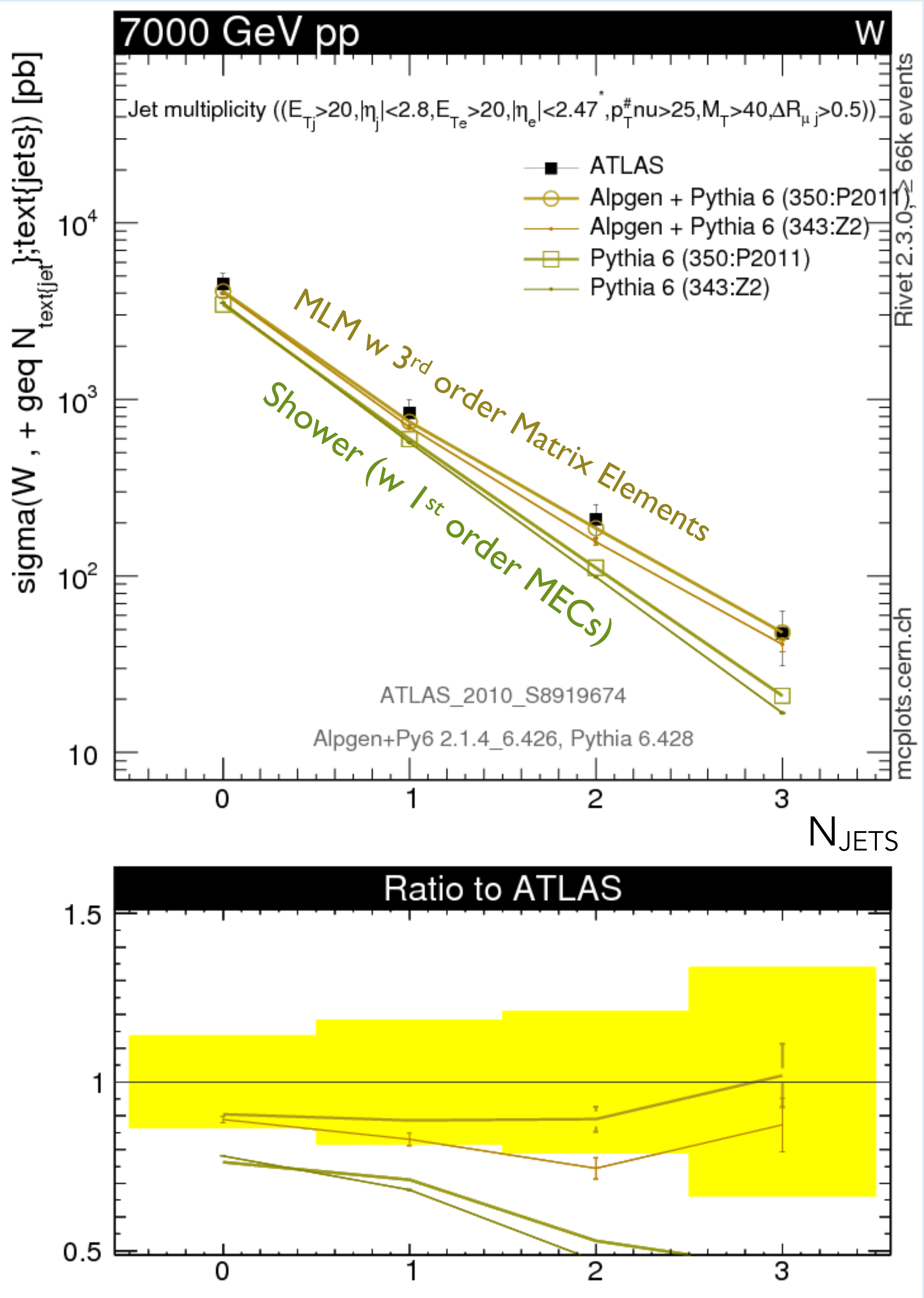
Shower approximation is set to zero above some scale:

- Due to "dead-cone" regions (as it occurs in Herwig).
- Veto some emissions above some matching scale.

Multi-jet merging algorithms such as CKKW-L and MLM tend to fill this empty region by

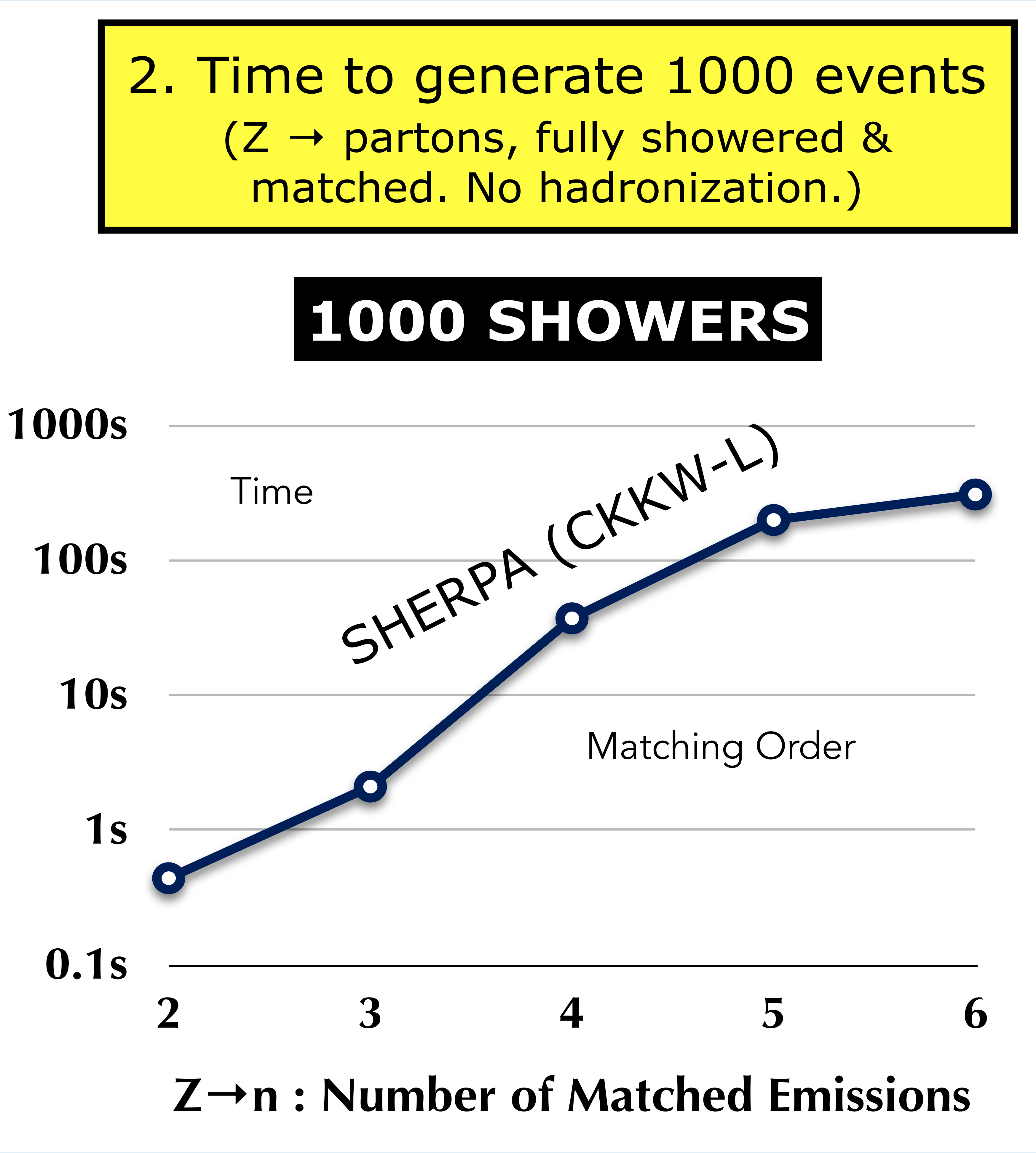
- Generating multi-jet samples corresponding to high-multiplicity tree-level matrix elements.
- The multi-jet samples must be associated with Sudakov form factors (to ensure smooth transitions).

Example: LHC$_7$: W + 20-GeV Jets



Plot from mcplots.cern.ch; see arXiv:1306.3436

Example: e+e- → Z → Jets



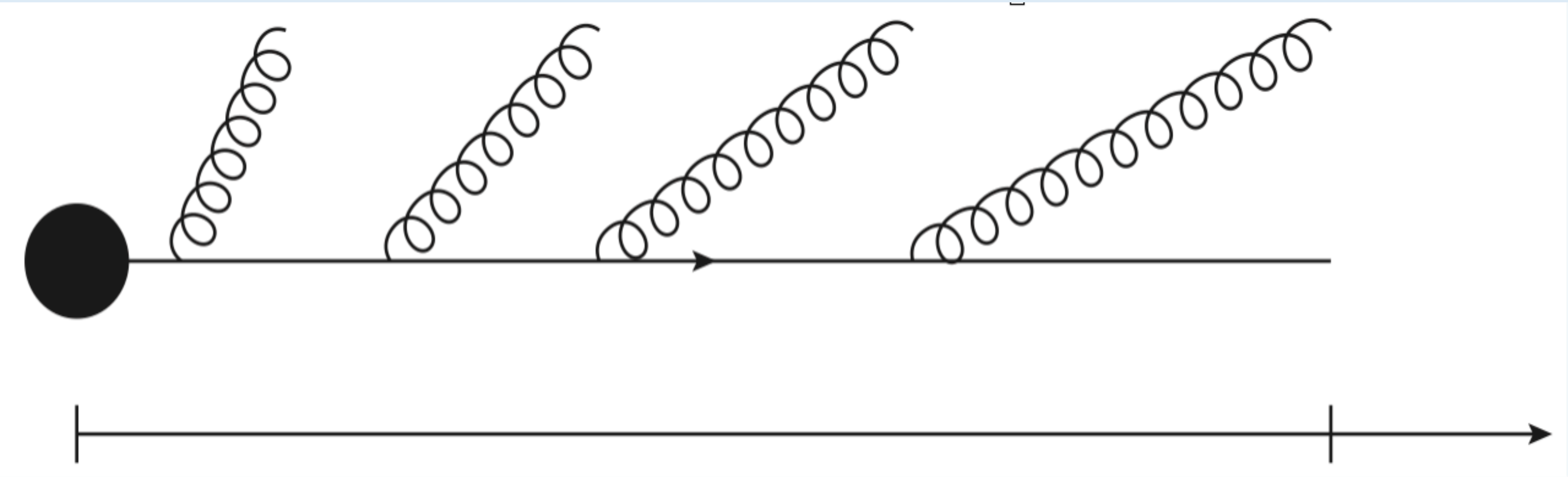
See e.g. Lopez-Villarejo & Skands, arXiv:1109.3608

From Partons to Hadrons

Parton starts at a high factorization scale
$Q = Q_F = Q_{\text{hard}}$

It showers (bremsstrahlung)

It ends up at a low effective factorization scale
$Q \approx m_\rho \approx 1\ \text{GeV}$



Q_{hard}

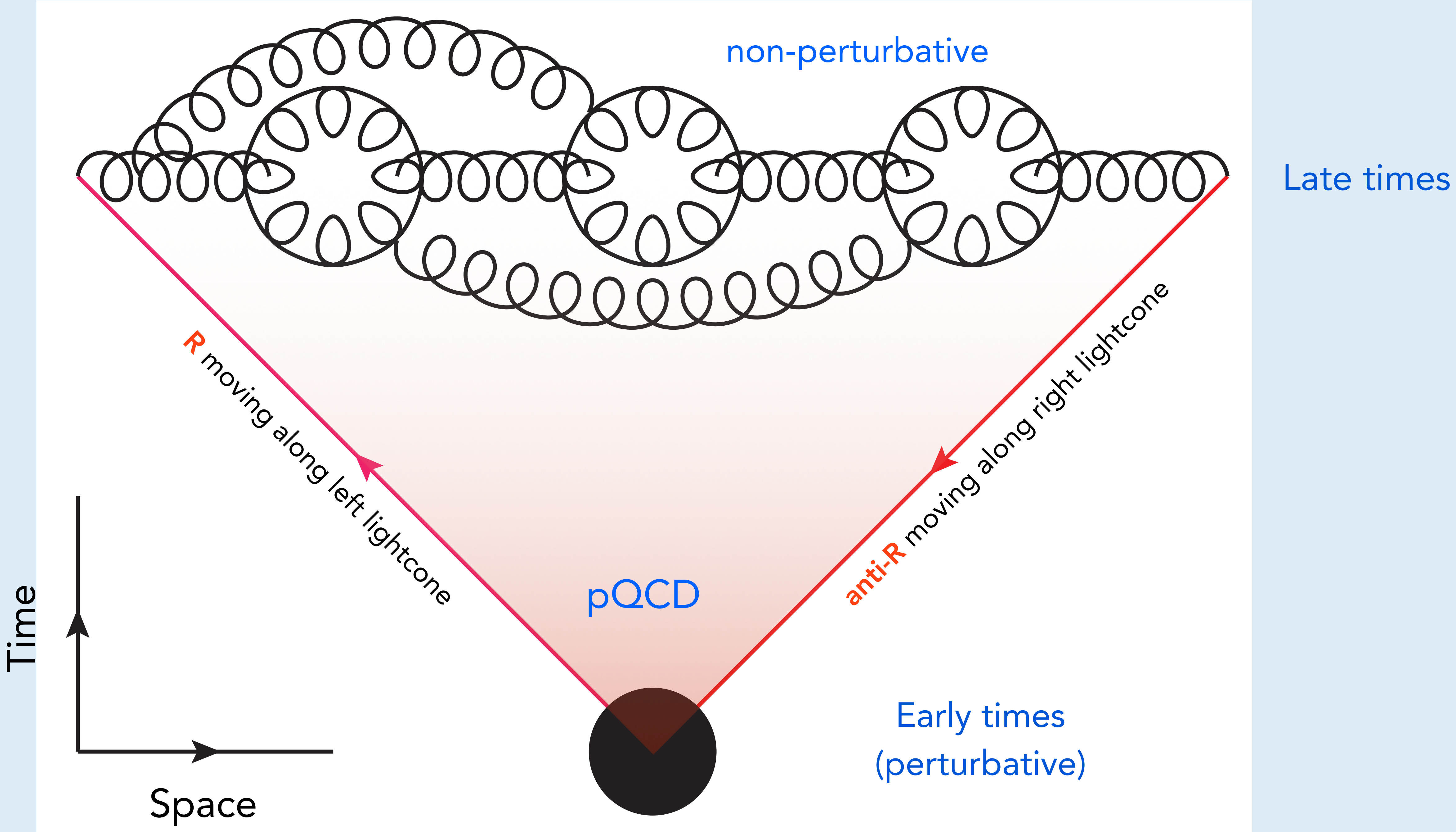
$Q_{\text{cut}} \approx 1\ \text{GeV}$

Colour Neutralisation

A **physical** hadronization model

Should involve at least **two** partons, with opposite color charges

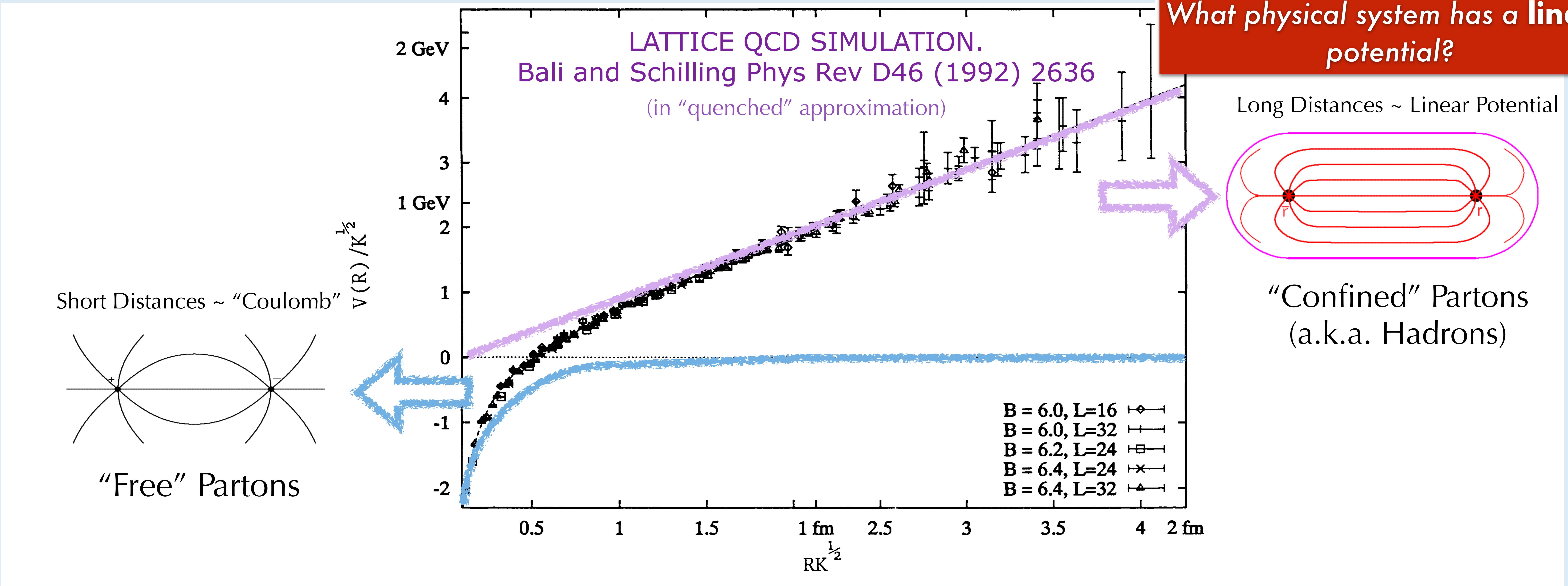
A strong **confining field** emerges between the two when their separation ≳ 1fm



Linear confinement

Explicit computer simulations of QCD on a 4D "lattice" (lattice QCD) can provide the potential of the colour-singlet $q\bar{q}$ system

What physical system has a linear potential?



"Cornell Potential" fit: $$V(r) = -\frac{a}{r} + \kappa r$$ with $\kappa \sim 1$ GeV/fm (→ could lift a 16-ton truck)

Hadronisation: Lund string model

The Lund string model is based on the following symmetric function

Causality and Lorentz invariance $\Longrightarrow$ $$f(z, m_{\perp h}) \equiv N \frac{(1-z)^a}{z} \exp\left(- \frac{-bm_{\perp h}^2}{z} \right)$$

This function gives

the probability to produce a hadron with energy fraction z and transverse mass $m_{\perp h}$.

This function depends on:

a and b are tunable parameters with the former controls the number of high energy hadrons while the latter controls the number of low energy hadrons. (Plus about few tens of others which control flavors…etc).

Properties of the Lund symmetric function

- If $f(z)$ is peaked around 1, then the QCD jet consists of few hadrons each carrying a high fraction of the parent energy.
- If $f(z)$ is peaked around 0, then the QCD jet consists of many hadrons each carrying a very low fraction of the parent energy.

Iterative string break-ups