mixed QCD-EW corrections to $pp \to l^+ \nu_l + X$

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(based on the paper ***PhysRevD.103.114012***, in collaboration with *L. Buonocore, M. Grazzini, S. Kallweit, F. Tramontano*)

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Universität Zürich UZH

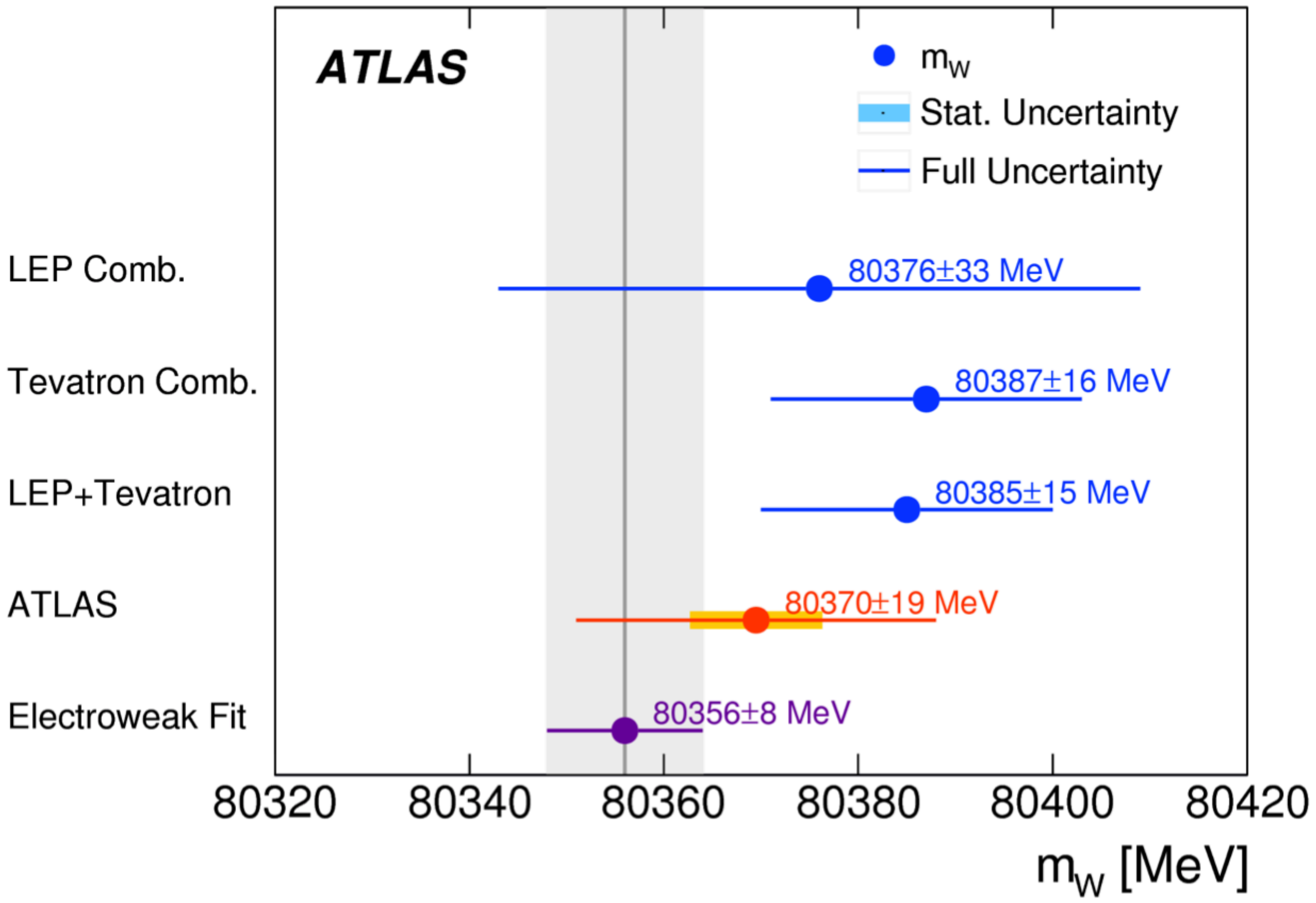
Outline

- *Introduction*
- *Handling of IR singularities:* q_T - subtraction formalism
- *Hard-virtual coefficient:* pole approximation technique
- *Numerical results*
- *Conclusions*

Introduction

Motivations :

- Drell-Yan process is a **"standard candle"** thanks to its large production rates and clean experimental signatures
- it is important for testing SM and QCD predictions, PDF fit etc.
- new physics searches at high invariant mass
- it is used to extract the precise measurements of EW parameters like the **W-boson mass**



- the accuracy expected on the experimental side $\mathcal{O}(10\,MeV)$ requires a better control of the theoretical predictions !

☑ **QCD corrections** are dominant effects:

- NNLO for differential cross sections (including leptonic decay)

  [Anastasiu, Dixon, Melnikov, Petriello (2003)] , [Melnikov, Petriello (2006)] , [Catani, Cieri, Ferrera, de Florian, Grazzini (2009)] , [Catani, Ferrera, Grazzini (2010)]

- N3LO for inclusive cross section (γ^* and W boson production) and di-lepton rapidity distribution

  [Duhr, Dulat, Mistlberger (2020)], [Chen, Gehrmann, Glover, Huss, Yang and Zhu (2021)]

see Tonghzi's talk

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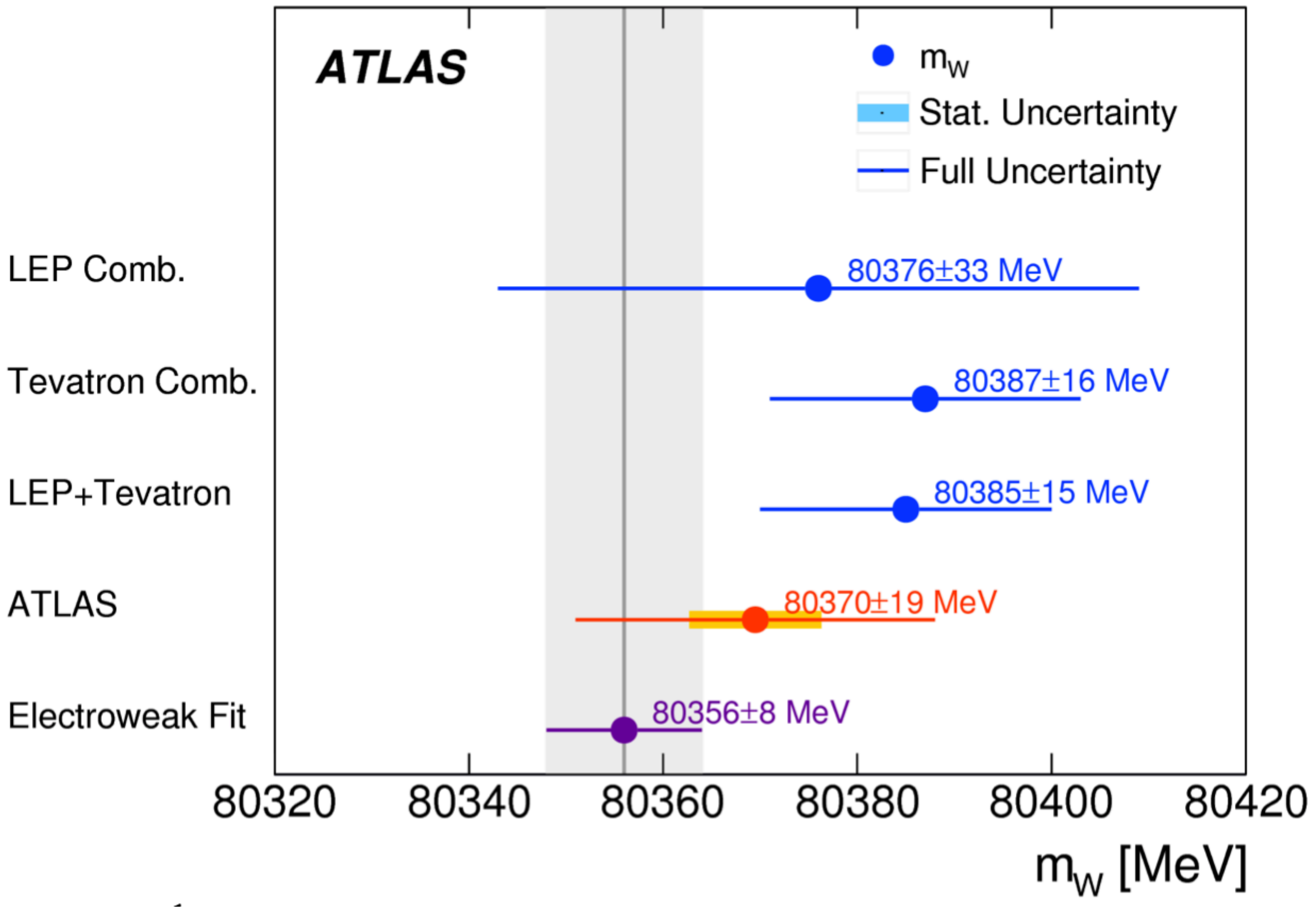

☑ **NLO EW corrections** are known:

[Dittmaier and Kramer (2002)] , [Baur et al (2002, 2004)] ,
[Zykunov (2006. 2007)] , [Arbuzov et al (2006, 2007)], [Carloni, Calame et al (2006, 2007)]

→ mixed QCD-EW corrections become relevant and their inclusion is highly desirable

Mixed QCD-EW corrections: state of art

The computation of fully differential mixed QCD-EW corrections is a **complicated task**.

the computation of $2 \to 2$ two-loop virtual amplitudes with different masses and scales is at the frontier of the current techniques!

- **On-shell Z/W production ($2 \to 1$ process):**
  - analytical mixed *QCD-QED* corrections to the *inclusive* production of an on-shell Z

    [de Florian, Der, Fabre (2018)]
  - *fully differential* mixed *QCD-QED* corrections to the production of an on-shell Z

    [Delto, Jaquier, Melnikov, Rontsch (2019)]
  - *total* Z production cross section in fully analytical form including exact NNLO *QCD-EW* corrections

    [Bonciani, Buccioni, Rana, Vicini (2020)]
  - *fully differential* on-shell Z and W production including exact NNLO *QCD-EW* corrections

    [Bonciani, Caola, Delto, Jaquier, Melnikov, Röntsch (2020)] , [Behring, Buccioni, Caola, Delto, Jaquier, Melnikov, Röntsch (2020)]

- **Beyond on-shell computations:**
  - dominant mixed *QCD-EW* corrections in *Pole Approximation* for neutral- and charged- DY processes

    [Dittmaier and Kramer (2002)] , [Baur et al (2002, 2004)]
  - approximate corrections available in *parton showers* based on a *factorised approach*

    [Balossini et al (2010)] , [Bernaciak, Wackeroth (2012)] , [Barzè et al (2012, 2013)], [Calame et al (2017)]
  - neutrino-pair production including NNLO *QCD-QED* corrections

    [Cieri, Der, de Florian, Mazzitelli (2020)]

it is here where this talk comes into play!

Mixed QCD-EW corrections: this talk

The computation of fully differential mixed QCD-EW corrections is a **complicated task**.

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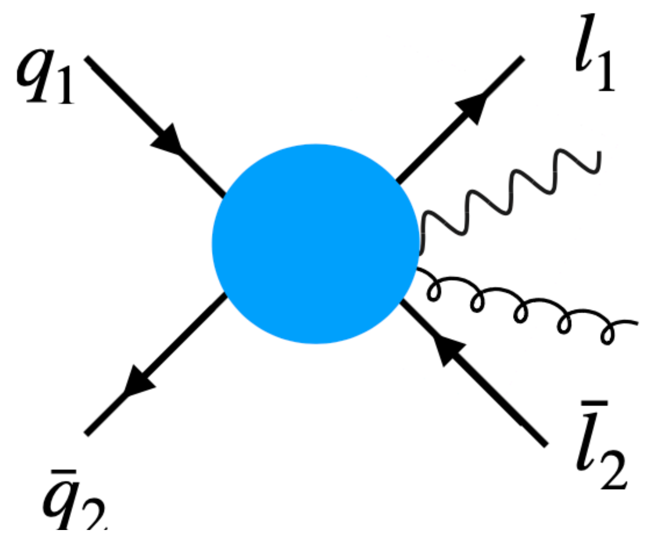
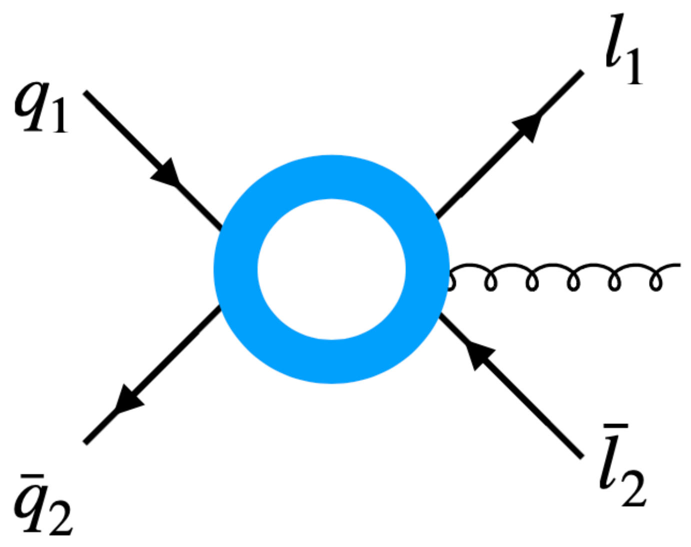
☑ **first (almost) exact fully differential** computation of the mixed QCD-EW corrections to the $2 \to 2$ *charged current* DY process
[Buonocore, Grazzini, Kallweit, CS, Tramontano (2021)]

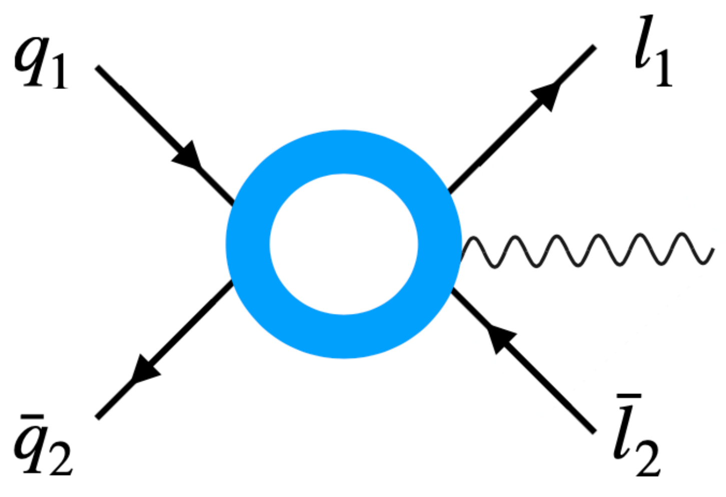
The needed ingredients are:

- tree-level diagrams with one gluon and one photon emission

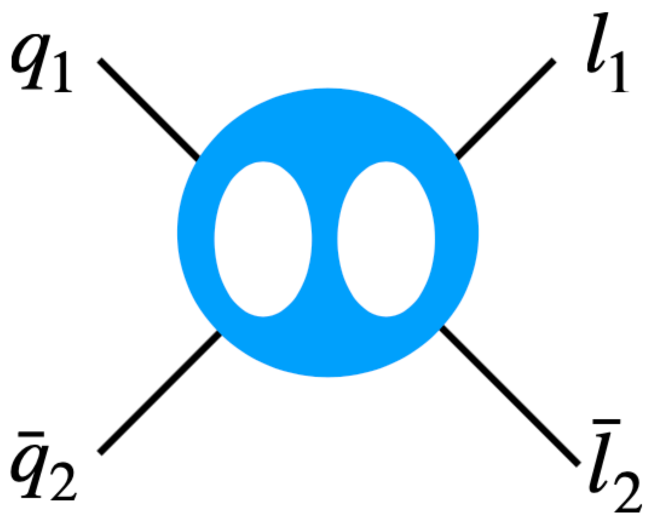


- one-loop diagrams with one gluon or one photon emission





- two-loop purely virtual diagrams (plus one-loop squared)



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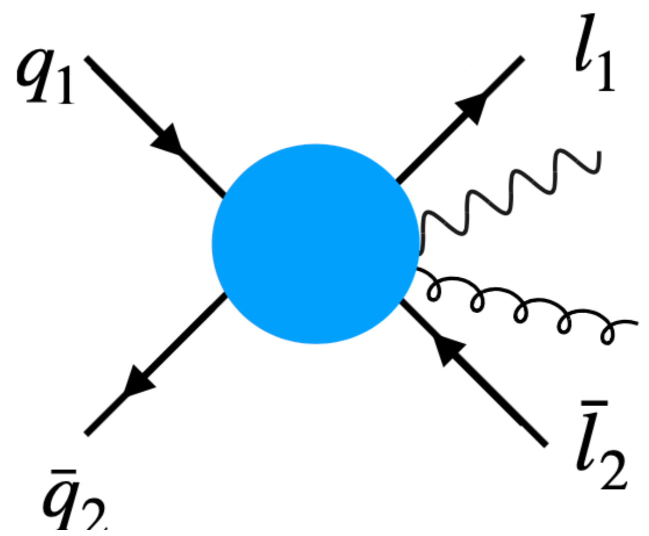

✓ automated tools like OpenLoops and Recola

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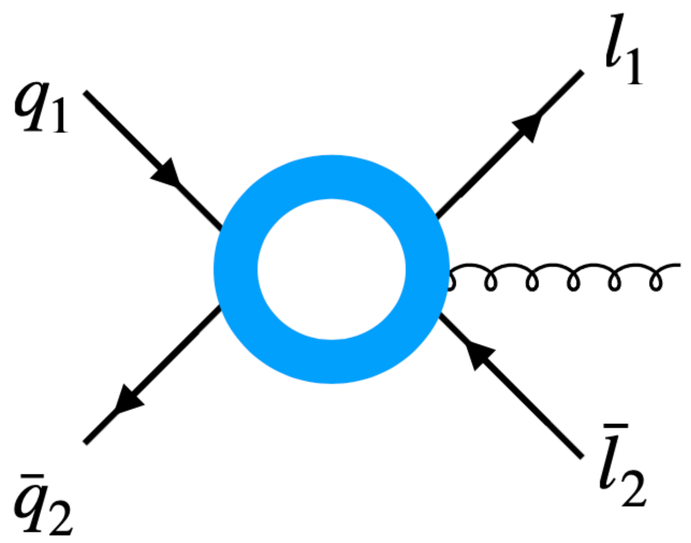

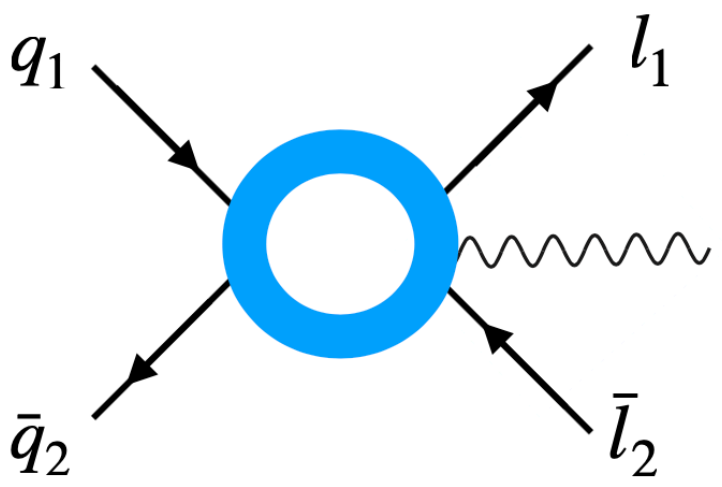

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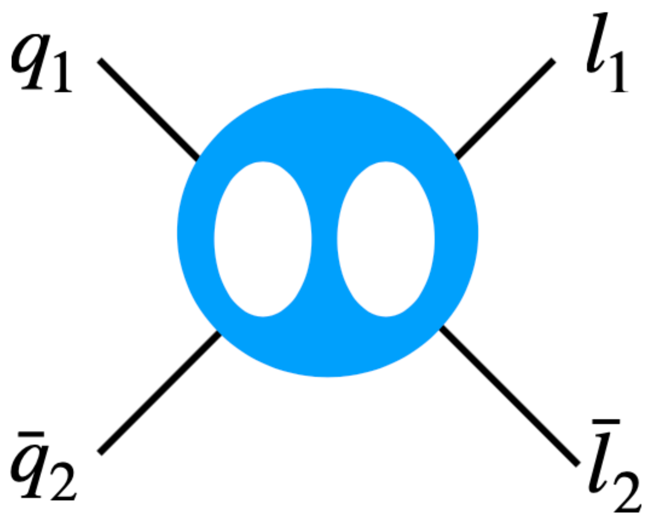

bottleneck

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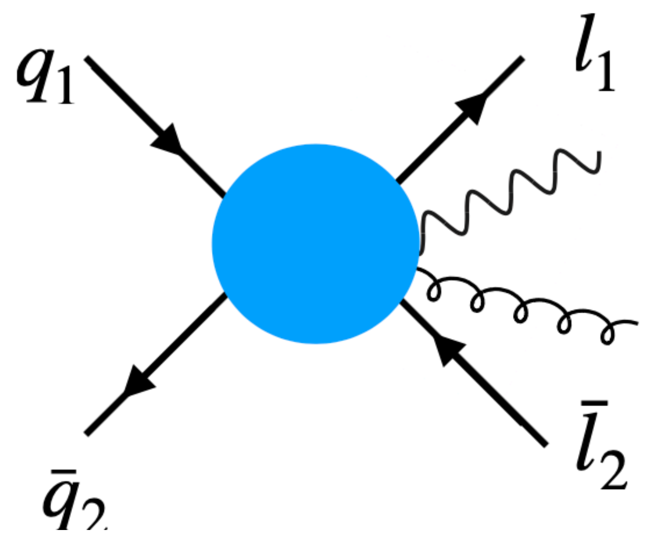

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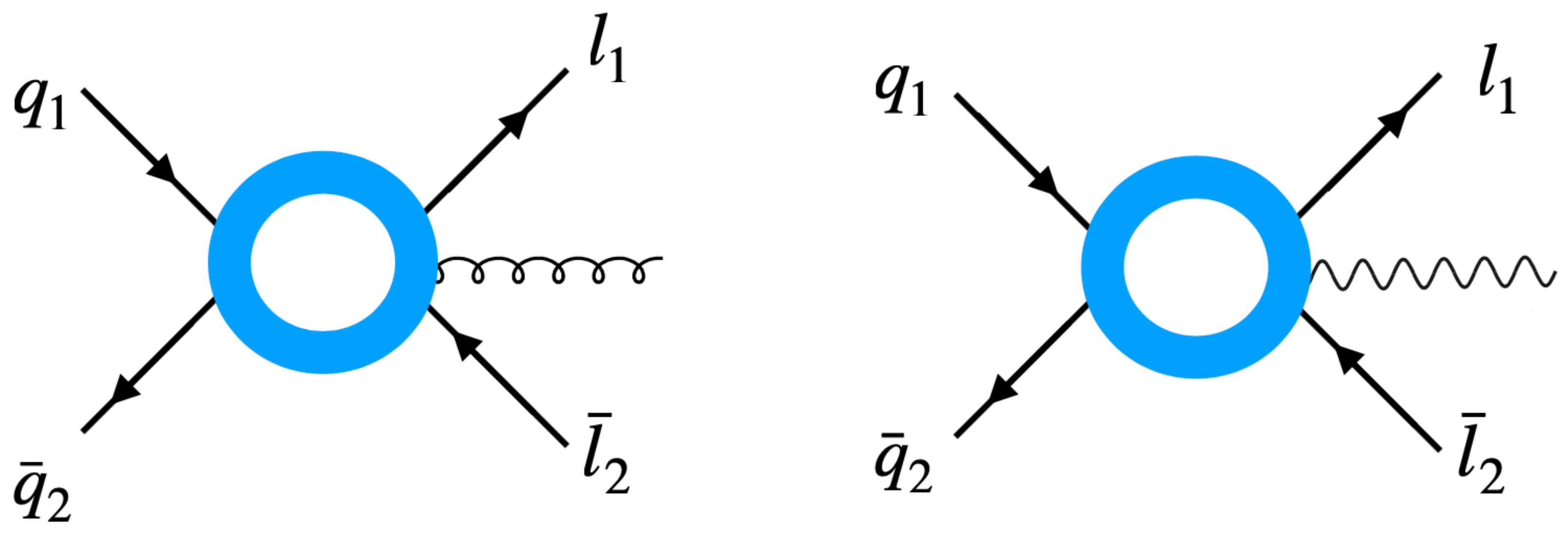

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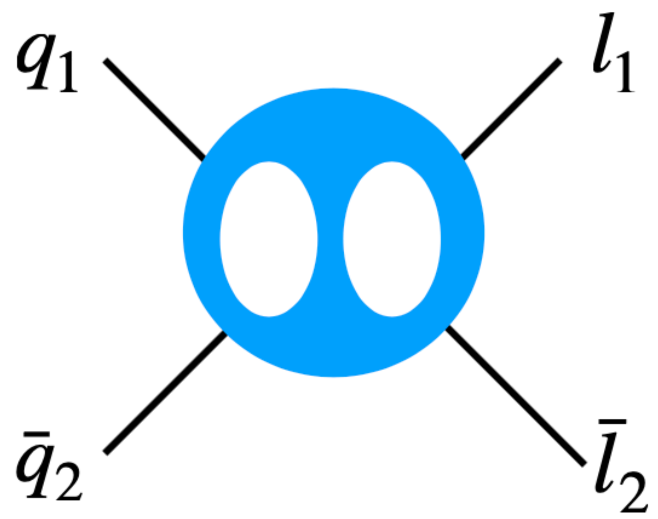

bottleneck

the finite part obtained with the **pole approximation technique**

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The needed ingredients are:

- tree-level diagrams with one gluon and one photon emission
- one-loop diagrams with one gluon or one photon emission
- two-loop purely virtual diagrams (plus one-loop squared)

All contributions listed above are separately infrared (IR) divergent

suitable subtraction scheme: q_T - **subtraction formalism**

☑ **first exact fully differential** computation of the mixed QCD-EW corrections to the $2 \to 2$ *neutral current* DY process

confirmation of the validity of our approximated procedure, applied in the charged current case

[Bonciani, Buonocore, Grazzini, Kallweit, Rana, Tramontano, Vicini (2021)]

Handling of IR singularities

General strategy :

- we start from the well-established q_T - **subtraction formalism** at NNLO QCD
- we require **massive leptons** in order to avoid collinear final state singularities that will spoil the method → bare muons
- the IR structure is associated only to QCD-QED subpart:

  we can recycle the NNLO QCD results via a careful **abelianisation procedure**

  [de Florian, Rodrigo, Sborlini (2016)], [de Florian, Der, Fabre (2018)]

- the FS muon is **colour neutral** and **massive**:

  we can use the recent developments to heavy quarks and, moreover, the purely soft structure is much more simple (only $\mathcal{O}(\alpha)$ coefficients)

  [Catani, Grazzini (2007)], [Catani, Torre, Grazzini (2014)]

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- the **master formula** for mixed QCD-EW corrections is:

$$d\sigma^{(1,1)} = \mathcal{H}^{(1,1)} \otimes d\sigma_{LO} + [d\sigma^{(1,1)}_{real} - d\sigma^{(1,1)}_{ctrm}]_{q_t/Q > r_{cut}}$$

q_t = transverse momentum of the dilepton system
Q = invariant mass of the dilepton system

where the superscript (m, n) stands for the contribution of order $\alpha_s^m \alpha^n$.

q_T-subtraction in a nutshell

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- **double real + real-virt** contributions subtracted with CS dipole subtraction and evaluated above the slicing cut
- **finite** contribution in 4-dim
- sensitivity to **logarithms** of the cutoff parameter r_{cut} (associated to double unresolved limits)

q_T-subtraction in a nutshell

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- **non-local counterterm** (born-like kinematics) evaluated above the slicing cut
- it is derived by considering the IR limits of the real matrix element
- completely known from the NNLO computation of heavy-quark production [Catani, Devoto, Grazzini, Kallweit, Mazzitelli,Sargsyan (2019)]
- it contains the **logarithms** of the **cutoff parameter** r_{cut} that cancel the ones from the resolved real contribution

power corrections remain: we need small values of the slicing cutoff !

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where the superscript (m, n) stands for the contribution of order $\alpha_s^m \alpha^n$.

- **hard-collinear coefficient** living at $q_T = 0$
- in order to expose the *irreducible* virtual contribution, we introduce the following decomposition

$$\mathscr{H}^{(1,1)} = H^{(1,1)}\delta(1 - z_1)\delta(1 - z_2) + \delta\mathscr{H}^{(1,1)}(z_1, z_2)$$

where $$H^{(1,1)} = \frac{2\Re(\mathscr{M}^{(1,1)}_{fin}\mathscr{M}^{(0,0)*})}{|\mathscr{M}^{(0,0)}|^2}$$

- it contains the genuine **two-loop virtual contribution** plus finite reminders that restore unitarity

bottleneck: only missing ingredient!

Hard-virtual coefficient: Pole Approximation

solution: approximate the genuine two-loop virtual contribution by exploiting the formalism of **Pole Approximation (PA)**

We applied it at NLO EW order (as a validation of our procedure) and at mixed QCD-EW order (our final goal).

Remarks: at variance with the computation carried out in [Dittmaier, Huss and Schwinn (2015)]

- we use PA **only for the (double virtual)-(tree level) interference**
- we include all factorisable and non-factorisable contributions, in order to reproduce the **correct structure of IR singularities**

General features of the Pole Approximation (PA)

PA can be applied to processes that contain one or more resonances. For simplicity, we would concentrate on processes involving a single resonance structure (Z/W intermediate bosons) in order to explain how PA works.

- PA is a **systematic expansion** around the resonance pole wrt the parameter Γ_V/M_V . Or in other words, only the parts of a scattering amplitude enhanced by a resonance factor are taken into account:

$$\mathcal{M} = \boxed{\frac{W(p_V^2)}{p_V^2 - M_V^2 + \Sigma(p_V^2)}} + \boxed{N(p_V^2)}$$

non-resonant contribution neglected !

factorisable + non-factorisable corrections

- The improvements wrt NWA are the following:

1. the **dominant logarithmic terms** in Γ_V/M_V are kept (since the propagator is always evaluated in the off-shell kinematics);

2. the **structure of the IR poles** resembles the one of the exact computation.

- The prescription to compute the leading contribution of this expansion is well defined and requires the construction of an **on-shell projection** of the off-shell kinematics.

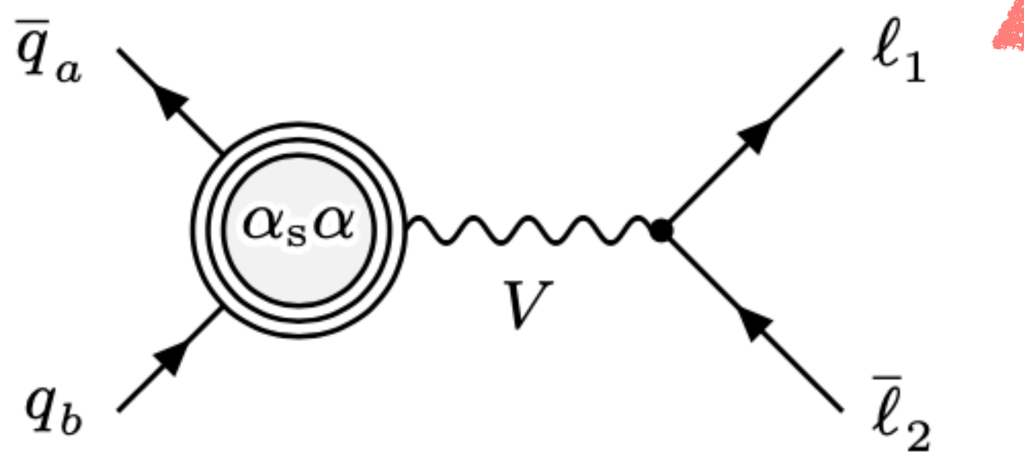
Hard-virtual coefficient: Pole Approximation

Application of PA to the mixed QCD-EW hard-virtual coefficient

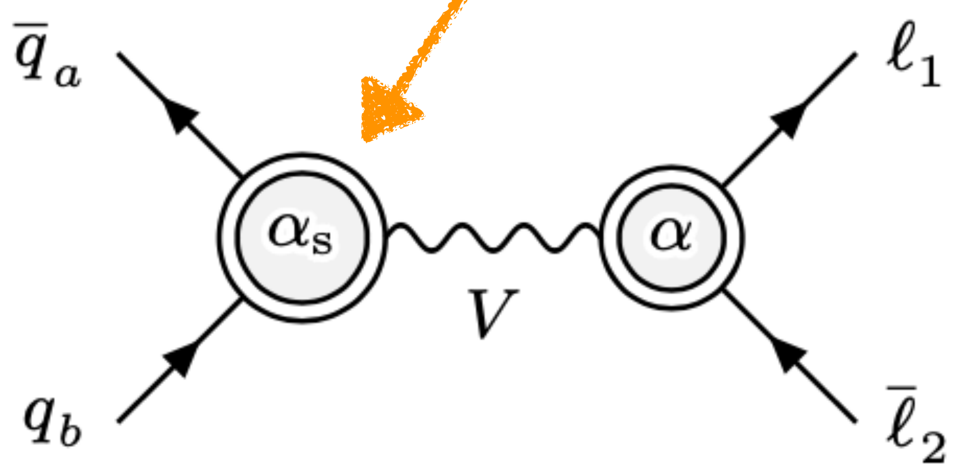
- the (already introduced) mixed two-loop contribution is: $H^{(1,1)}_{PA} = \dfrac{2\Re(\mathcal{M}^{(1,1)}_{fin}\mathcal{M}^{(0,0)*})_{PA}}{|\mathcal{M}^{(0,0)}|^2}$ with $\mathcal{M}^{(1,1)}_{fin,PA} = \boxed{\mathcal{M}^{(1,1)}_{fact}} + \boxed{\mathcal{M}^{(1,1)}_{nfact}}$

- for the construction of the **factorisable** corrections we need the following contributions:

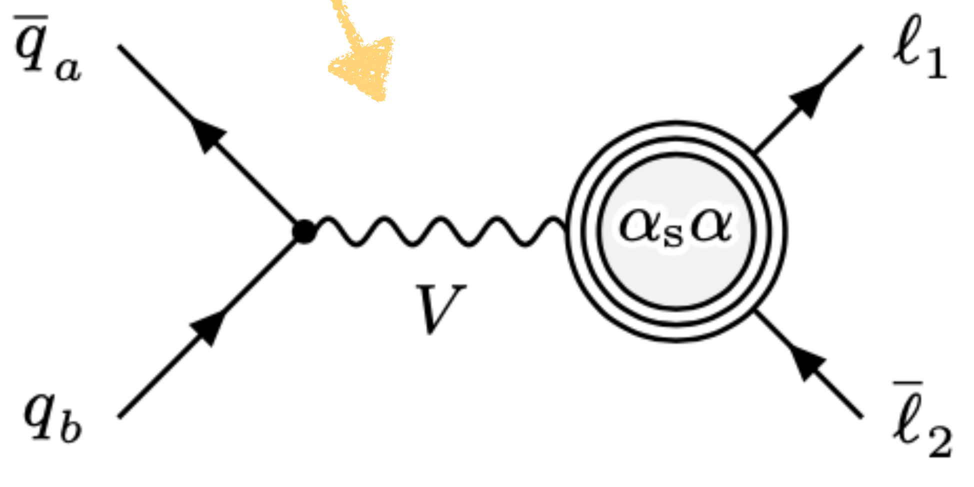
$$\boxed{\mathcal{M}^{(1,1)}_{fact}} = \sum_{\lambda_W} \frac{\boxed{\mathcal{M}^{(1,1)}_{prod}}(\lambda_W)\,\mathcal{M}^{(0)}_{dec}(\lambda_W) + \boxed{\mathcal{M}^{(1,0)}_{prod}(\lambda_W)\,\mathcal{M}^{(0,1)}_{dec}(\lambda_W)} + \mathcal{M}^{(0)}_{prod}(\lambda_W)\,\boxed{\mathcal{M}^{(1,1)}_{dec}}(\lambda_W)}{p_W^2 - \mu_W^2}$$



[Behring, Buccioni, Caola, Delto, Jaquier, Melnikov, Röntsch (2020)]



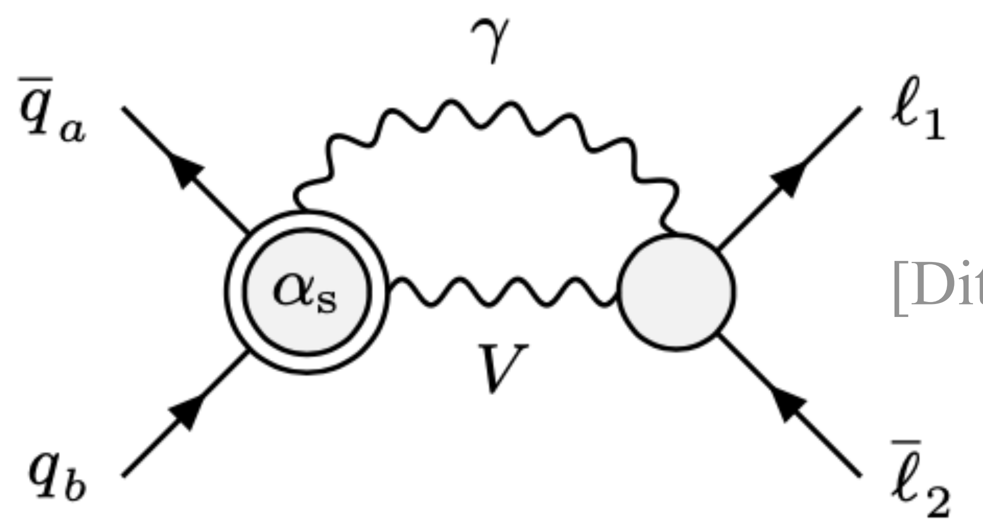
computed using *Recola*



[Dittmaier, Huss, Schwinn (2015)]

- for the construction of the **non-factorisable** corrections we need:

$$\boxed{\mathcal{M}^{(1,1)}_{nfact}} = \boxed{\delta^{(0,1)}_{nfact}\delta^{(1,0)}}\,\mathcal{M}^{(0,0)}_{PA}$$



[Dittmaier, Huss, Schwinn (2014)]

Hard-virtual coefficient: Pole Approximation

Re-weighting procedure

$$H_{PA}^{(m,n)} = \frac{2\Re(\mathcal{M}_{fin}^{(m,n)}\mathcal{M}^{(0,0)*})_{PA}}{|\mathcal{M}^{(0,0)}|^2}\,, \qquad \text{for m = 0, 1 and n = 1}$$

here the couplings are included in the definition of $H^{(m,n)}$

Remark: in the q_T -subtraction formalism, the hard virtual term is finally multiplied by the born matrix element

- in order to improve our approximation, we considered **alternative definitions**:
  - at *NLO EW* (m = 0 and n = 1)

$$H_{PA,rwg}^{(0,1)} = \frac{2\Re(\mathcal{M}_{fin}^{(0,1)}\mathcal{M}^{(0,0)*})_{PA}}{|\mathcal{M}_{PA}^{(0,0)}|^2} = H_{PA}^{(0,1)}\boxed{\frac{|\mathcal{M}^{(0,0)}|^2}{|\mathcal{M}_{PA}^{(0,0)}|^2}}$$

  - at *mixed QCD-EW* (m = n = 1)

$$H_{PA,rwg_B}^{(1,1)} = \frac{2\Re(\mathcal{M}_{fin}^{(1,1)}\mathcal{M}^{(0,0)*})_{PA}}{|\mathcal{M}_{PA}^{(0,0)}|^2} = H_{PA}^{(1,1)}\boxed{\frac{|\mathcal{M}^{(0,0)}|^2}{|\mathcal{M}_{PA}^{(0,0)}|^2}}$$

$$H_{PA,rwg_V}^{(1,1)} = H_{PA}^{(1,1)}\frac{H^{(0,1)}}{H_{PA}^{(0,1)}} = H_{PA}^{(1,1)}\boxed{\frac{2\Re(\mathcal{M}_{fin}^{(0,1)}\mathcal{M}^{(0,0)*})}{2\Re(\mathcal{M}_{fin}^{(0,1)}\mathcal{M}^{(0,0)*})_{PA}}}$$

effective re-weight of the virtual with a ratio between the exact born and the born in PA

effective re-weight of the virtual with a ratio between the exact NLO EW and the NLO EW in PA

Implementation and setup

We consider the charged current DY process $pp \to \mu^+ \nu_\mu + X$ at the LHC with $\sqrt{s} = 14\,TeV$

Implementation

The implementation has been carried out in the MATRIX framework: [Grazzini, Kallweit, Wieseman (2017)]

- efficient **multi-channel** generator Munich
- interface with automated tools for the evaluation of tree-level and one-loop amplitudes (OpenLoops and Recola)
- automatic implementation of **dipole subtraction**
- implementation of q_T **-subtraction** as a slicing method

Setup

[Dittmaier, Huss, Schwinn (2015)]

- $G_F = 1.1663787 \times 10^{-5}\,GeV^{-2}$ (G_μ *scheme* to treat the EW input parameters)
- $M_{W,OS} = 80.385\,GeV$ $M_{Z,OS} = 91.1876\,GeV$ → unstable particles treated in the complex mass scheme (**CMS**)
- $\Gamma_{W,OS} = 2.085\,GeV$ $\Gamma_{Z,OS} = 2.4952\,GeV$
- $m_\mu = 105.658369\,MeV$ → no lepton-photon recombination: **bare muons**
- $m_t = 173.3\,GeV$ $m_H = 125.9\,GeV$

- factorisation and renormalisation scales: $\mu_F = \mu_R = M_{W,OS}$
- pdf set: NNPDF31_nnlo_as_0118_luxqed

- fiducial cuts: $p_{T,\mu} > 25\,GeV$ $|y_\mu| < 2.5$ $p_{T,\nu_\mu} > 25\,GeV$

Numerical results

[Validation of PA at mixed QCD-EW order]()

reminder:

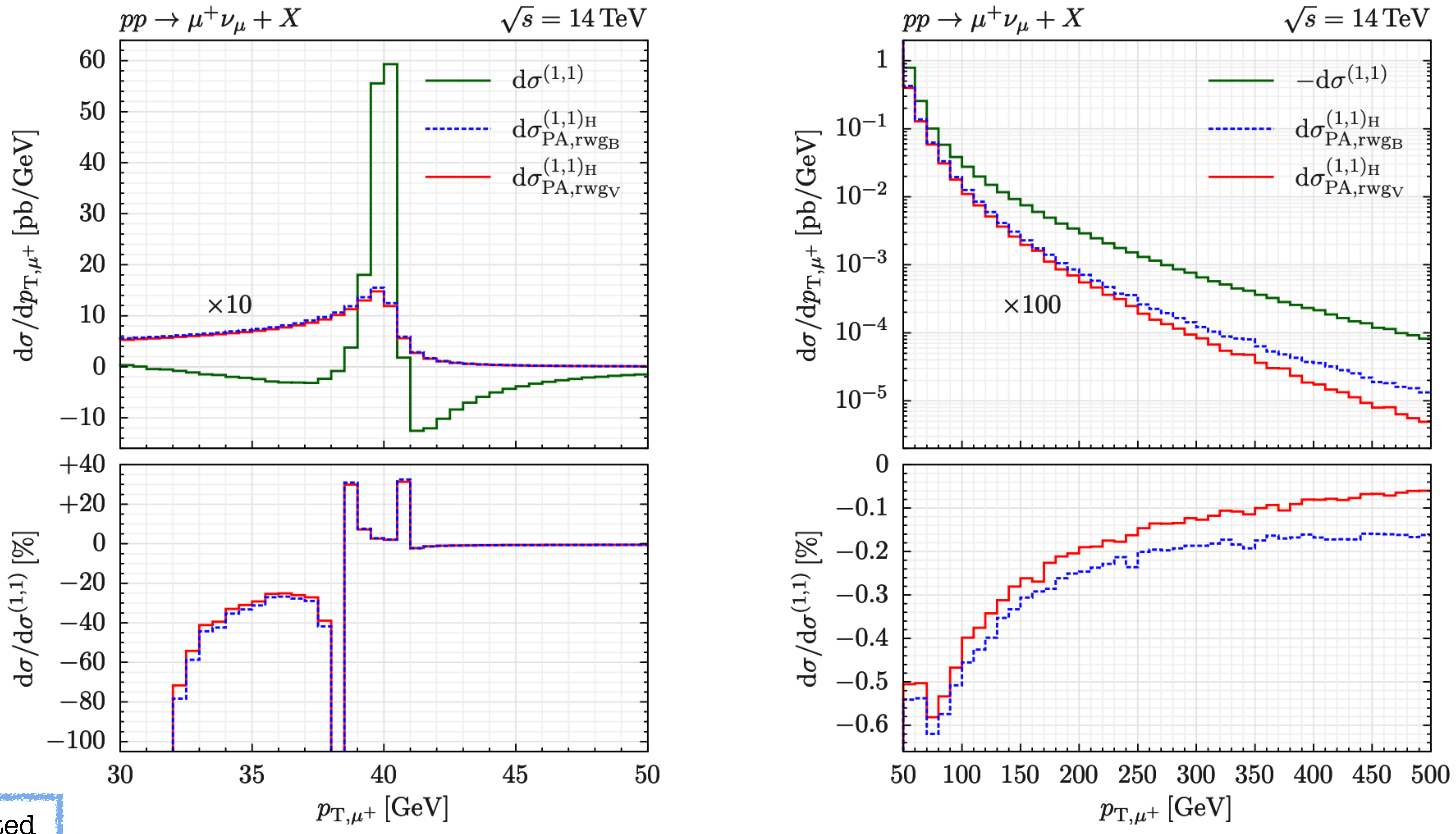
$$H^{(1,1)}_{PA,rwg_B} = \frac{2\Re(\mathcal{M}^{(1,1)}_{fin}\mathcal{M}^{(0,0)*})_{PA}}{|\mathcal{M}^{(0,0)}_{PA}|^2} = H^{(1,1)}_{PA}\frac{|\mathcal{M}^{(0,0)}|^2}{|\mathcal{M}^{(0,0)}_{PA}|^2}$$

$$H^{(1,1)}_{PA,rwg_V} = H^{(1,1)}_{PA}\frac{H^{(0,1)}}{H^{(0,1)}_{PA}}$$

take-home messages:

1. in the **peak region**, the two approximations are very close to each other
2. in the **high - p_T region**, the impact of the $H^{(1,1)}$ coefficient is smaller than 1% of the entire $\mathcal{O}(\alpha_s\alpha)$ correction
3. the contribution of $H^{(1,1)}$ becomes even smaller as p_T increases (**suppression of resonant born-like topologies**, dominant contribution from gq-channel)
4. softer spectrum of $H^{(1,1)}_{PA,rwg_V}$ at large p_T since **Sudakov logs** are **effectively included**

the **extent** to which our approximation is expected to work depends on the **impact** that $H^{(1,1)}$ has on the full mixed QCD-EW correction

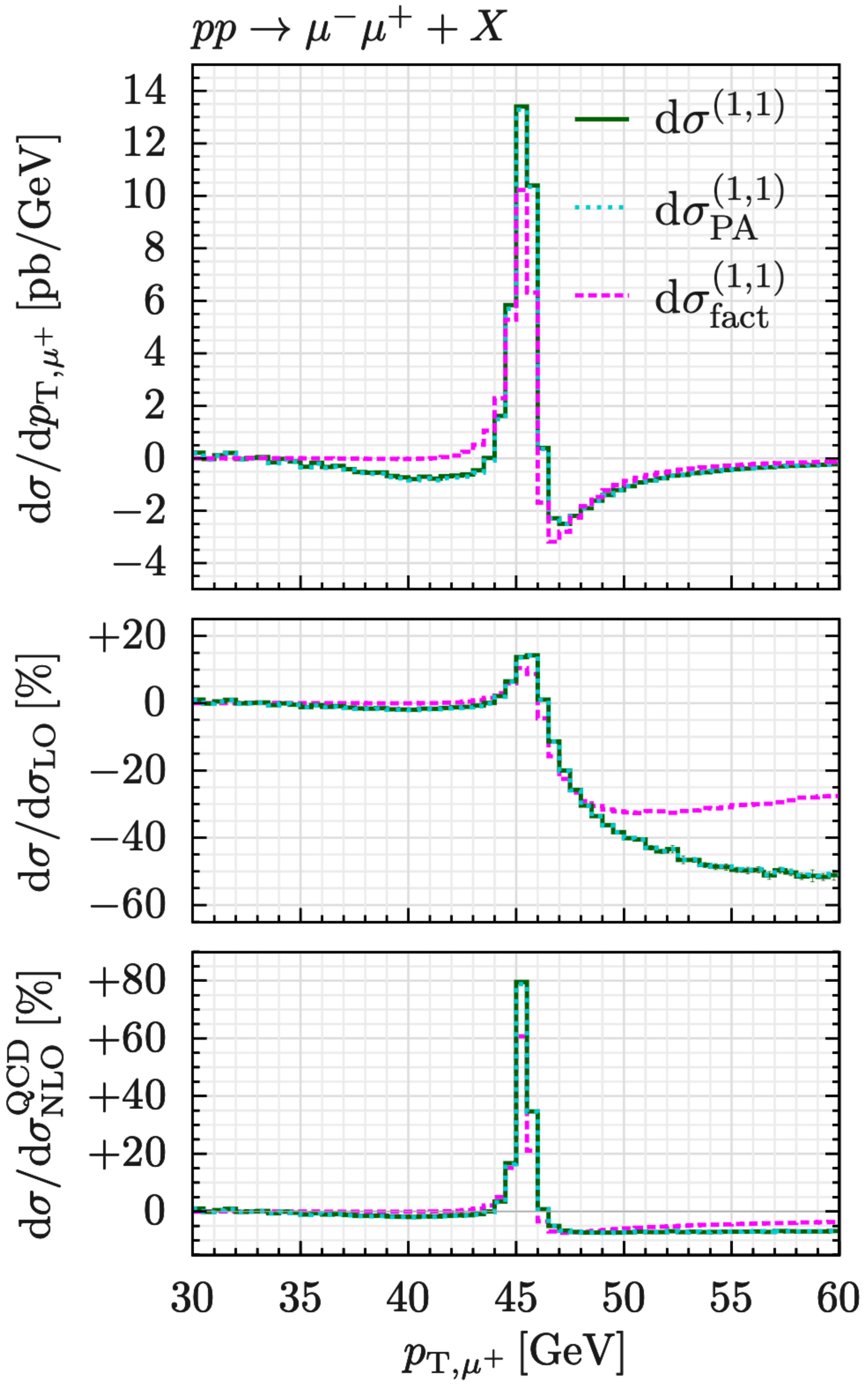


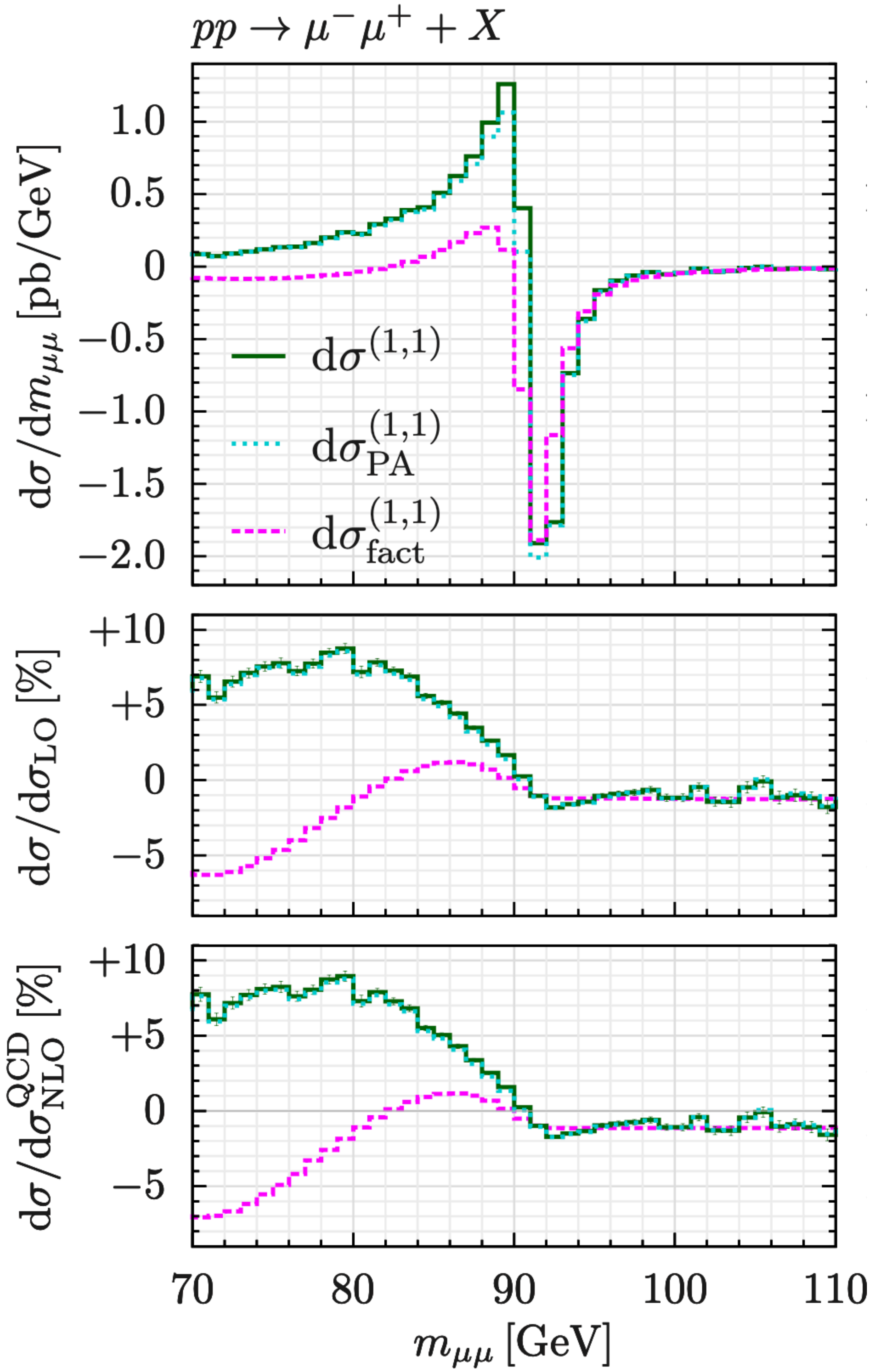
Numerical results

[Validation of PA at mixed QCD-EW order: neutral current (NC) case]()

[Bonciani, Buonocore, Grazzini, Kallweit, Rana, Tramontano, Vicini (2021)]

first **exact** fully differential computation





$p_{T,\mu} > 25\,GeV$

$|y_\mu| < 2.5$

$m_{\mu\mu} > 50\,GeV$

perfect agreement between exact and PA distributions

Numerical results

Fiducial results for the transverse momentum of the muon

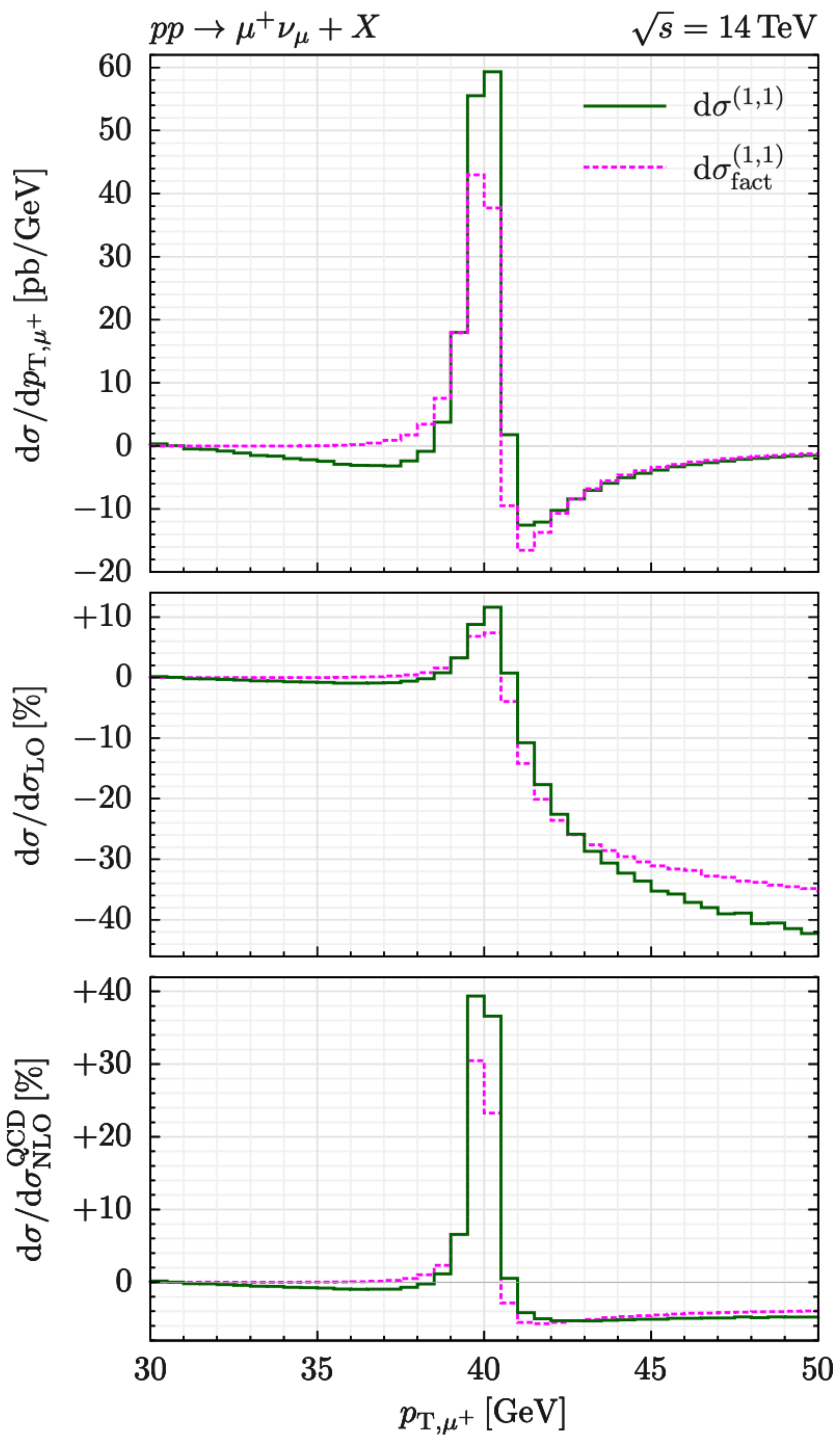
after the validation, we consider $H^{(1,1)}_{PA,rwg_V}$ as our best estimation of the exact (double virtual)-(tree level) interference

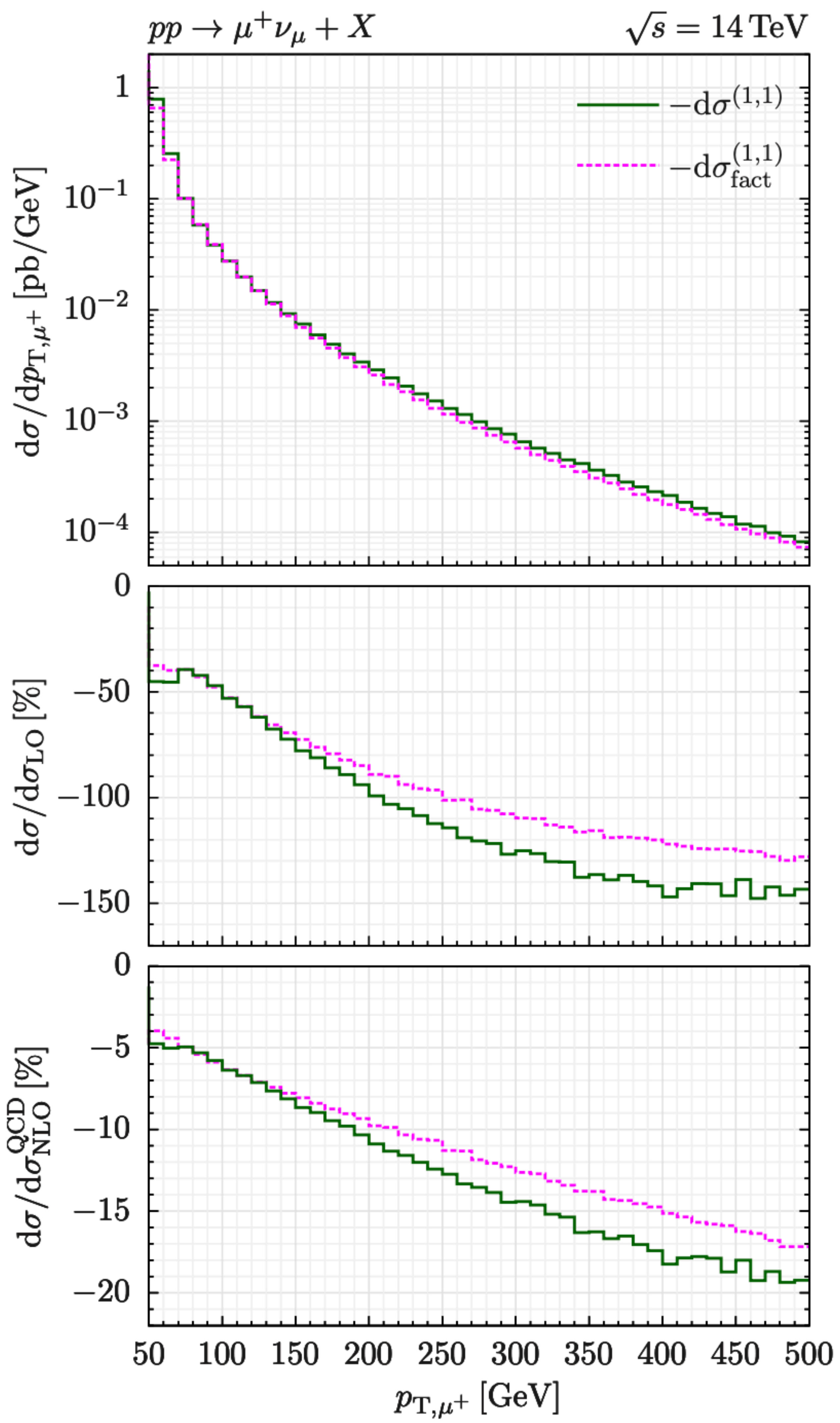
- Predictions are presented as:

1. absolute correction (upper panels)
2. correction normalised to the LO cross section (central panels)
3. correction normalised to the NLO QCD cross section (lower panels)

- They are compared with a naive factorised approach:

$$\frac{d\sigma^{(1,1)}_{fact}}{dp_T} = \left(\frac{d\sigma^{(1,0)}}{dp_T}\right) \times \left(\frac{d\sigma^{(0,1)}_{q\bar{q}}}{dp_T}\right) \times \left(\frac{d\sigma_{LO}}{dp_T}\right)^{-1}$$





take-home messages:

1. negative corrections in the tail of the p_T -spectum
2. the factorised approach decently reproduces the complete result both in the peak region an in the tail of the distribution

Conclusions

- We have presented a new computation of the **mixed QCD-EW** corrections to the $2 \to 2$ **charged DY process** with massive charged lepton.
- **All contributions** are computed **exactly** except for the finite part of the **two-loop amplitude**. To overcome this issue, we have exploited the **pole approximation** technique via a suitable **re-weighting** procedure.
- The cancellation of IR singularities is achieved with an extension of q_T **- subtraction** (abelianisation procedure of the NNLO QCD results for heavy quarks).
- We have shown first results for the **muon transverse momentum** distribution (peak and tail regions).
- The reliability of our approximation has been confirmed by a subsequent work [Bonciani, Buonocore, Grazzini, Kallweit, Rana, Tramontano, Vicini (2021)] on the neutral DY process, for which the exact two-loop amplitudes are now available.

work in progress on the computation of the exact two-loop virtual amplitudes for the charged DY process and more phenomenological studies to come!
STAY TUNED!

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Thank you for the attention!

Backup slides

Abelianisation procedure

- *Abelianisation* refers to a procedure aiming to determine the abelian subset of the QCD computation, in order to extract the results for the EW/QED case.
- In our case, it was applied in order to find the q_T -subtraction formula for the mixed QCD-EW corrections, starting from the well-known results at NNLO QCD order for heavy quarks.
- The main idea is to consider all QCD Feynman diagrams with two gluon emissions and replace (when it is possible) a gluon with a photon. This allows to find proper replacement rules. For processes initiated by *same flavour quark-antiquark collisions*, the replacement rules are the following ones:

$$\boxed{C_A \to 0 \qquad T_R \to 0 \qquad C_F^2 \to 2C_F e_f^2}$$

where e_f is the electric charge of the quark (antiquark).

- In the W-boson case, we have to take into account that the IS partons have different charges: $\boxed{e_f^2 \to \dfrac{e_u^2 + e_d^2}{2}}$
- By so doing, it is possible to convert pure NNLO QCD contributions of type $X_1^{(1,0)} \times X_2^{(1,0)}$ or $X^{(2,0)}$ (where X, X_1, X_2 stand for any resummation coefficient, collinear function, Altarelli-Parisi kernel or hard-virtual coefficient function in the q_T -subtraction formula) into mixed QCD-EW ones:
  - $X_1^{(1,0)} \times X_2^{(1,0)} \to X_1^{(1,0)} \times X_2^{(0,1)} + X_1^{(0,1)} \times X_2^{(1,0)}$
  - $X^{(2,0)} \to X^{(1,1)}$

IR structure and finite amplitudes

$$\Gamma_t = -\frac{1}{4}\left\{ e_\ell^2(1-i\pi) + \sum_{i=1,2} e_i e_3 \ln\frac{(2p_i\cdot p_3)^2}{Q^2 m_\ell^2} \right\}$$

UV renormalised but still IR divergent

$$\mathcal{M}^{(1,0)}_{\rm fin} = \boxed{\mathcal{M}^{(1,0)}} + \frac{1}{2}\left(\frac{\alpha_s}{\pi}\right) C_F \left[\frac{1}{\epsilon^2} + \left(\frac{3}{2}+i\pi\right)\frac{1}{\epsilon} - \frac{\pi^2}{12}\right]\mathcal{M}^{(0)}$$

$$\mathcal{M}^{(0,1)}_{\rm fin} = \boxed{\mathcal{M}^{(0,1)}} + \frac{1}{2}\left(\frac{\alpha}{\pi}\right)\left\{\left[\frac{1}{\epsilon^2} + \left(\frac{3}{2}+i\pi\right)\frac{1}{\epsilon} - \frac{\pi^2}{12}\right]\frac{e_u^2+e_d^2}{2} - \frac{2\Gamma_t}{\epsilon}\right\}\mathcal{M}^{(0)}$$

$$\begin{aligned}\mathcal{M}^{(1,1)}_{\rm fin} = \boxed{\mathcal{M}^{(1,1)}} &- \left(\frac{\alpha_s}{\pi}\right)\left(\frac{\alpha}{\pi}\right)\left\{\frac{1}{8\epsilon^4}(e_u^2+e_d^2)C_F + \frac{1}{2\epsilon^3}C_F\left[\left(\frac{3}{2}+i\pi\right)\frac{e_u^2+e_d^2}{2} - \Gamma_t\right]\right\}\mathcal{M}^{(0)} \\
&+\frac{1}{2\epsilon^2}\left\{\left(\frac{\alpha}{\pi}\right)\frac{e_u^2+e_d^2}{2}\mathcal{M}^{(1,0)}_{\rm fin} + C_F\left(\frac{\alpha_s}{\pi}\right)\mathcal{M}^{(0,1)}_{\rm fin}\right. \\
&\qquad \left. + C_F\left(\frac{\alpha_s}{\pi}\right)\left(\frac{\alpha}{\pi}\right)\left[\left(\frac{7}{12}\pi^2 - \frac{9}{8} - \frac{3}{2}i\pi\right)\frac{e_u^2+e_d^2}{2} + \left(\frac{3}{2}+i\pi\right)\Gamma_t\right]\mathcal{M}^{(0)}\right\} \\
&+\frac{1}{2\epsilon}\left\{\left(\frac{\alpha}{\pi}\right)\left[\left(\frac{3}{2}+i\pi\right)\frac{e_u^2+e_d^2}{2} - 2\Gamma_t\right]\mathcal{M}^{(1,0)}_{\rm fin} + \left(\frac{\alpha_s}{\pi}\right)C_F\left[\frac{3}{2}+i\pi\right]\mathcal{M}^{(0,1)}_{\rm fin}\right. \\
&\qquad \left. + \frac{1}{8}C_F\left(\frac{\alpha_s}{\pi}\right)\left(\frac{\alpha}{\pi}\right)\left[\left(\frac{3}{2} - \pi^2 + 24\zeta(3) + \frac{2}{3}i\pi^3\right)\frac{e_u^2+e_d^2}{2} - \frac{2}{3}\pi^2\Gamma_t\right]\mathcal{M}^{(0)}\right\}\end{aligned}$$

finite remainders defined according to q_T- subtraction convention

Hard-virtual coefficient: Pole Approximation

Main contributions in PA

- The two main contributions are called **FACTORISABLE** and **NON-FACTORISABLE** corrections (both gauge-invariant):

$$\mathcal{M} = \boxed{\frac{W(\mu_V^2)}{p_V^2-\mu_V^2}\frac{1}{1+\frac{d}{dp_V^2}\Sigma(\mu_V^2)}} + \left[\frac{W(p_V^2)}{p_V^2-M_V^2+\Sigma(p_V^2)} - \frac{W(\mu_V^2)}{p_V^2-\mu_V^2}\frac{1}{1+\frac{d}{dp_V^2}\Sigma(\mu_V^2)}\right]$$

FACTORISABLE CORRECTIONS:

- they **regard IS and FS separately**.

The only connection between production and decay stages is the kinematic of the vector boson and its helicity state that is responsible for spin correlations:

$$\mathcal{M}_{fact} = \sum_{\lambda_V} \frac{\mathcal{M}^{virt}_{prod}(\lambda_V)\,\mathcal{M}^{born}_{decay}(\lambda_V) + \mathcal{M}^{virt}_{decay}(\lambda_V)\,\mathcal{M}^{born}_{prod}(\lambda_V)}{p_V^2-\mu_V^2}$$

where the sum is over the helicity states of the vector boson. The previous formula holds at fixed helicity configuration of the external particles.

- production and decay sub-amplitudes appearing on either side of the resonance propagator are evaluated with the **on-shell kinematics** of the vector boson while the squared momentum in the **propagator is kept exact**.

Hard-virtual coefficient: Pole Approximation

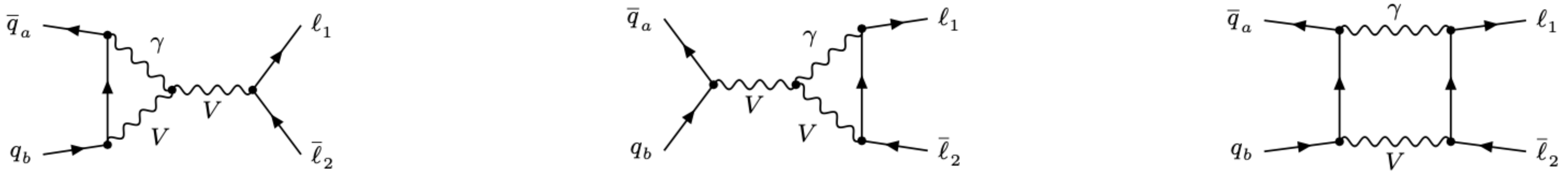
Main contributions in PA

- The two main contributions are called **FACTORISABLE** and **NON-FACTORISABLE** corrections (both gauge-invariant):

$$\mathcal{M} = \frac{W(\mu_V^2)}{p_V^2 - \mu_V^2} \frac{1}{1 + \frac{d}{dp_V^2}\Sigma(\mu_V^2)} + \boxed{\left[\frac{W(p_V^2)}{p_V^2 - M_V^2 + \Sigma(p_V^2)} - \frac{W(\mu_V^2)}{p_V^2 - \mu_V^2} \frac{1}{1 + \frac{d}{dp_V^2}\Sigma(\mu_V^2)}\right]}$$

NON-FACTORISABLE CORRECTIONS:

- they involve contributions in which production and decay subprocesses do not proceed independently (**box diagrams**, triangle diagrams connecting the external legs with the resonance propagator etc.)



- they are formally defined as the difference between the full contribution of a diagram (in which the on-shell expansion is performed after the loop-integration) and the factorisable contribution (computed by setting $p_V^2 = M_V^2$ at the integrand level). They are not vanishing only if the two steps of loop integration and on-shell limit do not commute.

- they produce a resonance factor only if a **soft photon** is exchanged. Since they are of soft origin, they can always be written as:

$$\mathcal{M}_{nfact} = \boxed{\delta_{nfact}} \mathcal{M}_{PA}^{born}$$

form factor containing the poles plus the finite reminders

Hard-virtual coefficient: on-shell projection

case of W-boson: one massive charged lepton and a massless neutrino

The Mandelstam invariants are rescaled in order to ensure the on-shell condition for the vector boson :

$$s \to \frac{M_W^2}{s} s = M_W^2 \equiv \hat{s}_m \qquad t \to \frac{M_W^2}{s}(t - m_l^2/2) + m_l^2/2 \equiv \hat{t}_m \qquad u \to \frac{M_W^2}{s}(u - m_l^2/2) + m_l^2/2 \equiv \hat{u}_m$$

The on-shell kinematics of the process $\bar{q}_a(\hat{p}_1) + q_b(\hat{p}_2) \to W^- \to l^-(\hat{p}_3) + \bar{\nu}_l(\hat{p}_4)$ is given by :

convention: all momenta OUTGOING

$$\hat{p}_1 = \frac{M_W}{2}(-1,0,0,-1)$$

$$\hat{p}_2 = \frac{M_W}{2}(-1,0,0,1)$$

$$\hat{p}_3 = \frac{m_l^2 + M_W^2}{2M_W}(1, \beta\sqrt{1 - cos^2\theta}\, cos\phi, \beta\sqrt{1 - cos^2\theta}\, sen\phi, \beta cos\theta)$$

$$\hat{p}_4 = \frac{m_l^2 + M_W^2}{2M_W}(\beta, -\beta\sqrt{1 - cos^2\theta}\, cos\phi, -\beta\sqrt{1 - cos^2\theta}\, sen\phi, -\beta cos\theta)$$

with $\beta = \frac{M_W^2 - m_l^2}{m_l^2 + M_W^2}$ and $\beta cos\theta = 1 + (\hat{t}_m - m_l^2)\frac{2}{m_l^2 + M_W^2}$

Numerical results

Validation of PA at NLO EW order

reminder:

$$H_{PA}^{(0,1)} = \frac{2\Re(\mathscr{M}_{fin}^{(0,1)}\mathscr{M}^{(0,0)*})_{PA}}{|\mathscr{M}^{(0,0)}|^2}$$

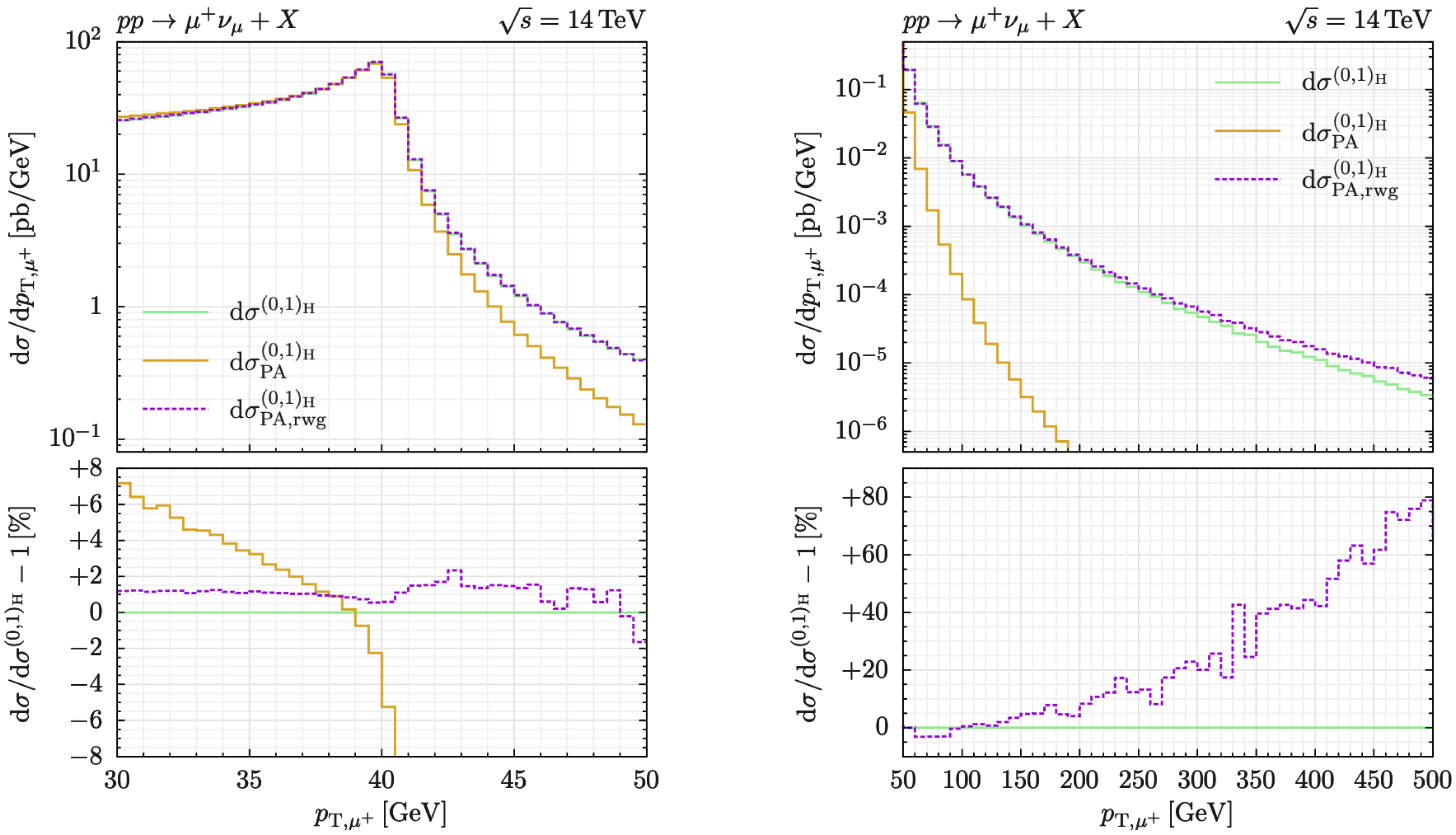
$$H_{PA,rwg}^{(0,1)} = \frac{2\Re(\mathscr{M}_{fin}^{(0,1)}\mathscr{M}^{(0,0)*})_{PA}}{|\mathscr{M}^{(0,0)}|_{PA}^2}$$

take-home messages:

1. the coefficients, approximated in PA with and without re-weighting, **agree** in the region **around the peak**

2. the **re-weighting** procedure allows us to model the exact behaviour at the **percent level** both below and above the peak

3. in the tail of the p_T - distribution the exact hard-virtual contribution is systematically overestimated due to the **lack of Sudakov logarithms**



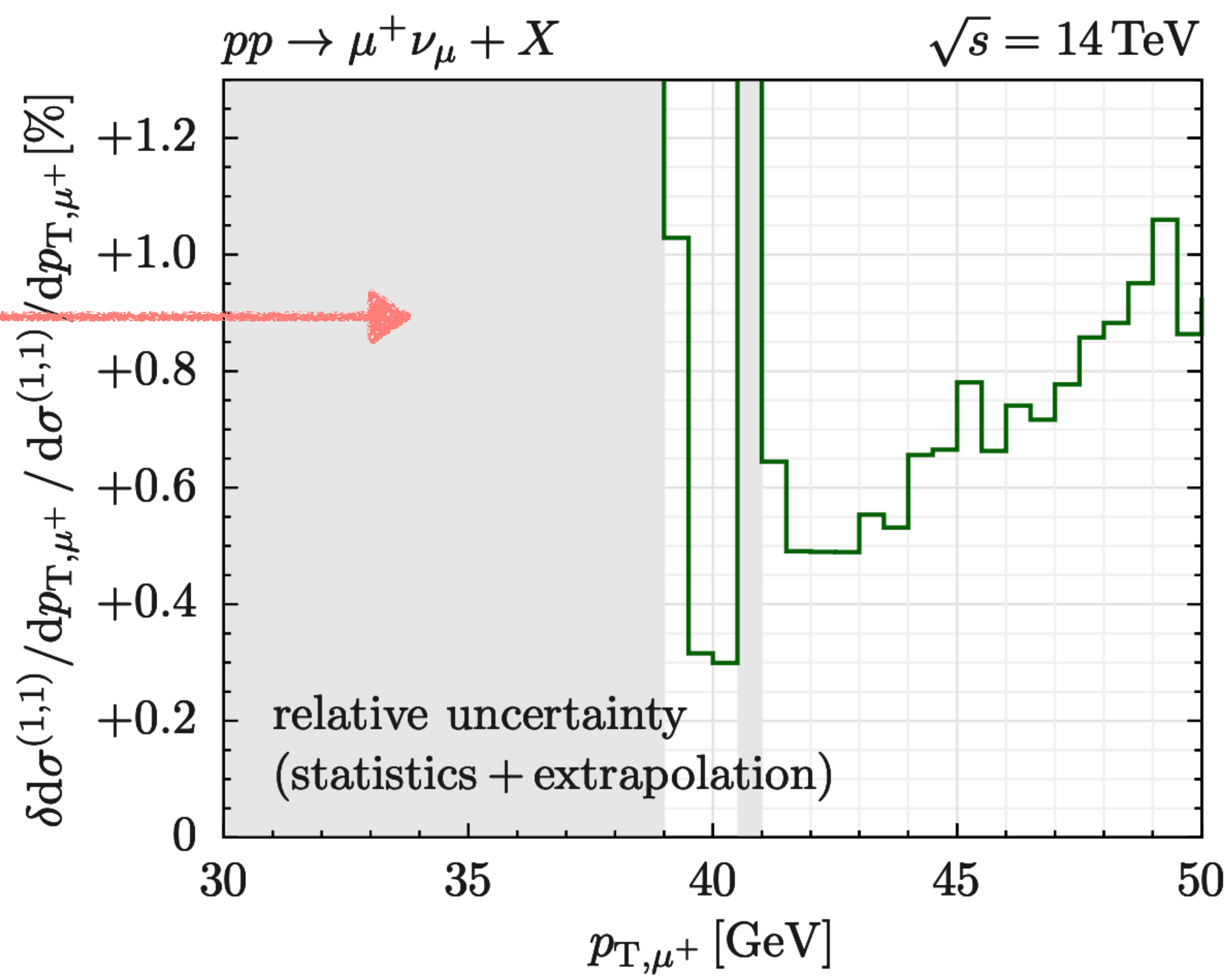
Numerical results: fiducial cross sections

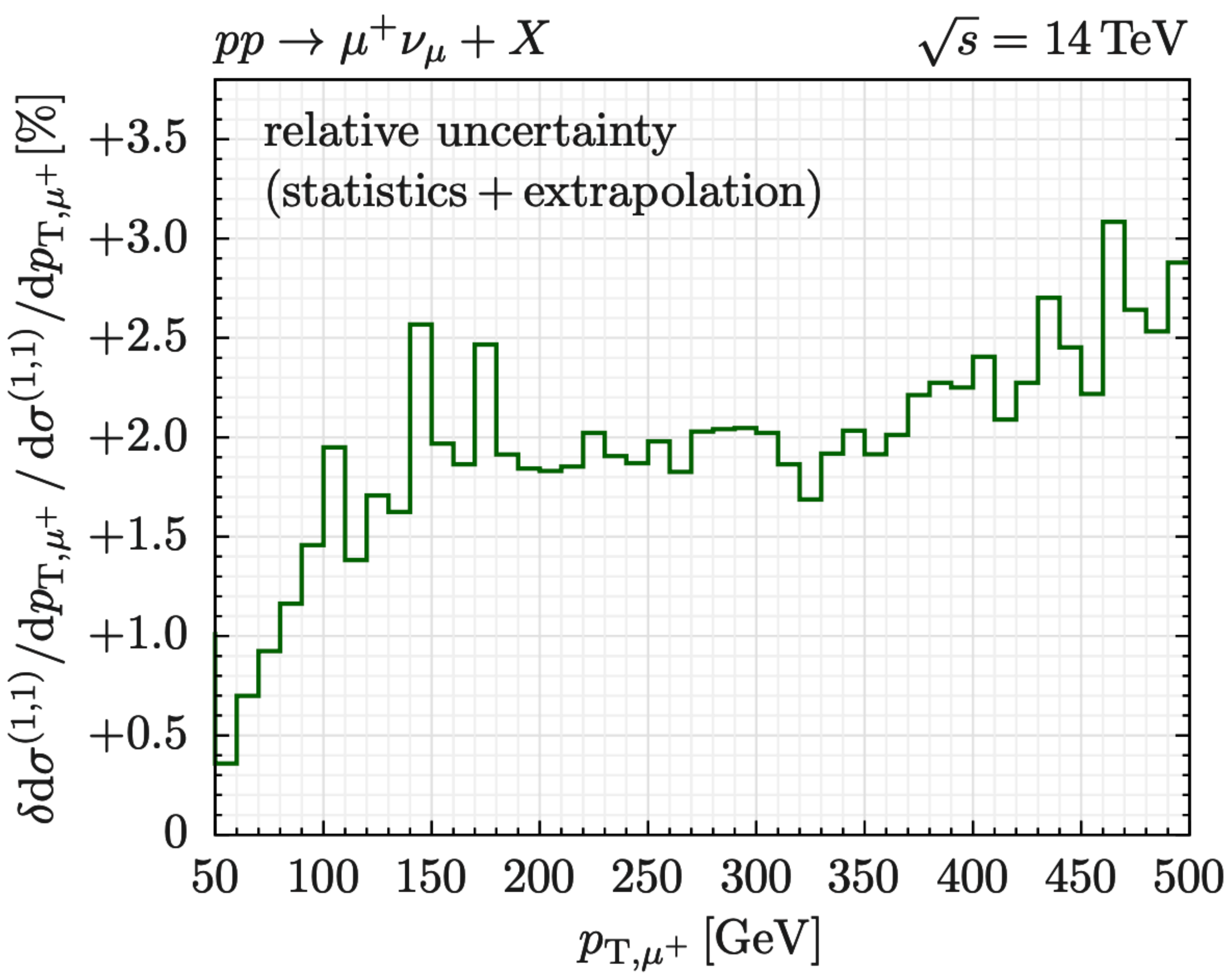
σ [pb]	$\sigma_{\rm LO}$	$\sigma^{(1,0)}$	$\sigma^{(0,1)}$	$\sigma^{(2,0)}$	$\sigma^{(1,1)}$
$q\bar{q}$	5029.2	970.5(3)	−143.61(15)	251(4)	− 7.0(1.2)
qg	—	−1079.86(12)	—	−377(3)	39.0(4)
$q(g)\gamma$	—	—	2.823(1)	—	0.055(5)
$q(\bar{q})q'$	—	—	—	44.2(7)	1.2382(3)
gg	—	—	—	100.8(8)	—
tot	5029.2	−109.4(4)	−140.8(2)	19(5)	33.3(1.3)
$\sigma^{(m,n)}/\sigma_{\rm LO}$		−2.2 %	−2.8 %	+0.4 %	+0.6 %

- NLO and NNLO QCD corrections to the individual channels are responsible for a *large cancellation* when the channels are summed up (especially between $q\bar{q}$ and qg).
- NLO QCD and NLO EW corrections are of the *same order.*
- Mixed QCD-EW corrections are of the same order as the NNLO QCD ones and they are dominated by the qg channel.
- This pattern is sensitive to the choice of the factorisation and renormalisation scales ($\mu_F = \mu_R = M_W$).

Numerical results: systematic uncertainties

shaded regions: the mixed correction is almost vanishing and the corresponding uncertainty is meaningless





- q_T -subtraction is implemented as a *slicing method*: power corrections in the slicing cut-off $r_{cut} = \dfrac{q_{T,cut}}{Q}$ remain.
- Our best prediction is obtained in the limit $r_{cut} \to 0$, via an *extrapolation procedure* applied both at the level of the total cross section and at the level of individual bins of kinematical distributions. The distribution is calculated at fixed $r_{cut} \in [0.01\,\%\,, r_{max}]$ and a quadratic least χ^2 fit is performed for different $r_{max} \in [0.5\,\%\,, 1\%]$.
- The best prediction corresponds to the minimum χ^2/d.o.f and the corresponding uncertainty is estimated by comparing the results obtained through the different fits.
- Systematic and statistical uncertainties are combined. We can argue that we have a rather good control of the systematic (below few percents) both in the peak and the the tail regions.