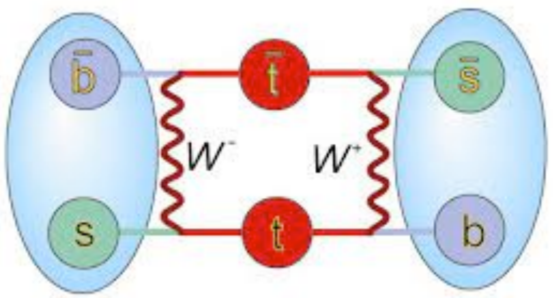
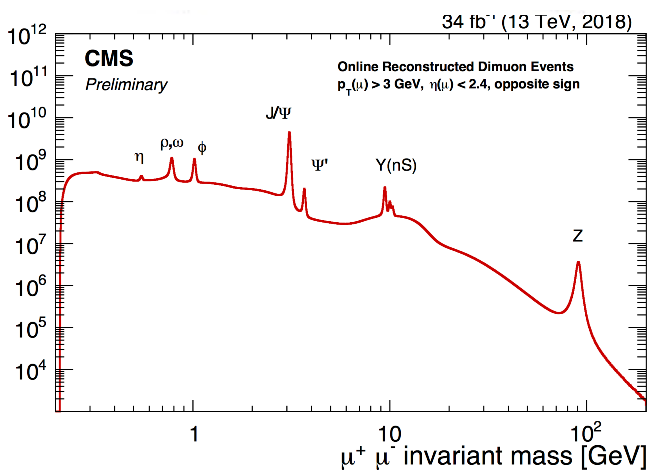
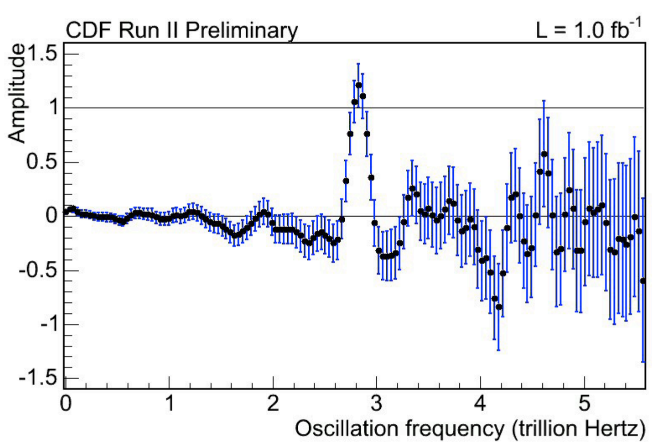
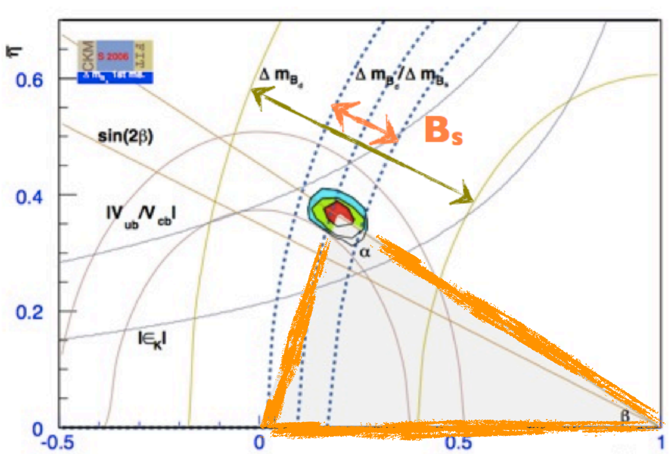




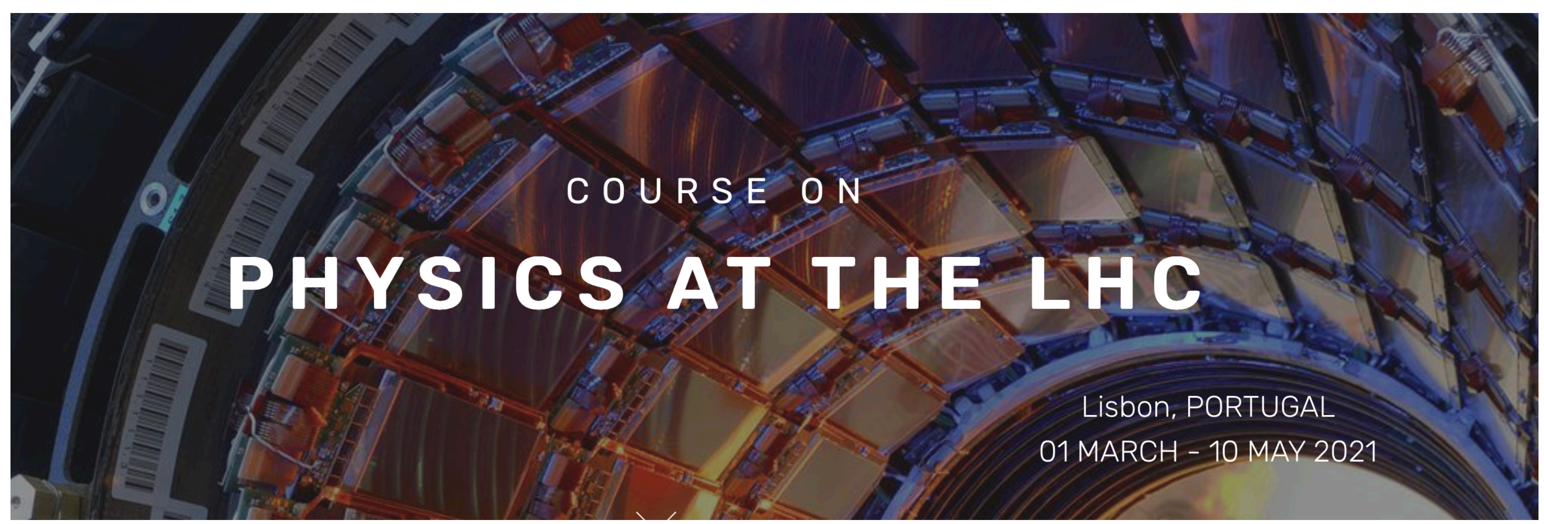




Lecture on

(heavy) flavor physics

Nuno Leonardo
LIP & IST, nuno@cern.ch



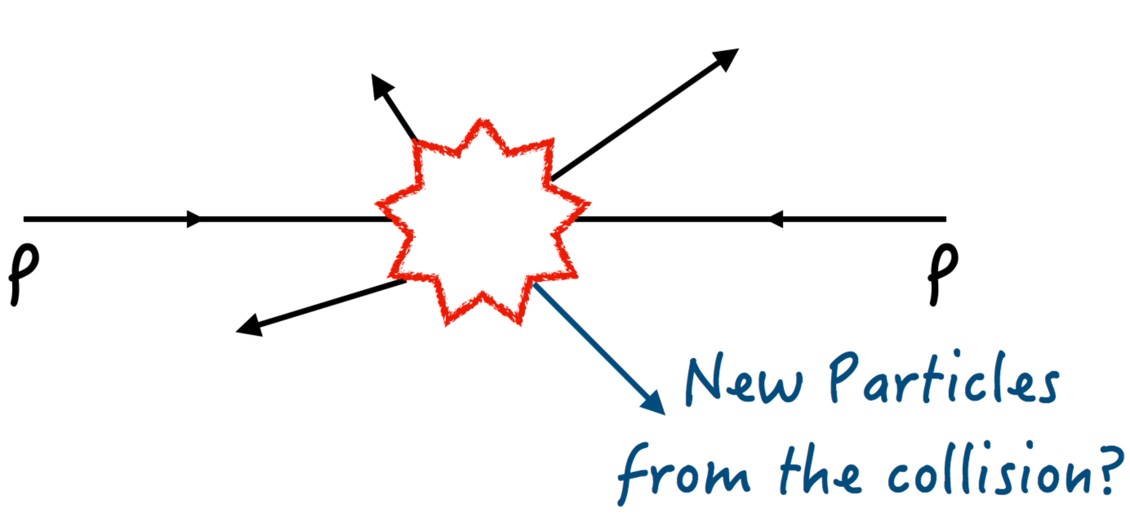
LABORATÓRIO DE INSTRUMENTAÇÃO E FÍSICA EXPERIMENTAL DE PARTICULAS

Fundação para a Ciência e a Tecnologia
MINISTÉRIO DA CIÊNCIA, TECNOLOGIA E ENSINO SUPERIOR

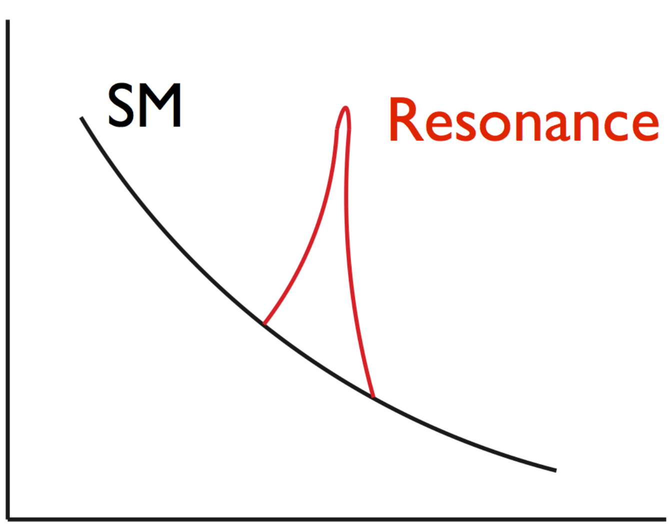
the role of flavor physics

- in searching for **New Physics**
  - discovery potential beyond energy frontier e.g. via searches for rare processes
- in understanding why the **SM** appears so fundamental
  - in that no phenomena beyond the SM has (yet) been detected at LHC
- in learning about standing mysteries of the **flavor structure** of SM (and BSM)
- in connecting **CP violation** to the matter-antimatter asymmetry in the observable universe
- in understanding **QCD**, and probing the properties of **deconfined** matter at high temperature and density
- extra: as an experimental tool & probe
  - serve as probe or a **dominant background** in SM measurements and BSM searches
  - used for **detector calibration** (e.g. material budget, magnetic field, detector performance)

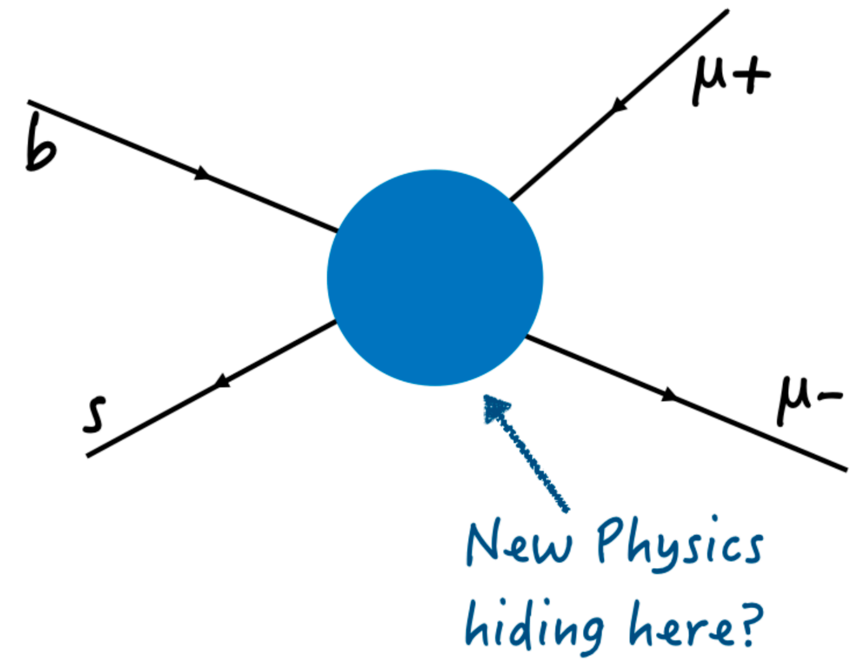
Direct search for NP



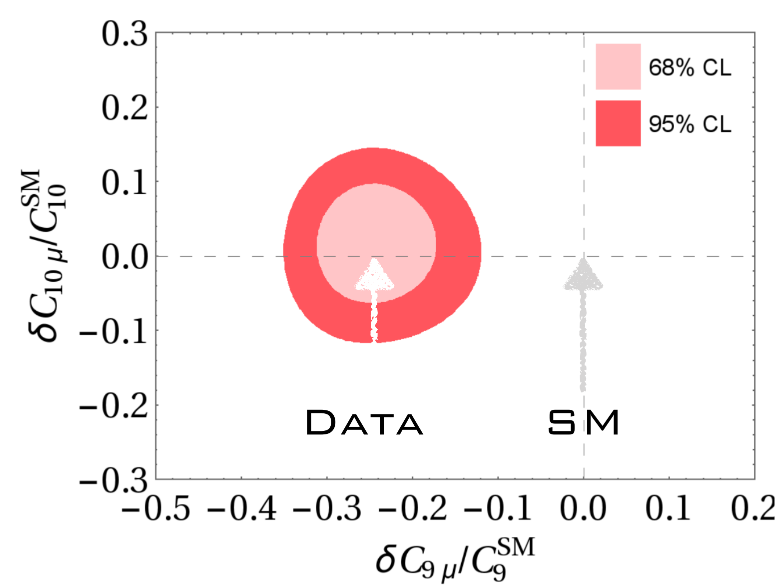
- searching for the decay products of NP particles produced in collision



Indirect search for NP



- searching for effects of NP particles running in quantum loops (virtual)



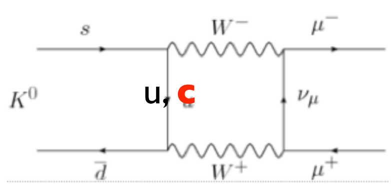
Flavour!

indirect discovery via precision

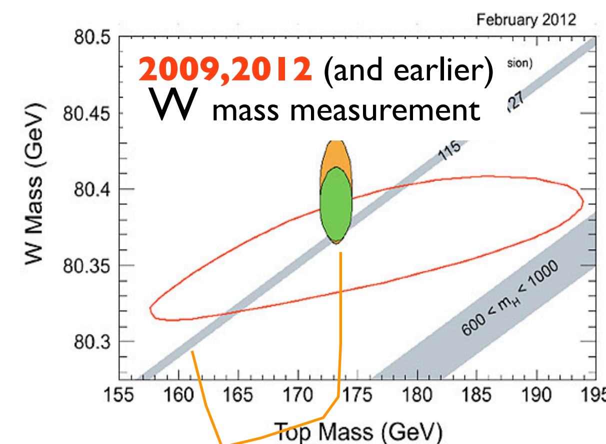
- new physics can show up at precision frontier before energy frontier
  - kaon (1947), Λ^0 (1950) led to discovery of strangeness
  - GIM mechanism (1970) before discovery of charm (1974)
  - CP violation (1964) before discovery of bottom (1977) & top (1995)
  - neutral current (1973) before discovery of Z (1983)
  - precision W and t mass meas. constrained Higgs mass, etc!

(note: quarks postulated 1964 [Gellman&Zweig], based on hadron classification ['eightfold way'], directly confirmed experimentally 1968 [DIS])

1970
GIM mechanism
(Glashow–Iliopoulos–Maiani)
required the fourth quark: **c**harm
e.g. $K^+ \to \mu^+\nu_\mu$ so why not $K^0 \to \mu^+\mu^-$?



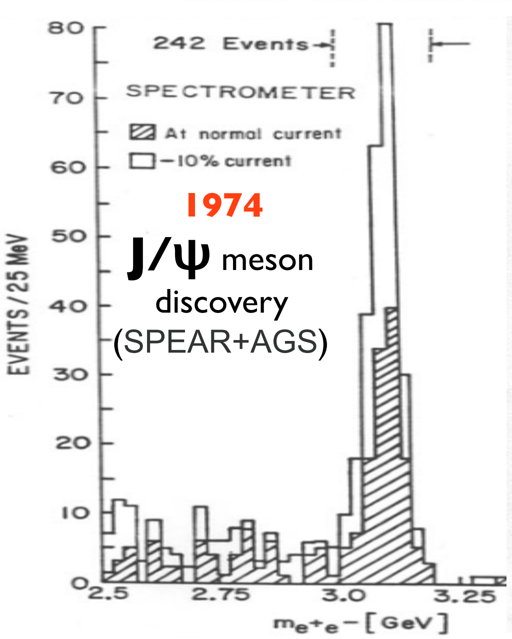
1973
3rd quark generation predicted
bottom and **t**op
by Kobayachi and Maskawa
to explain observed CP violation
(discovered in **1964** in the kaon system)

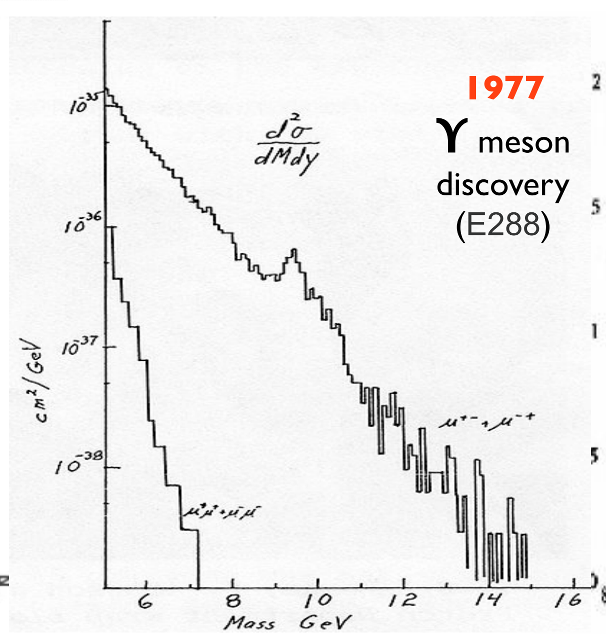


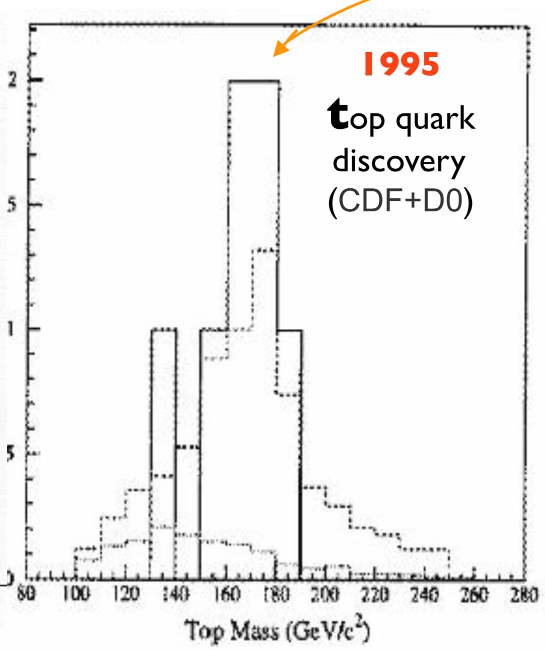
precision m_W and m_t
$\Rightarrow m_H < 145$GeV

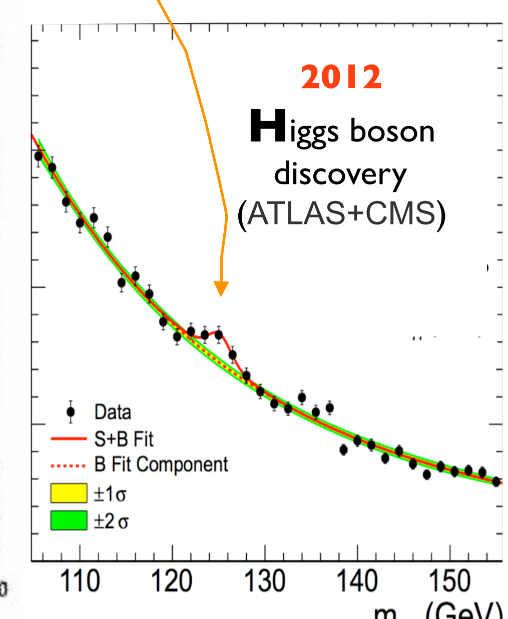
$\bar{\nu}_\mu e^- \to \bar{\nu}_\mu e^-$

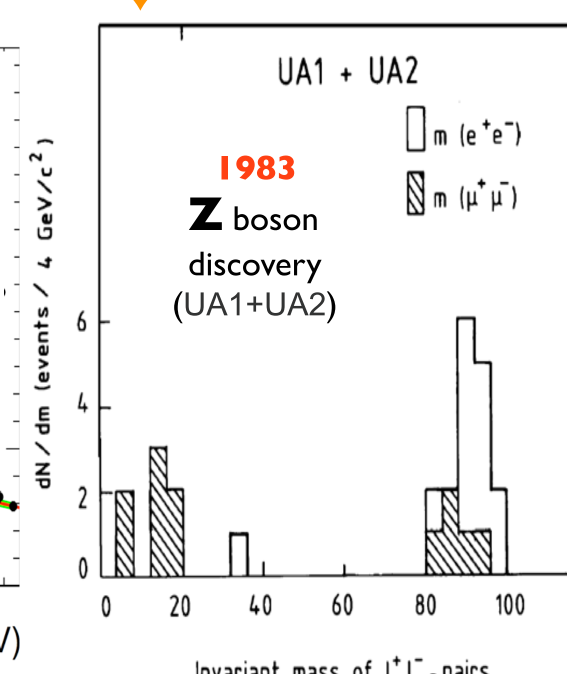
1973
neutral currents observed







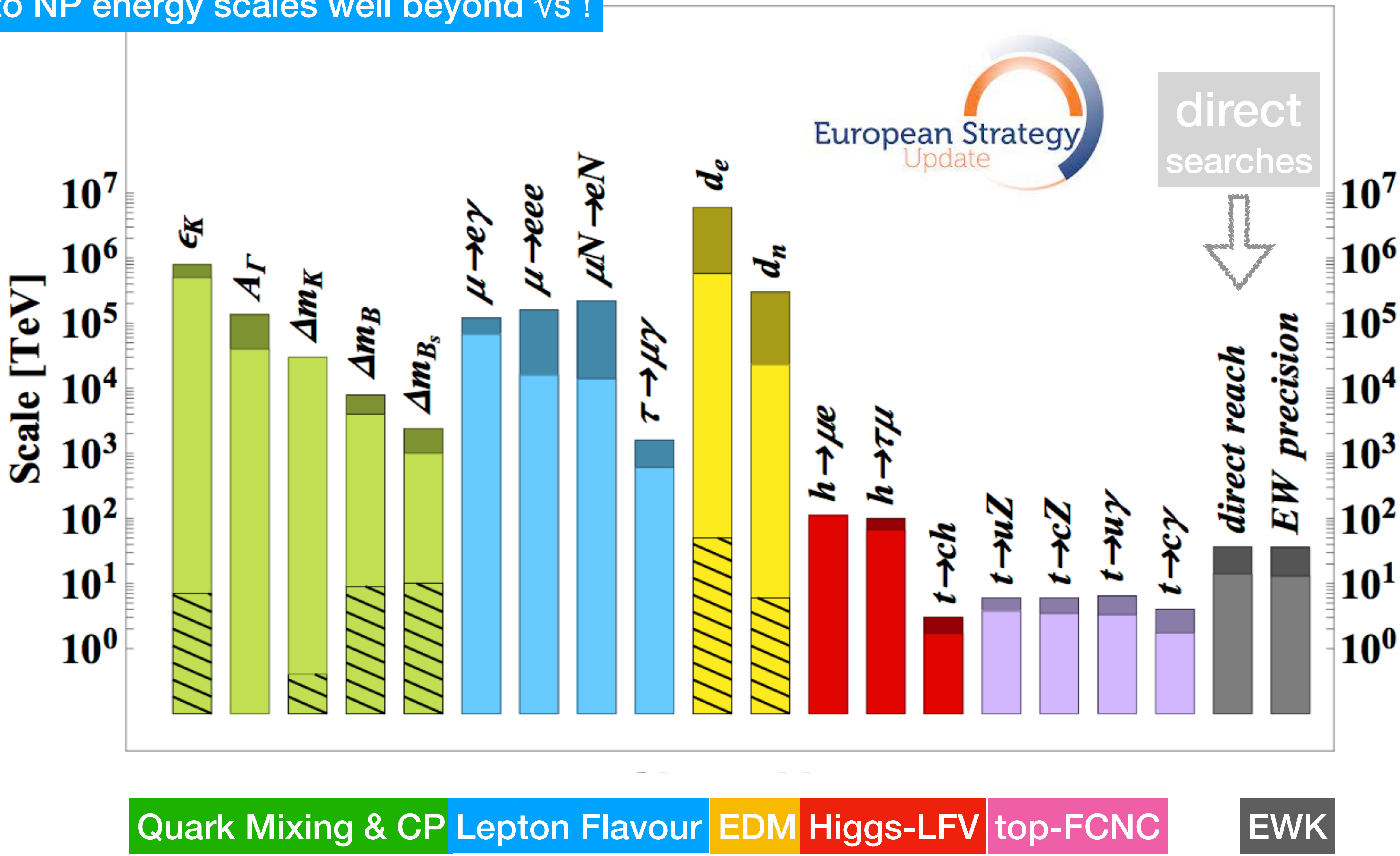




<u>bottom line</u>: historically, precision measurements at lower energies predicted the existence of new, heavier states

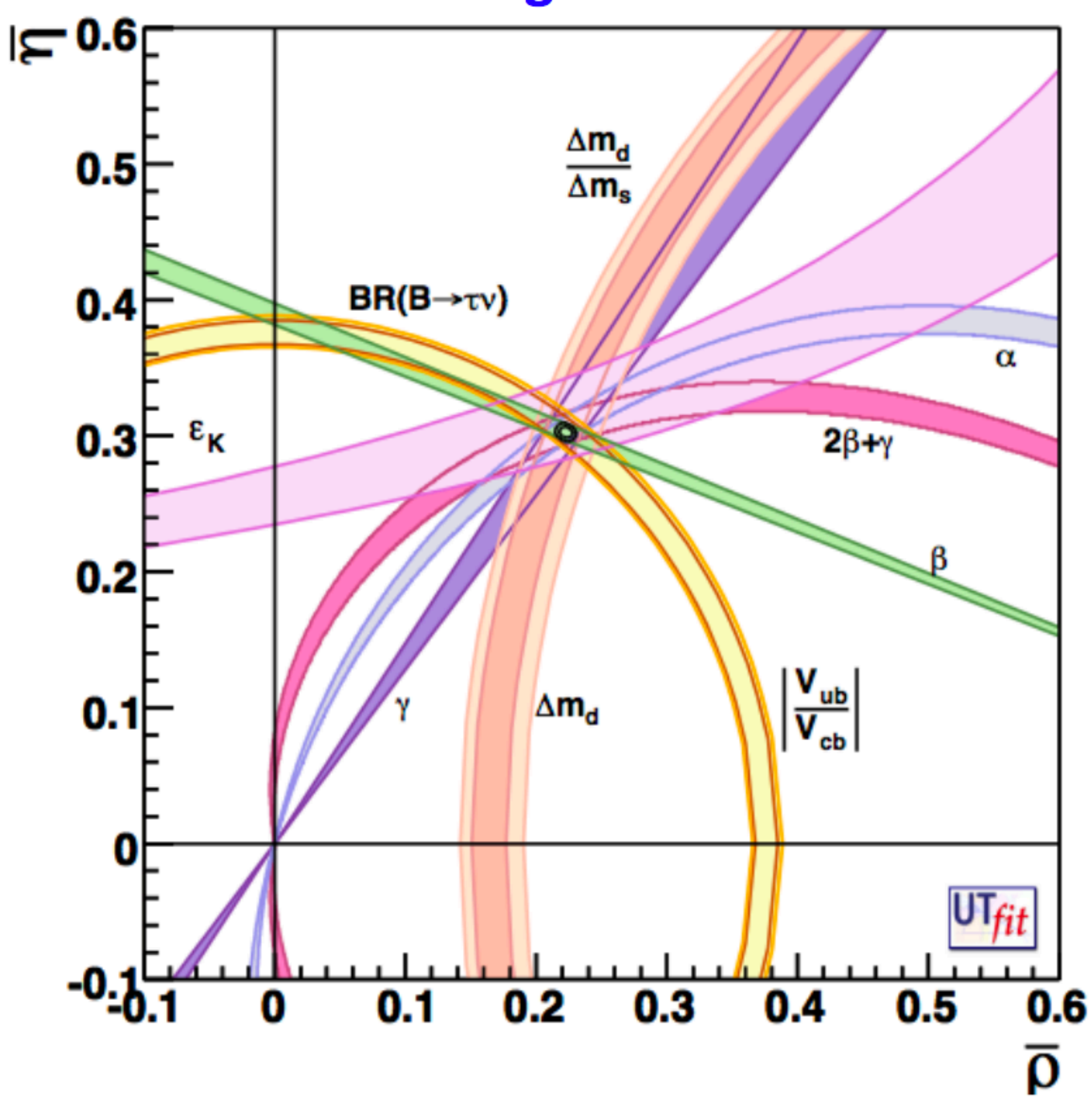
Indirect searches: fuelled by Quantum Mechanics

Flavour observables provide access to NP energy scales well beyond √s !

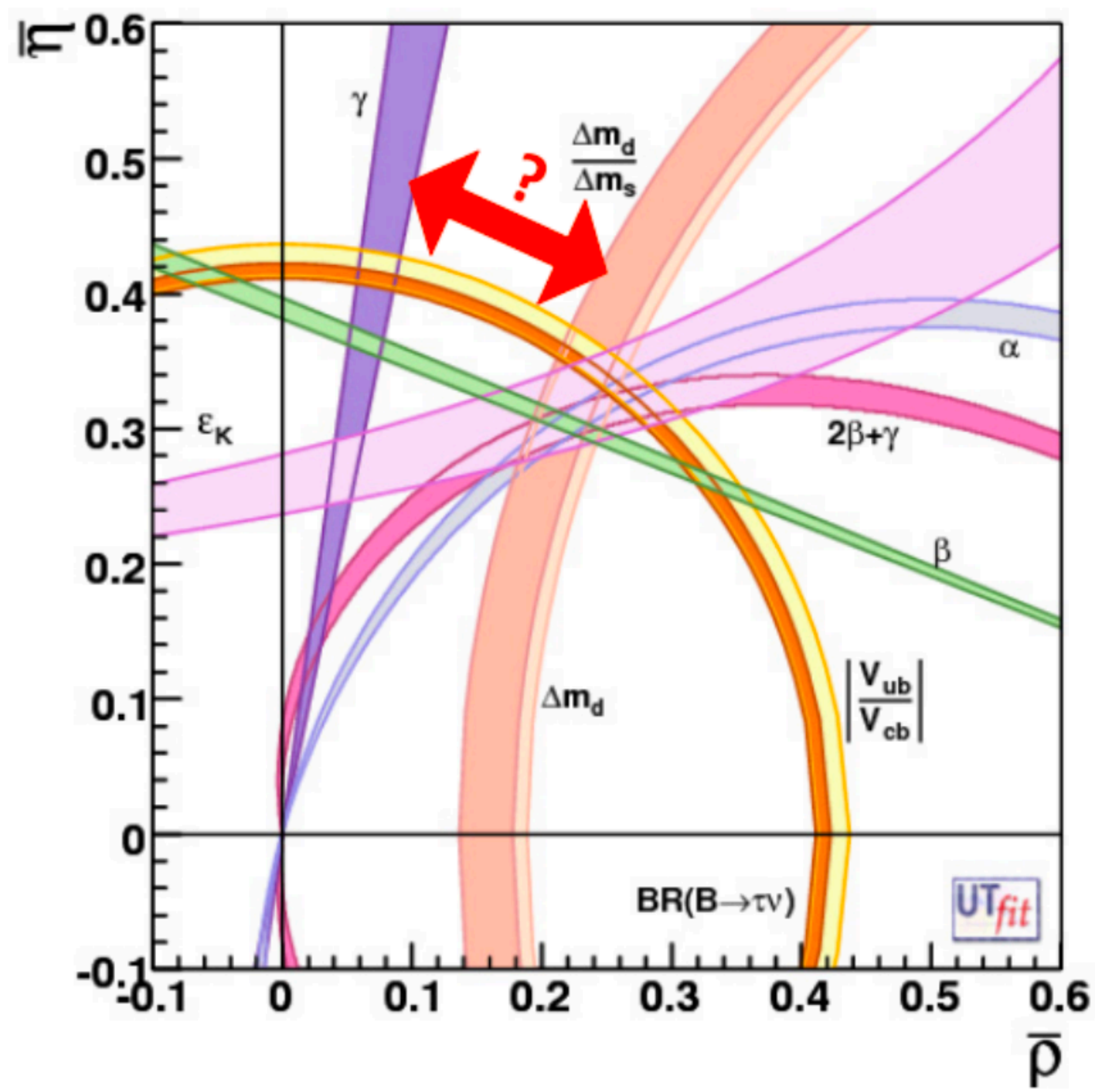


Precision tests of the SM

The "nightmare"

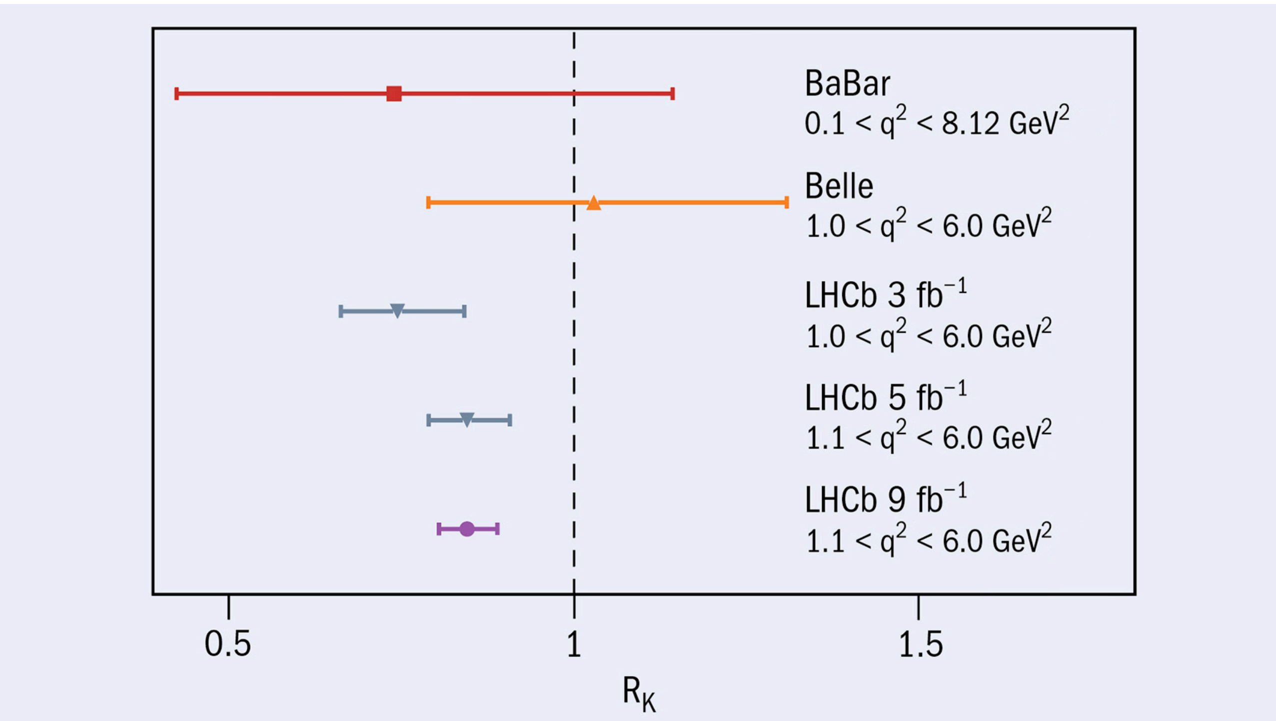


The "dream"



https://arxiv.org/abs/0710.3799

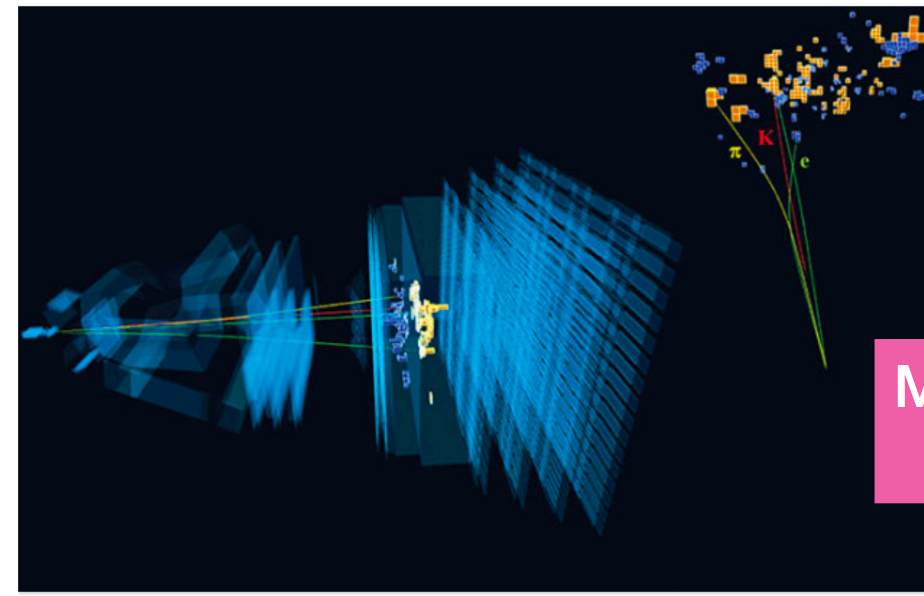
Anomalies !



Intriguing new result from the LHCb experiment at CERN

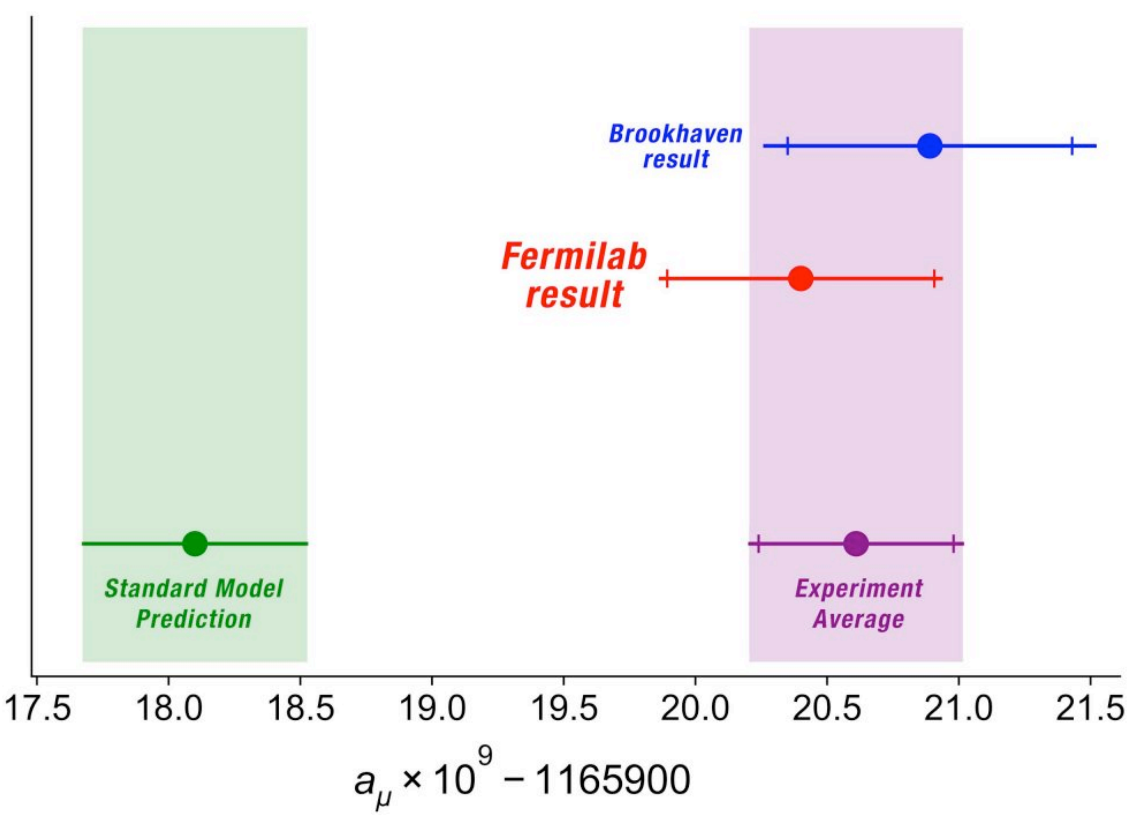
The LHCb results strengthen hints of a violation of lepton flavour universality

23 MARCH, 2021



Very rare decay of a beauty meson involving an electron and positron observed at LHCb (Image: CERN)

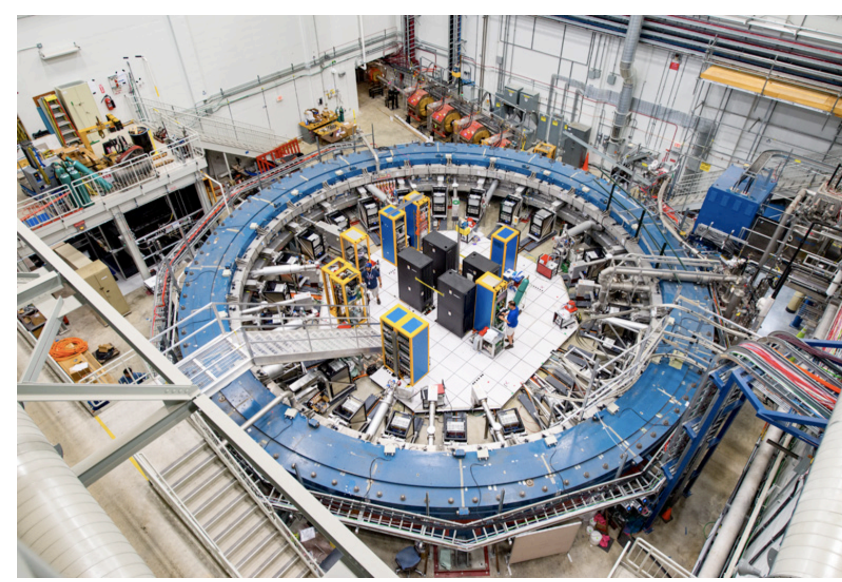
March’2021
@CERN



First results from Fermilab’s Muon g-2 experiment strengthen evidence of new physics

April 7, 2021

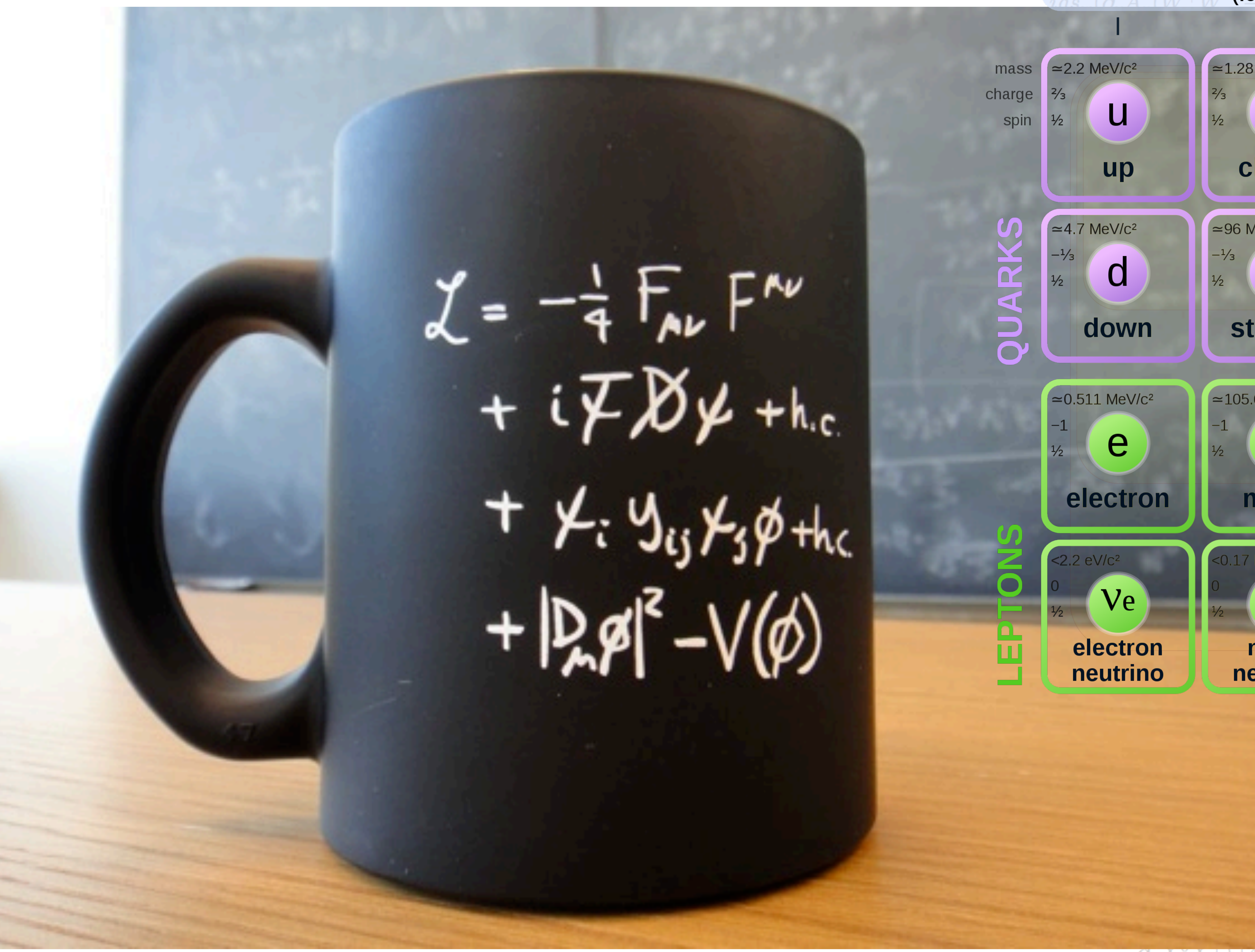
Share Tweet Email

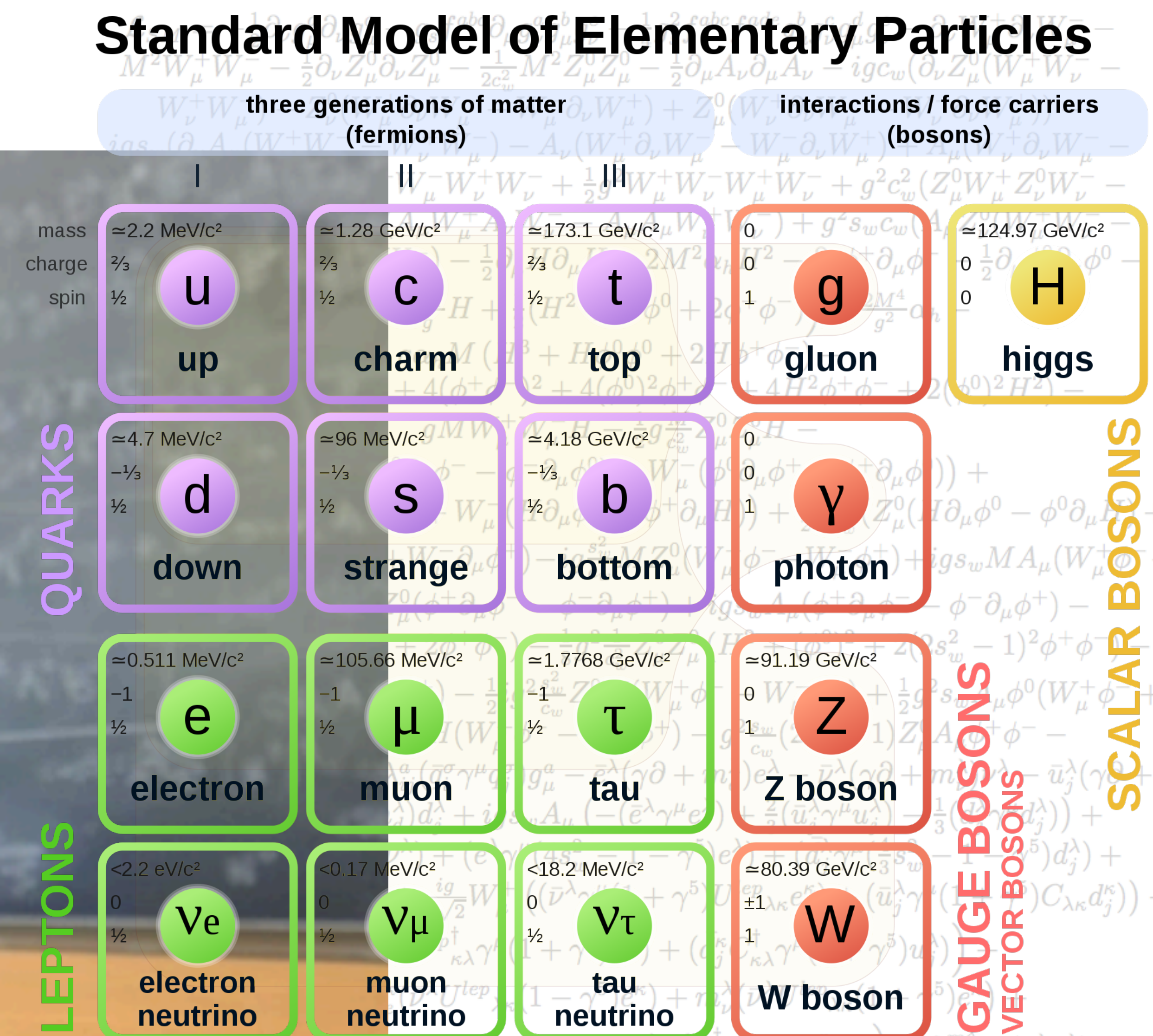


April’2021
@Fermilab

the standard model *(of particle physics)*

The SM Lagrangian

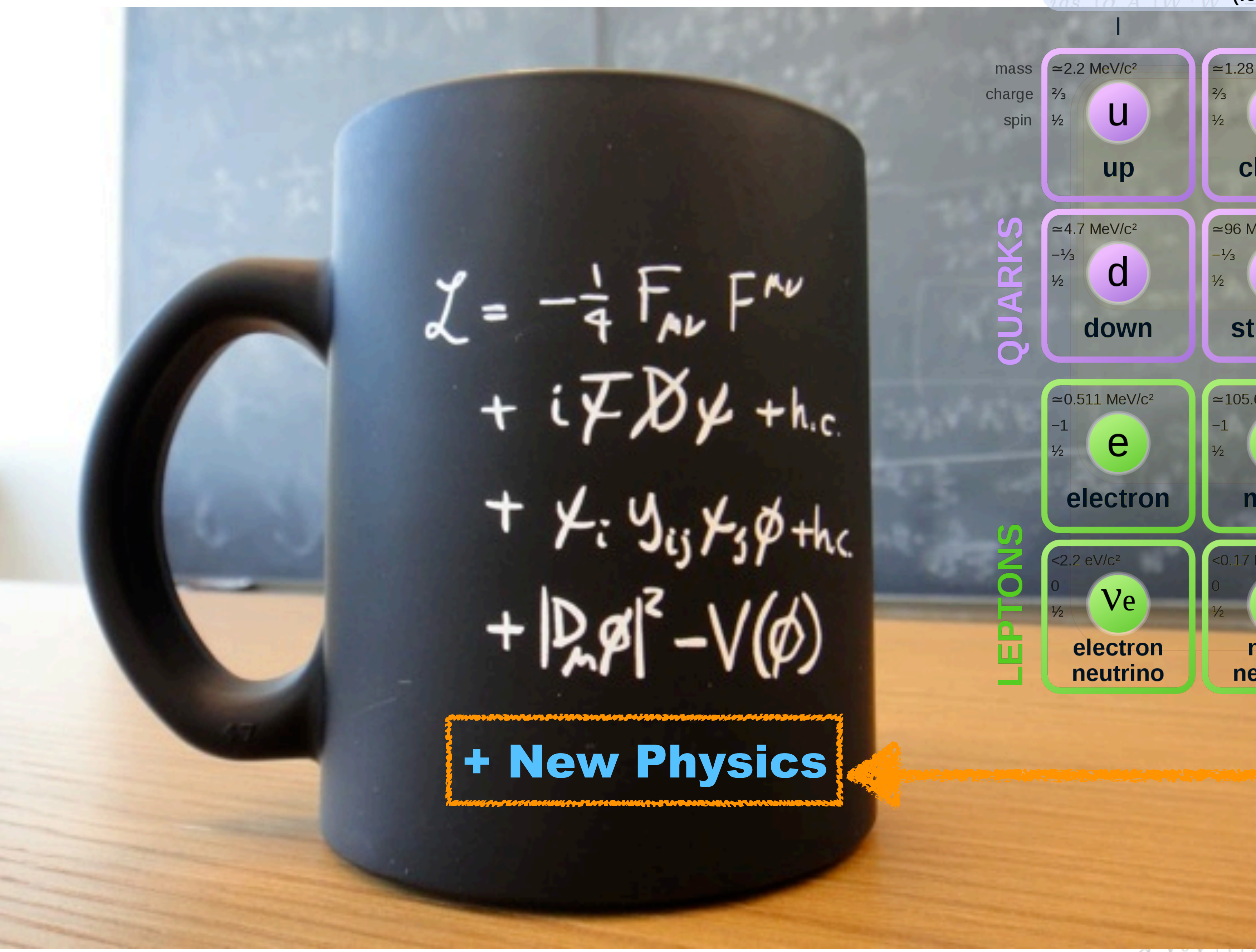


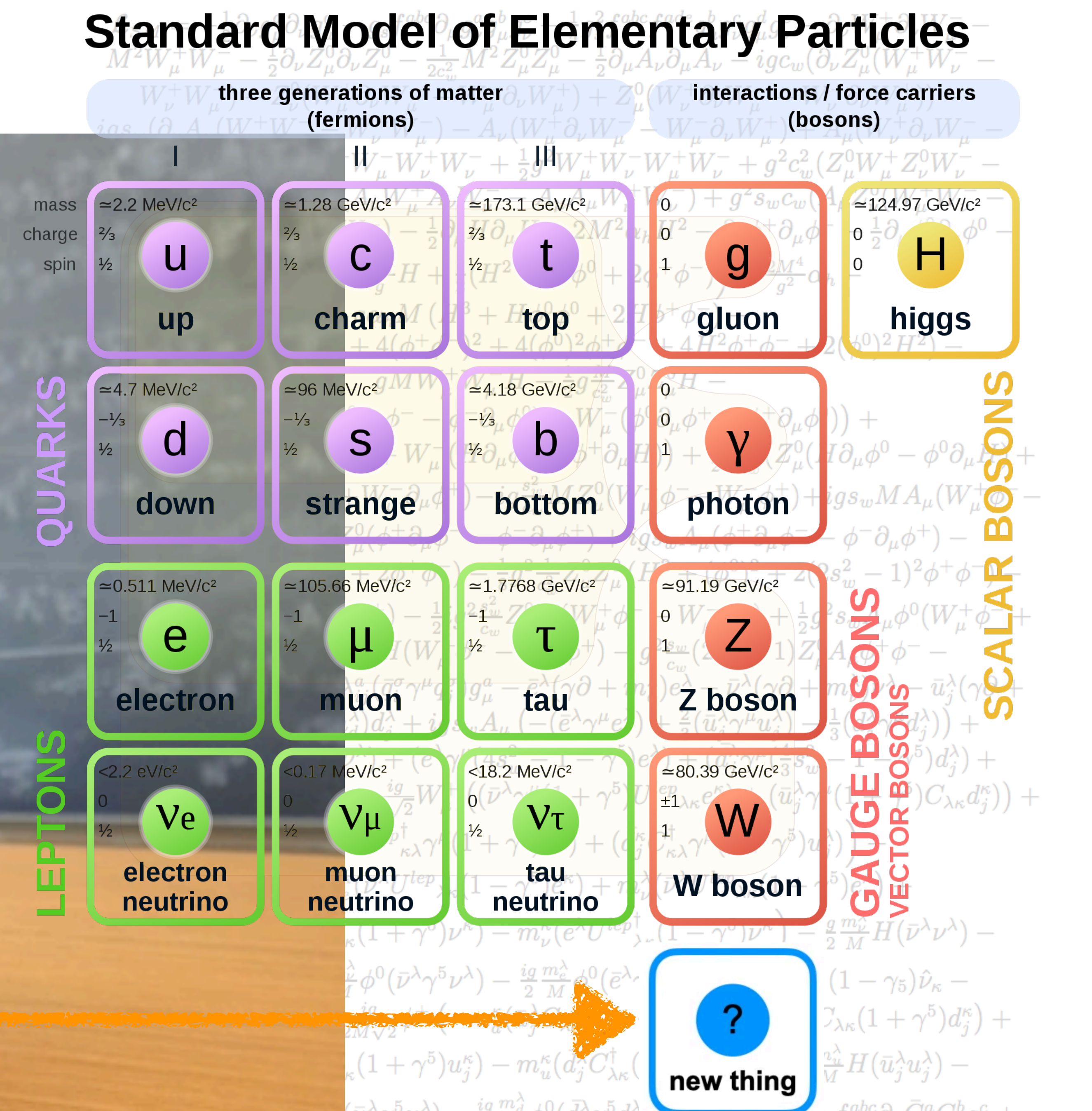


A great triumph of 20th century science.

the standard model *(of particle physics)*

The SM Lagrangian

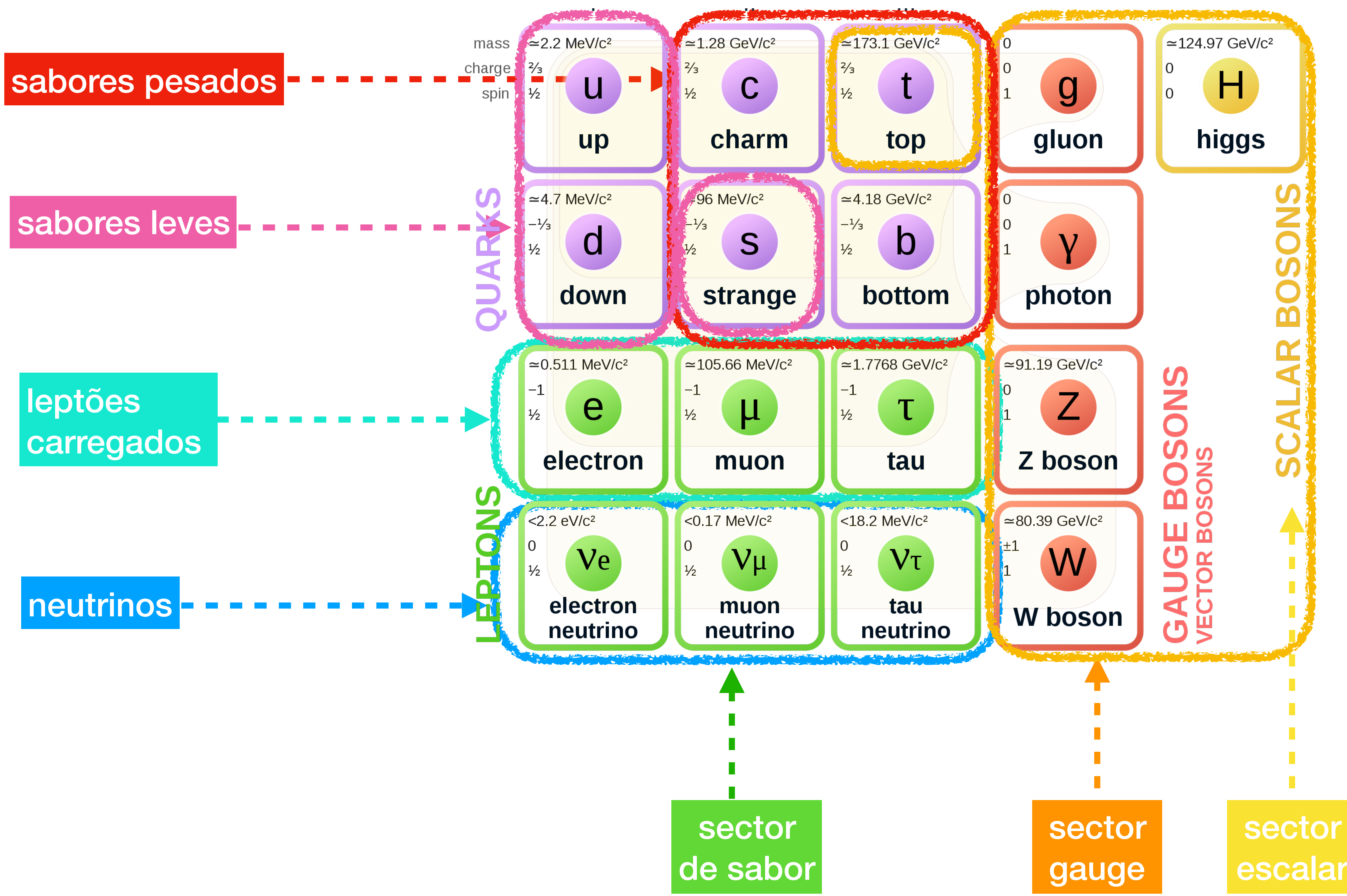




Going Beyond the SM — *New Physics*!

(heavy) flavor?

a tabela ‘periódica’ das partículas elementares



a tabela 'periódica' das partículas elementares
— testes de precisão e buscas de novas partículas

LHC, (LEP, Tevatron)

sabores pesados

LHC, BelleII, BESIII, (Belle, BaBar), . . .

sabores leves

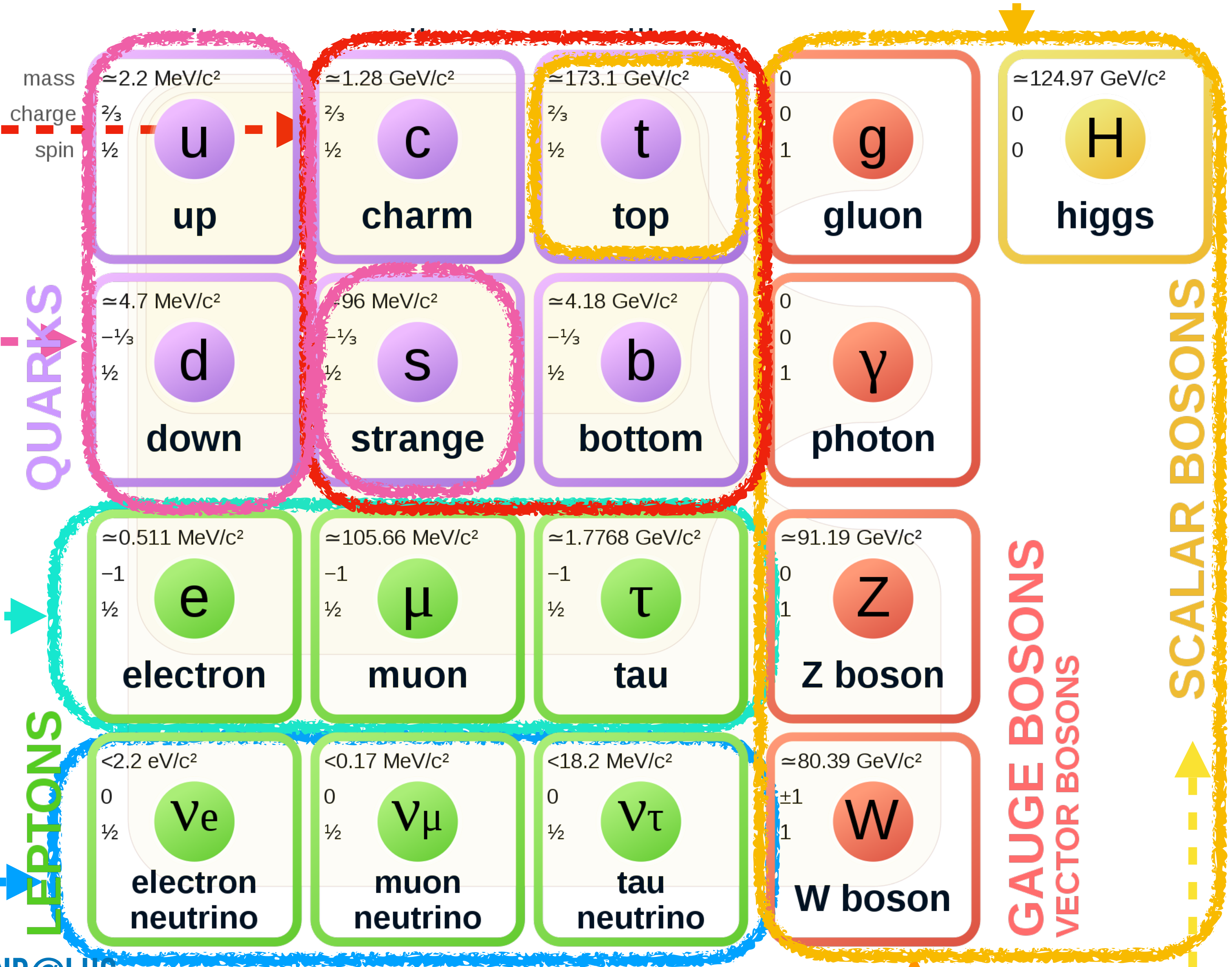
Compass, NA62, KOTO, (Amber, KLEVER), . . .

leptões carregados

MEG, Mu2e, Mu3e, BelleII, (TauFV), . . .

neutrinos

SNO, Minos, Nova, MicroBoone, DayBay, T2K, SK, Opera, ANTARES, BOREXINO, (DUNE, HK), SND@LHC



Ions: RHIC, (FAIR,EIC), . . .

Forward: Totem / PPS, Alfa / LHCf, (AFP), . . .

FIPs: MoEDAL, FASER, (Codex-b, MATHUSLA, MAPP, SHiP), . . .

Cosmic: AMS, Auger, IceCube, Axion, . . .

DM: LUX, ZEPLIN, DAMMA, CDMS, ADMX, CRESST, EDELWEISS, EURECA, XENON, DEAP, ArDM, WARP, DarkSide, PandaX, ...

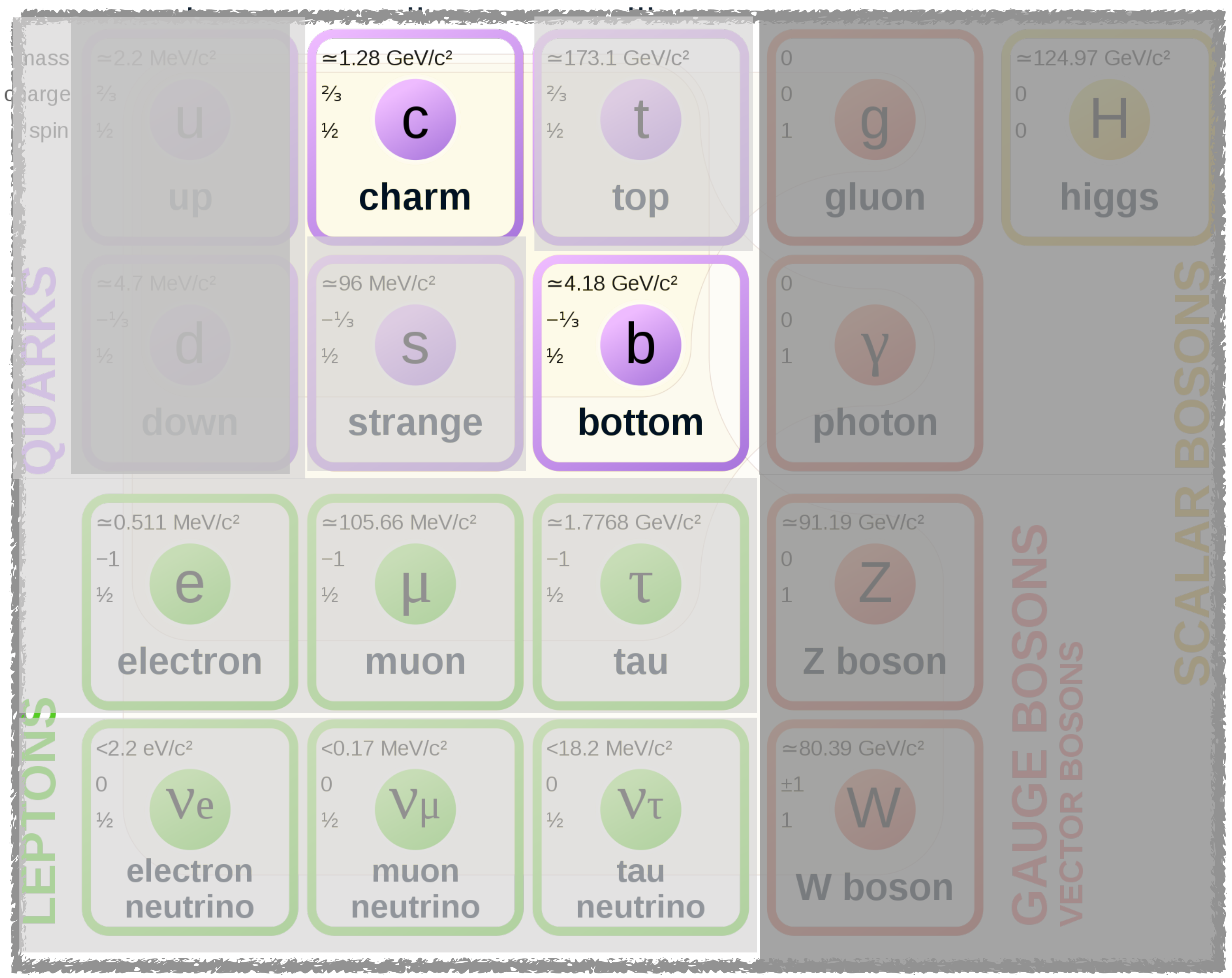
sector de sabor

sector gauge

sector escalar

a tabela ‘periÓdica’ das partículas elementares: flavour @ LHC

Just as ice cream has both color and flavour, so do quarks



quarks
(heavy)
b, c

&

leptons
(charged)
e, μ, τ

top: too heavy | does not hadronize | see in detain in previous lectures

neutrinos: interact too feebly | studied at LHC by dedicated experiments (e.g. SND)

a rich «flavor» phenomenology

- the SM flavor sector arises from interplay of fermion-weak–gauge and fermion-Higgs couplings

Electroweak symmetry breaking:
Masses for gauge bosons and fermions [Higgs mechanism]

Three generations of quarks and leptons

T = ½

Left-handed doublets: $\begin{pmatrix}\nu_e \\ e\end{pmatrix}_L, \begin{pmatrix}\nu_\mu \\ \mu\end{pmatrix}_L, \begin{pmatrix}\nu_\tau \\ \tau\end{pmatrix}_L, \quad \begin{pmatrix}u \\ d\end{pmatrix}_L, \begin{pmatrix}c \\ s\end{pmatrix}_L, \begin{pmatrix}t \\ b\end{pmatrix}_L$

Right-handed singlets: $e_R, \quad \mu_R, \quad \tau_R, \quad u_R, d_R, \; c_R, s_R, \; t_R, b_R$

T = 0

Rich flavor phenomenology ...

The parameters of the SM

- 3 gauge couplings
- 2 Higgs parameters
- strong CPV parameter, $\bar{\theta}$
- 6 quark masses
- 3 quark mixing angles + 1 phase (CKM)
- 3 (+3) lepton masses
- (3 lepton mixing angles + 1 phase (PMNS))

( ) = with Dirac neutrino masses

Out of the19 parameters of the SM (excluding neutrino masses/mixing), 14 arise from the flavor sector.

flavor parameters

flavor «puzzle»

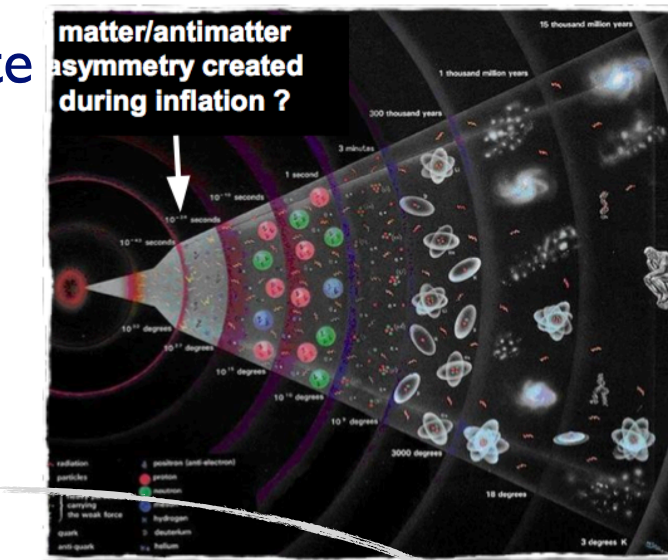
- there are standing mysteries intrinsic to the SM flavor sector
  - why are there so many free parameters
    - why do these parameters exhibit strong hierarchical structure spanning several orders of magnitude
  - why are there so many fermions
  - what is responsible for their organization into generations
    - and why are there 3 such generations each of leptons and quarks
  - why wide range of fermion couplings and masses
    - for example: $O(10^{-5}) \cdot m_t \sim m_u \sim m_\nu \cdot O(10^{+6})$, $|V_{ub}| \sim O(10^{-3}) \cdot |V_{td}|$
  - why are there flavor symmetries
    - and what breaks them
  - why is $\theta_{QCD} < 10^{-9}$
  - what is the origin of CP violation

- various solutions to this puzzle have been proposed (but not established), inevitably leading to beyond-the-SM scenarios
  - for within the SM these parameters can only be accommodated, not explained

another, related «puzzle»: BAU

(baryon asymmetry in the universe)

- Sakharov conditions (1967), necessary for dynamical evolution of matter dominated universe from symmetric initial state
  1. baryon number violation
  2. C & CP violation
  3. thermal inequilibrium



- no significant amounts of antimatter observed
  - $\Delta N_B/N_\gamma \equiv [N(\text{baryon})-N(\text{antibaryon})] / N_\gamma \sim 10^{-10}$
- amount of CP violation in SM not sufficient to explain BAU
  - CPV in quark sector (CKM) would yield an asymmetry of $O(10^{-17}) \ll 10^{-10}$
- more CPV is needed!
  - to create a larger asymmetry, require: new sources of CP violation ... that occur at higher energies
- where might it be found?
  - lepton sector: CPV in neutrino oscillations
  - quark sector: discrepancies with KM predictions
  - gauge/higgs sector; extra dimensions or other new physics?
  - *precision measurements of flavor observables sensitive to additions to SM*

«heavy» flavor, aka B Physics

light quarks: $m \lesssim \Lambda_{QCD}$
u, d: realm of nuclear physics
s: rare kaon decays test SM

$$m_u \approx 3\ MeV$$
$$m_d \approx 5\ MeV$$
$$m_s \approx 100\ MeV$$
$$m_c \approx 1300\ MeV$$
$$m_b \approx 4200\ MeV$$
$$m_t \approx 170000\ MeV$$

top (not that heavy!)
the top quark has its own phenomenology (since it does not hadronize)

$$m_{\nu_1} \lesssim 10^{-6}\ MeV$$
$$m_{\nu_2} \lesssim 10^{-6}\ MeV$$
$$m_{\nu_3} \lesssim 10^{-6}\ MeV$$
$$m_e \approx 0.5\ MeV$$
$$m_\mu \approx 100\ MeV$$
$$m_\tau \approx 1800\ MeV$$

neutrinos
have their own phenomenology, not detected (directly) at LHC

light charged leptons e.g. electric and magnetic dipole moments test SM

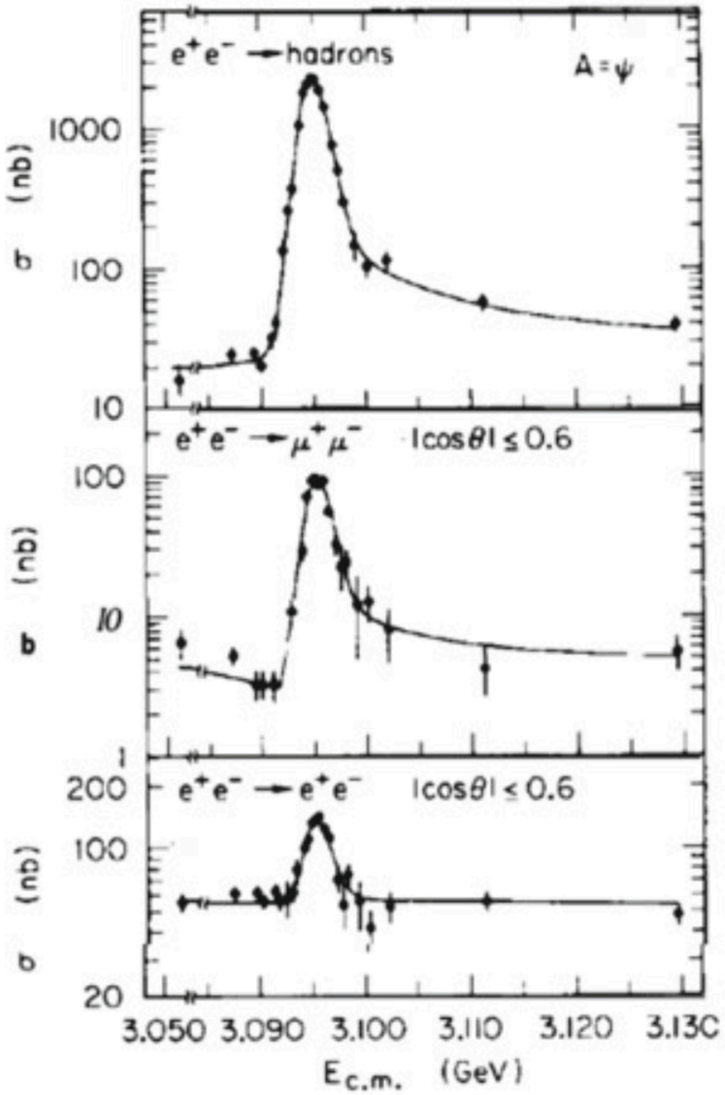
tau
e.g. searches for lepton flavor violation, $\tau \rightarrow \mu\mu\mu$

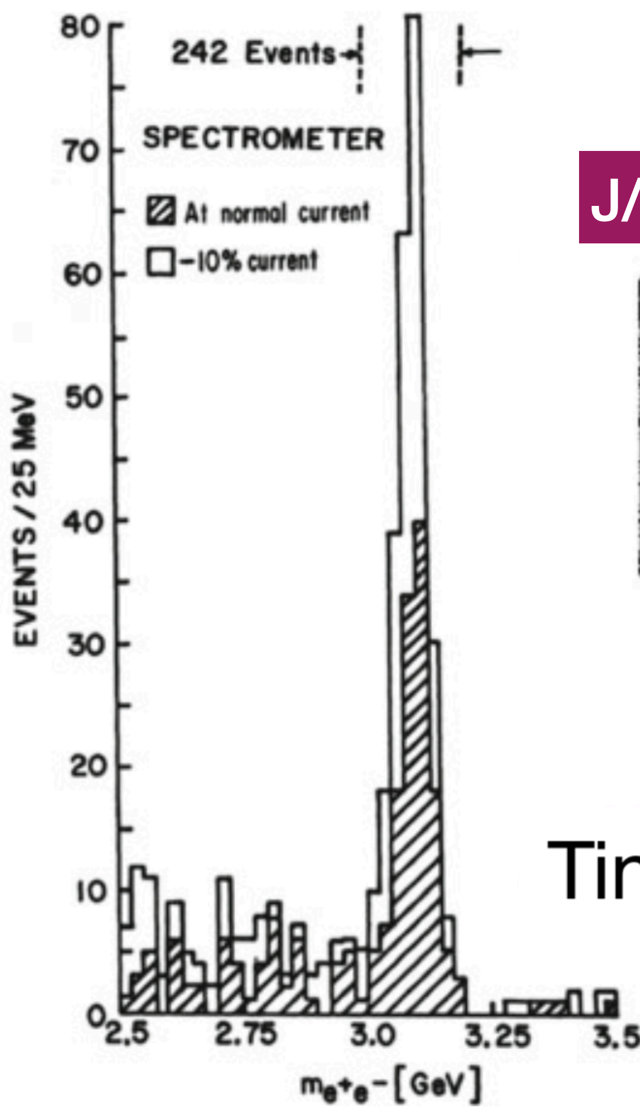
Study Beauty and Charm quarks
- **hidden** flavor aka quarkonia: ψ ($c\underline{c}$), Υ($b\underline{b}$), $X_{c,b}$
- **open** charm: D mesons
- open beauty, B mesons (B_u, B_d, B_s, B_c) and b-baryons (Λ_b, Ξ_b, Ω_b, ...)
- **exotic** hadrons: X,Y,Z states

note:
- «B physics» refers to study of flavor-changing interactions of b-quark mesons
  - heavier states accessible only at the LHC — Υ, χ_b, B_s, B_c, b-baryons

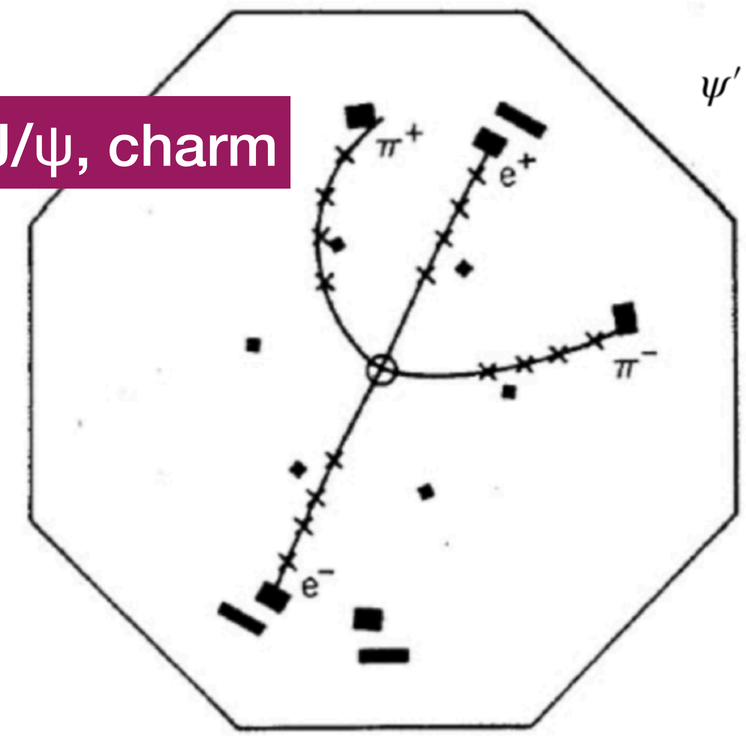
the SM discovery

1974





J/ψ, charm

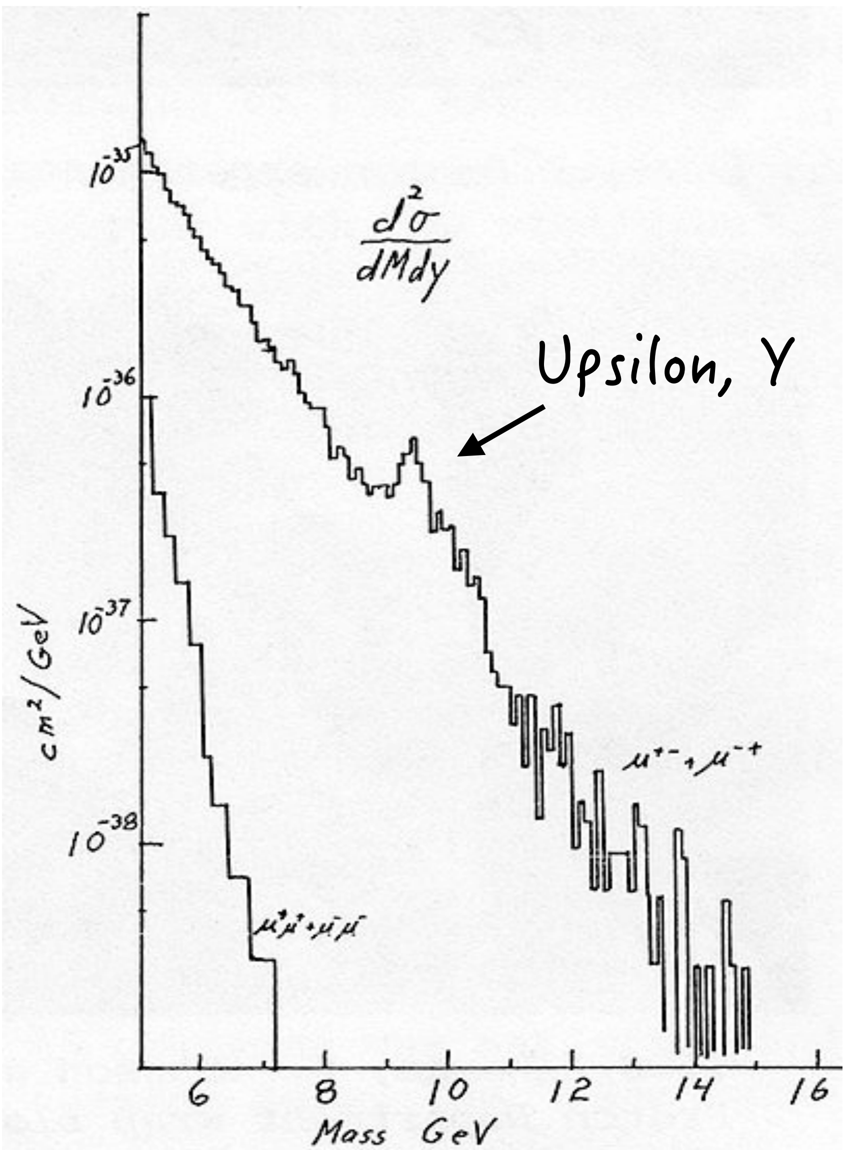


$\psi' \to J/\psi\,(\to e^+e^-)\,\pi^+\pi^-$

Ting&Richter, Nobel prize 1976

the SM discovery

1977

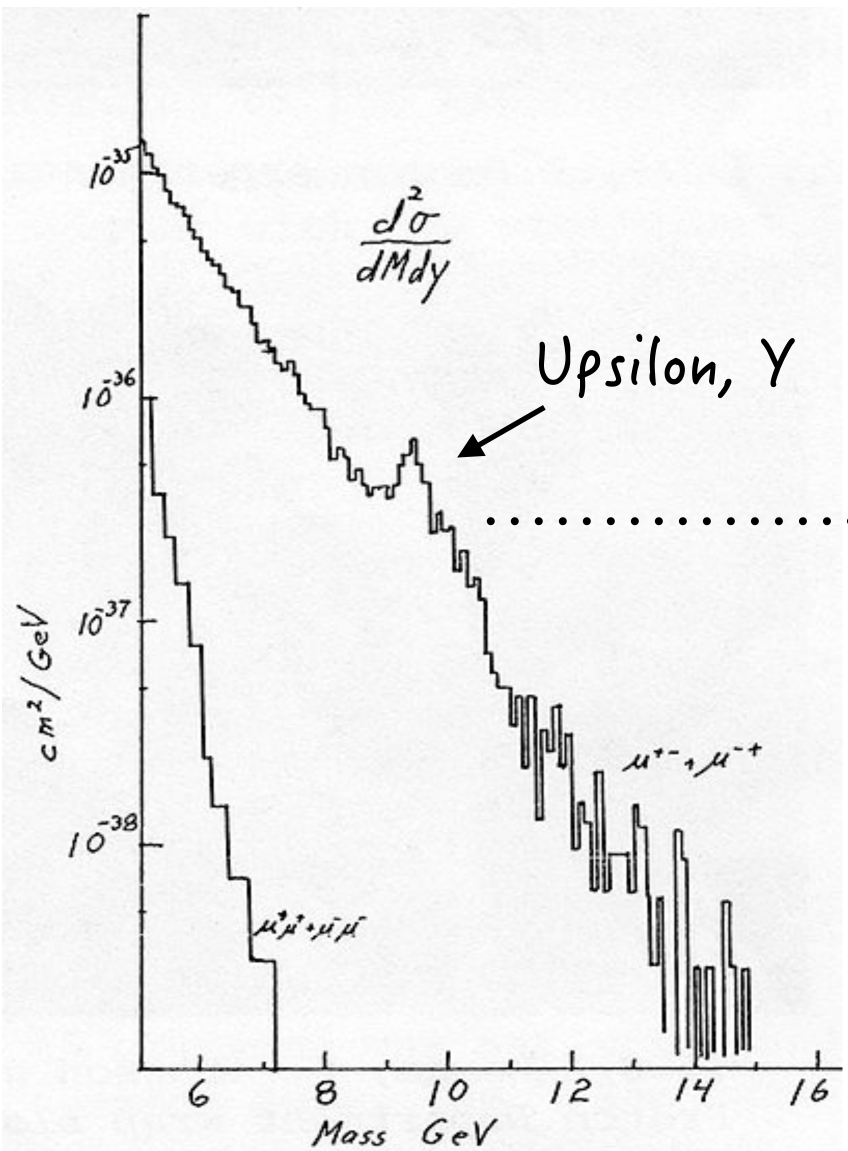


the discovery of the
b quark

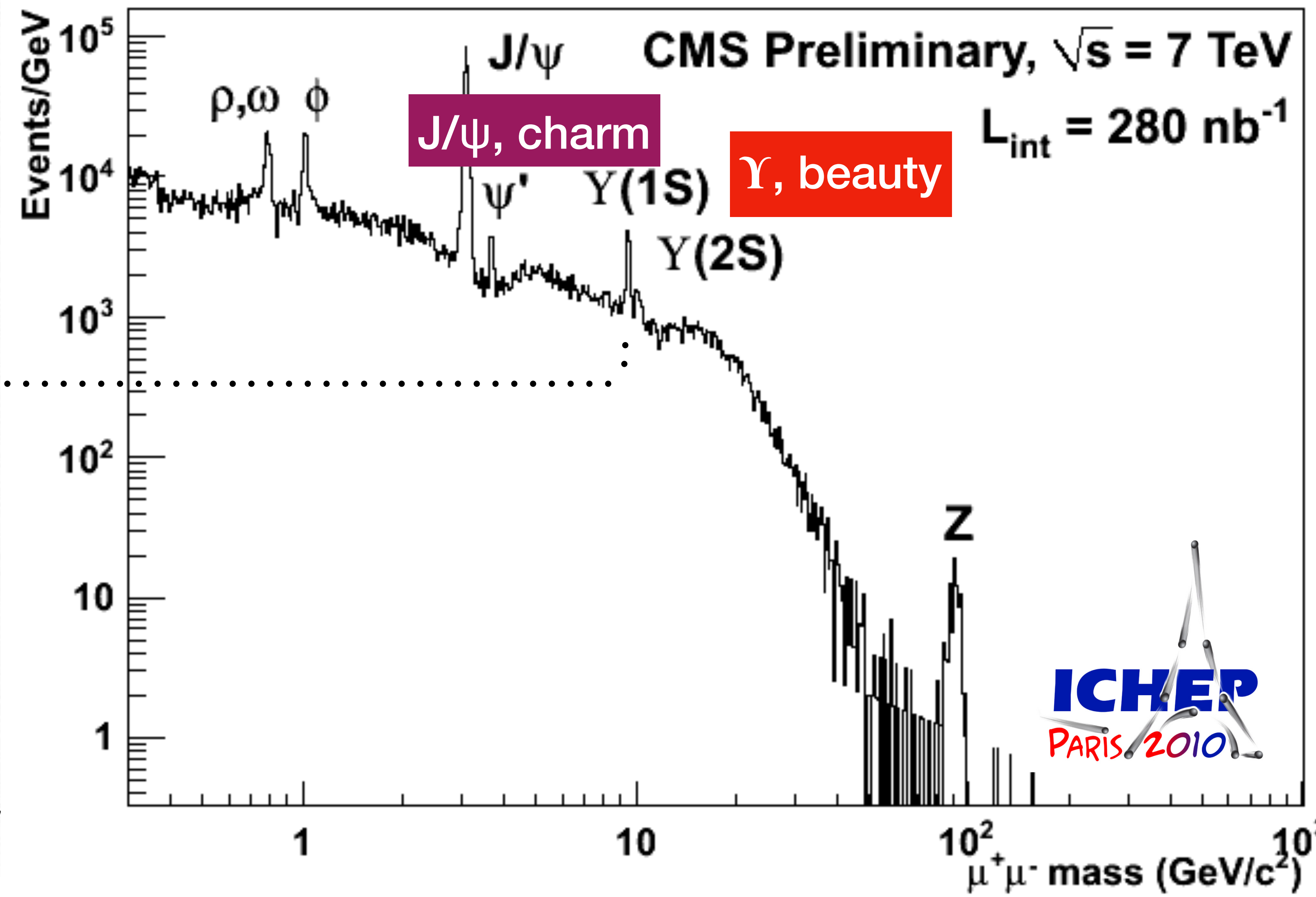
Υ, beauty

the SM *re*-discovery @ *LHC*

1977



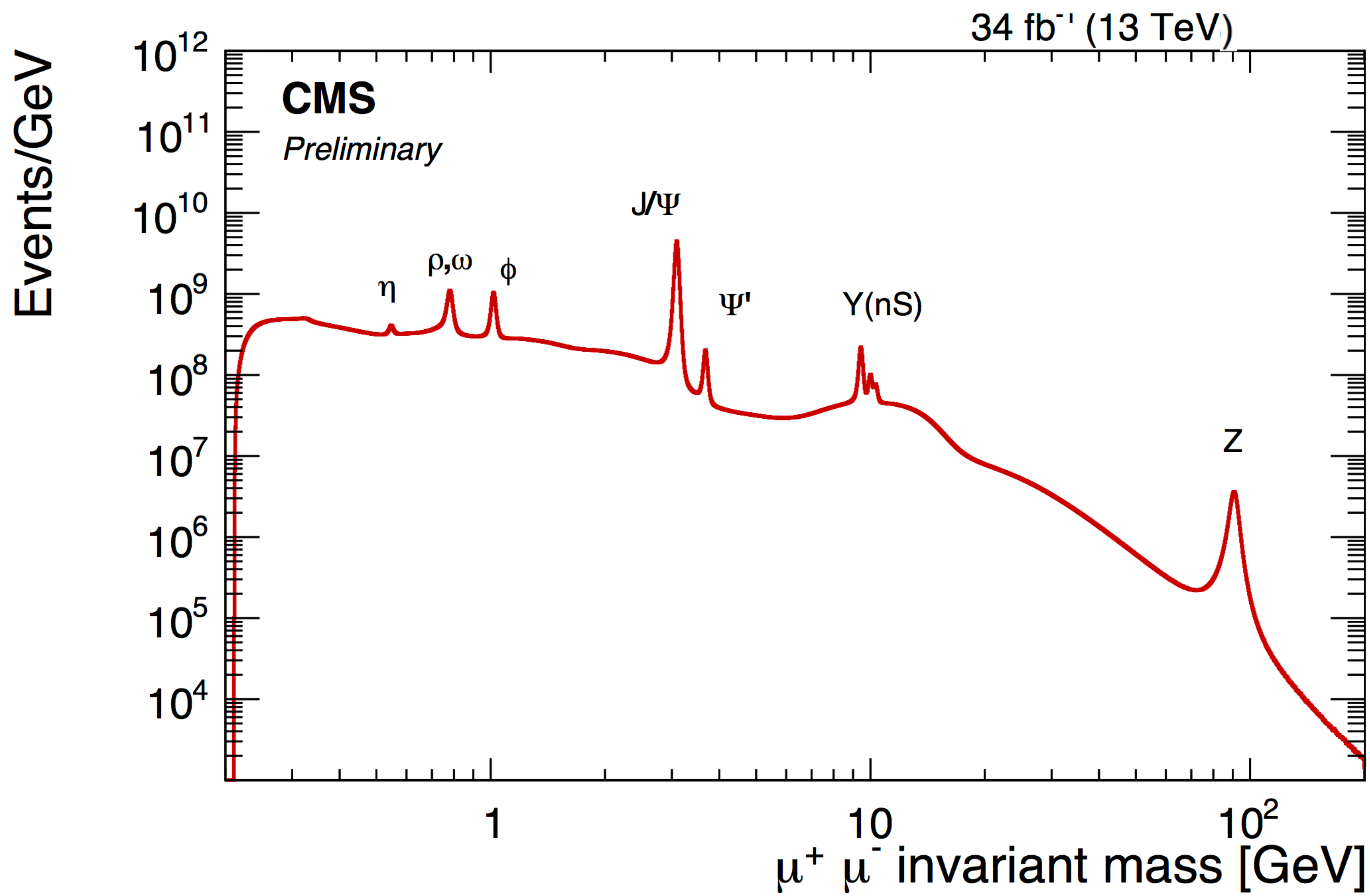
2010



the discovery of the
b quark

decades worth of **particle physics discovery** … in a single plot!

the SM *re-*discovery @ *LHC*



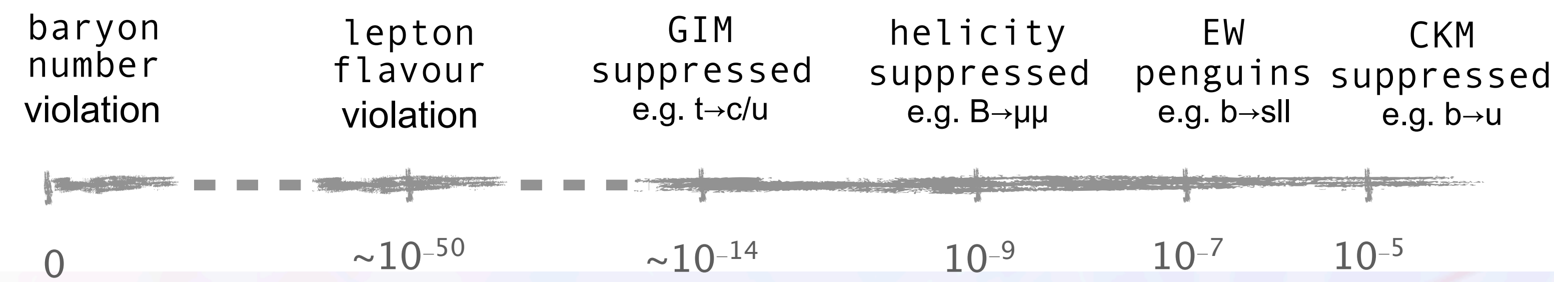
… with 100’000 times more data (➡ precision + new particles?)

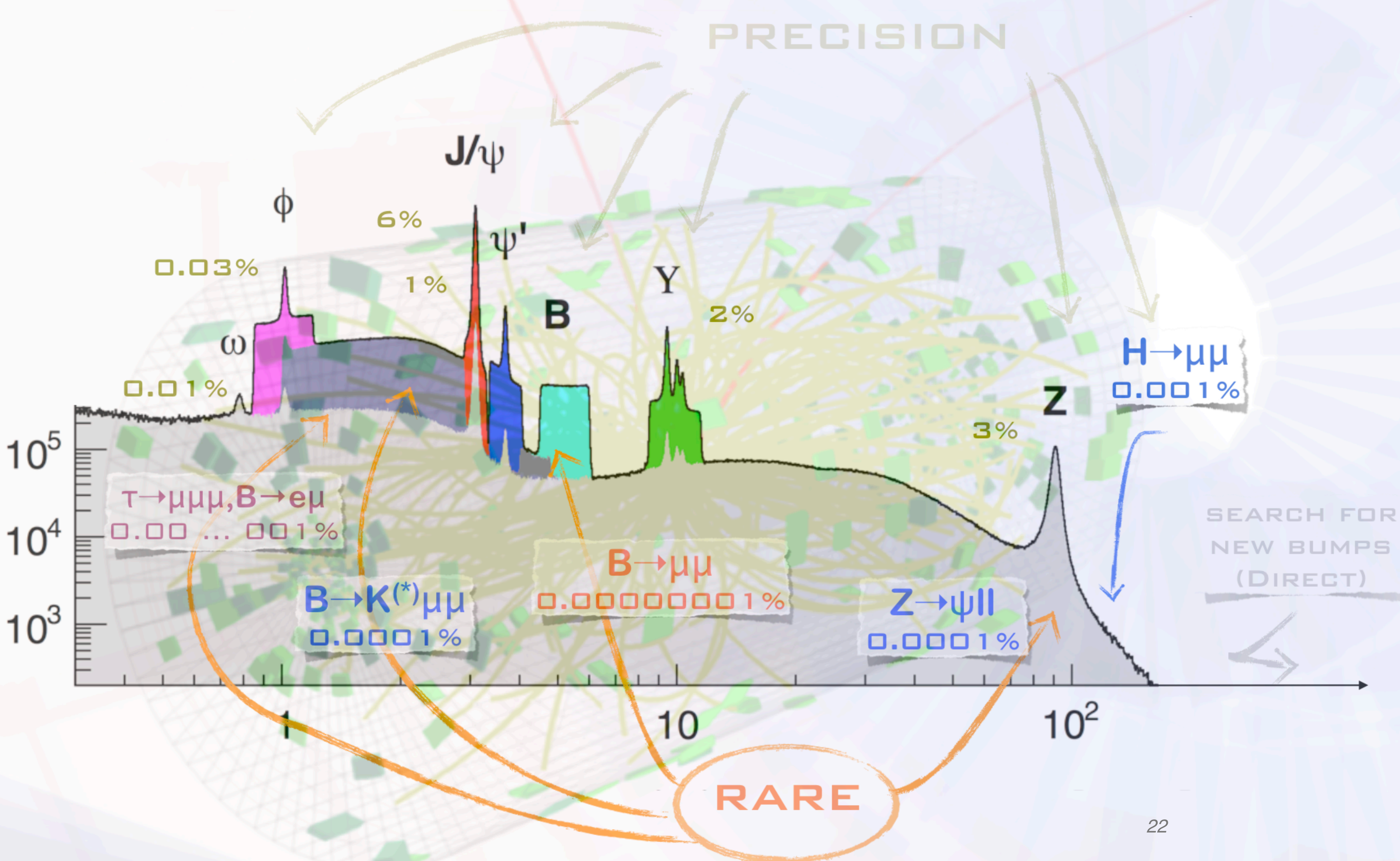
NOT-SO-RARE ➡ PRECISION!

MEDIUM RARE ⬅ EWK PENGUINS

VERY RARE ⬅ FCNC/GIM+HELICITY

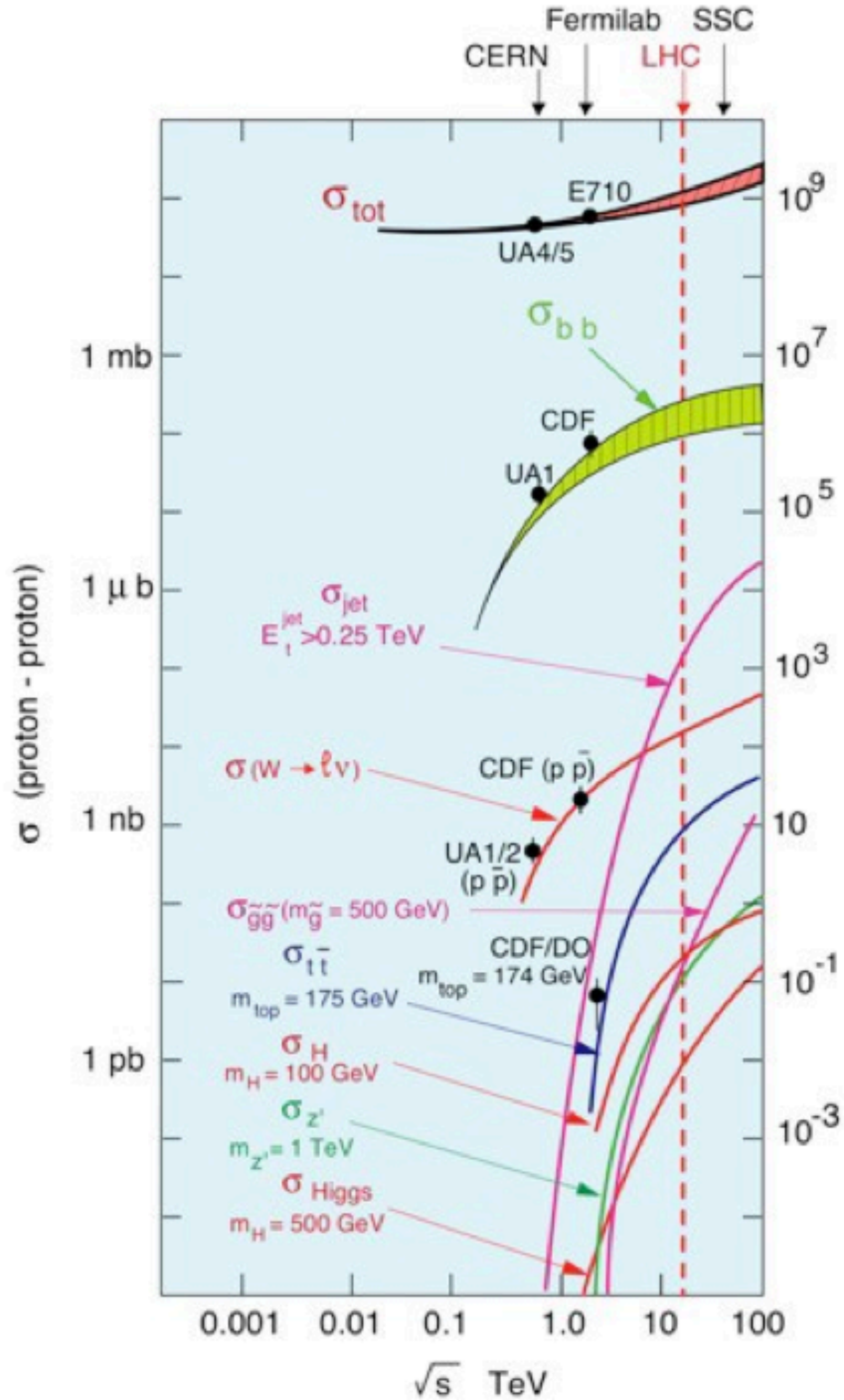
ULTRA RARE ⬅ LFV





production

HF production

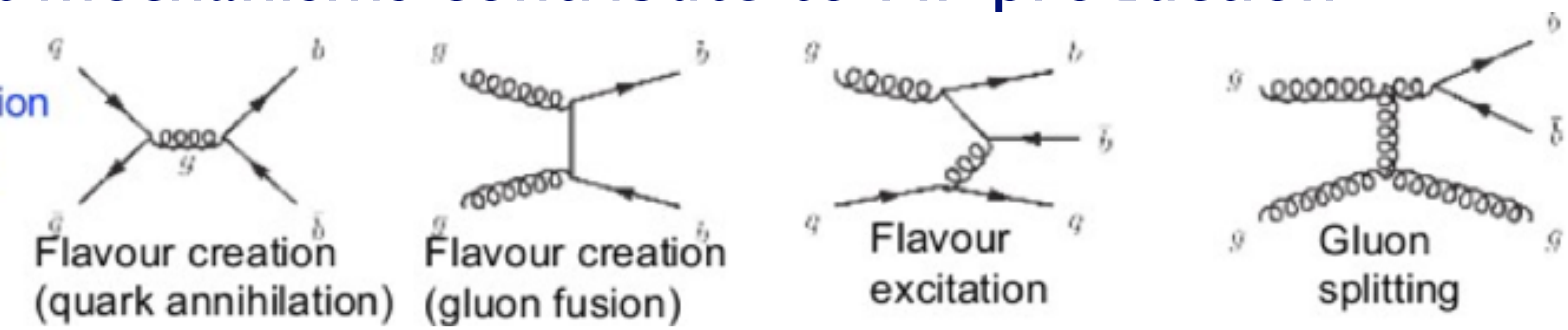


- high HF production rates at the LHC
  - very large production cross section (σ)
  - large accumulated luminosity (L)
- LHC: HF 'factory' (N=L.σ)
  - allow to perform precision measurements, as well as to search for very rare processes
- HF production is ubiquitous
  - forming backgrounds for many physics processes explored at the LHC
  - need to be thoroughly understood

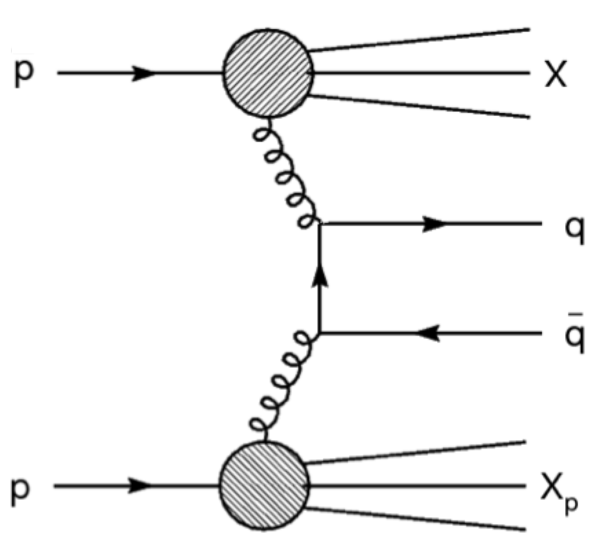
hadron production

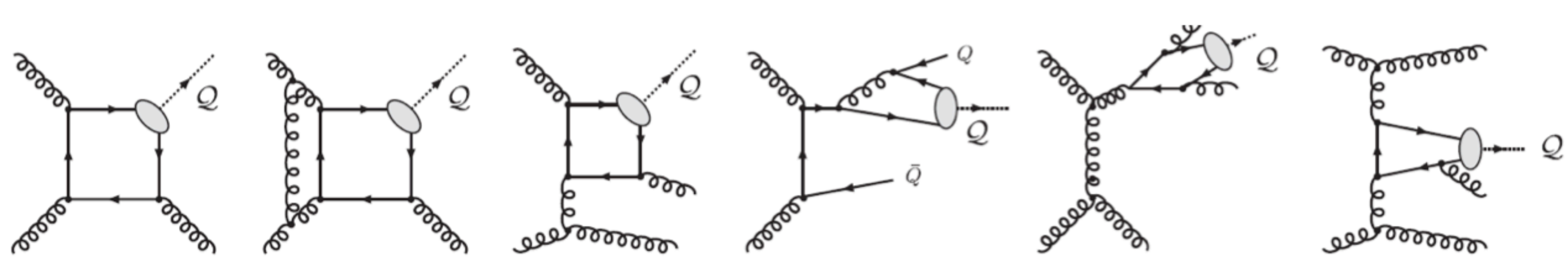
- different mechanisms contribute to HF production

bb production at hadron colliders



- produced quarks evolve into hadrons: known as fragmentation
  - involving short-distance/perturbative vs long-distance processes
- heavy quarkonia $Q\bar{Q}$=($b\bar{b}$, $c\bar{c}$) are an ideal laboratory in which to study the strong force and the mechanisms of hadron formation
  - non-perturbative evolution of $Q\bar{Q}$ pair into a quarkonium state
  - employ effective theories: e.g. non-relativistic QCD (NRQCD; CSM, CEM...)

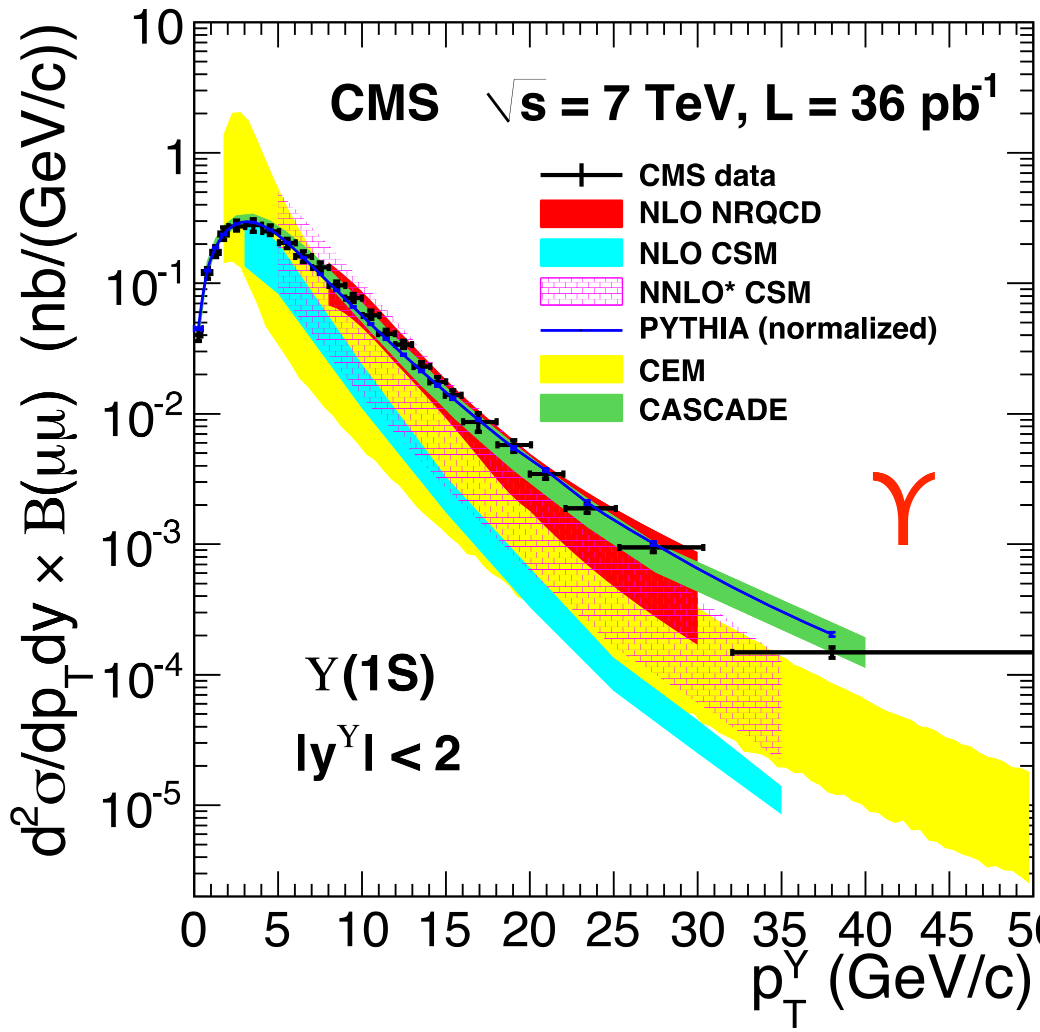


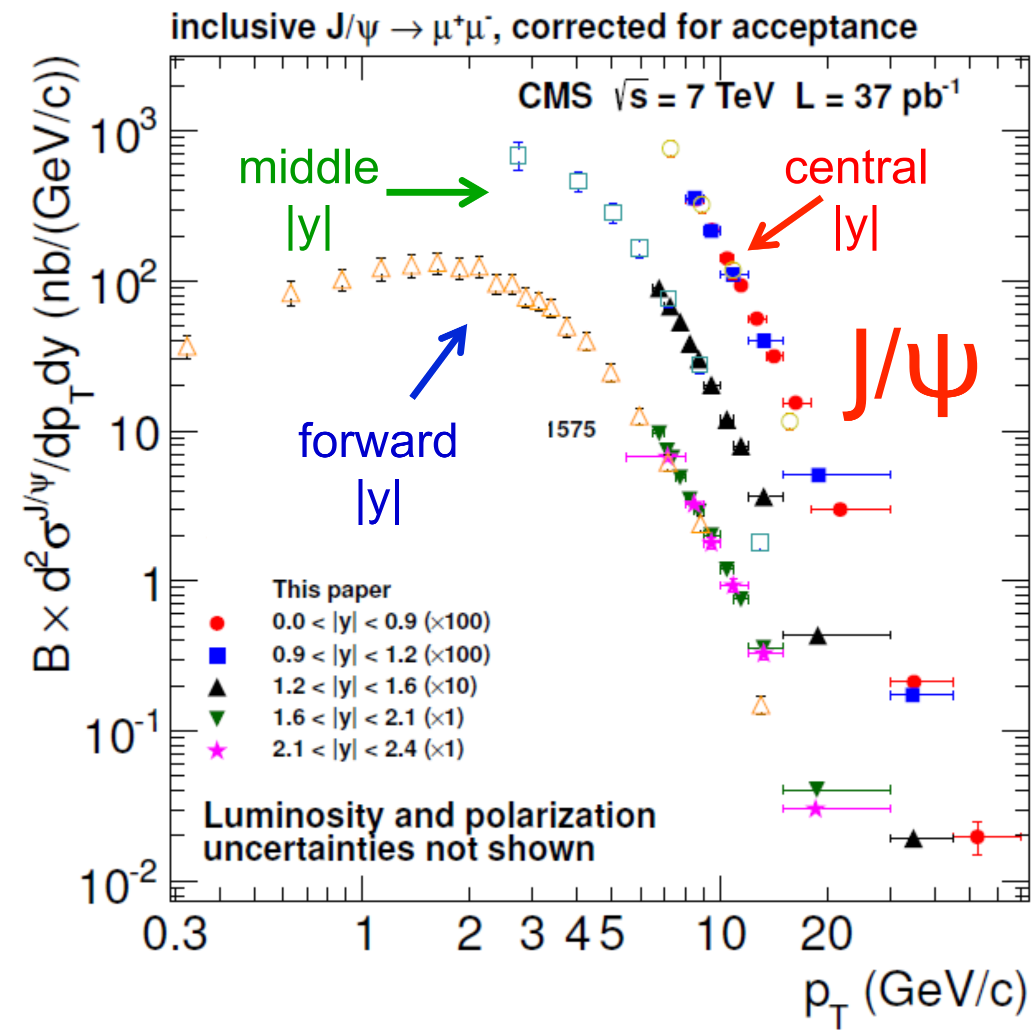


need to carry out detailed of HF production, including cross sections, polarizations, etc

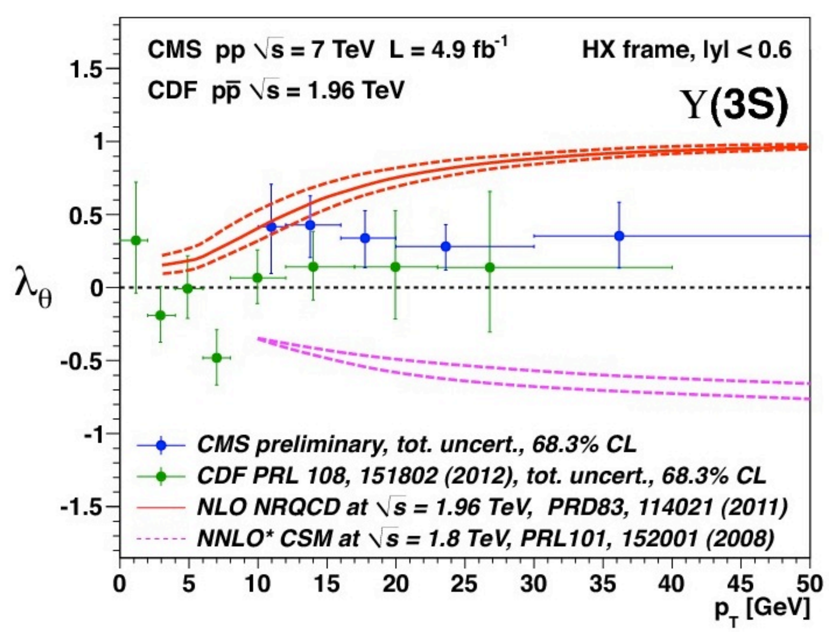
quarkonia production

cross sections





polarizations



- precision measurements of quarkonium production
- LHC allows to probe higher p_T region for the first time

cross section measurement, σ

$$\frac{d^2\sigma(Q\overline{Q})}{dp_T dy}\mathcal{B}\left(Q\overline{Q}\to\mu^+\mu^-\right)=\frac{N_{fit}(Q\overline{Q})}{\mathcal{L}\cdot\mathcal{A}\cdot\epsilon\cdot\Delta p_T\cdot\Delta y}$$

N = L x σ

$$\sigma=\frac{N}{L_{int}}$$

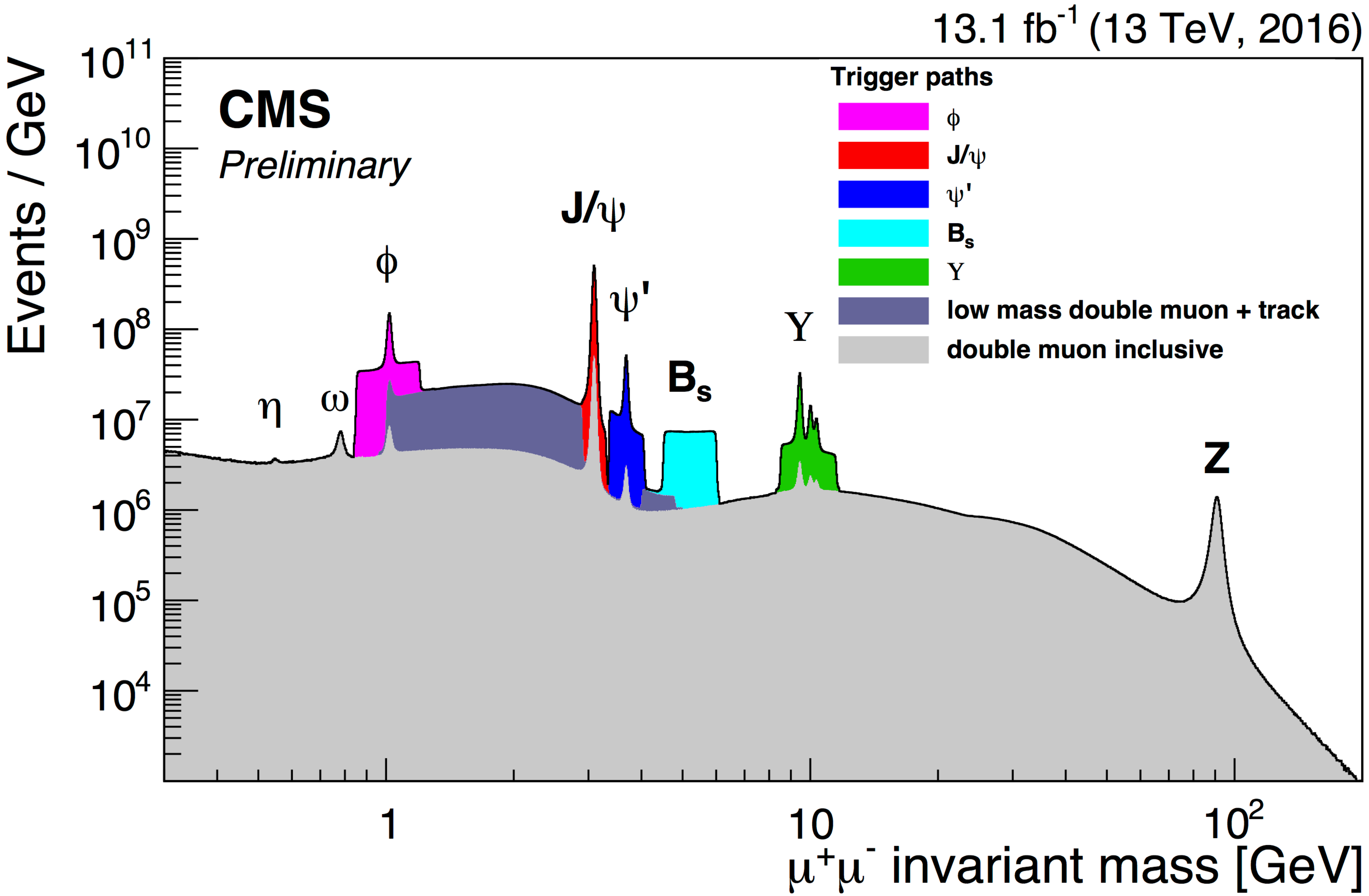
$$\sigma=\frac{N_{obs}}{A\,.\,\epsilon\,.\,L_{int}}$$

$$\sigma=\frac{N_{obs}-N_{bkg}}{A\,.\,\epsilon\,.\,L_{int}}$$

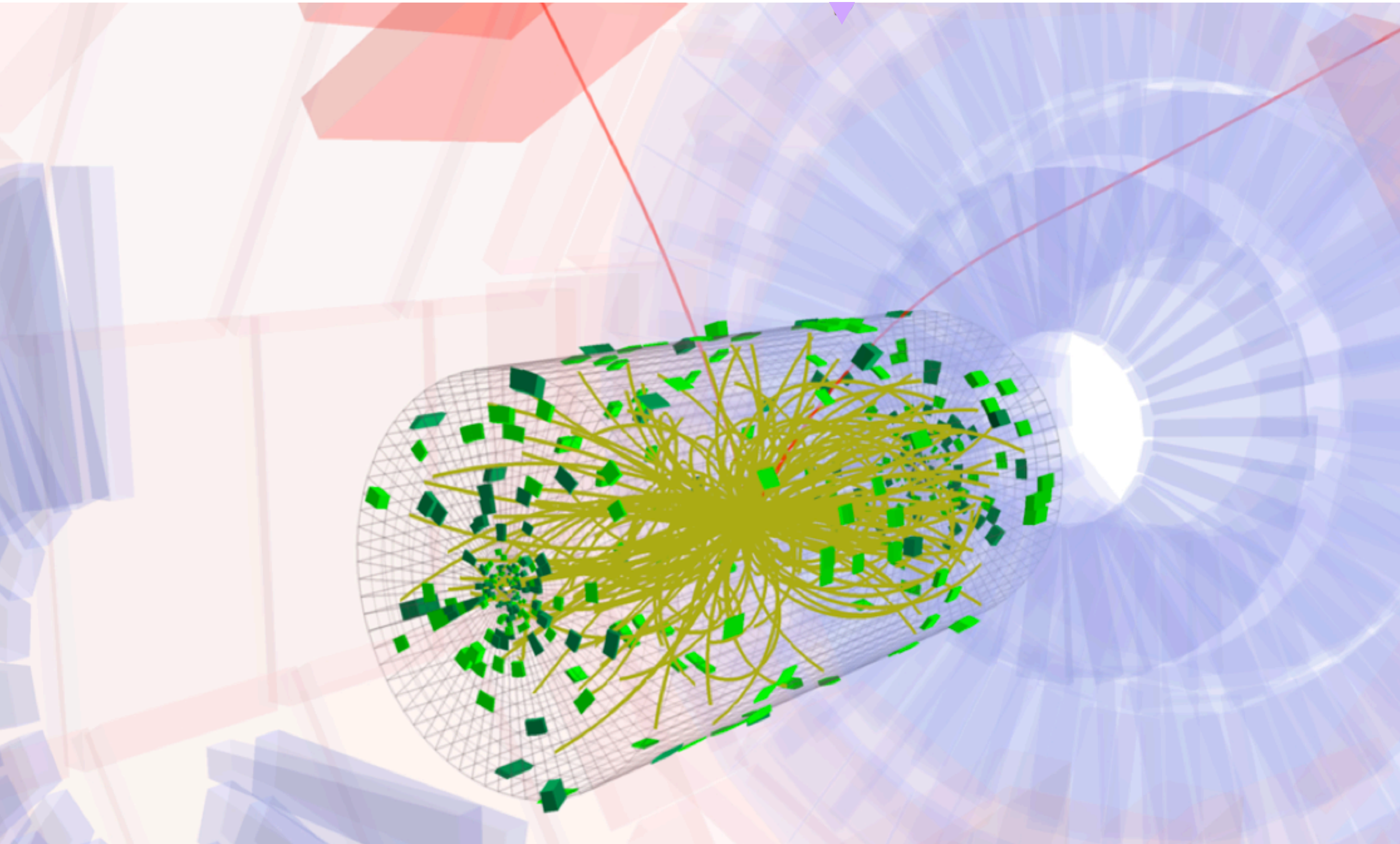
1. Trigger (online selection)
2. Muon identification and reconstruction
3. Υ meson candidate reconstruction
4. Extract signal (offline selection) N
5. Determine detector acceptance A and selection efficiency ε
6. Luminosity L and branching fraction B
7. Systematic uncertainties (on N, A, L, B)
8. Compare to theory predictions (NRQCD)

Trigger

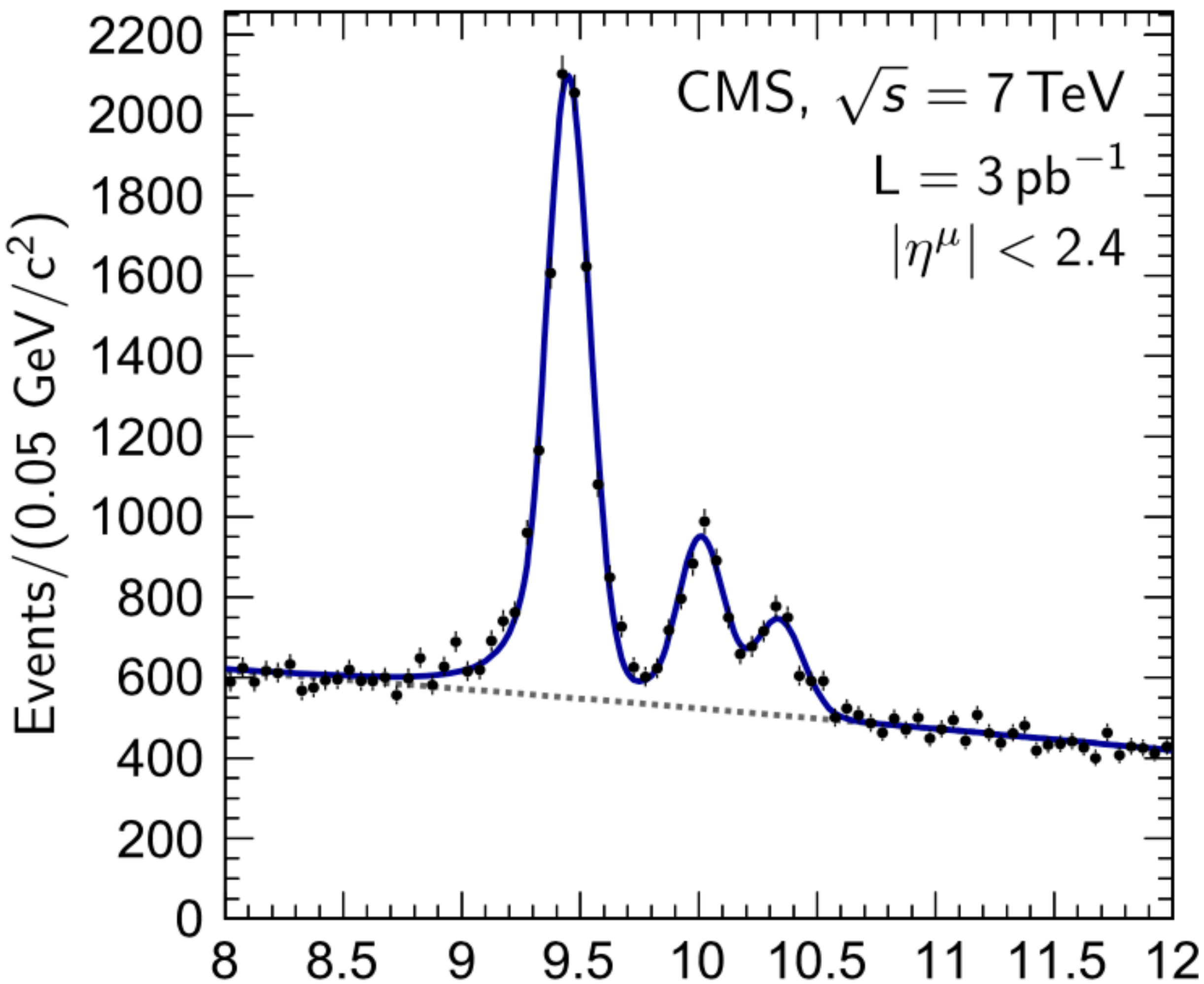
L1_DoubleMu_0
HLT_DoubleMu_0_MassY

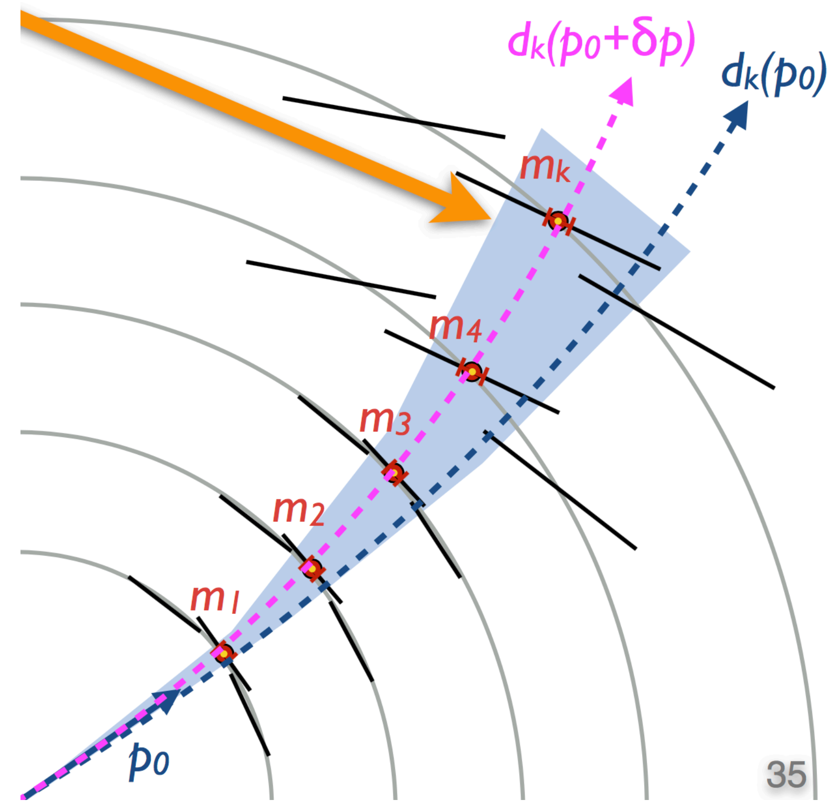


(Di)Muon signal in detector



di-muon Invariant Mass





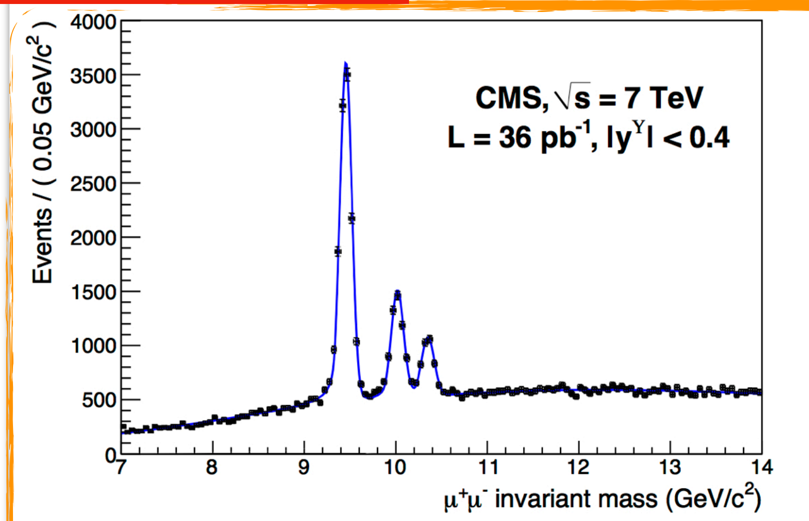
- Statistical procedure: extended unbinned maximum likelihood (EUML)

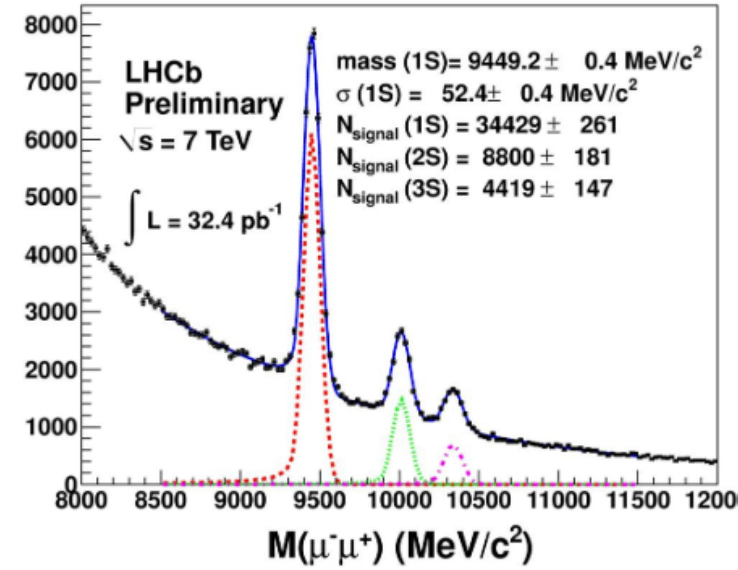
$$\mathcal{L}(\vec{\lambda}|\{m_i\}) = \left(\prod_{i=1}^{N_{\text{obs}}} \sum_{\alpha} N_\alpha \mathcal{P}_\alpha(m_i|\vec{\lambda}) \right) \times \frac{e^{-N} N^{N_{\text{obs}}}}{N_{\text{obs}}!}$$

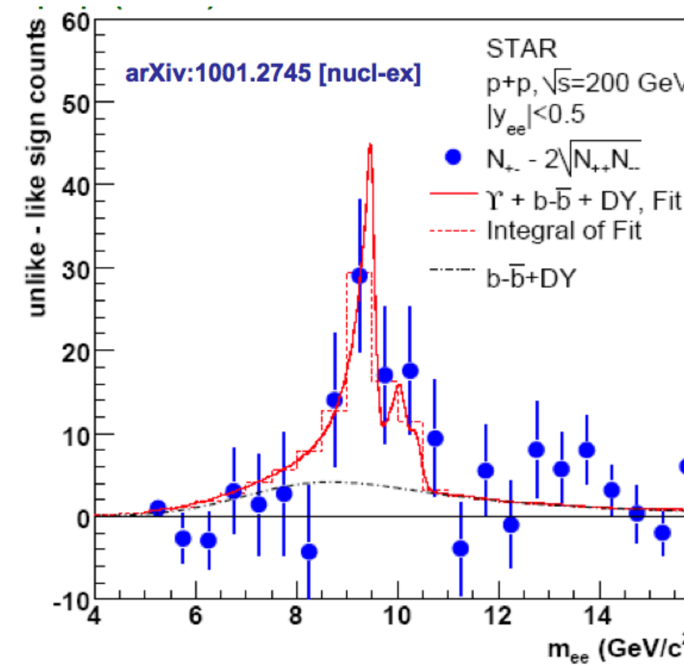
- Fit PDFs: Signal (3 x Crystal Ball) and background (polynomial) models
  - yield ($N \pm \sigma_{stat}$), mass ($m \pm \sigma_m$)

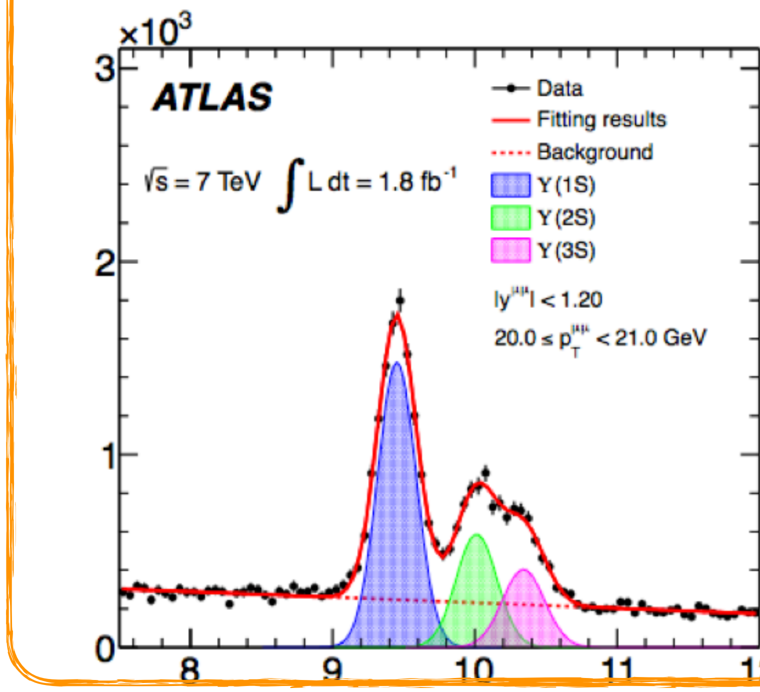
Detector resolution

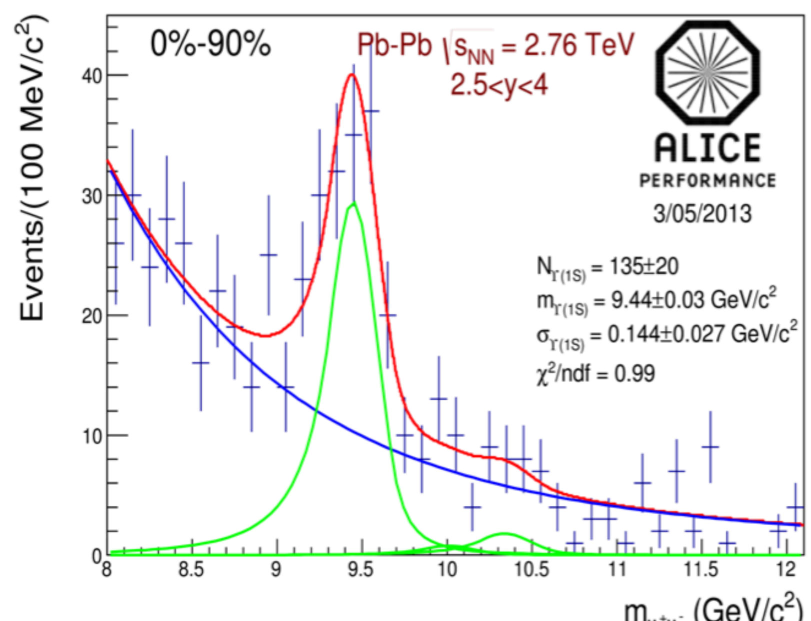
LHC

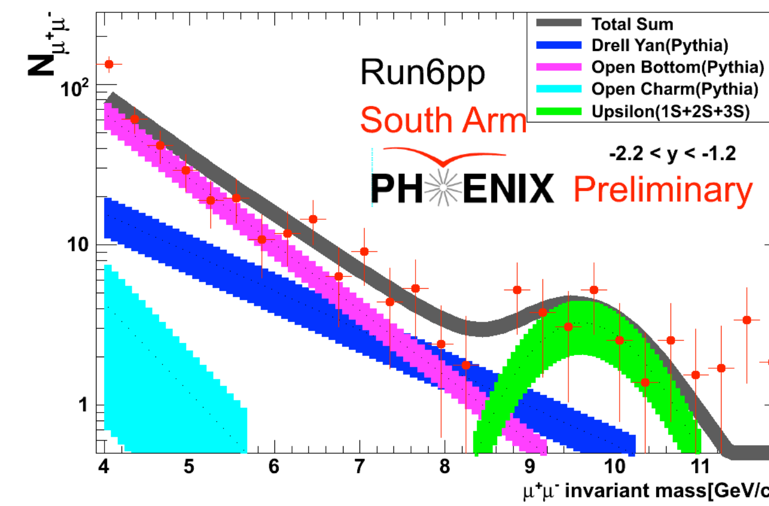


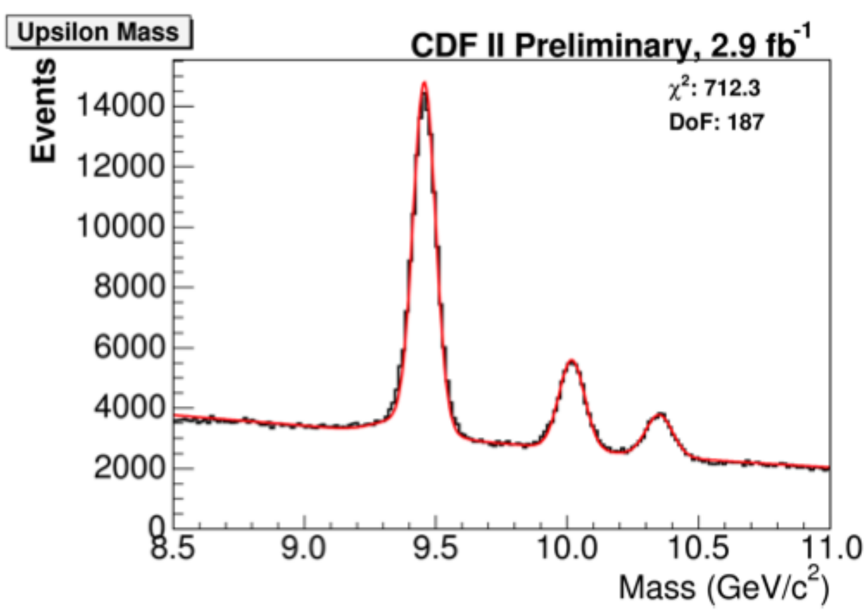


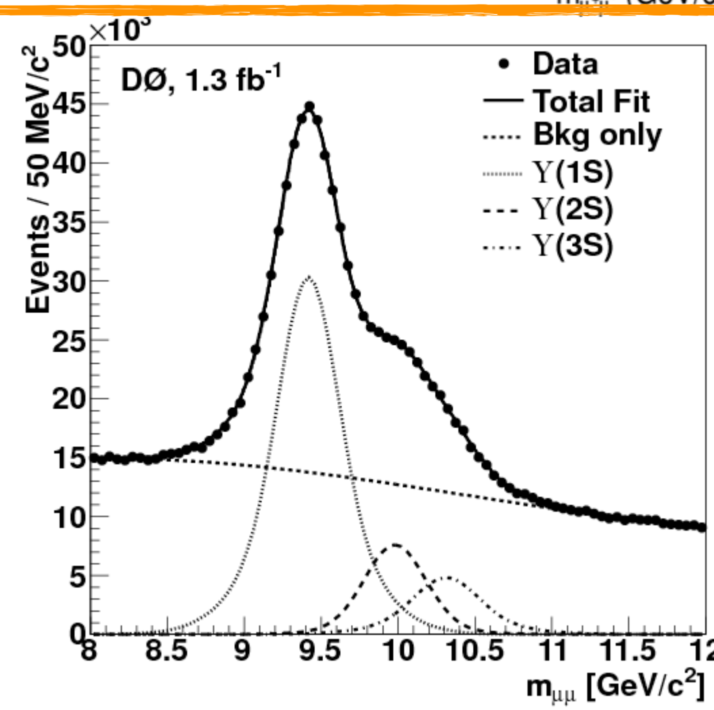


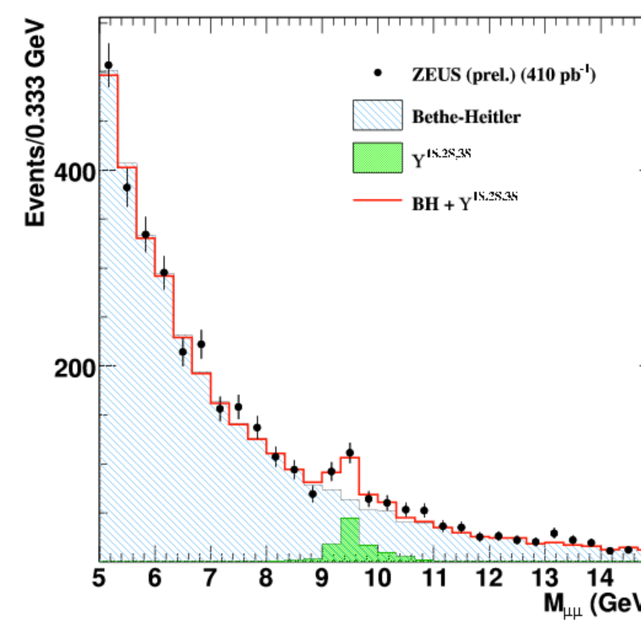


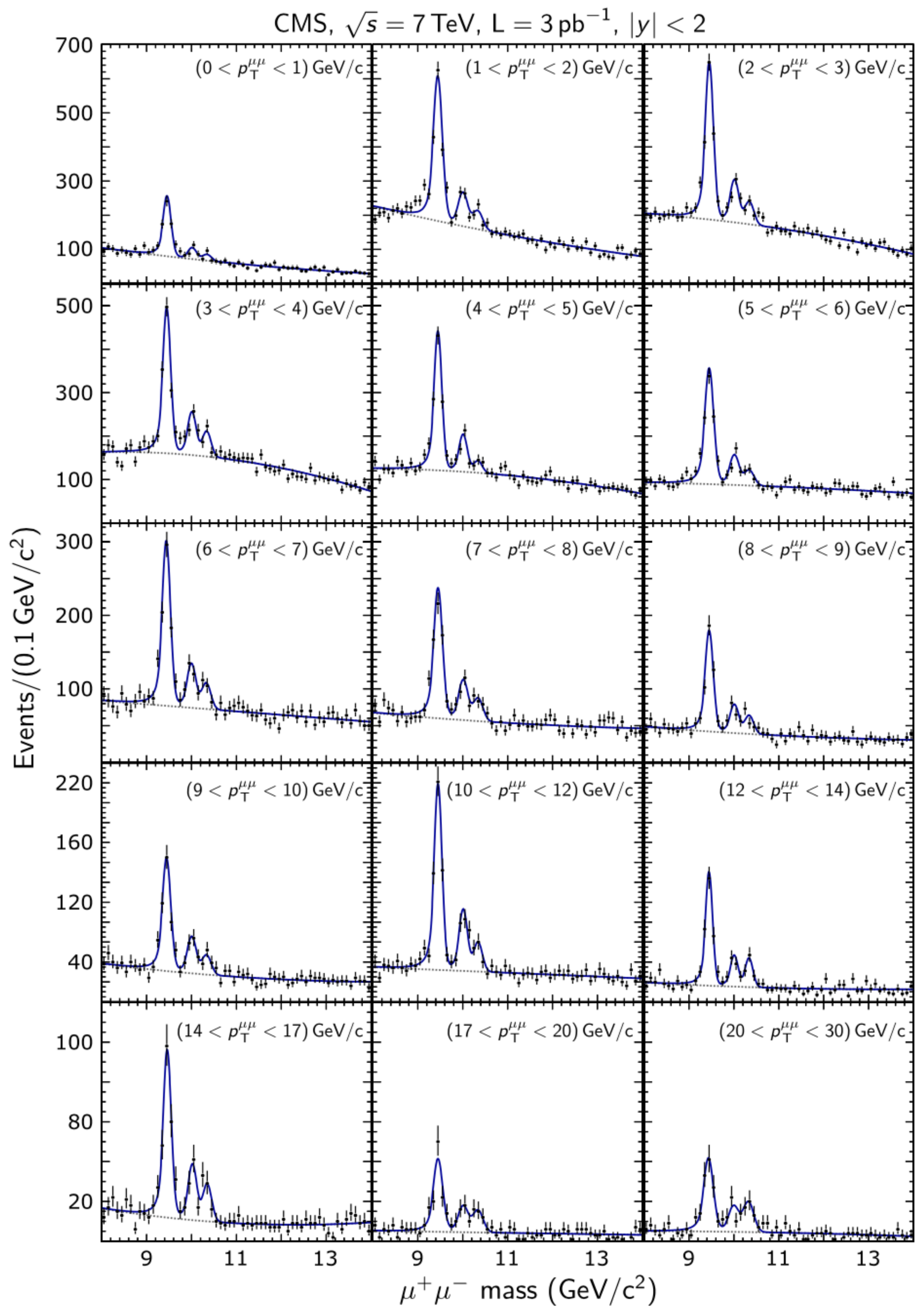












Differential
yield measurement

- Perform measurement as function of given observable, e.g. pT
- Split the dataset in ranges
- Perform analysis produce in each range

- note backgrounds levels, resolutions vary!

production cross section measurement, σ

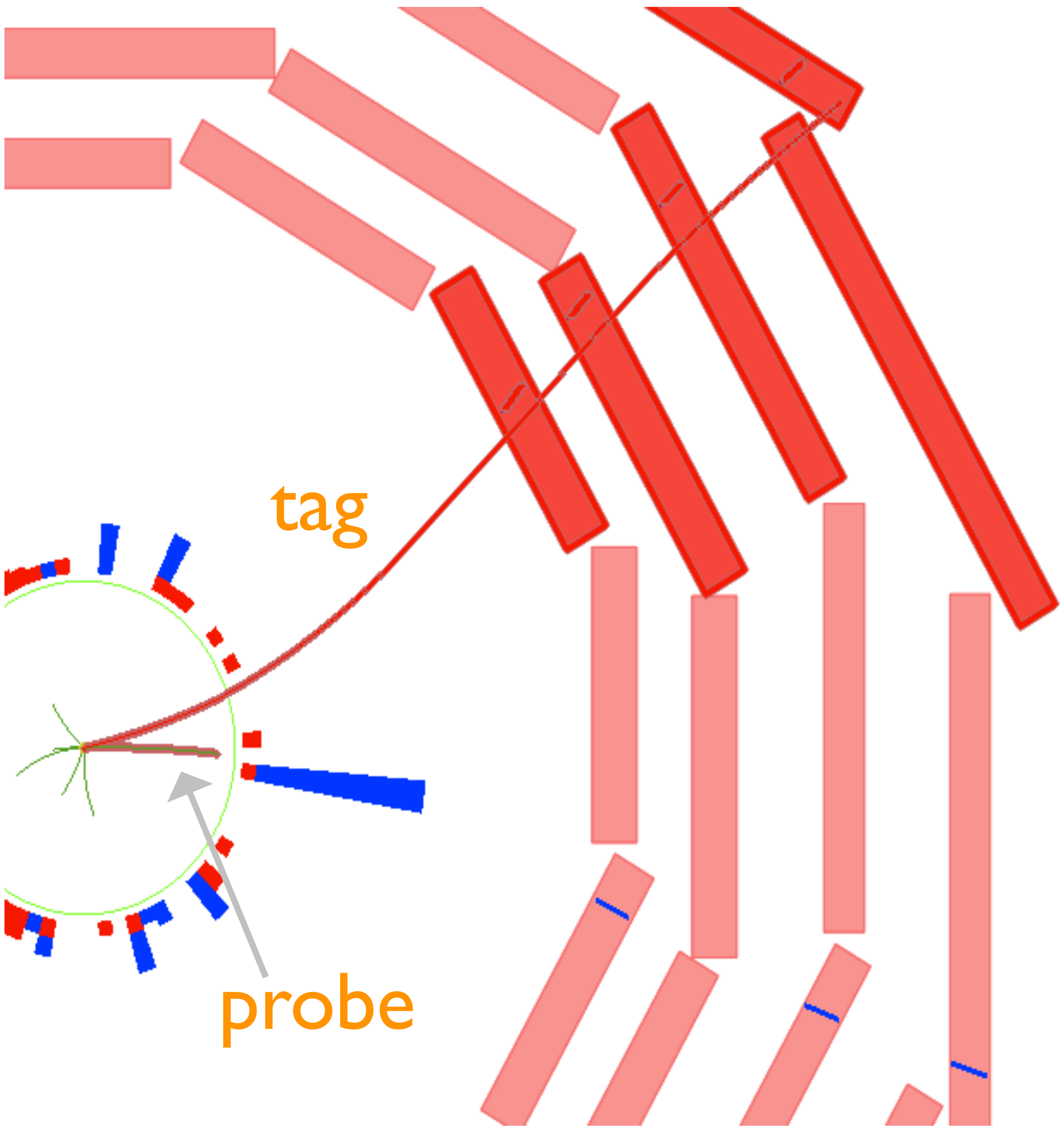
N = L x σ

$$\frac{d^2\sigma(Q\overline{Q})}{dp_T dy}\mathcal{B}\left(Q\overline{Q}\to\mu^+\mu^-\right) = \frac{N_{fit}(Q\overline{Q})}{\mathcal{L}\cdot\mathcal{A}\cdot\epsilon\cdot\Delta p_T\cdot\Delta y}$$

1. Trigger (online selection)
2. Muon identification and reconstruction
3. Y meson candidate reconstruction
4. Extract signal (offline selection) **N**
5. Determine detector acceptance A and selection efficiency ε
6. Luminosity L and branching fraction B <— external input
7. Systematic uncertainties (on N, A, ε, L, B)
8. Compare to theory predictions (NRQCD)

tag & probe

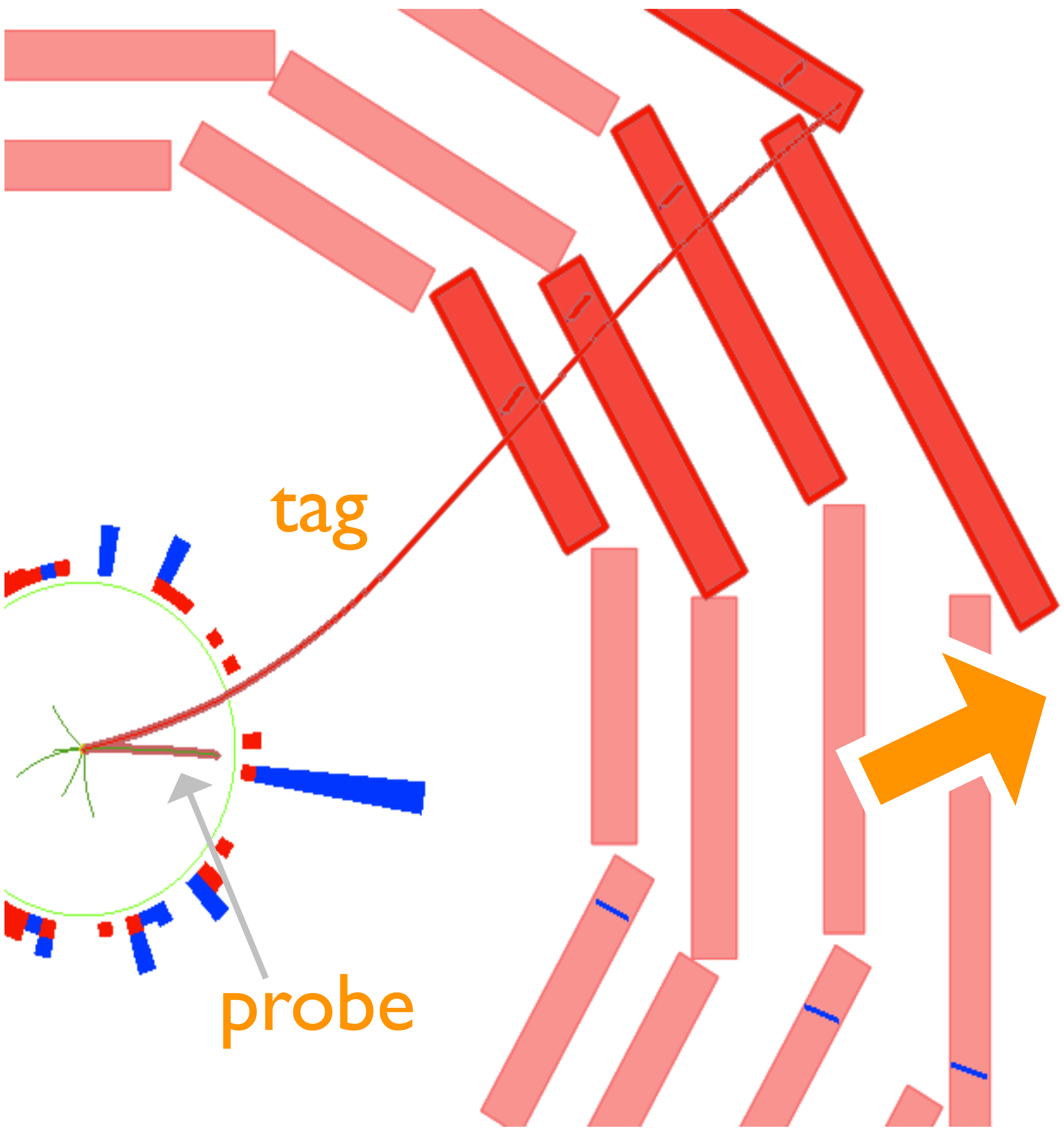
explore J/ψ, Υ, $Z \rightarrow \mu\mu$

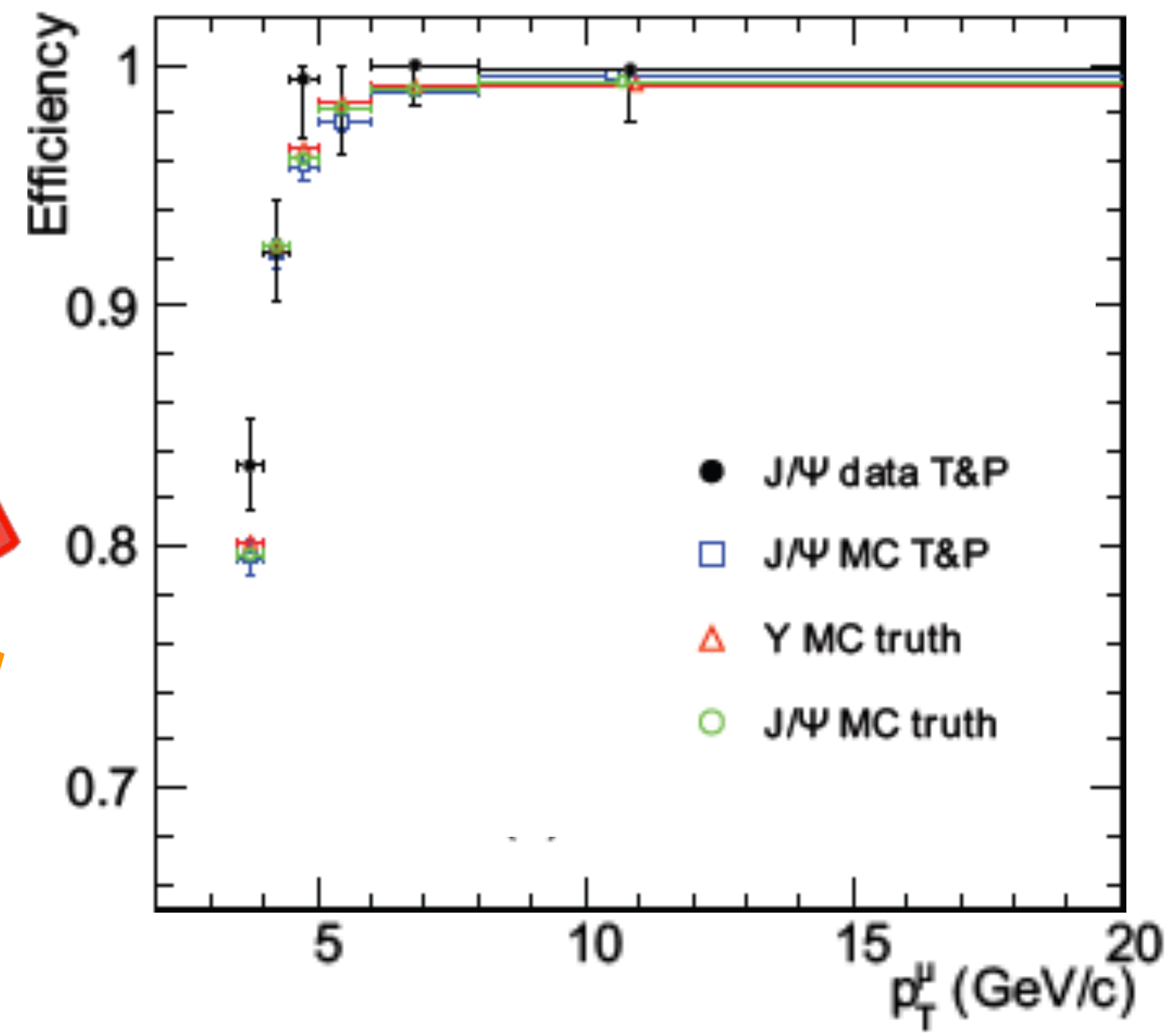


- example:
  μ-identification efficiency
- data:
  - single-muon trigger dataset
- tag
  - good global muon
  - matched to single-muon path to remove trigger bias
- probe
  - inner track
  - passing criteria: identified as muon

tag & probe

explore J/ψ, ϒ, Z→ μμ

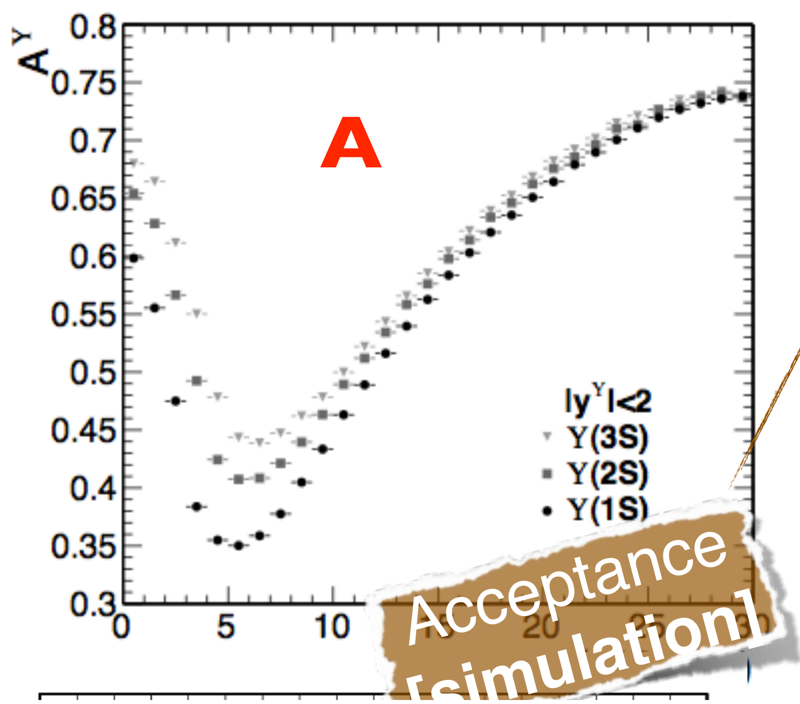


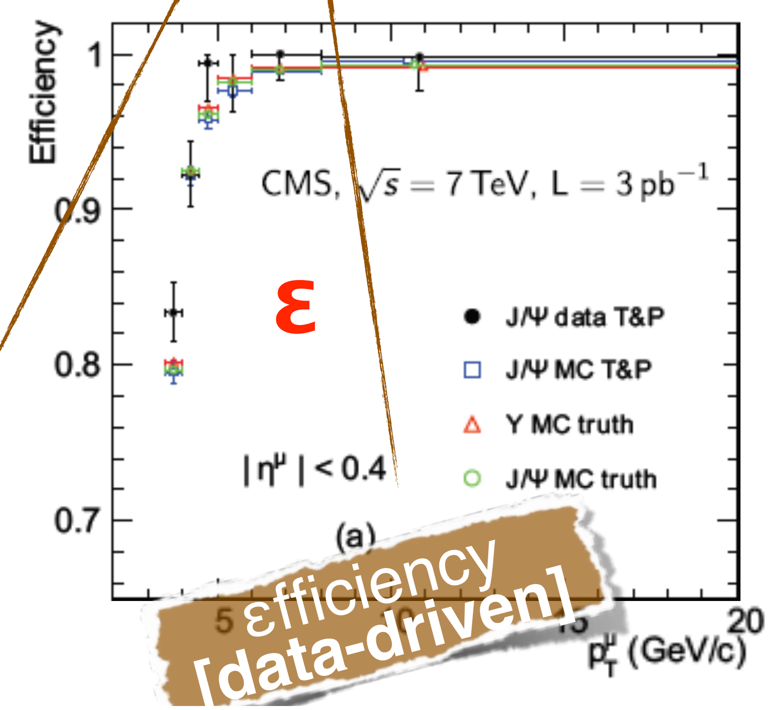


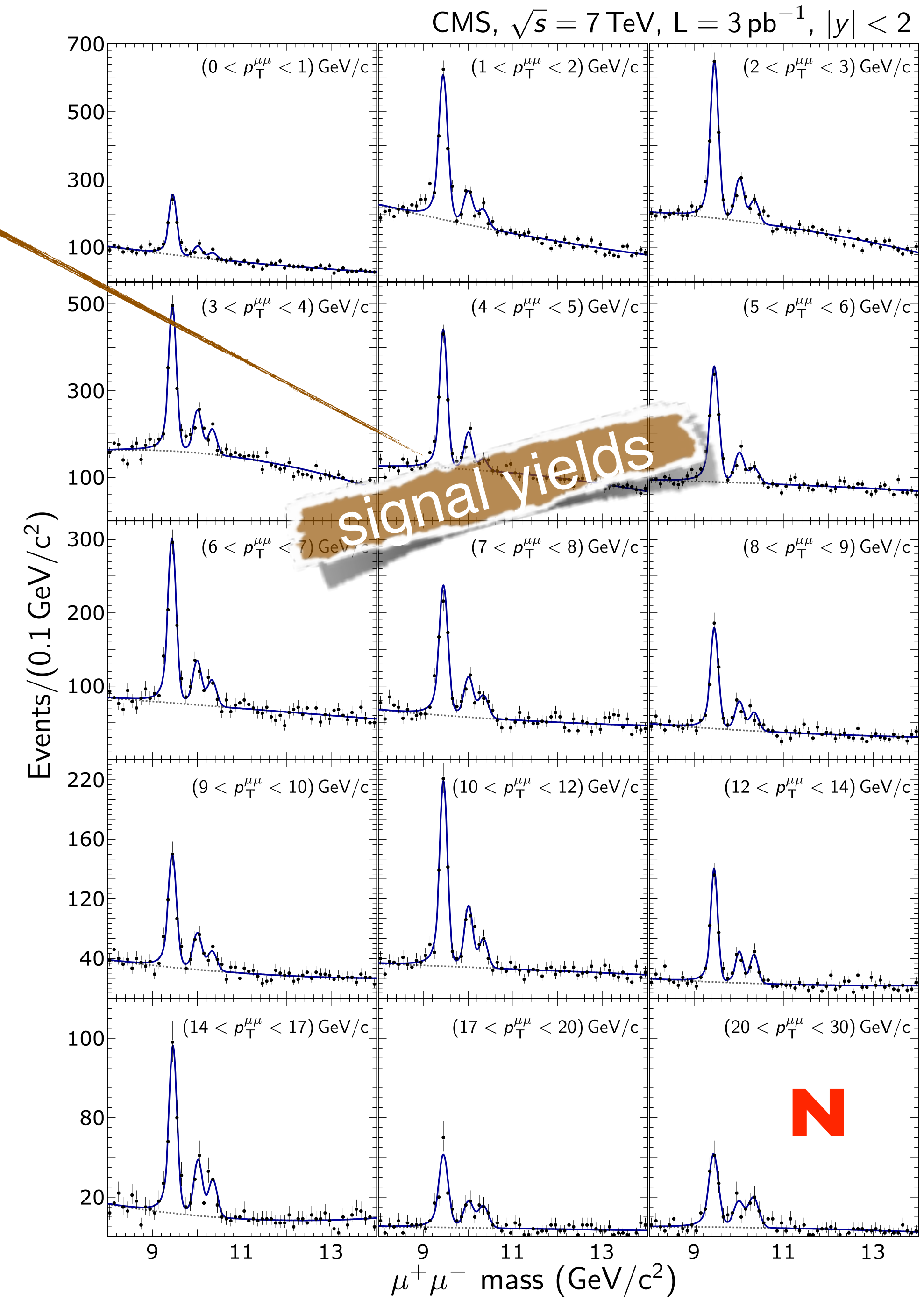
cross section

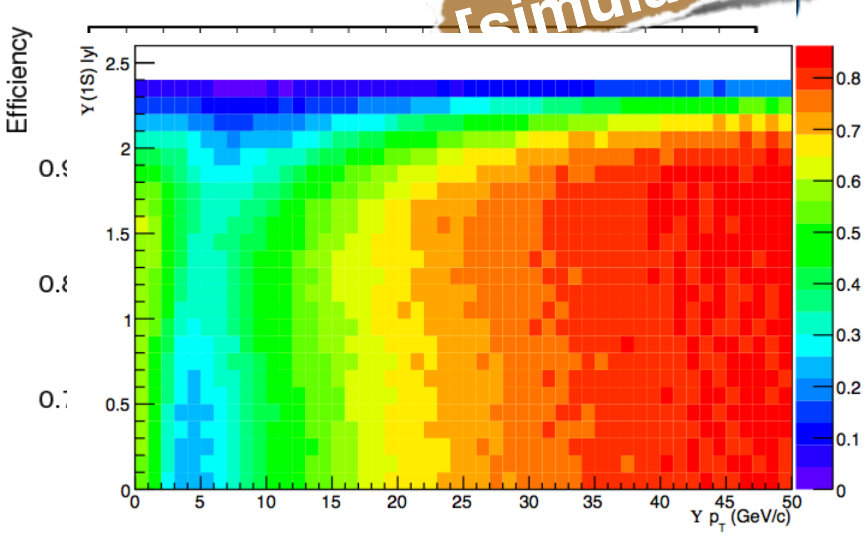
"N=L.σ"

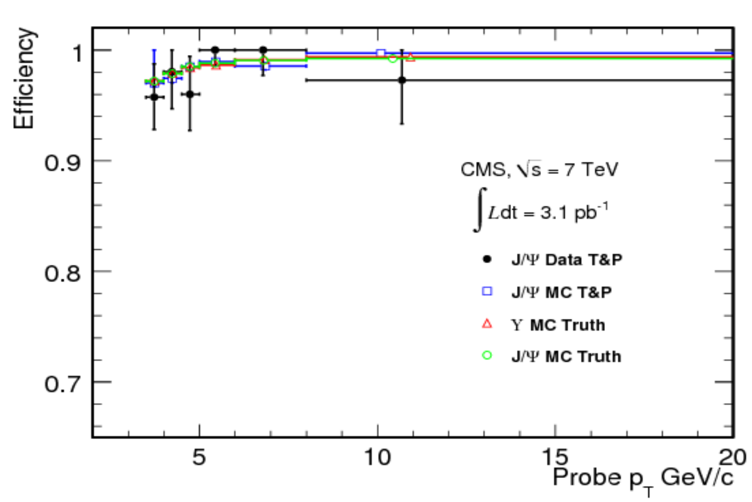
$$\frac{d^2\sigma(Q\overline{Q})}{dp_T\,dy}\mathcal{B}\left(Q\overline{Q}\rightarrow\mu^+\mu^-\right)=\frac{N_{fit}(Q\overline{Q})}{\mathcal{L}\;\cdot\mathcal{A}\cdot\epsilon\cdot\Delta p_T\cdot\Delta y}$$







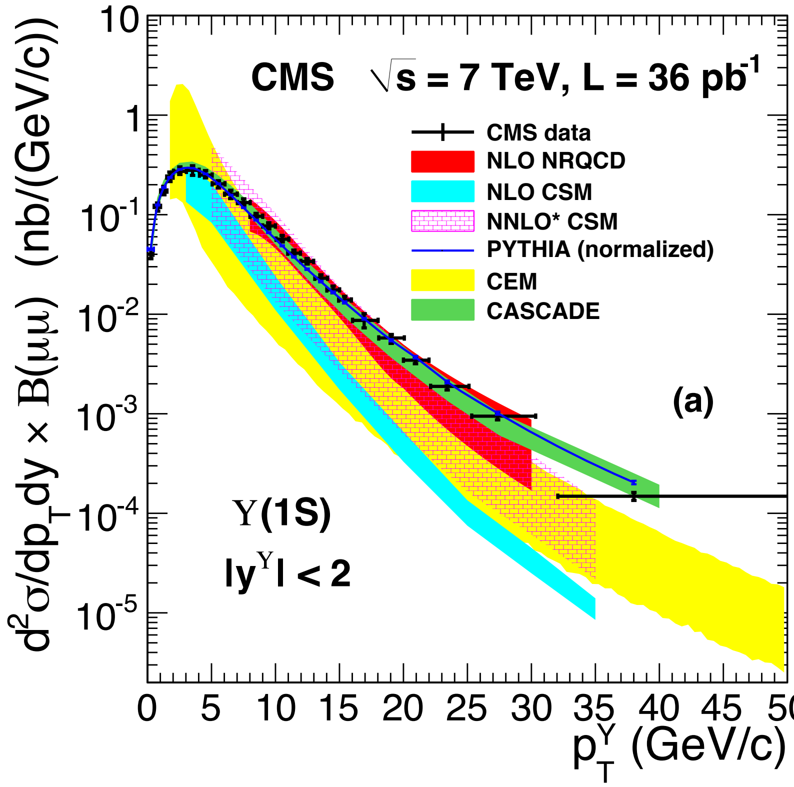




Systematics

- quantify all possible sources of systematic uncertainty
- signal yield, **N**
  - signal & background PDF
  - resolution, FSR, …
  - fit procedure, possible bias
- acceptance, **A**
  - polarization, limited MC statistics
- efficiency, ε
  - data vs MC, T&P stat and syst errors, limited size of T&P calibration sample
- luminosity, **L**
  - stat & syst err on L (measured separately) give syst error on x-section
- branching fraction
  - world average (PDG) uncertainty
- finally, can then quote Υ measured production cross section:
  - $N \pm \sigma_{stat} \pm \sigma_{syst}$

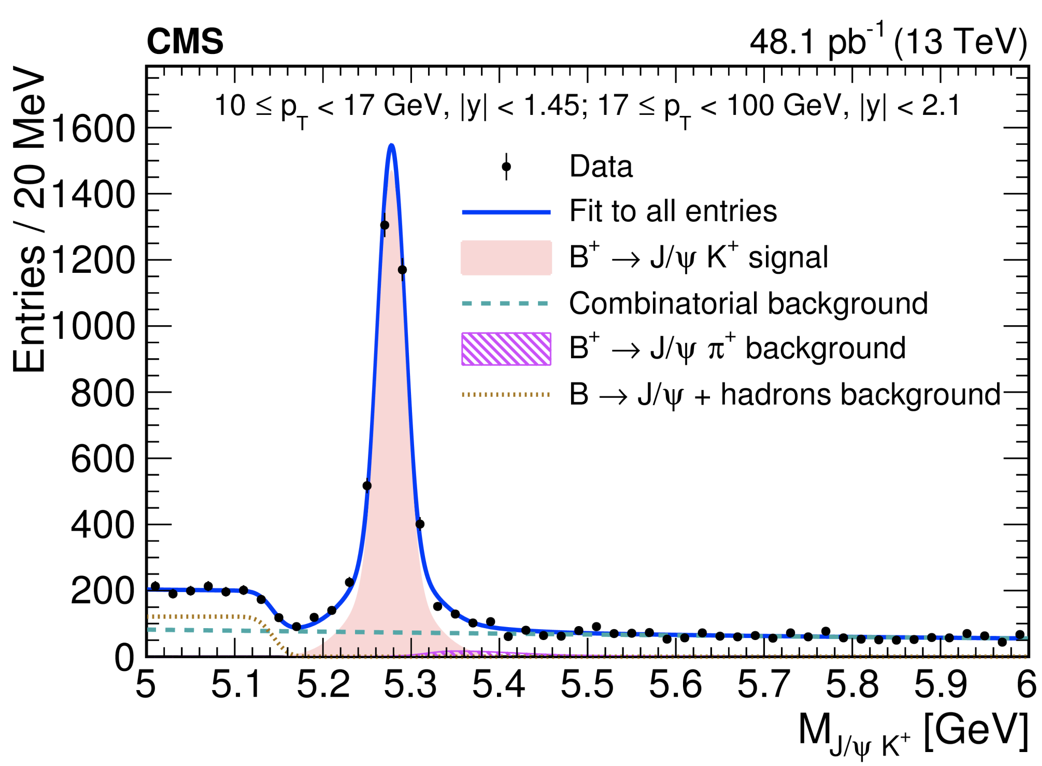
b cross sections

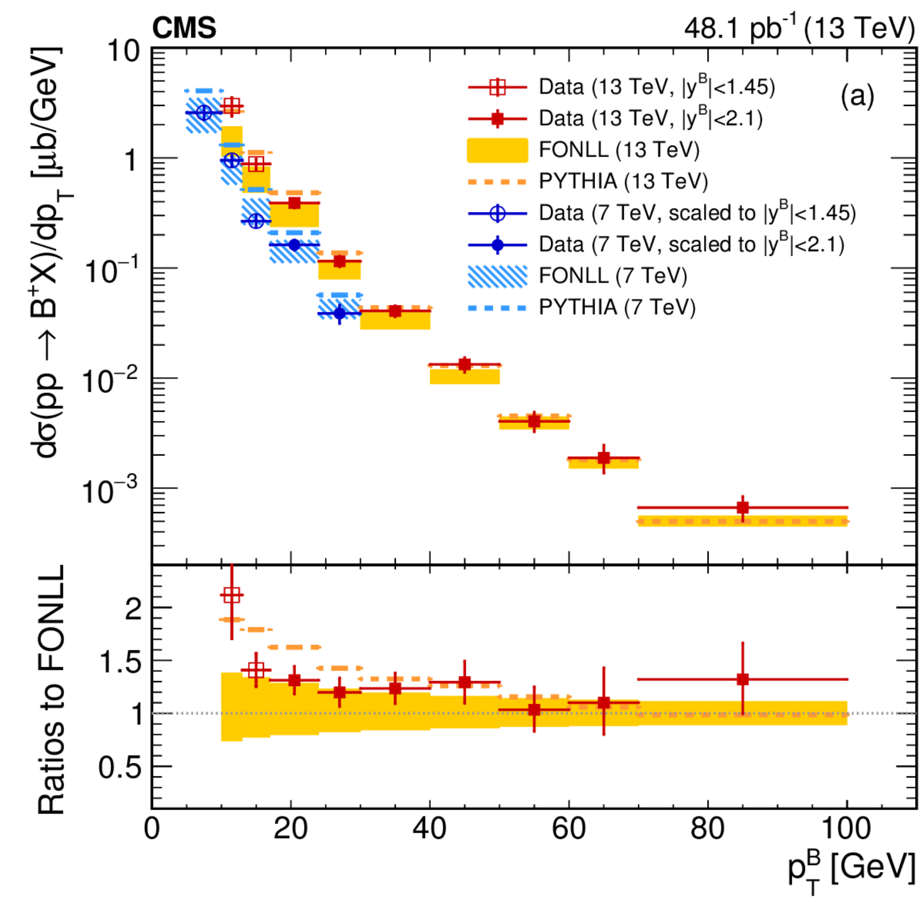


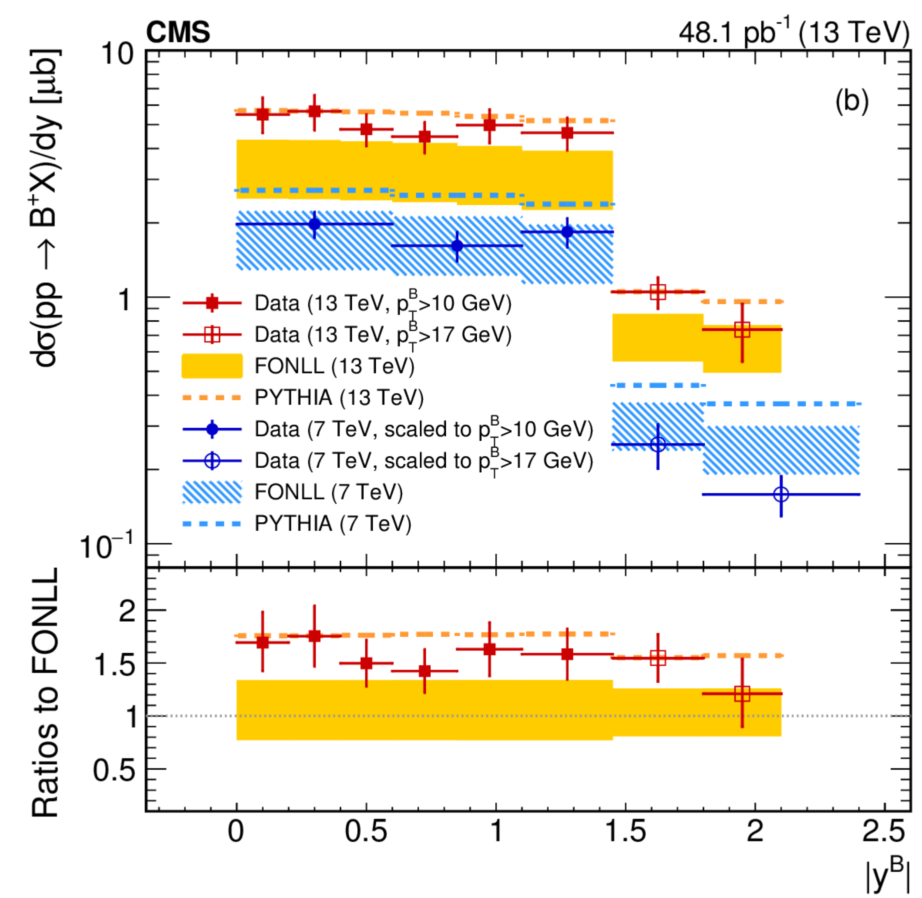
<- Hidden flavour

Open flavour ->

Cross sections
vs momentum ->
vs rapidity ->
vs √s ->

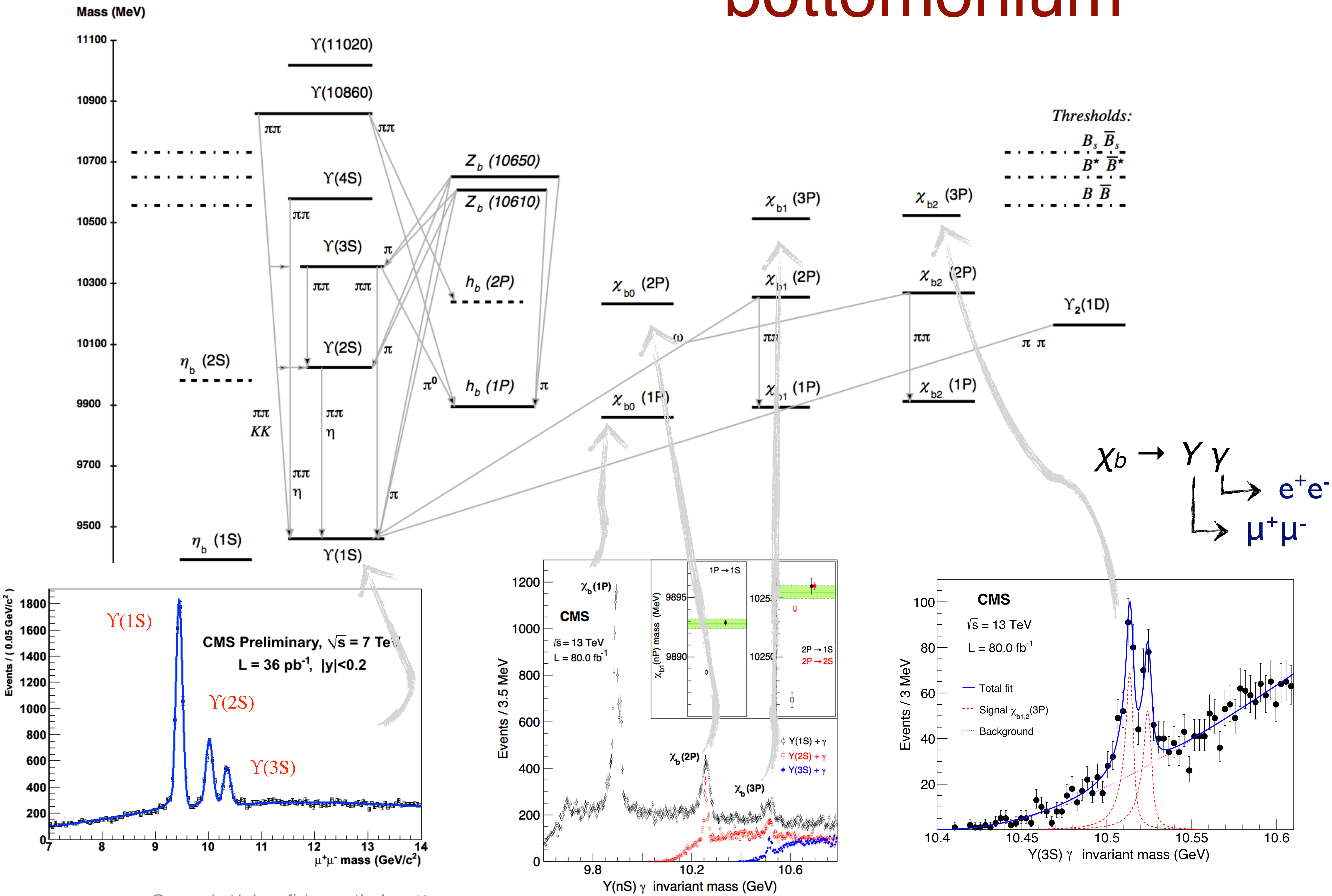






spectroscopy

bottomonium



Review of bottomonium measurements from CMS

Z. Hu
Fermi National Accelerator Laboratory, Batavia, IL 60510, USA
zhenhu@fnal.gov

N. T. Leonardo
Laboratório de Instrumentação e Física Experimental de Partículas
Lisboa 1649-003, Portugal
nuno.leonardo@cern.ch

T. Liu
Fermi National Accelerator Laboratory, Batavia, IL 60510, USA

M. Haytmyradov
The University of Iowa, Iowa City, IA 52242, USA

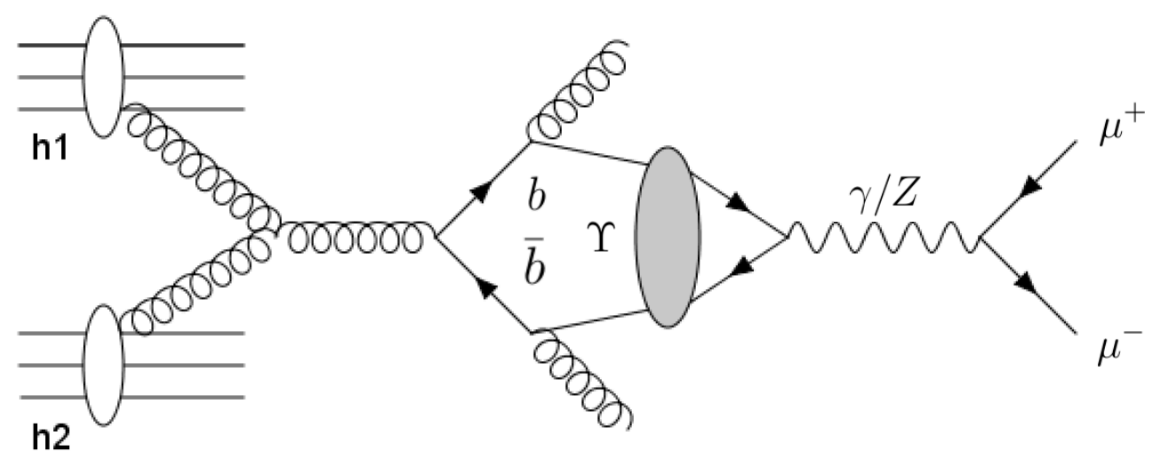
We review the results on the bottomonium system from the CMS experiment at the Large Hadron Collider. Measurements have been carried out at different center-of-mass energies in proton collisions and in collisions involving heavy ions. These include precision measurements of cross sections and polarizations, shedding light on hadroproduction mechanisms, and the observation of quarkonium sequential suppression, a notable indication of quark-gluon plasma formation. The observation of the production of bottomonium pairs is also reported along with searches for new states. We close with a brief outlook of the future physics program.

Keywords: Quarkonia; bottomonia; cross section; polarization; suppression; QGP; LHC.

PACS numbers: 14.40.Pq, 25.75.Nq, 13.85.Ni

arXiv:1708.02913

bottomonium



PHYSICAL REVIEW D **83,** 112004 (2011)

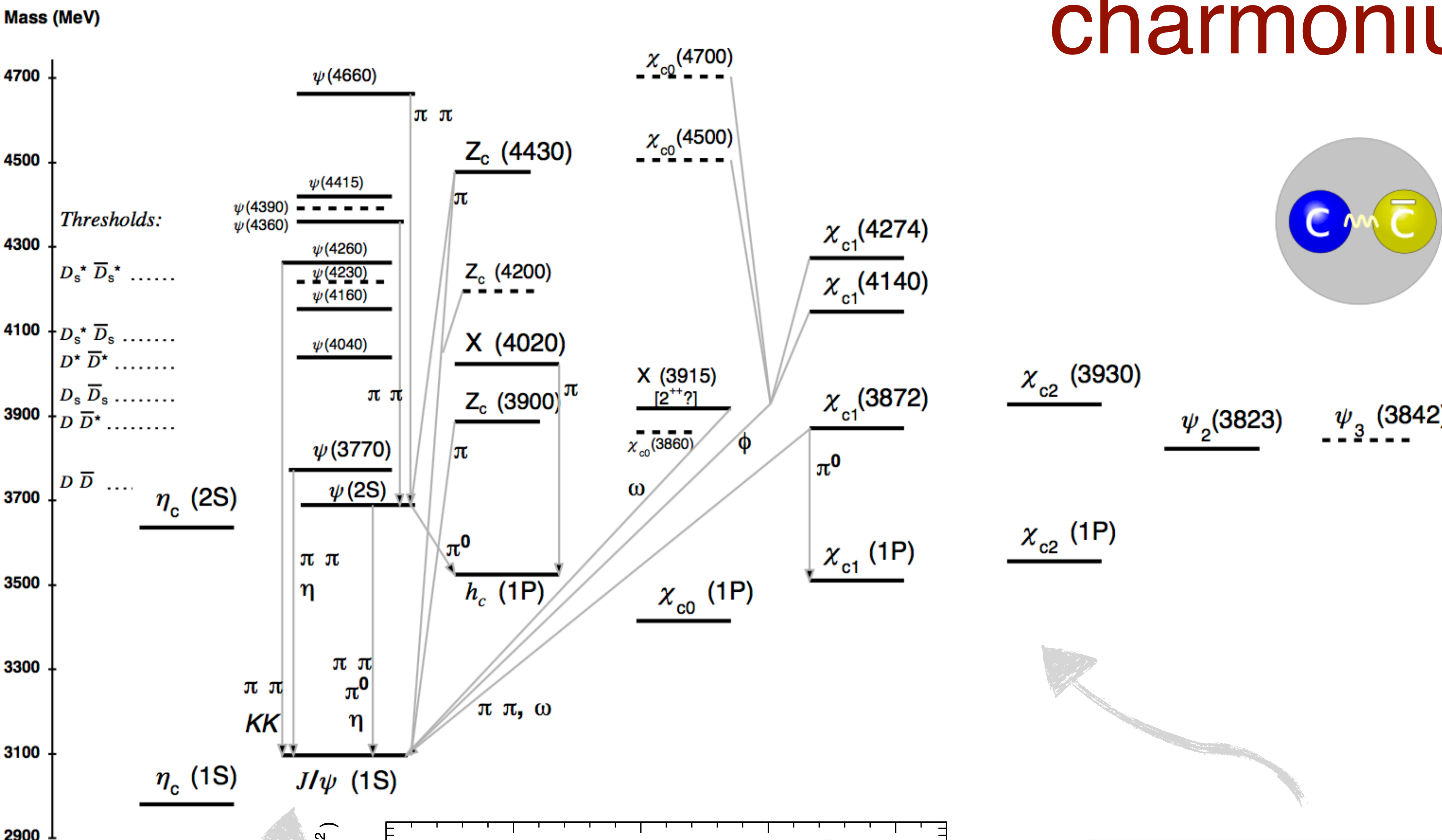
Upsilon production cross section in pp collisions at $\sqrt{s} = 7$ TeV

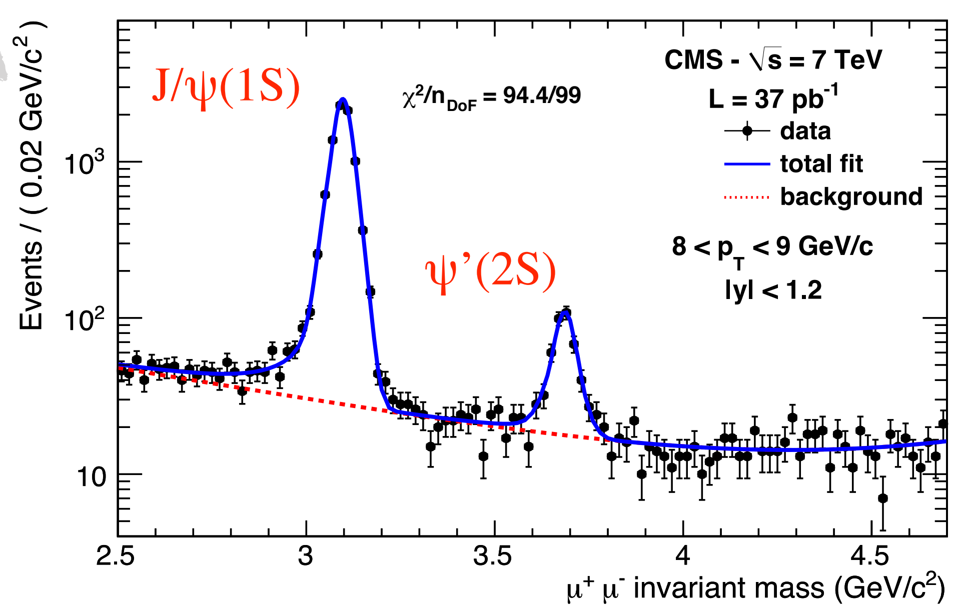
V. Khachatryan *et al.**
(CMS Collaboration)
(Received 27 December 2010; published 15 June 2011)

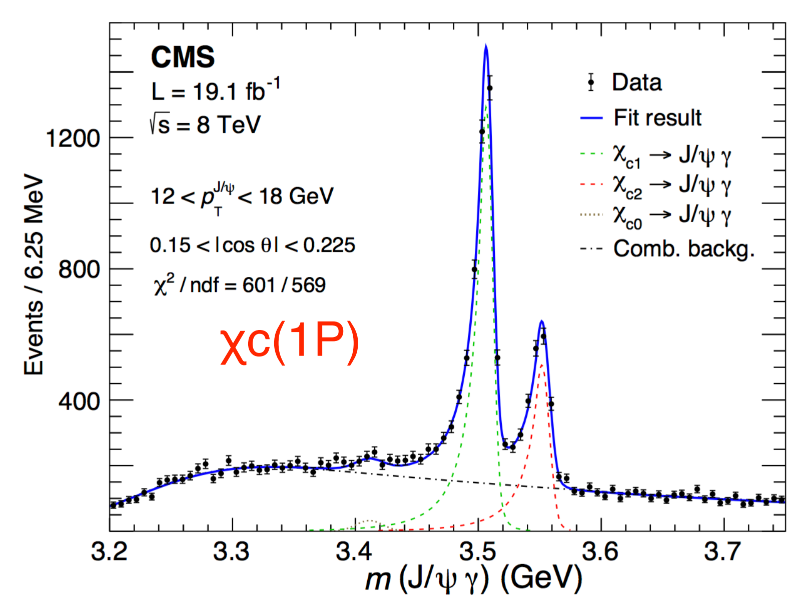
The $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$ production cross sections in proton-proton collisions at $\sqrt{s} = 7$ TeV are measured using a data sample collected with the CMS detector at the LHC, corresponding to an integrated luminosity of 3.1 ± 0.3 pb^{-1}. Integrated over the rapidity range $|y| < 2$, we find the product of the $\Upsilon(1S)$ production cross section and branching fraction to dimuons to be $\sigma(pp \to \Upsilon(1S)X) \cdot \mathcal{B}(\Upsilon(1S) \to \mu^+\mu^-) = 7.37 \pm 0.13^{+0.61}_{-0.42} \pm 0.81$ nb, where the first uncertainty is statistical, the second is systematic, and the third is associated with the estimation of the integrated luminosity of the data sample. This cross section is obtained assuming unpolarized $\Upsilon(1S)$ production. With the assumption of fully transverse or fully longitudinal production polarization, the measured cross section changes by about 20%. We also report the measurement of the $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$ differential cross sections as a function of transverse momentum and rapidity.

arXiv:1012.5545

charmonium

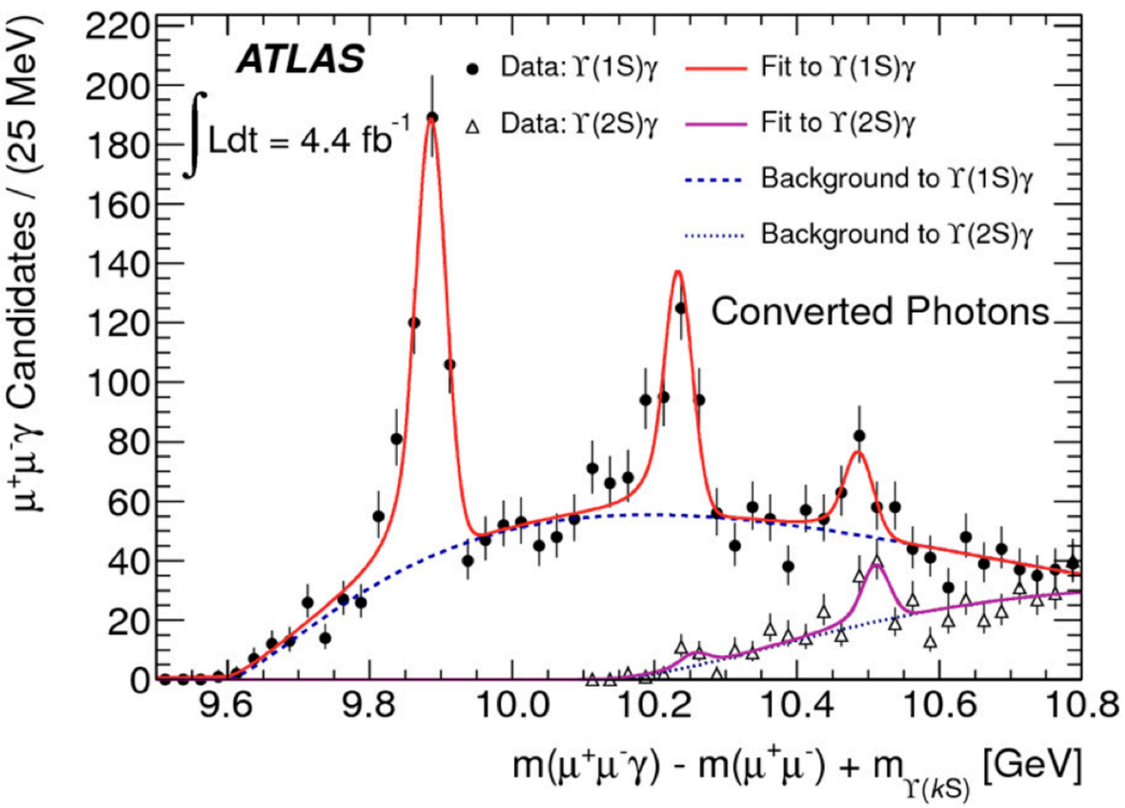

Mass (MeV)
4700
4500
4300
4100
3900
3700
3500
3300
3100
2900
Thresholds:
$D_s^* \bar{D}_s^*$
$D_s^* \bar{D}_s$
$D^* \bar{D}^*$
$D_s \bar{D}_s$
$D \bar{D}^*$
$D \bar{D}$
η_c (2S)
η_c (1S)
J/ψ (1S)
ψ(4660)
ψ(4415)
ψ(4390)
ψ(4360)
ψ(4260)
ψ(4230)
ψ(4160)
ψ(4040)
ψ(3770)
ψ(2S)
Z_c (4430)
Z_c (4200)
X (4020)
Z_c (3900)
h_c (1P)
χ_{c0}(4700)
χ_{c0}(4500)
X (3915)
[2++?]
χ_{c0}(3860)
χ_{c0} (1P)
χ_{c1}(4274)
χ_{c1}(4140)
χ_{c1}(3872)
χ_{c1} (1P)
χ_{c2} (3930)
χ_{c2} (1P)
ψ_2(3823)
ψ_3 (3842)
π π
π
π⁰
ω
ϕ
η
KK
π π, ω
c
$\bar{c}$


J/ψ(1S)
ψ'(2S)
χ^2/n_{DoF} = 94.4/99
CMS - $\sqrt{s}$ = 7 TeV
L = 37 pb^{-1}
data
total fit
background
8 < p_T < 9 GeV/c
|y| < 1.2
Events / (0.02 GeV/c^2)
10^3
10^2
10
2.5
3
3.5
4
4.5
$\mu^+ \mu^-$ invariant mass (GeV/c^2)

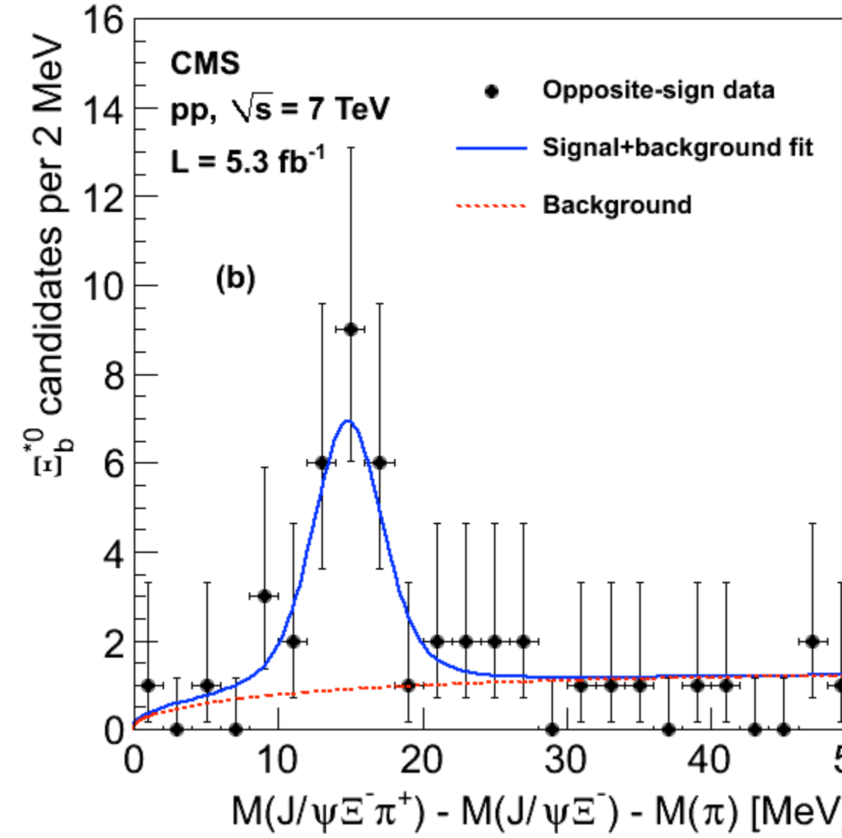

CMS
L = 19.1 fb^{-1}
$\sqrt{s}$ = 8 TeV
12 < $p_T^{J/\psi}$ < 18 GeV
0.15 < |cos θ| < 0.225
χ^2 / ndf = 601 / 569
Data
Fit result
$\chi_{c1} \to J/\psi\ \gamma$
$\chi_{c2} \to J/\psi\ \gamma$
$\chi_{c0} \to J/\psi\ \gamma$
Comb. backg.
χc(1P)
Events / 6.25 MeV
1200
800
400
3.2
3.3
3.4
3.5
3.6
3.7
m (J/ψ γ) (GeV)

the first new particles found at LHC

$\chi_b(3P)$ **meson**



Ξ_b^* **baryon**



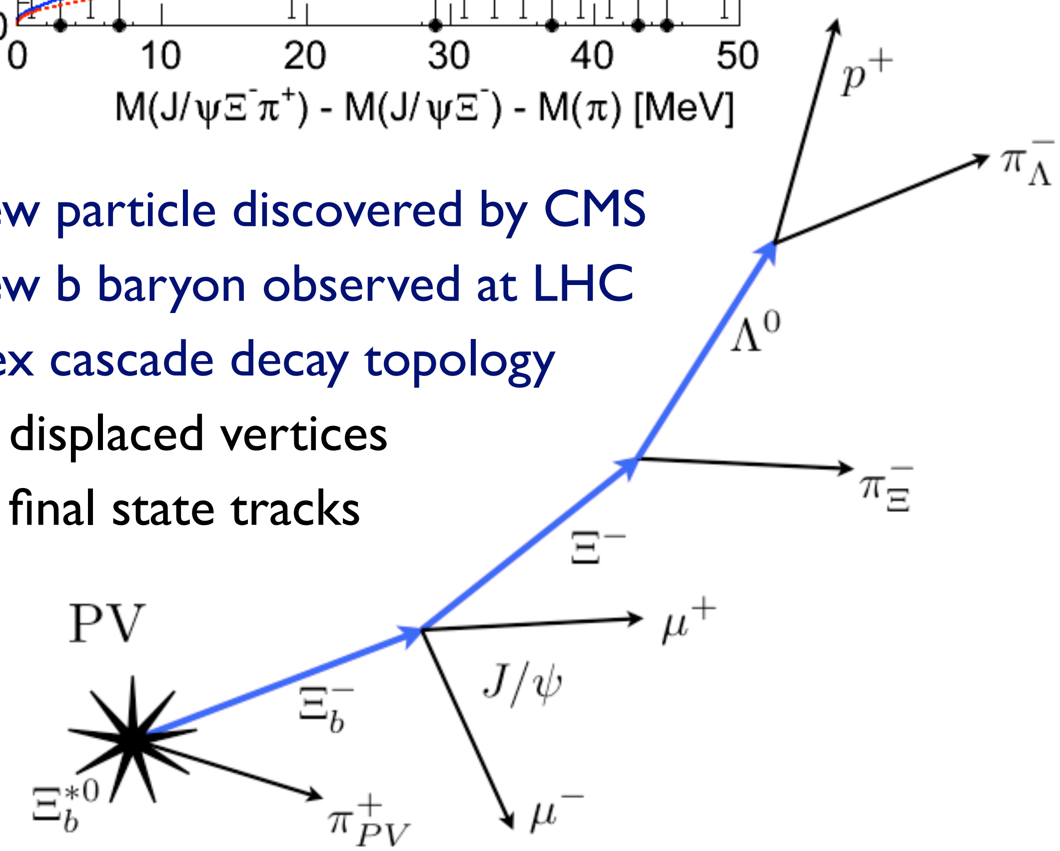
- first new particle discovered by ATLAS
- reconstruct the radiative bottomonium decay by exploring photon conversions in tracker material

$\chi_b \rightarrow \Upsilon\ \gamma$

$\gamma \rightarrow e^+e^-$

$\Upsilon \rightarrow \mu^+\mu^-$

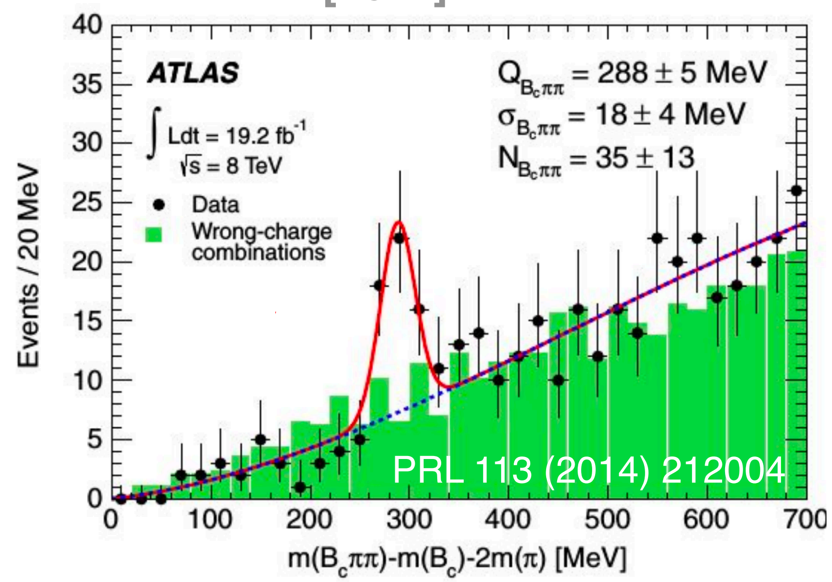
- first new particle discovered by CMS
- first new b baryon observed at LHC
- complex cascade decay topology
  - 4 displaced vertices
  - 6 final state tracks



➡ note: these (orthogonal capabilities) further illustrate the ability of general purpose detectors to make flavor discoveries

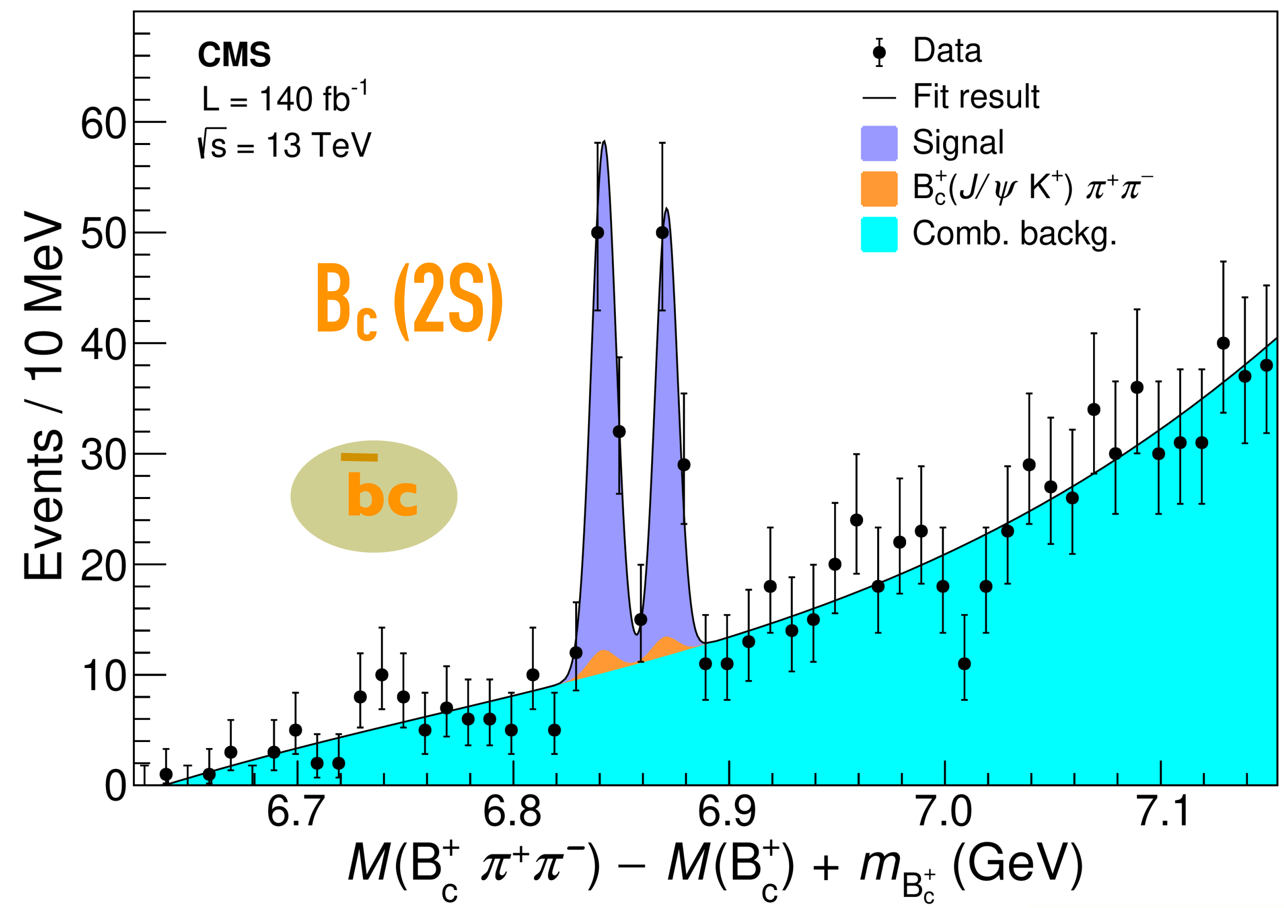
the latest new particles found @LHC

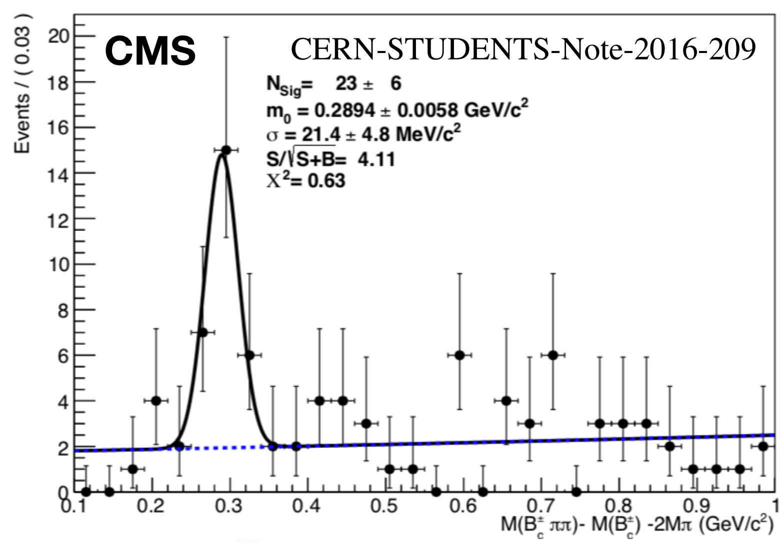
[Run1]

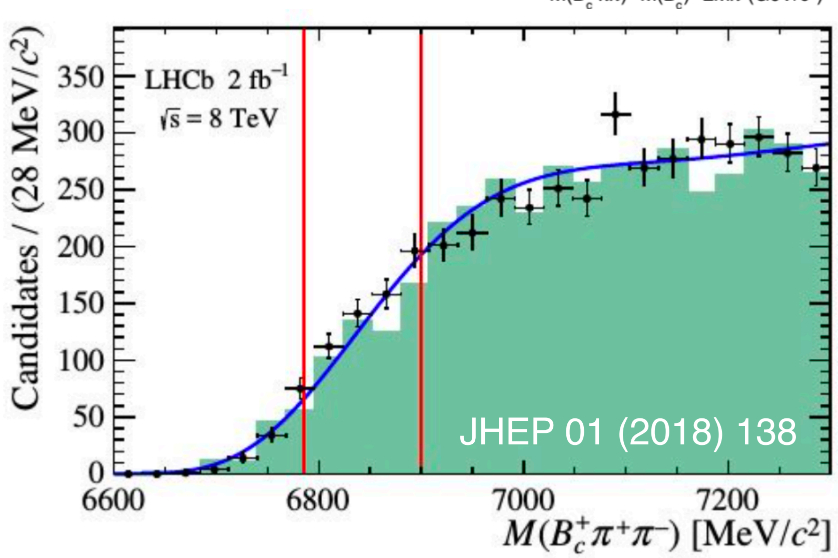


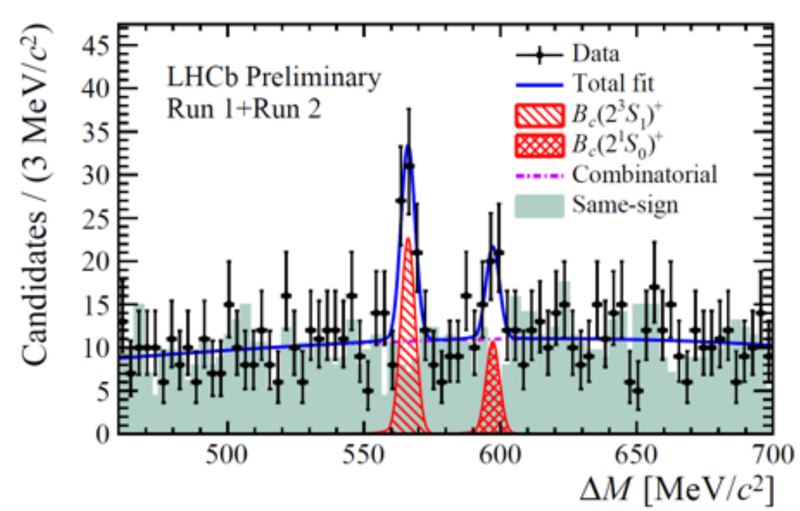
First LHC result with full Run2 dataset!

(2015+2016+2017+2018 ~140fb^{-1})

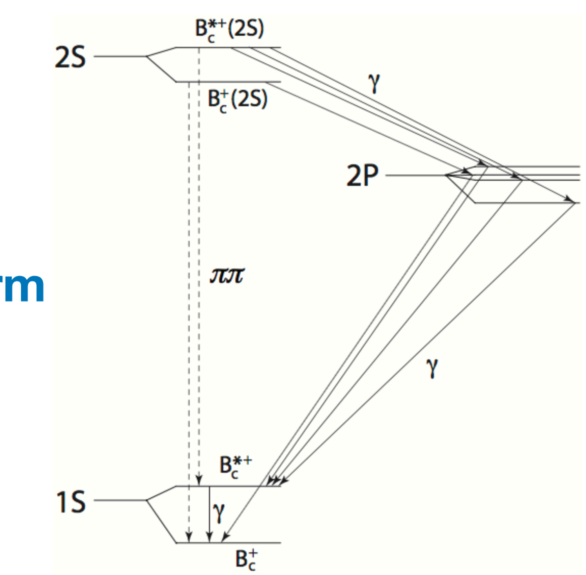






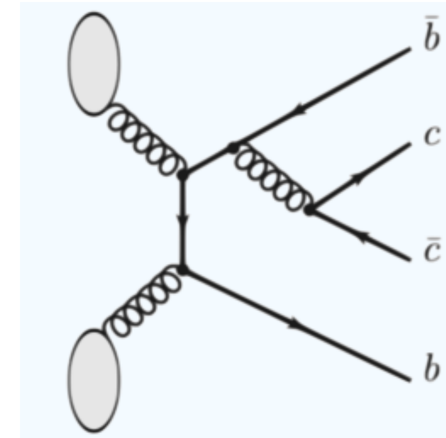


two new bottom-charm states



B_c

- meson with different heavy flavors -- unique in SM
  - sometimes also referred to as 'quarkonium': similar non-relativist potential techniques used to predict properties
  - formed of b̲+c quarks: the heaviest quark flavors expected to form mesons
  - b and c may both decay weakly
    ➠ much shorter lifetime than other B mesons
- state by now observed in several modes
  - no excited states observed yet (many expected)



$\bar{b} \to \bar{c}$ **transition**

$B_c^+ \to J/\psi\, \ell^+ \nu_\ell$

$B_c^+ \to J/\psi\, \pi^+$

$c \to s$ **transition**

$B_c^+ \to B_s^0 \pi^+$

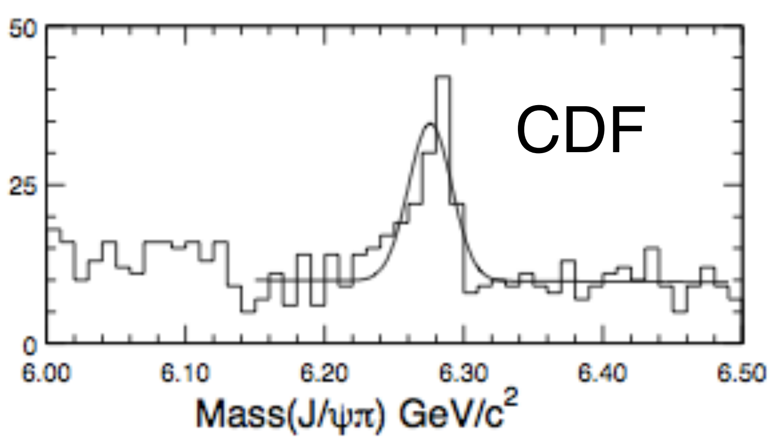
$B_c^+ \to B_s^0 \ell^+ \nu_\ell$

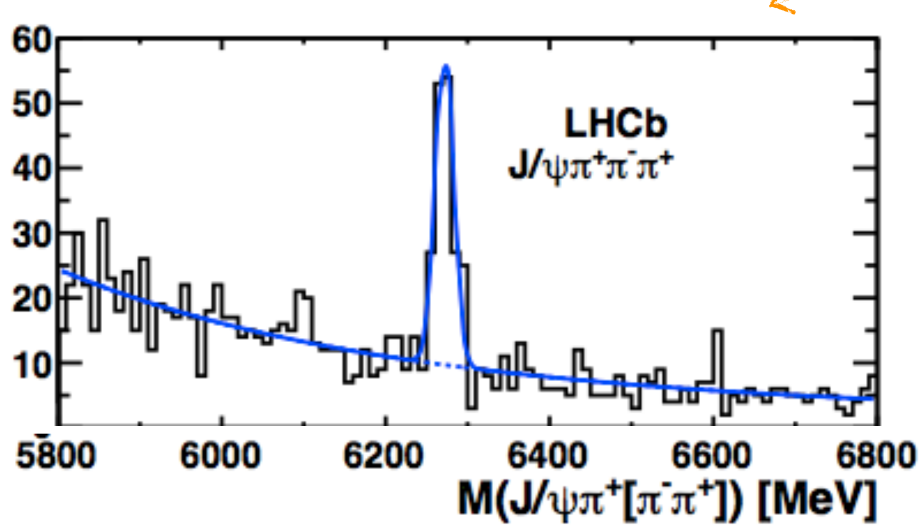
$c\bar{b} \to W^+$ **transition**

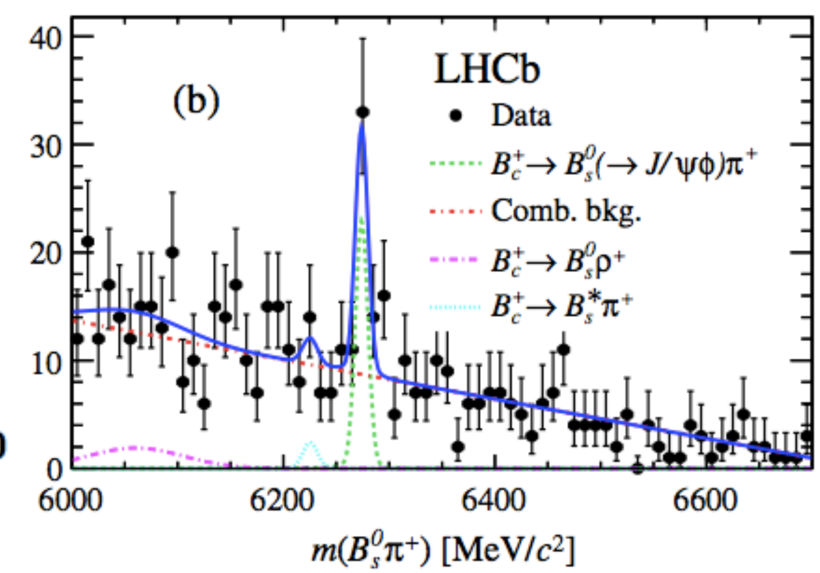
$B_c^+ \to \bar{K}^{*0} K^+$

$B_c^+ \to \phi K^+$

$B_c^+ \to \tau^+ \nu_\tau$

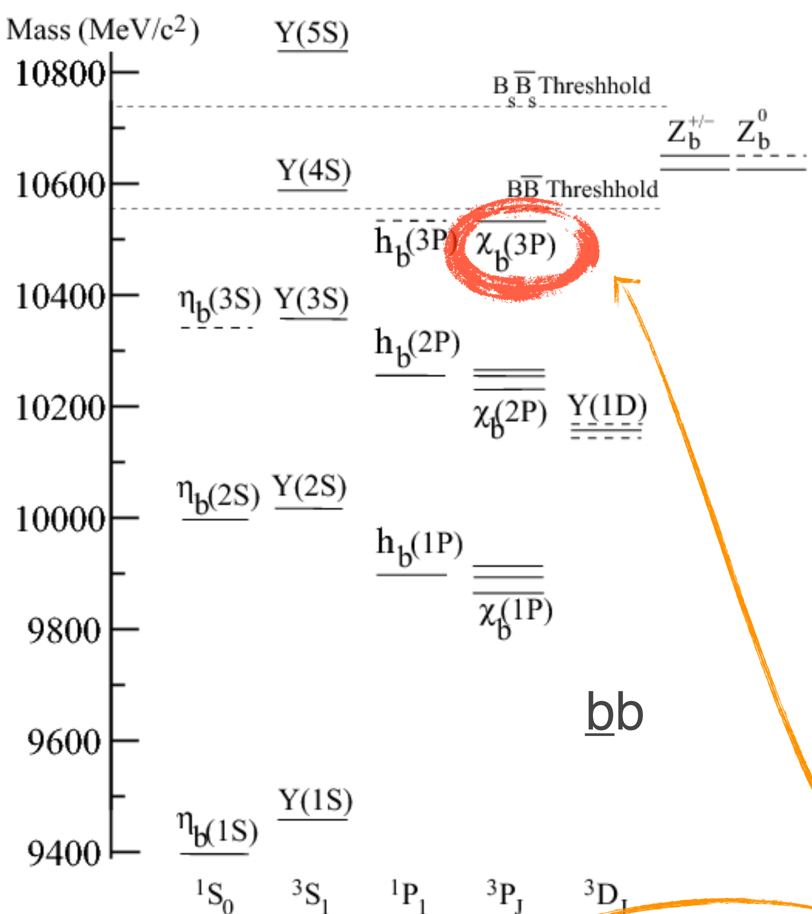




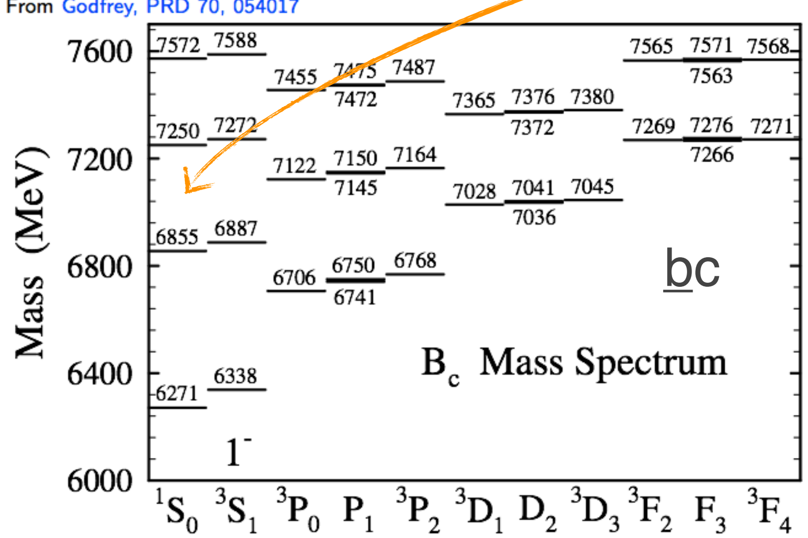


beauty spectroscopy

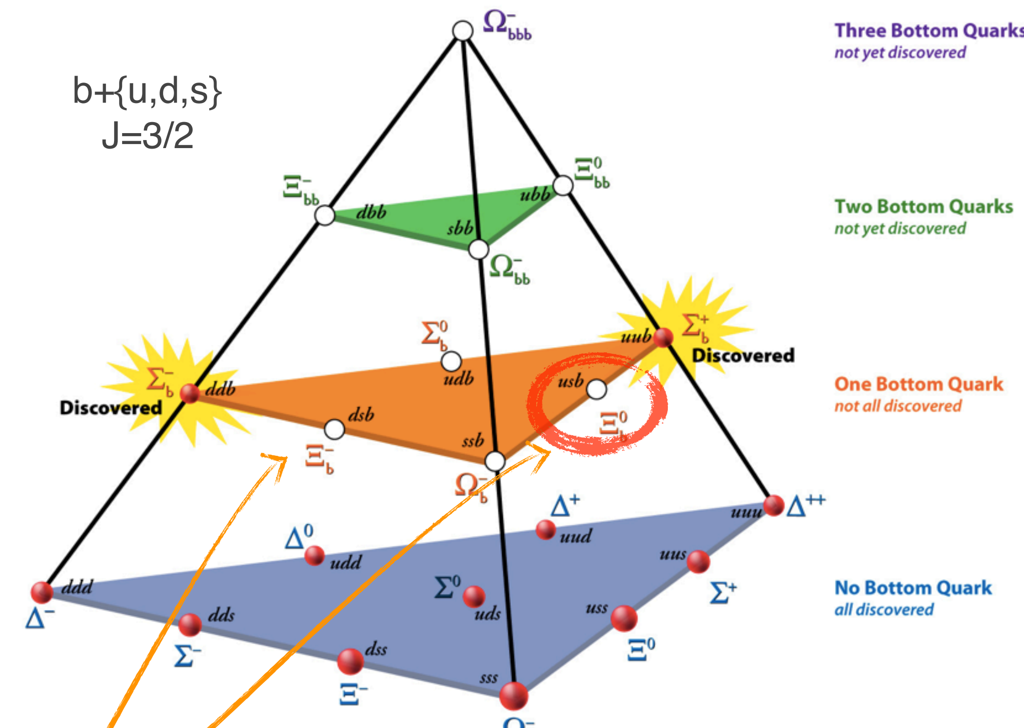
mesons

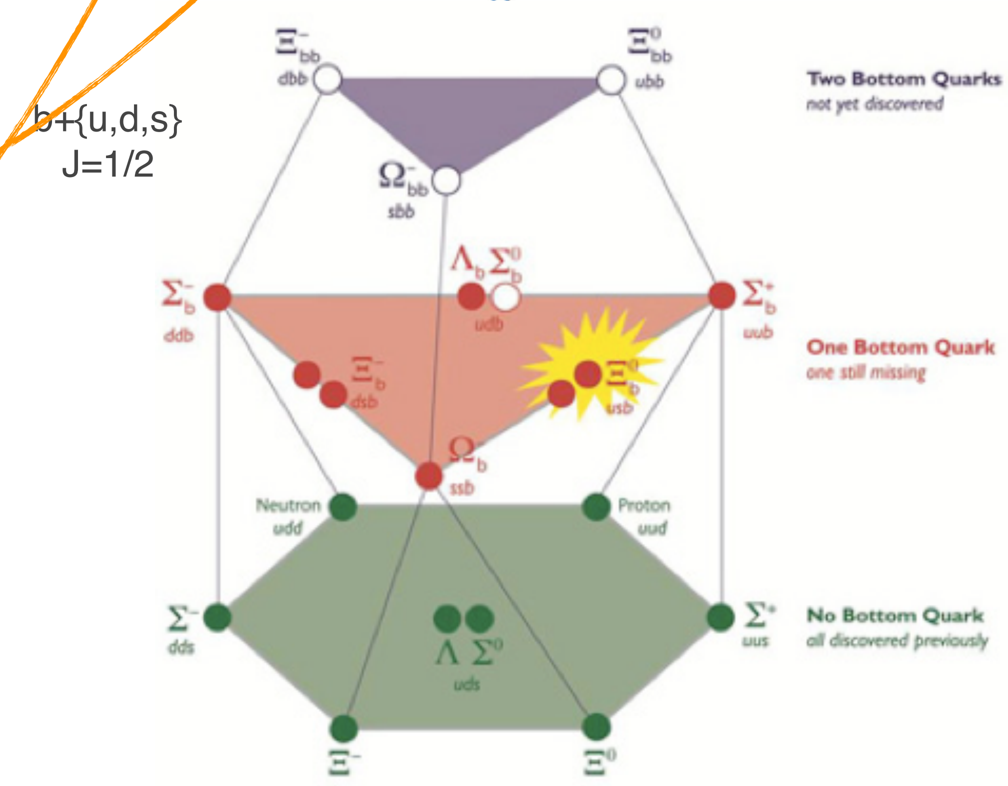


From Godfrey, PRD 70, 054017



baryons





Before 2006, only one b baryon had been seen: Λ_b

CDF and D0 contributed several such discoveries:

- Σ_b^- (2006)
- Ξ_b- (2007)
- Ω_b (2008)

LHC:

- χ_b(3P) (ATLAS' 2011)
- Ξ_b^{*0} (CMS'2012)
- B_c(2S) (ATLAS'2014)
- Ξ_b^{*-} $\Xi_b^{'-}$ (LHCb'2014)

Several other composite **particles awaiting to be discovered**!

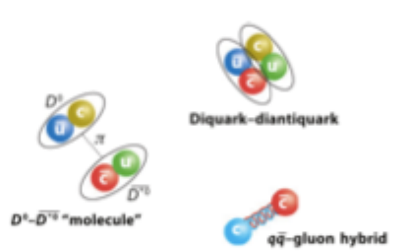
exotic spectroscopy

- while not all of the predicted states have been observed yet... many unexpected ones already have
- referred to as XYZ states
- all started with the discovery of the X(3872) state by Belle in 2003
  - quickly confirmed by Babar, CDF, D0
  - other unconventional states popped up

State	m (MeV)	Γ (MeV)	J^{PC}	Process (mode)
$X(3872)$	3871.52 ± 0.20	1.3 ± 0.6 (<2.2)	$1^{++}/2^{-+}$	$B \to K(\pi^+\pi^- J/\psi)$ $p\bar{p} \to (\pi^+\pi^- J/\psi) + ...$ $B \to K(\omega J/\psi)$ $B \to K(D^{*0}\bar{D}^0)$ $B \to K(\gamma J/\psi)$ $B \to K(\gamma\psi(2S))$
$X(3915)$	3915.6 ± 3.1	28 ± 10	$0/2^{?+}$	$B \to K(\omega J/\psi)$ $e^+e^- \to e^+e^-(\omega J/\psi)$
$X(3940)$	3942^{+9}_{-8}	37^{+27}_{-17}	$?^{?+}$	$e^+e^- \to J/\psi(D\bar{D}^*)$ $e^+e^- \to J/\psi\ (...)$
$G(3900)$	3943 ± 21	52 ± 11	1^{--}	$e^+e^- \to \gamma(D\bar{D})$
$Y(4008)$	$4008^{+121}_{-\ 49}$	226 ± 97	1^{--}	$e^+e^- \to \gamma(\pi^+\pi^- J/\psi)$
$Z_1(4050)^+$	4051^{+24}_{-43}	82^{+51}_{-55}	$?$	$B \to K(\pi^+\chi_{c1}(1P))$
$Y(4140)$	4143.4 ± 3.0	$15^{+11}_{-\ 7}$	$?^{?+}$	$B \to K(\phi J/\psi)$
$X(4160)$	4156^{+29}_{-25}	139^{+113}_{-65}	$?^{?+}$	$e^+e^- \to J/\psi(D\bar{D}^*)$
$Z_2(4250)^+$	$4248^{+185}_{-\ 45}$	$177^{+321}_{-\ 72}$	$?$	$B \to K(\pi^+\chi_{c1}(1P))$
$Y(4260)$	4263 ± 5	108 ± 14	1^{--}	$e^+e^- \to \gamma(\pi^+\pi^- J/\psi)$
				$e^+e^- \to (\pi^+\pi^- J/\psi)$ $e^+e^- \to (\pi^0\pi^0 J/\psi)$
$Y(4274)$	$4274.4^{+8.4}_{-6.7}$	32^{+22}_{-15}	$?^{?+}$	$B \to K(\phi J/\psi)$
$X(4350)$	$4350.6^{+4.6}_{-5.1}$	$13.3^{+18.4}_{-10.0}$	$0,2^{++}$	$e^+e^- \to e^+e^-(\phi J/\psi)$
$Y(4360)$	4353 ± 11	96 ± 42	1^{--}	$e^+e^- \to \gamma(\pi^+\pi^-\psi(2S))$
$Z(4430)^+$	4443^{+24}_{-18}	$107^{+113}_{-\ 71}$	$?$	$B \to K(\pi^+\psi(2S))$
$X(4630)$	$4634^{+\ 9}_{-11}$	92^{+41}_{-32}	1^{--}	$e^+e^- \to \gamma(\Lambda_c^+\Lambda_c^-)$
$Y(4660)$	4664 ± 12	48 ± 15	1^{--}	$e^+e^- \to \gamma(\pi^+\pi^-\psi(2S))$
$Y_b(10888)$	10888.4 ± 3.0	$30.7^{+8.9}_{-7.7}$	1^{--}	$e^+e^- \to (\pi^+\pi^-\Upsilon(nS))$

[Eur.Phys.J.C71:1534,2011]

Many theoretical interpretations in discussion:

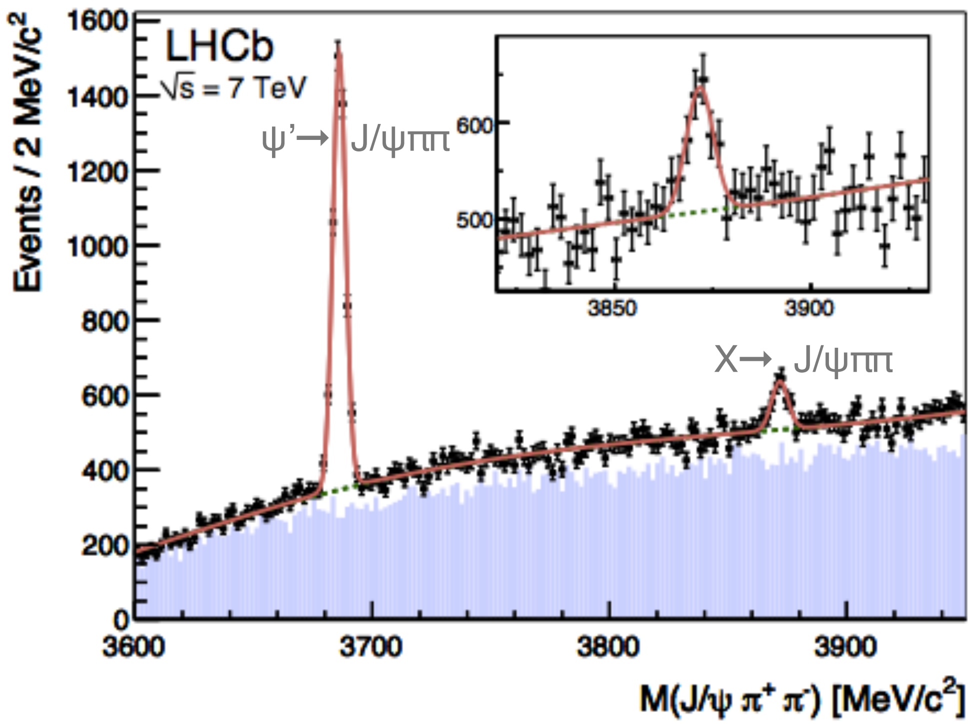


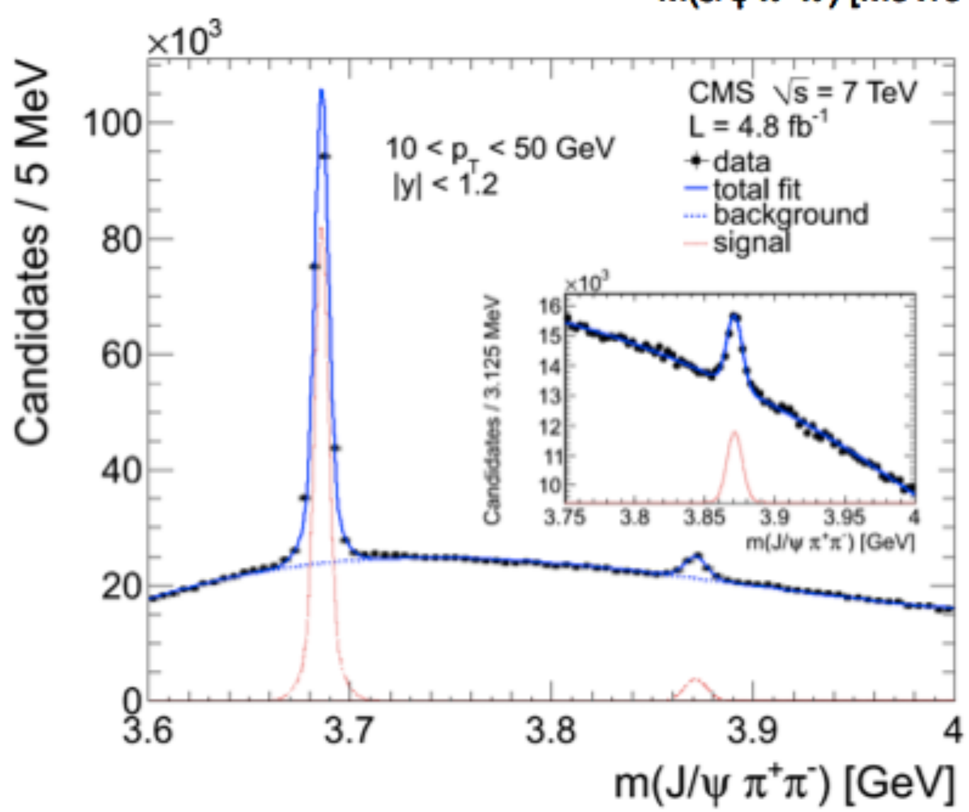
- conventional quarkonia;
- tetra-quarks states;
- meson-molecules;
- hybrid mesons;
- threshold effects;

➡ properties do not well fit the quarkonia picture

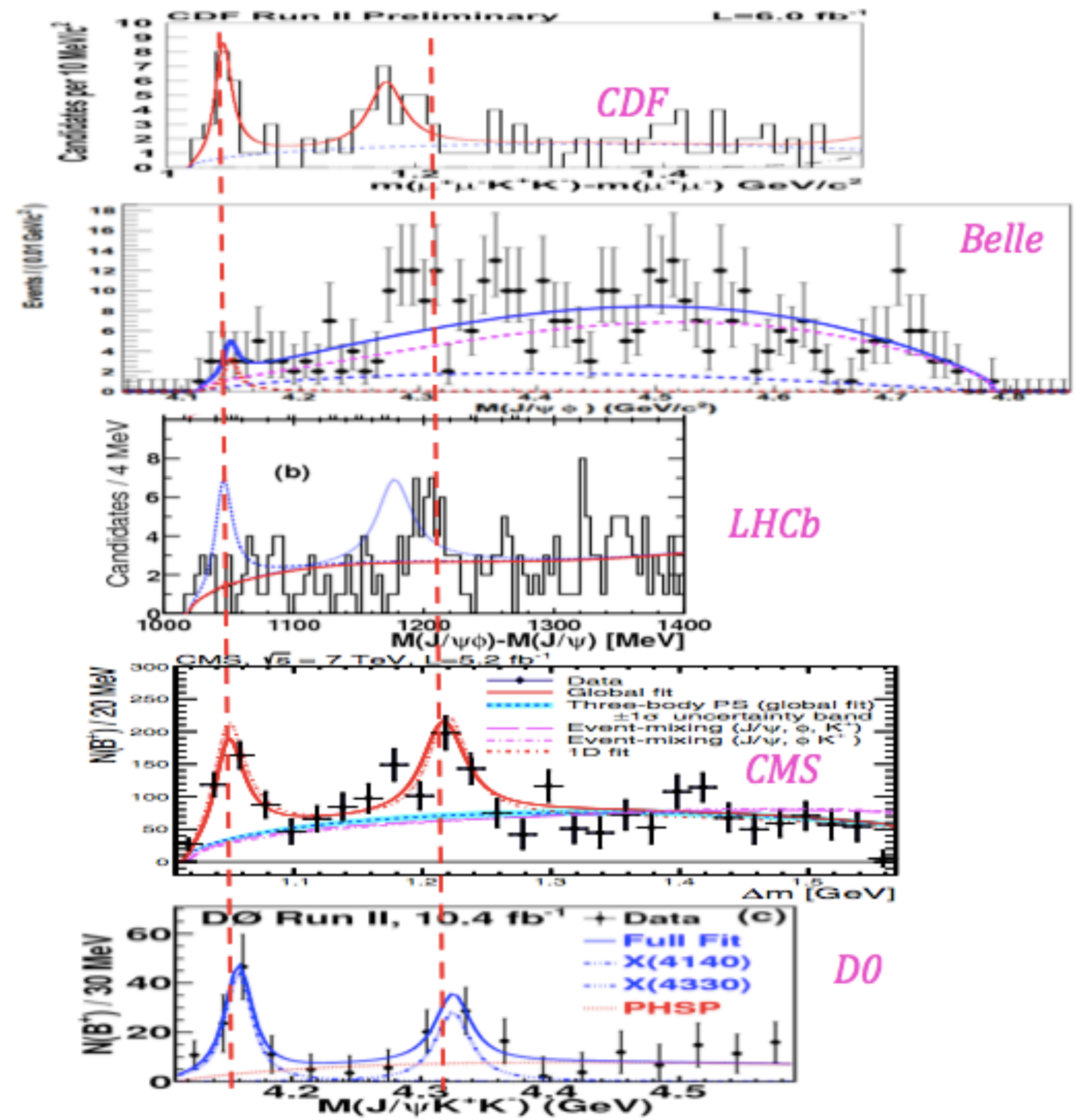
XYZ states

X(3872) →J/ψππ

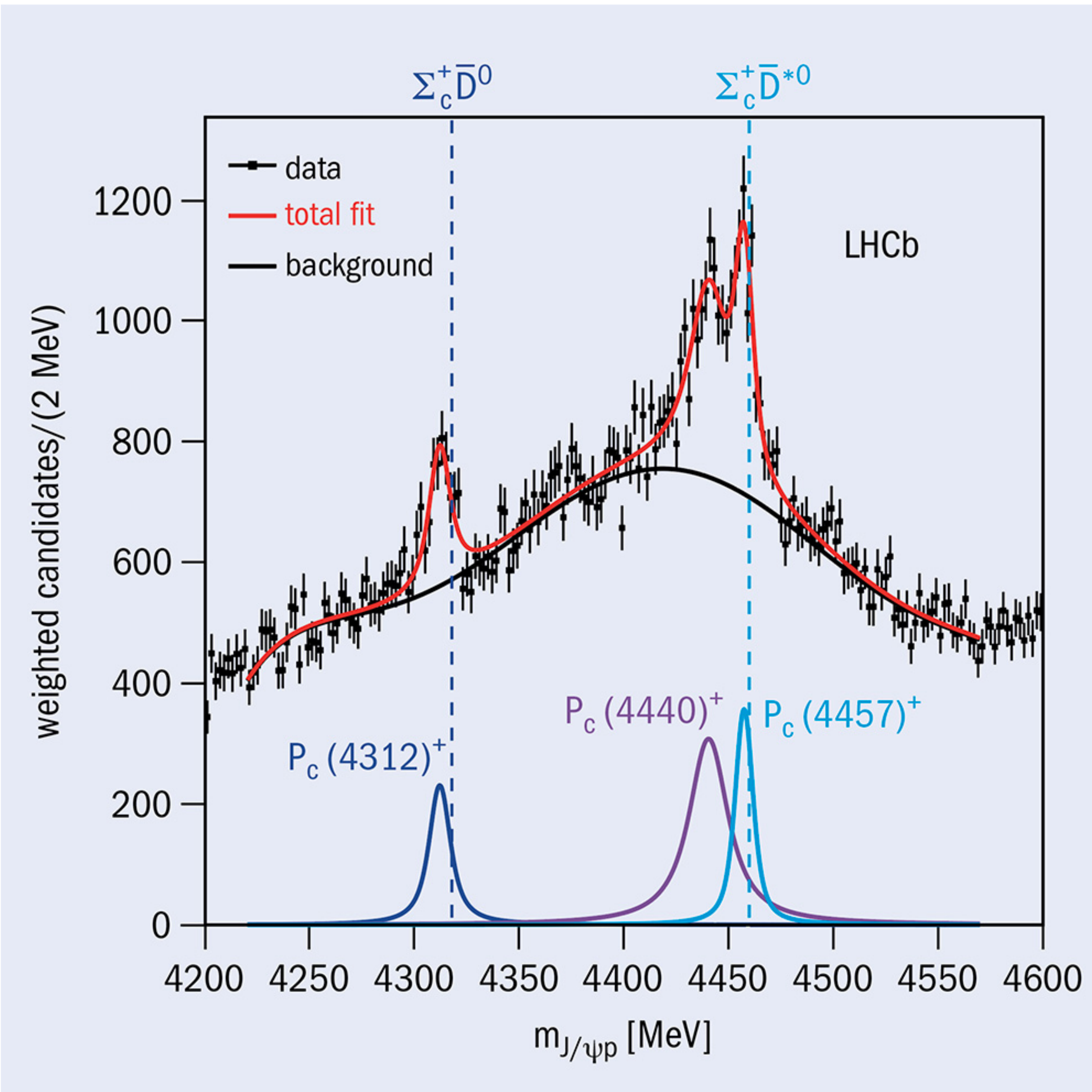


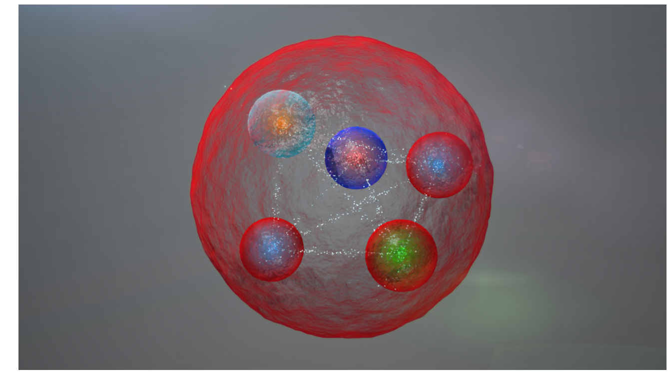


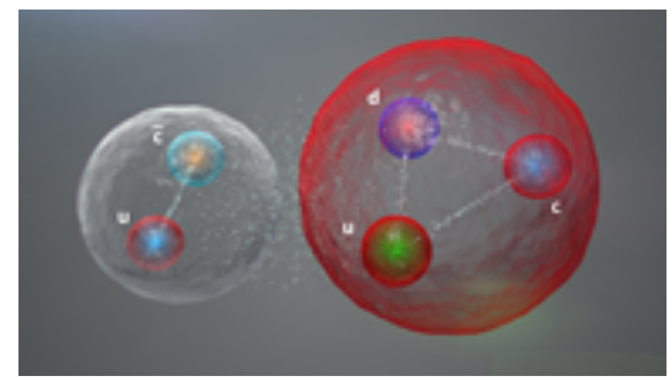
Y(4140) in $B_u \to X[J/\psi\phi]K$ decays

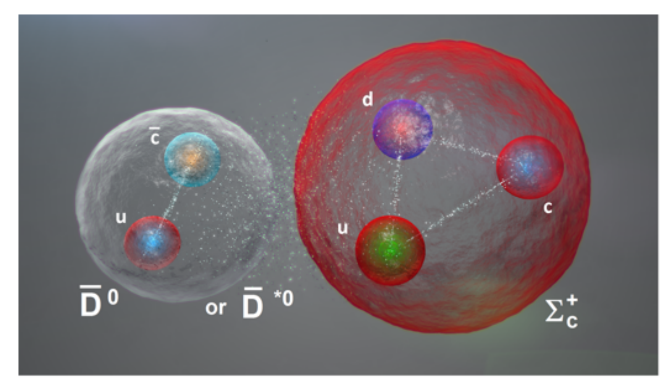


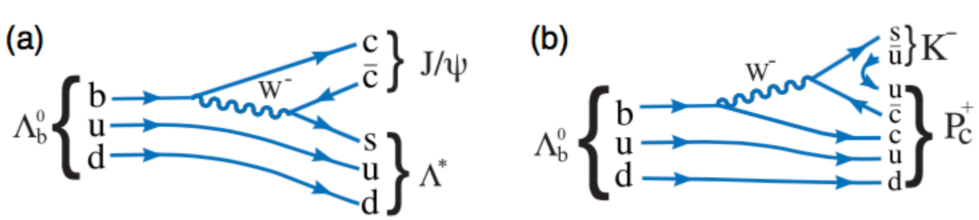
pentaquarks





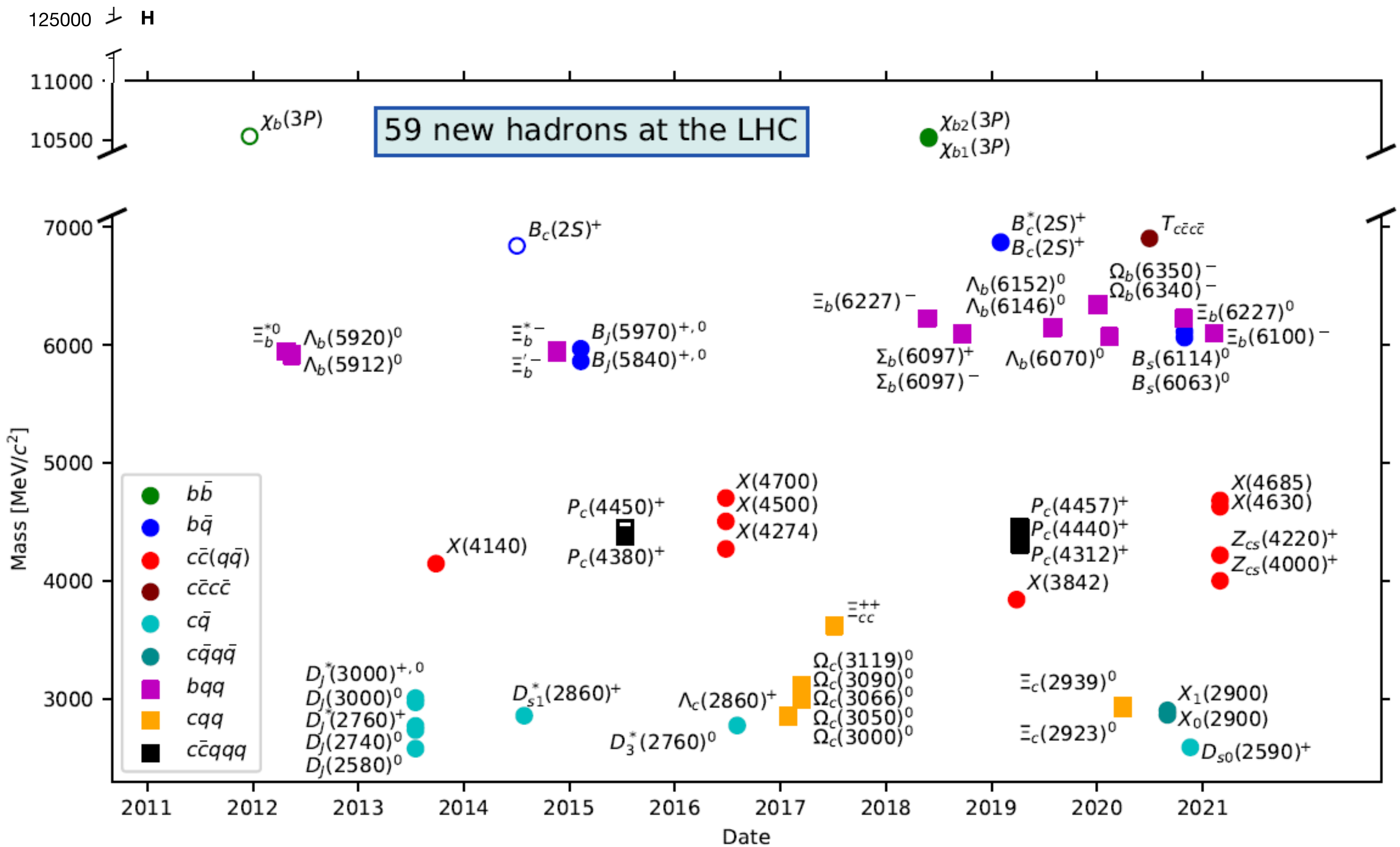






New particles discovered@LHC?

New particles discovered@LHC? 59+1



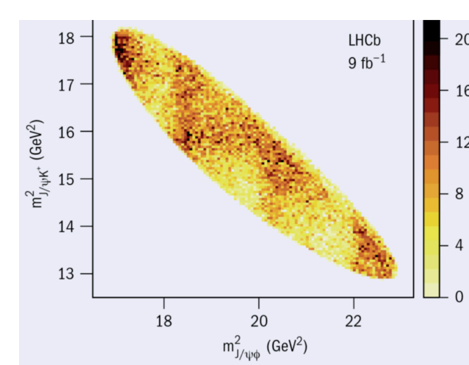
FLAVOUR PHYSICS | NEWS

news

LHCb observes four new tetraquarks

3 March 2021

March'2021 @CERN

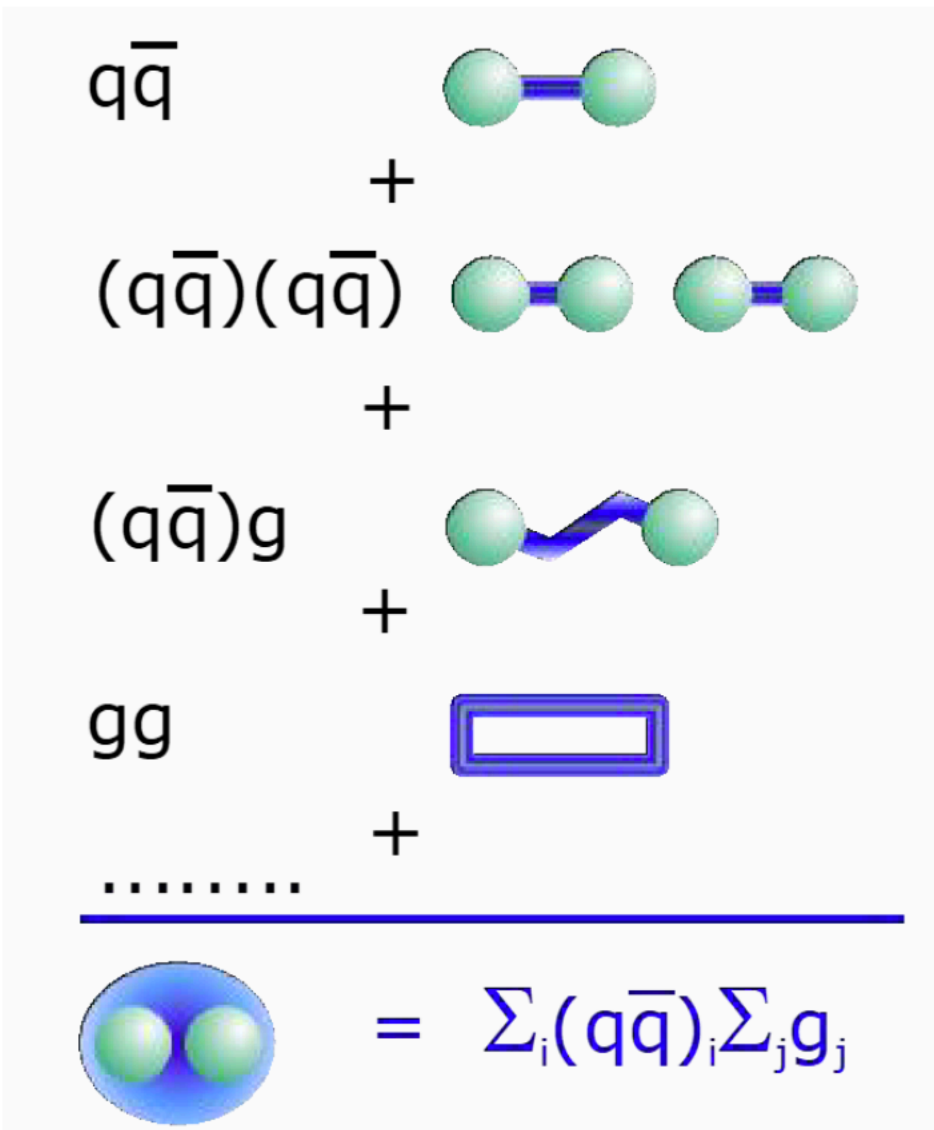


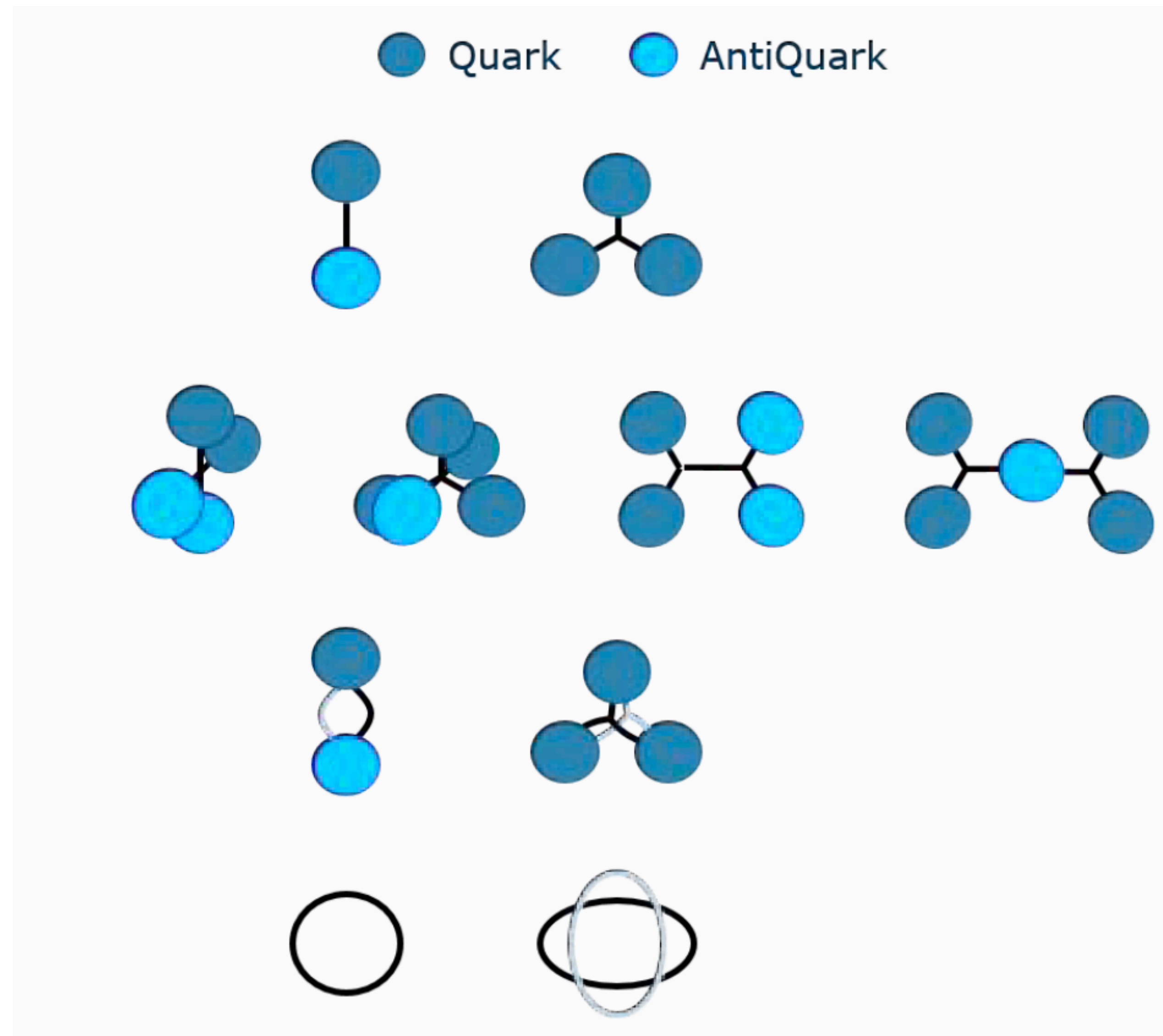
$B^+ \to J/\psi\phi K^+$

($c\bar{c}u\bar{s}$): $Z_{cs}(4000)^+$, $Z_{cs}(4220)^+$

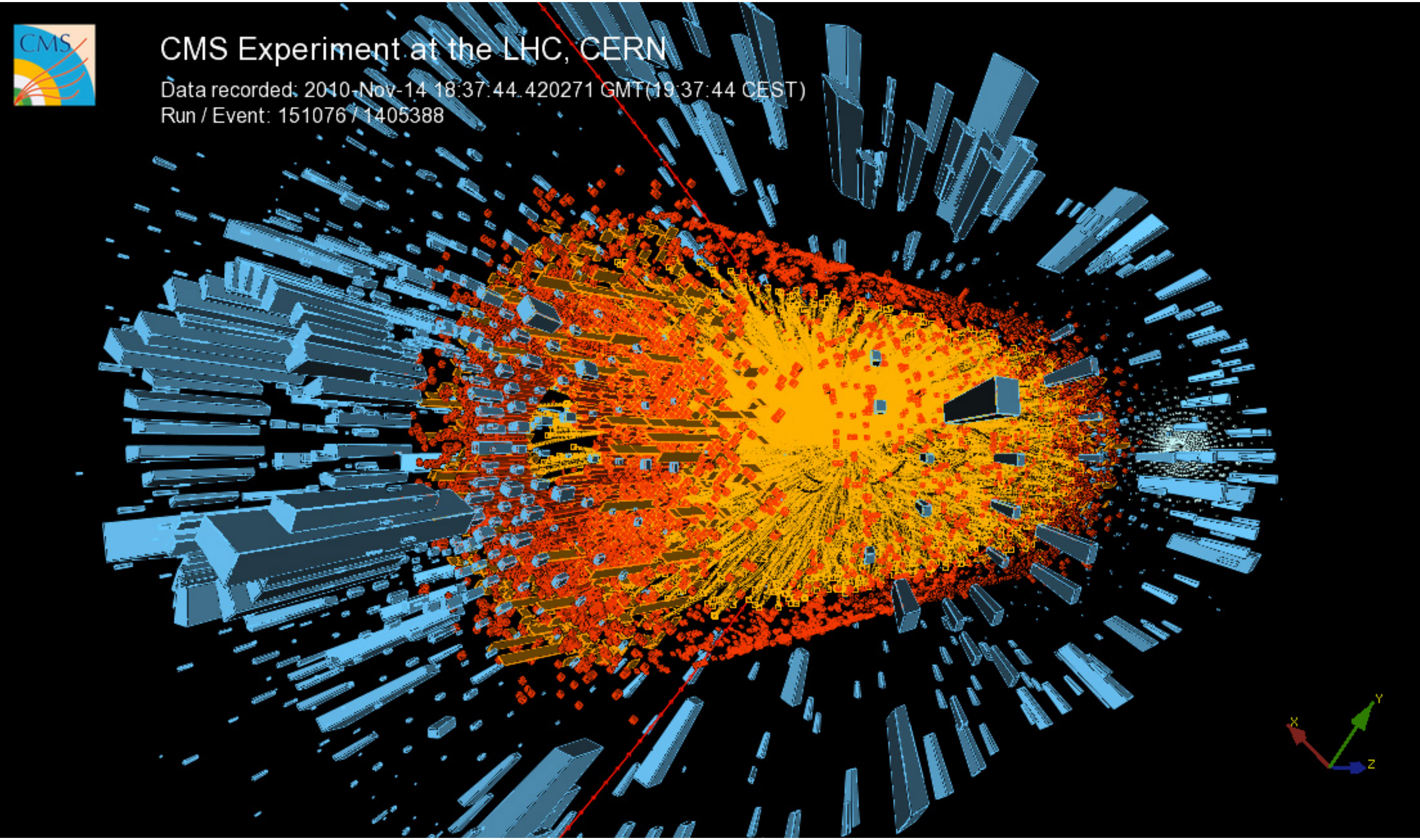
($ccss$): $X(4685)$, $X(4630)$

nature of hadrons, a tricky business





Nature of several particles unknown… exotic hadrons heavy ion collisions


CMS
CMS Experiment at the LHC, CERN
Data recorded: 2010-Nov-14 18:37:44.420271 GMT(19:37:44 CEST)
Run / Event: 151076 / 1405388
X
Y
Z

➡ heavy flavor studies at the LHC are opening up new research lines in nuclear physics, benefitting from the exquisite capability of the detectors and unprecedented collision energies at the LHC

➡ several ground-breaking results already delivered, many more to come

	proton-proton			heavy-ion		
$\sqrt{s_{NN}}$	1.96 TeV Tevatron (Run II)	$7_{(8)}$ TeV LHC Run I	$13_{(14)}$ TeV LHC Run II	0.2 TeV RHIC	2.76 TeV LHC Run I	5.5 TeV LHC Run II

x7

x27

energy

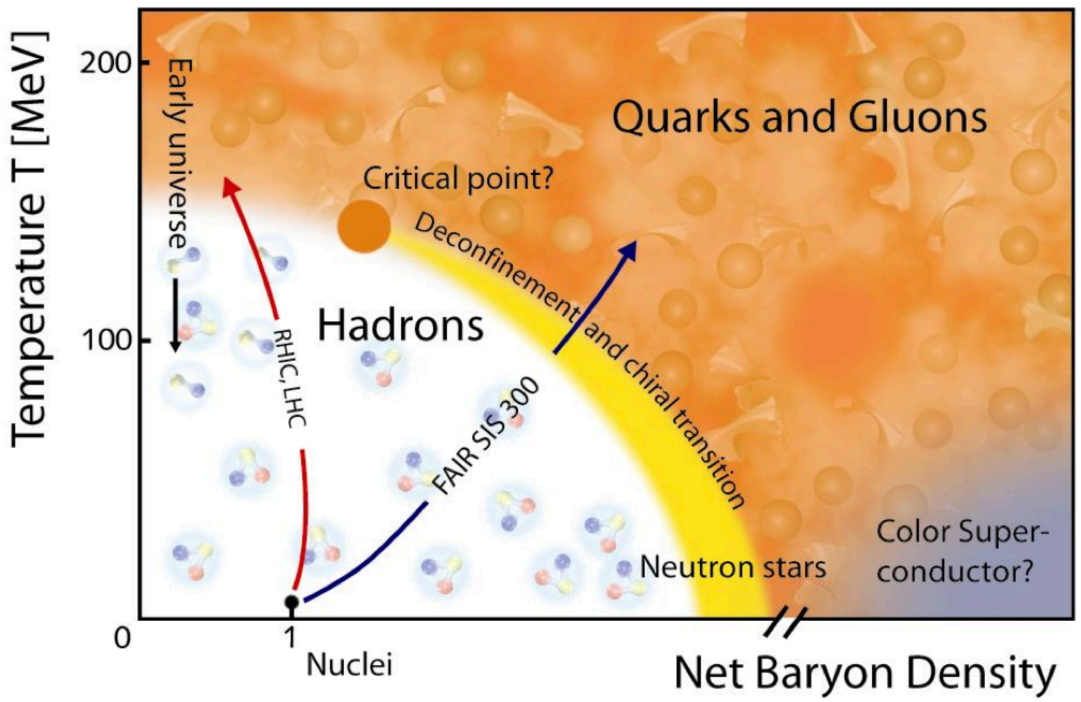
$\mathcal{L}$ very large datasets are being accumulated at the LHC

intensity

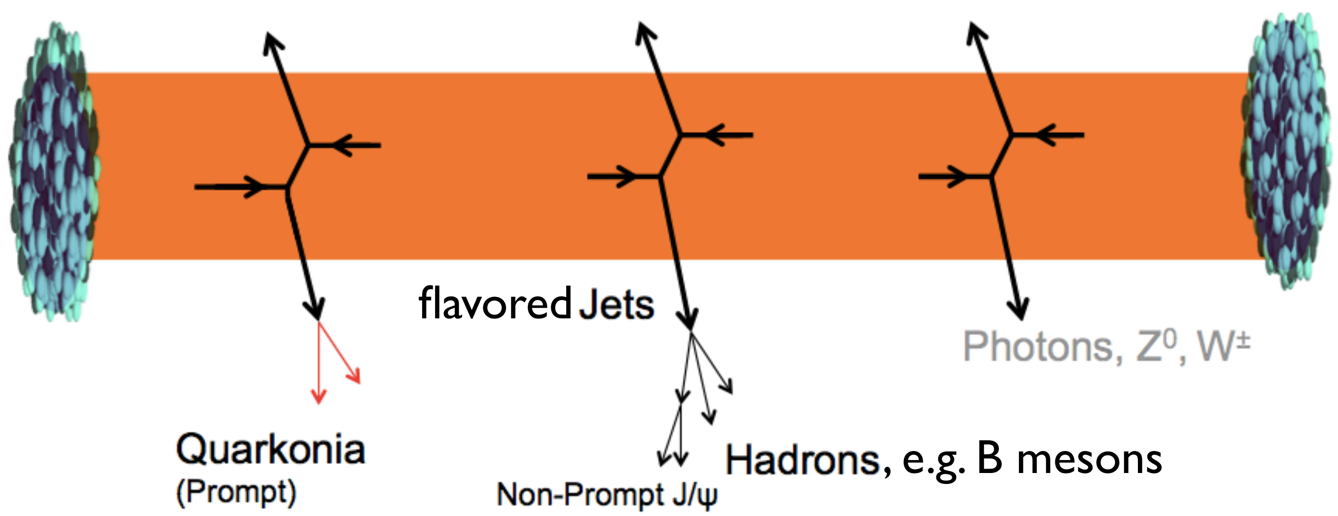
➡ large HF production cross section + precision HF detection capability

➡ at large energy densities, QCD predicts the existence of a deconfined state of quarks and gluons -- the quark gluon plasma (QGP)

- studied in heavy ion collisions
- the goal is to characterize and quantify the properties of the dense and hot medium produced at the unprecedented LHC energies

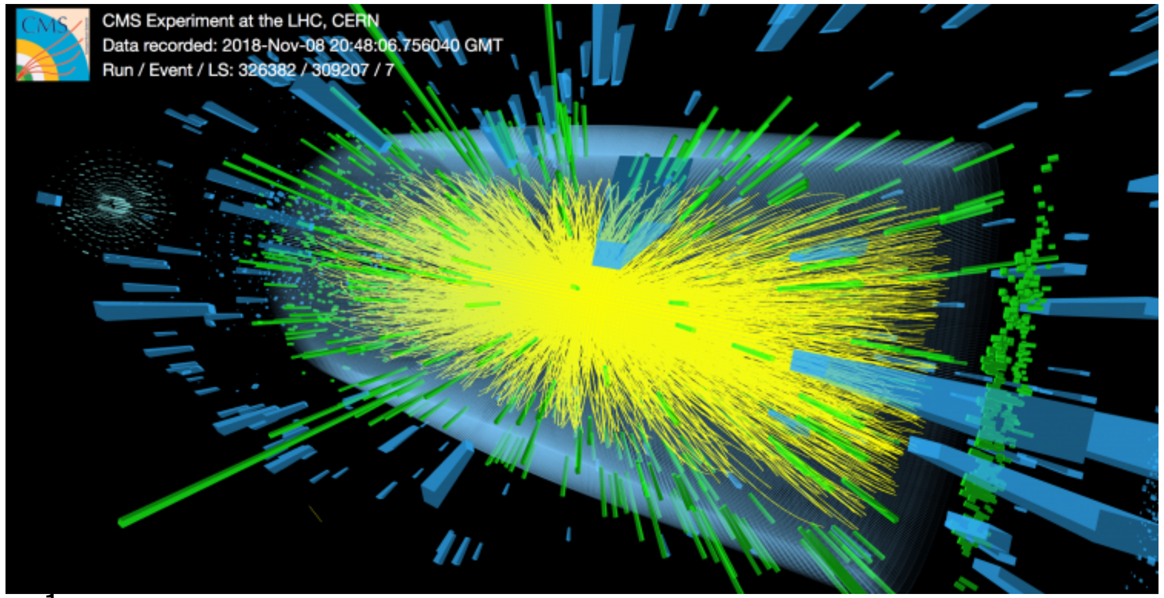


- heavy-flavor states are ideal “hard probes” for studying the properties of the created medium

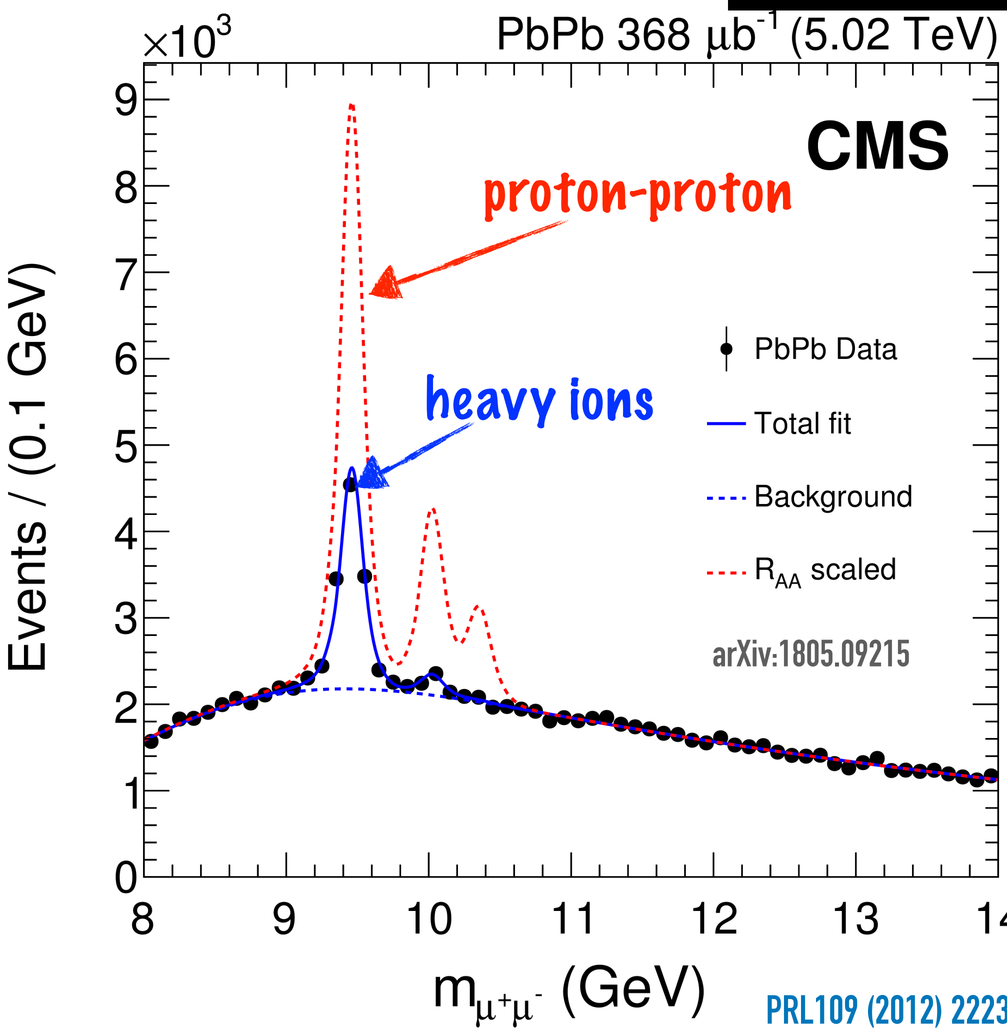


probing matter at extreme conditions

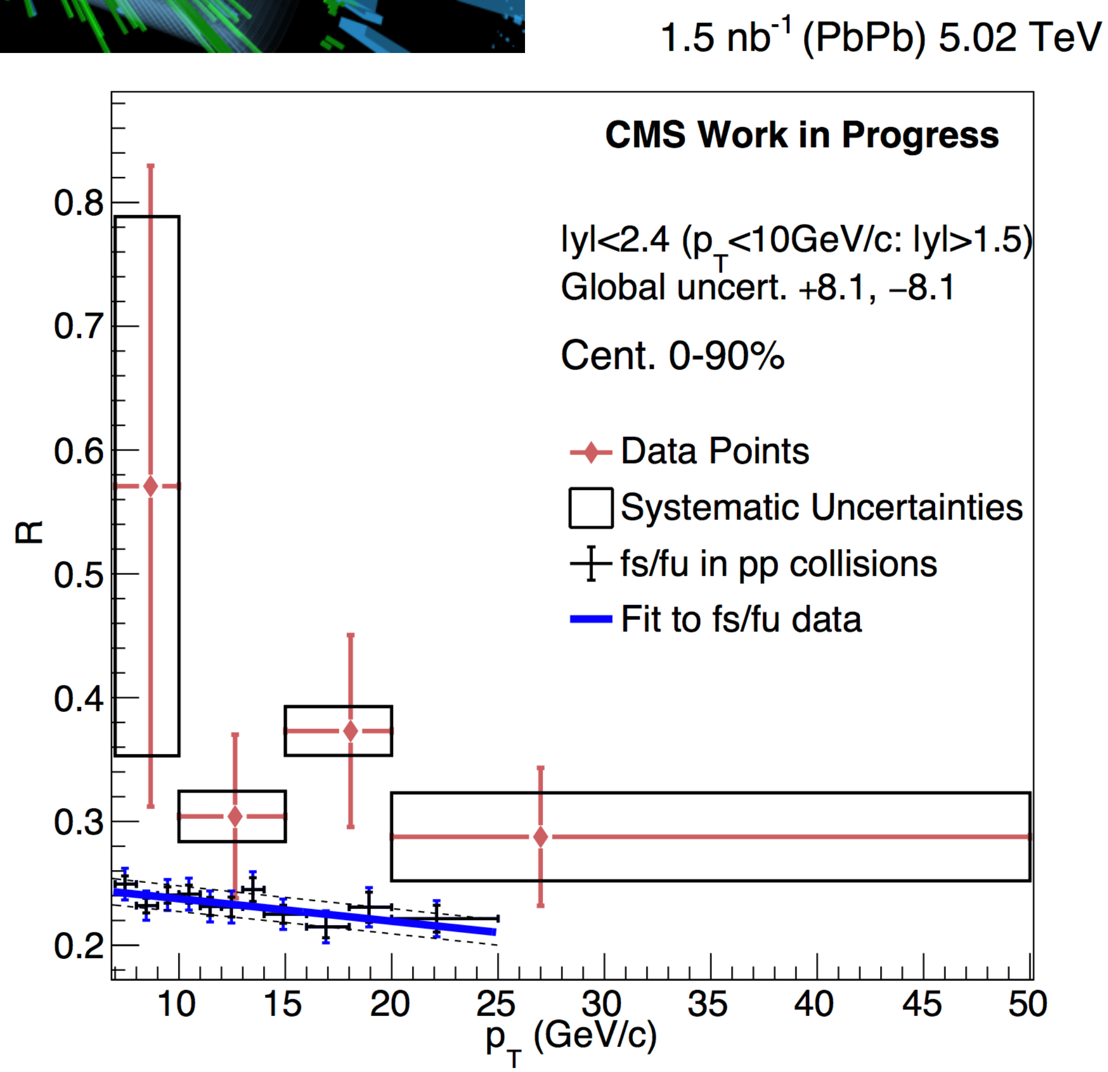
Particles melt in the QGP ! (sequentially)



Particles loose energy in the QGP ! (sequentially)

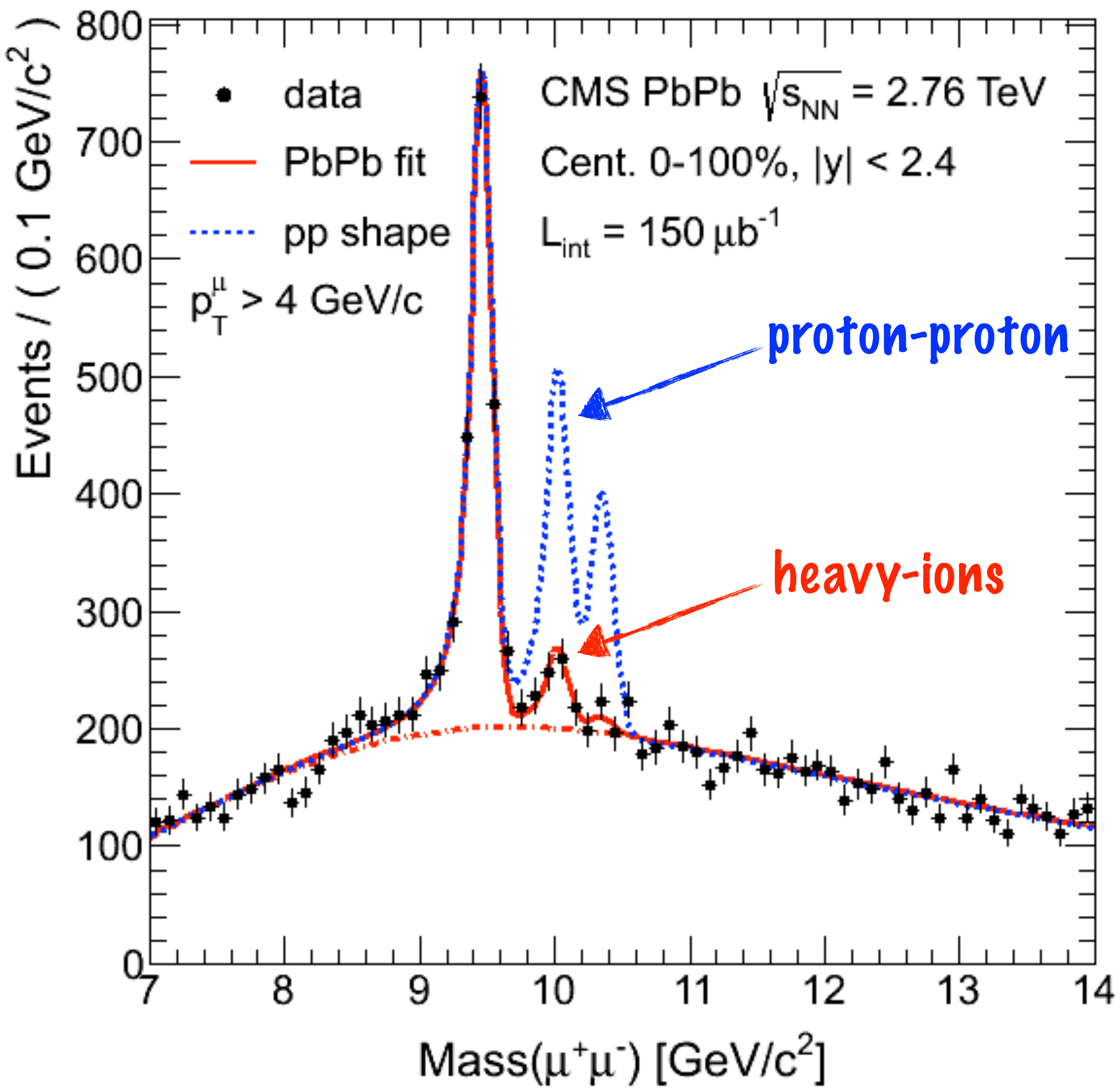


PRL109 (2012) 222301



CERN-THESIS-2019-256

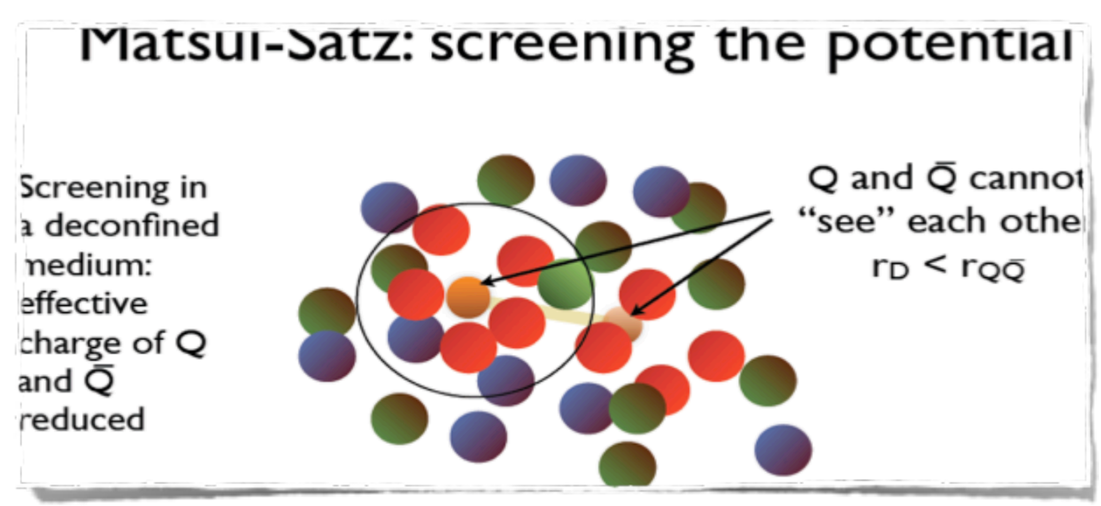
quarkonium suppression *[Pb-Pb]*



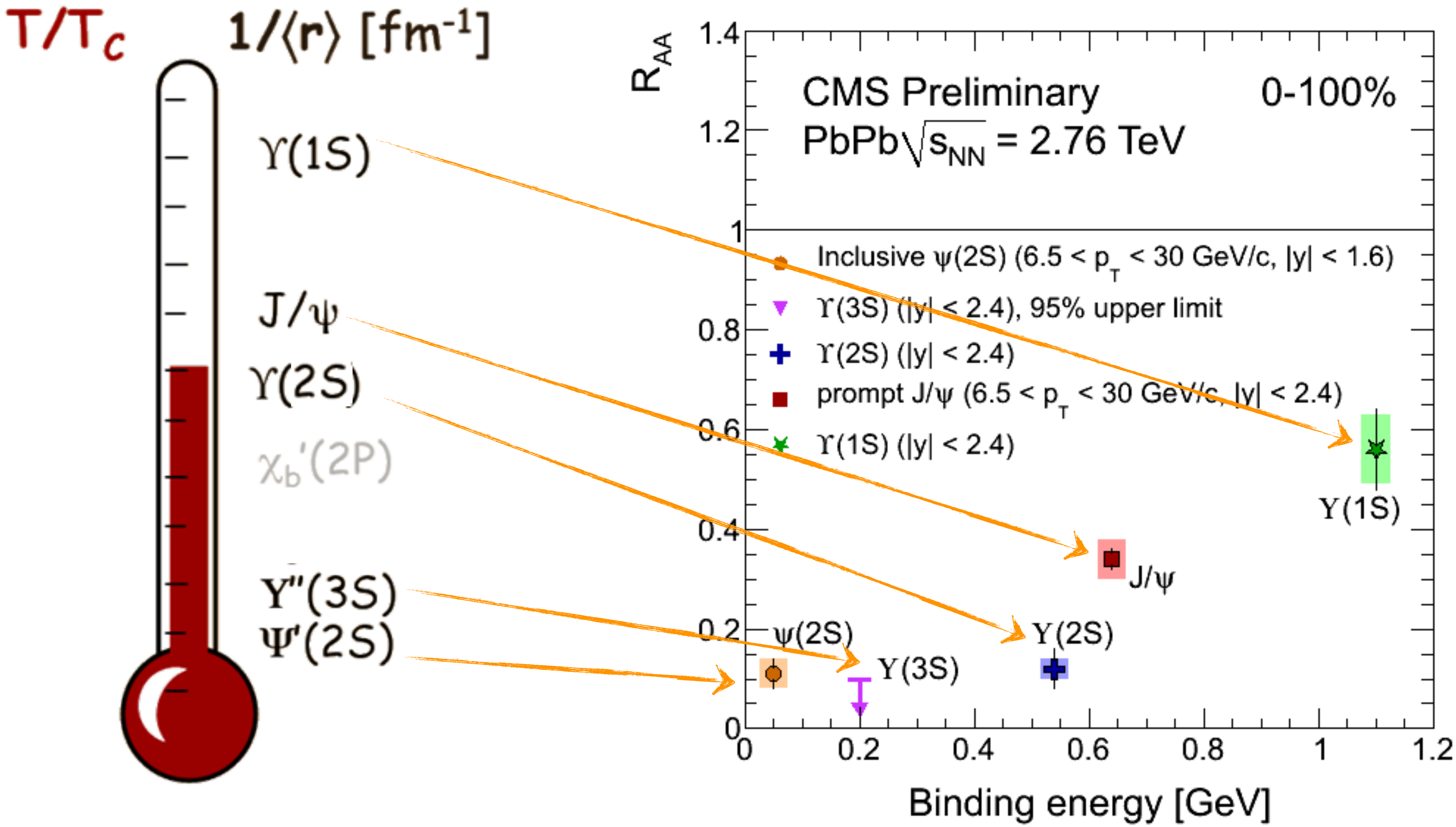
"the LHC heavy-ion **text book** result"

Exercise: the excited states being suppressed, what may be expected also of the observed ground state (hint: nS→1S feed-down)

- first (quantitative) measurements of the Υ(nS) states in HI collisions
- unprecedented resolutions, allowing to separate the three states
  - experimentally and theoretically robust
- excited states observed (>5σ) to be more suppressed than ground state
- spectacular indication of formation of **Q**uark **G**luon **P**lasma in heavy ion coll.

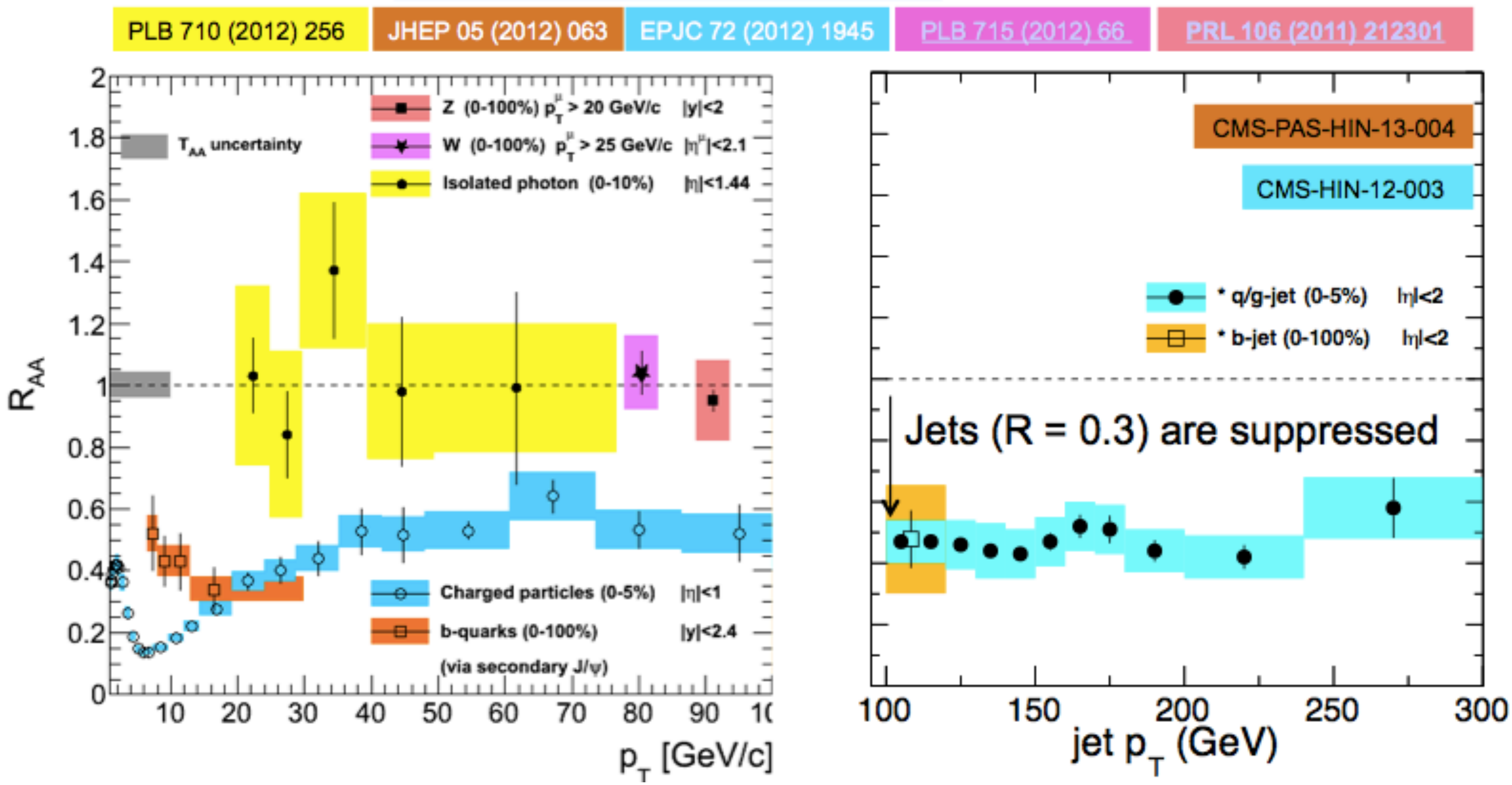


quarkonium sequential suppression



quarkonia suppression pattern experimentally established:
less tightly bound states are more suppressed in the medium

in medium hadron suppression

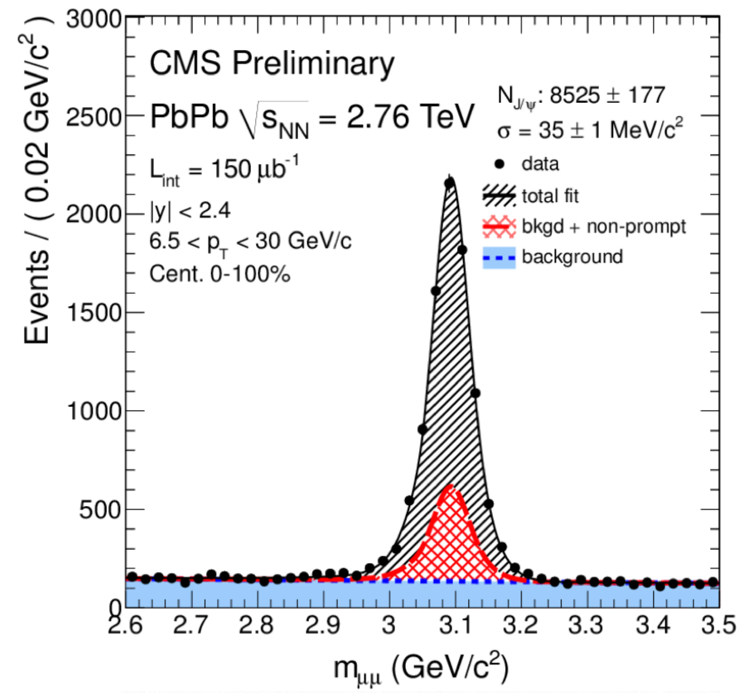


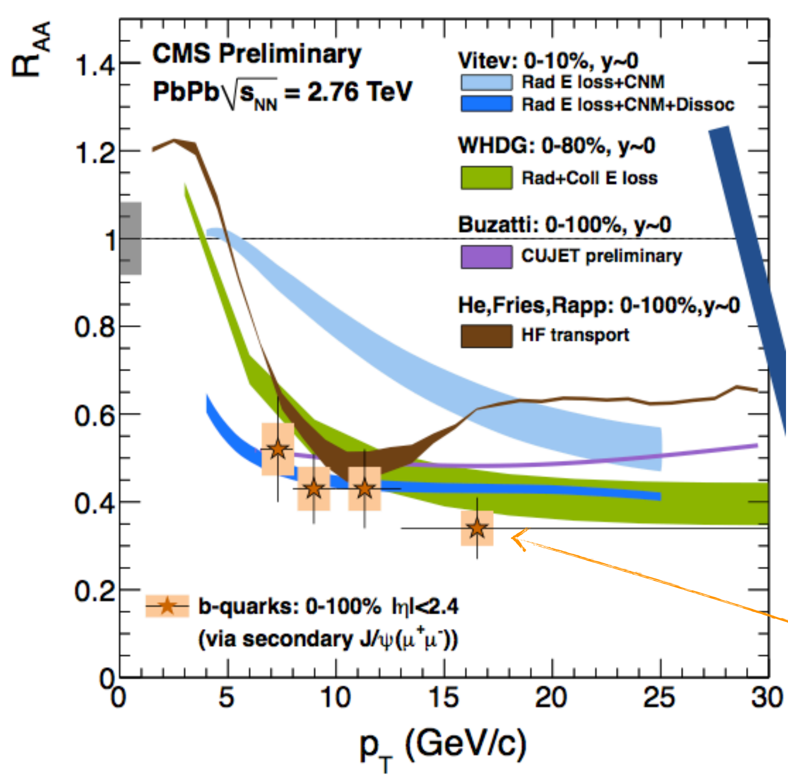
- measure of suppression, R_{AA} (nuclear modification factor)
  - cross section ration in PbPb vs pp, scaled by number of binary collisions
- different particle species undergo different energy loss in the medium
  - colorless probes (W,Z,γ) are not suppressed (R_{AA} ~ 1)

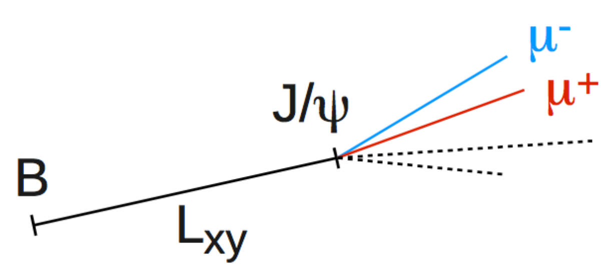
➡study flavor dependence of energy loss

b-hadron detection

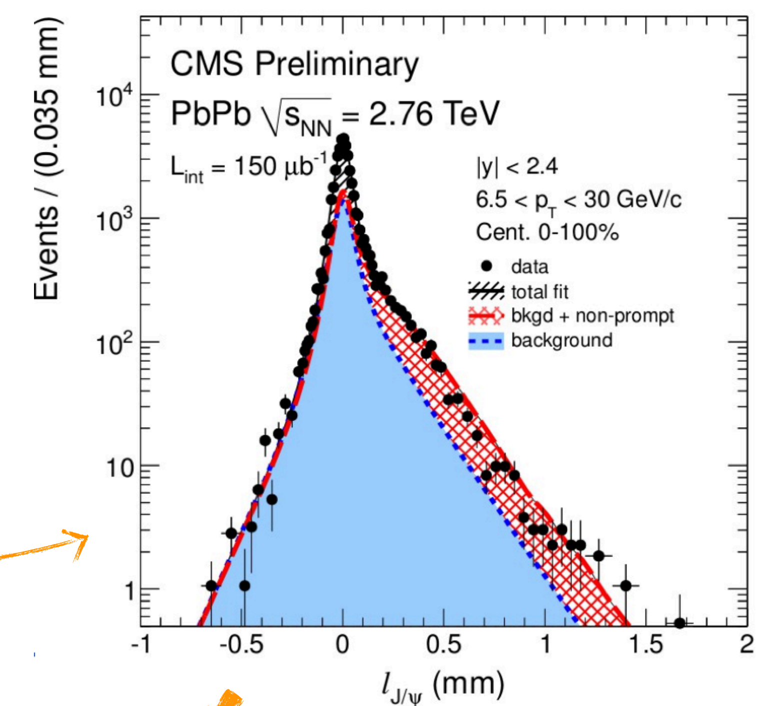
- prior to LHC, b-hadron detection was pursued mostly through inclusive-lepton (B→lX) and inclusive-charmonia (B→J/ψX) studies
- with LHC, moved to a new class of more reliable and precise new measurements
  - through non-prompt charmonia: remove prompt contribution through lifetime analysis [see next section]
  - through exclusive state reconstruction [see next slide]
  - ➡ both achieved for the first time at the LHC





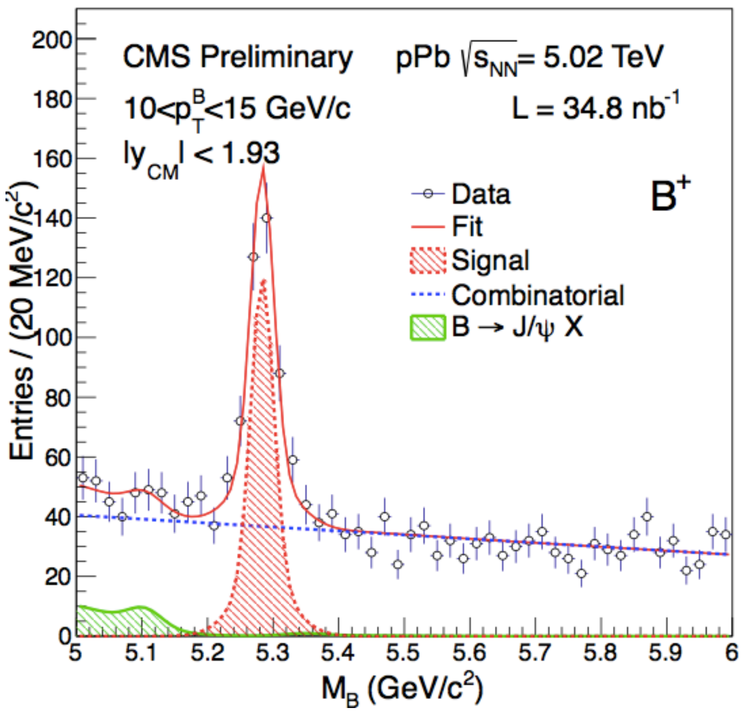


$$l_{J/\psi} = L_{xy}\frac{m_{J/\psi}}{p_T}$$

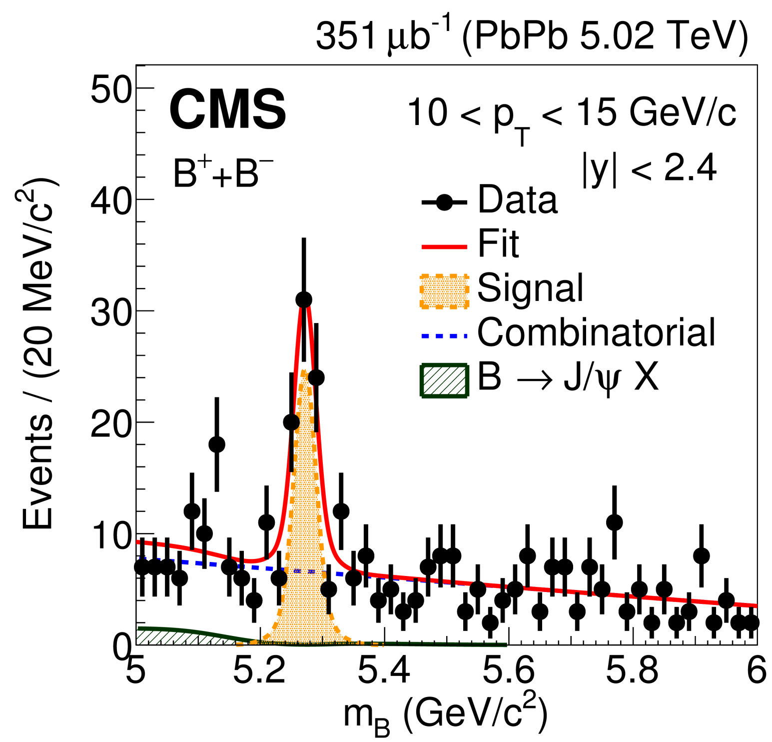


first! B mesons in ion collisions

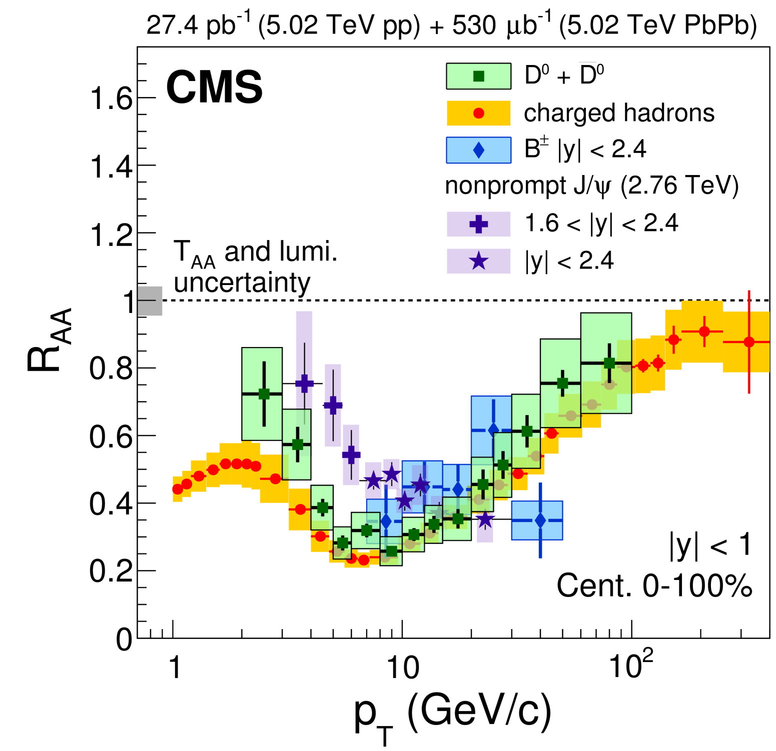
- first B meson decays (fully) reconstructed in collisions involving heavy ions
- novel probes of the QGP!



B+ in p+Pb



B+ in PbPb



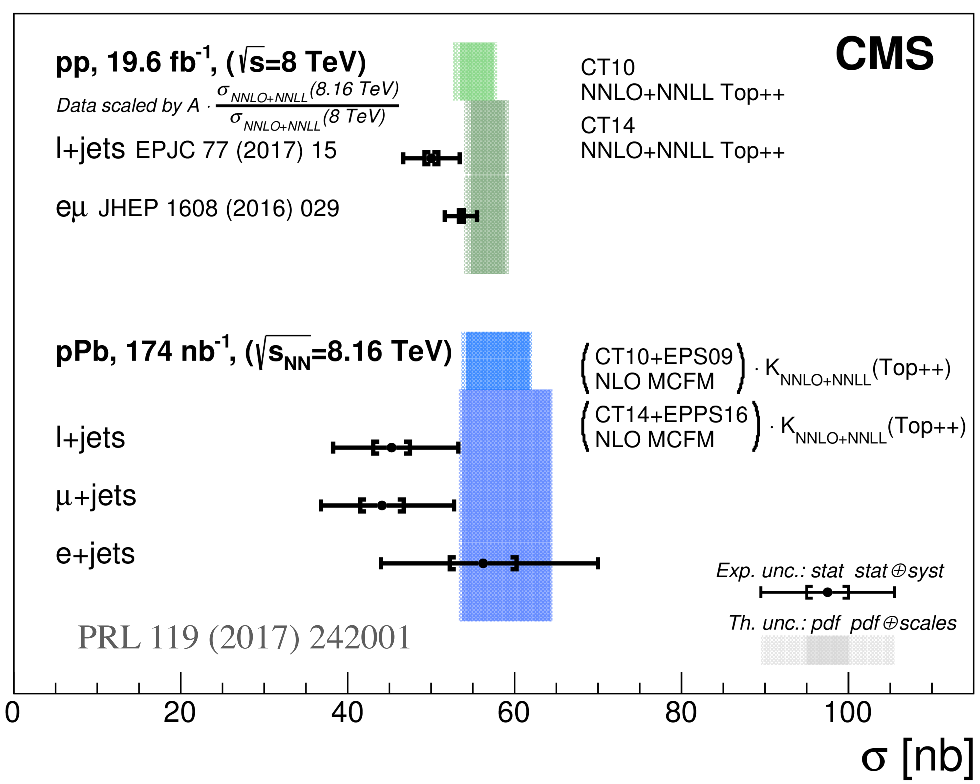
Measured suppression of different hadron species

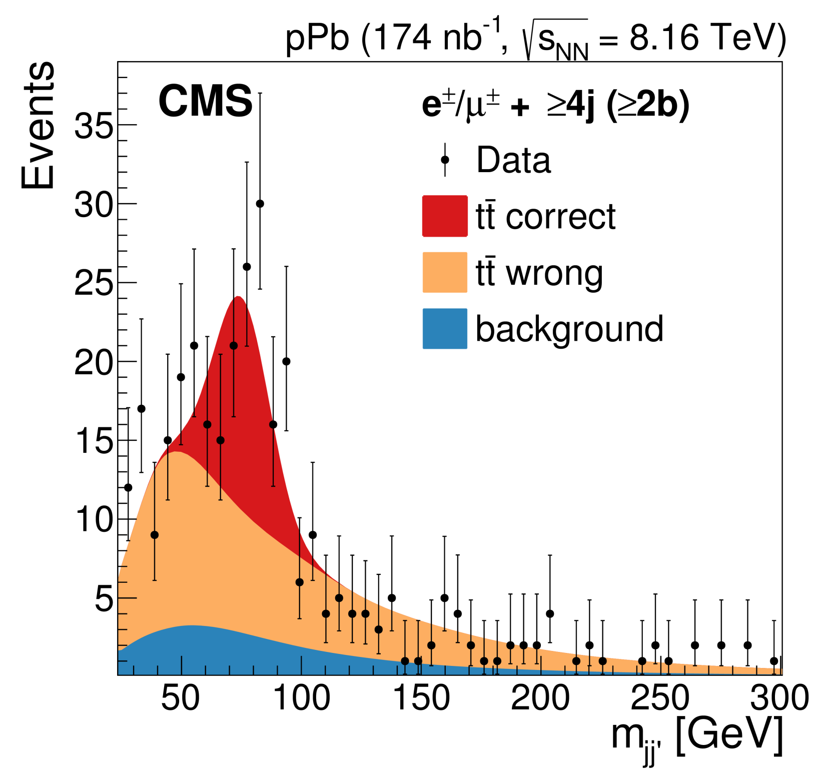
these systems constitute precise handles that will facilitate a much improved understanding of the mechanisms of energy loss of hadrons in the deconfined ('hot') and nuclear ('cold') media -- and of its flavor dependence

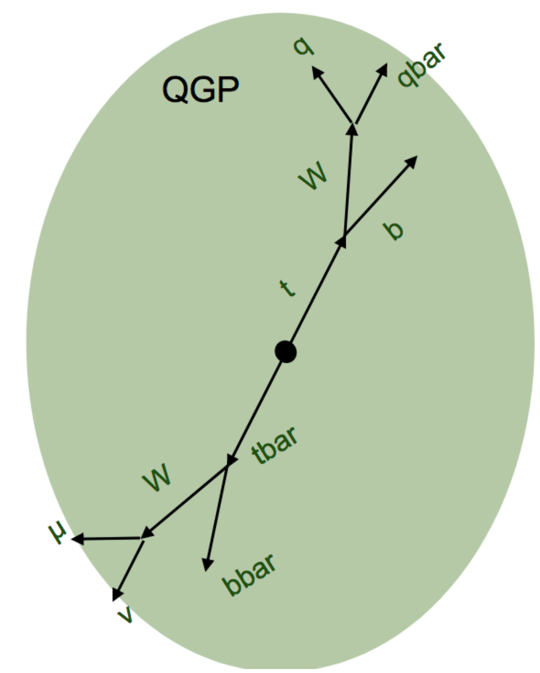
currently actively searching for Bs and Bc, using dataset collected November 2018

top quark in ion collisions

- top observed already in p+Pb collisions at 8TeV
  - used signature: e or muon plus >=4 jets
- yet another novel probe of the QGP
- may be used to resolve time dependence of jet quenching effects

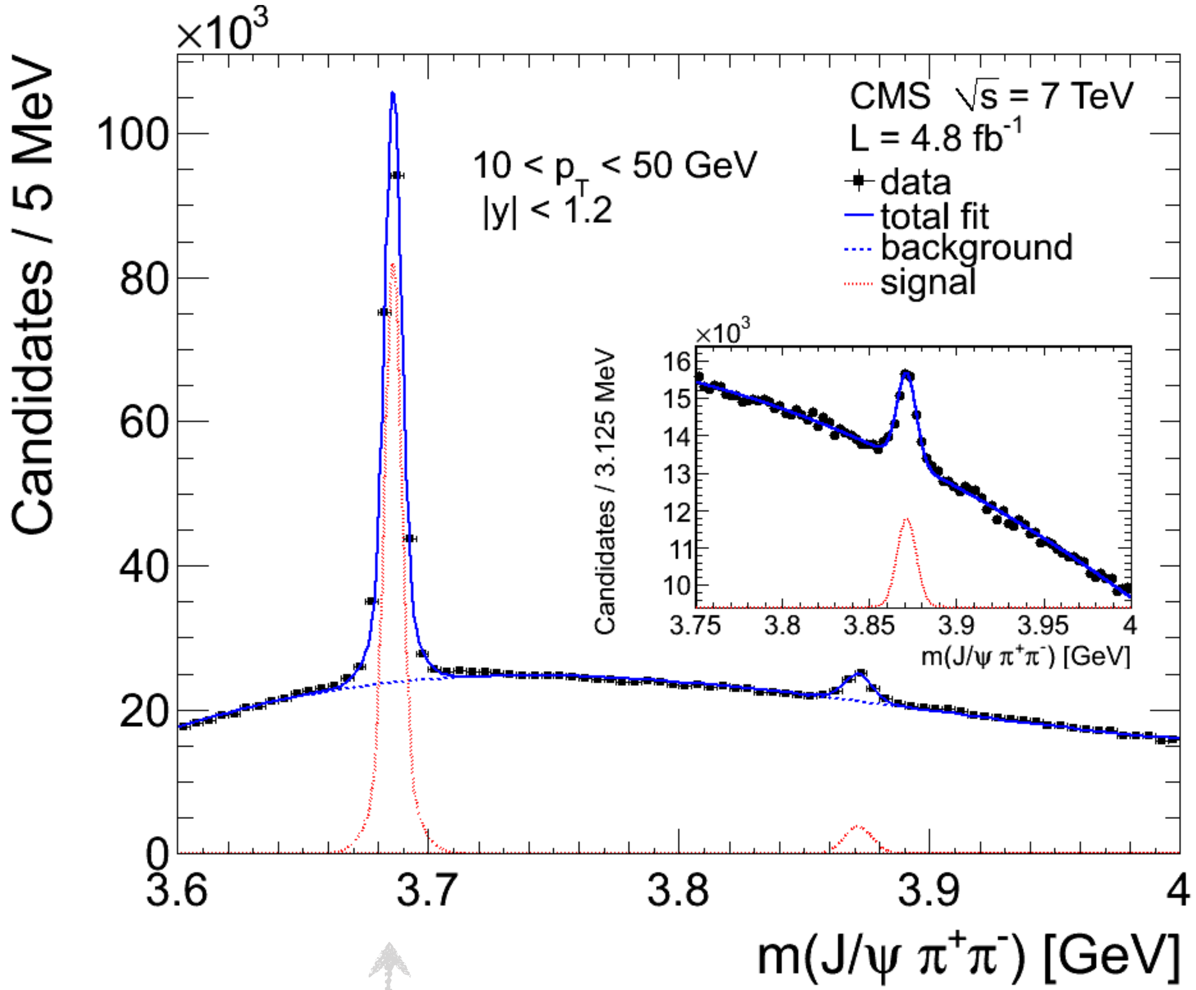


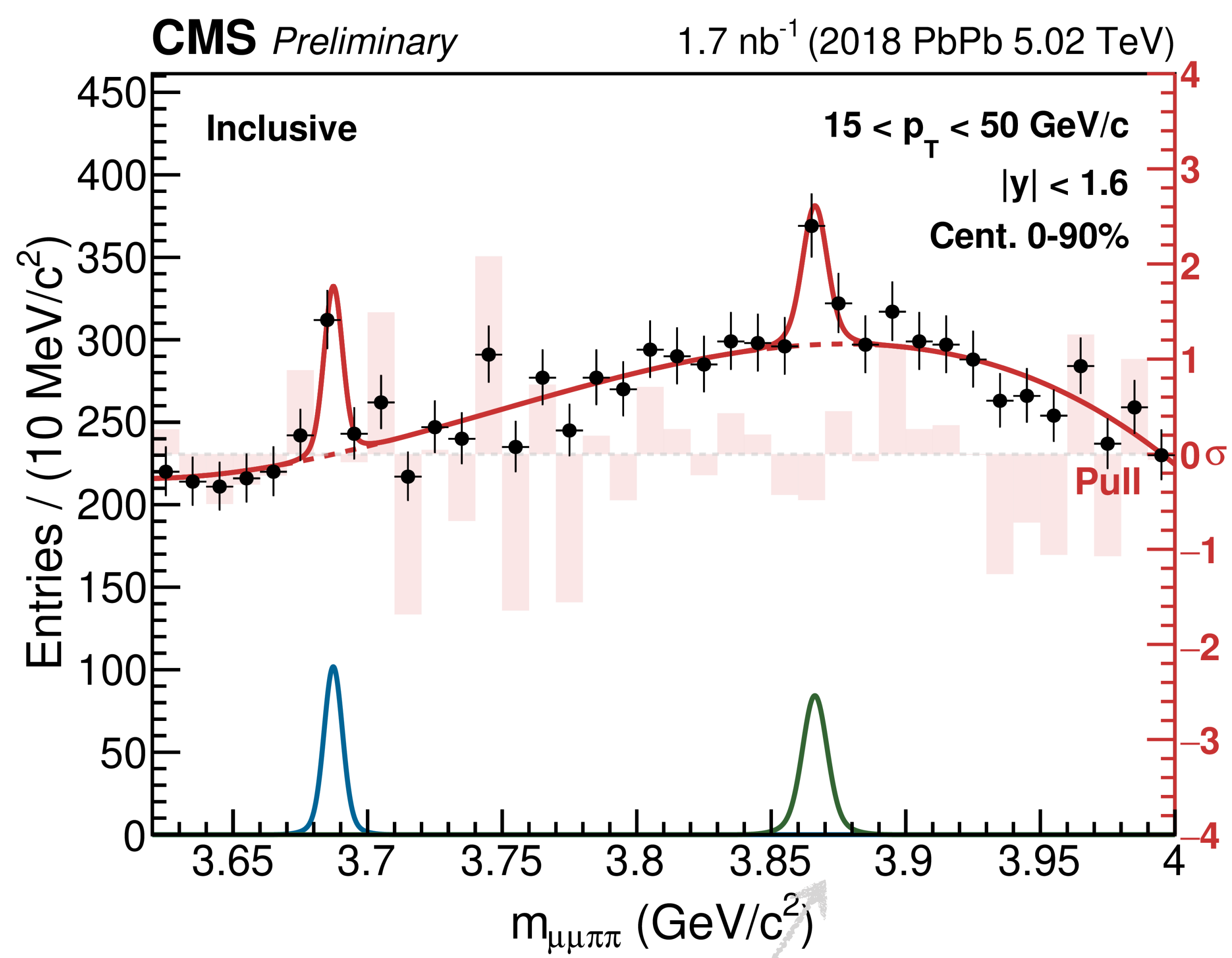




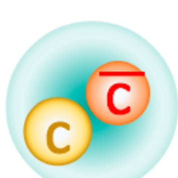
- next step: search for the top in PbPb at 5TeV collected in 2018

particle enhancement !?

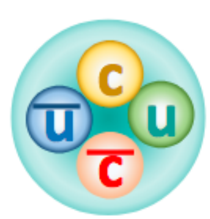


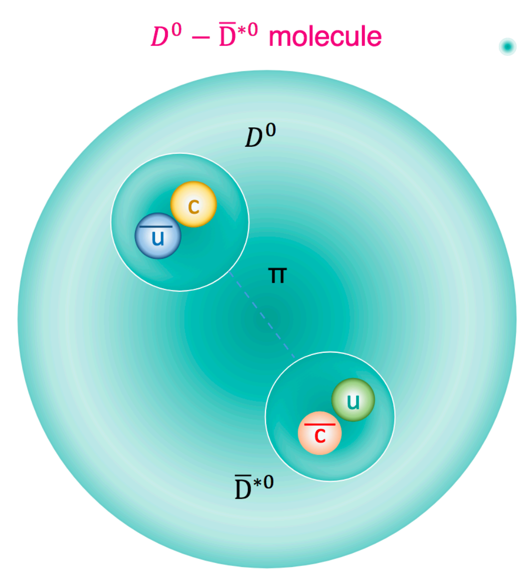


Charmonium

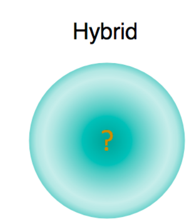


Tetraquark (4q)





Glueballs / Hybrid hadrons

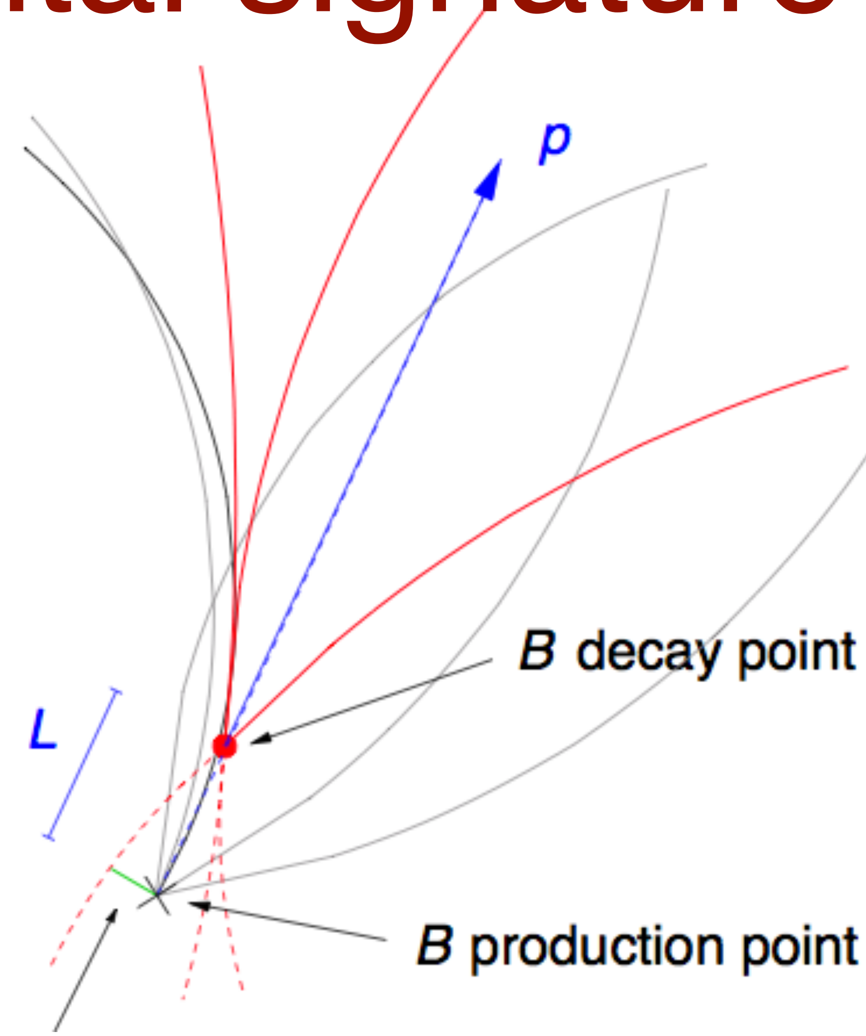


?

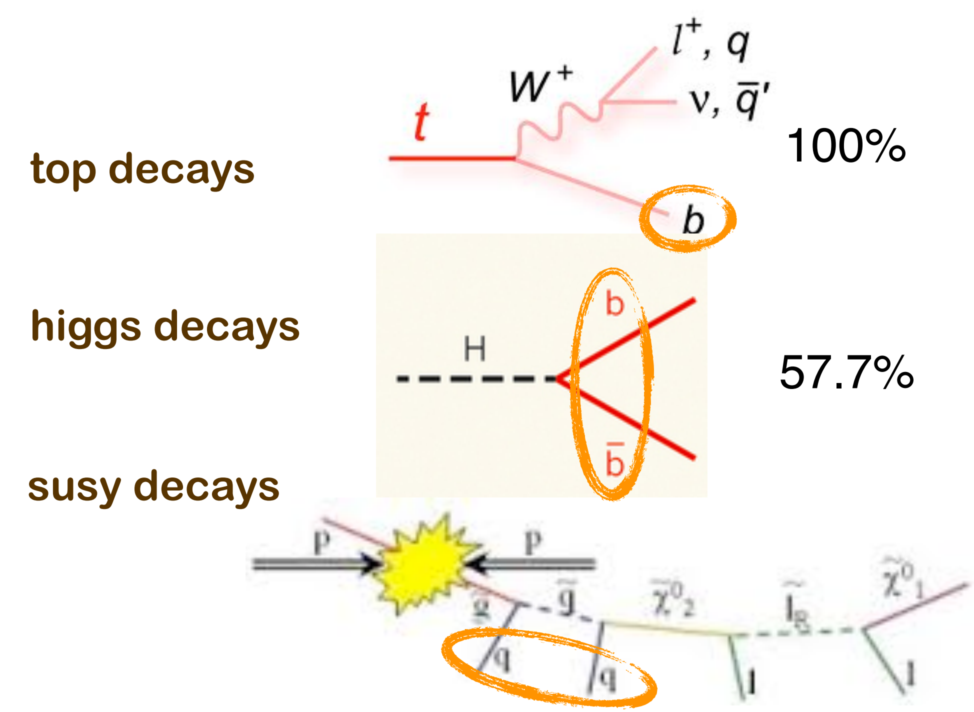
lifetime

a distinctive experimental signature

- bottom and charm hadrons live longer than the other unstable particles
  - $\tau(D) \sim 0.5\text{-}1\,\text{ps}$, $\tau(B) \sim 1.5$ ps
  - they travel macroscopic (i.e. measurable) distances in the detector before decaying, producing a displaced vertex topology



- extensively explored
  - in heavy-flavor analyses themselves
  - b-jet tagging: discriminate b-jets from the lighter quark jets
  - in SM measurements and BSM searches: to detect signal HF components (e.g. $t\to Wb$, $H\to b\bar{b}$,...) or control HF backgrounds (e.g. $b\bar{b}$ dijets,...)



quantum mechanics (i)

- an unstable particle may be described by an effective hamiltonian

$$\mathcal{H} = m - \frac{i}{2}\Gamma$$

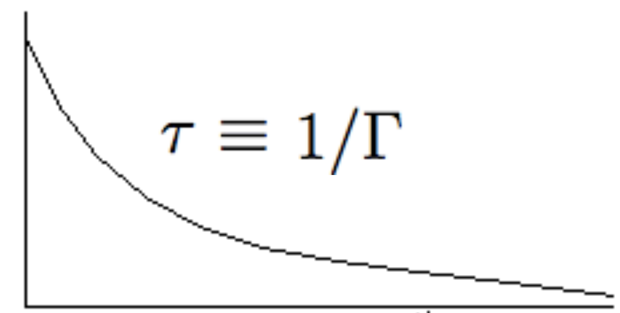
- through the non-relativistic Schrodinger equation

$$i\partial_t \psi = \mathcal{H}\psi$$

$$|\psi\rangle_t = e^{-imt} e^{-\frac{1}{2}\Gamma t} |\psi_0\rangle$$

$$|\langle\psi_0|\psi\rangle_t|^2 = e^{-\Gamma t},$$

- the solution reproduces the law of radioactive decay



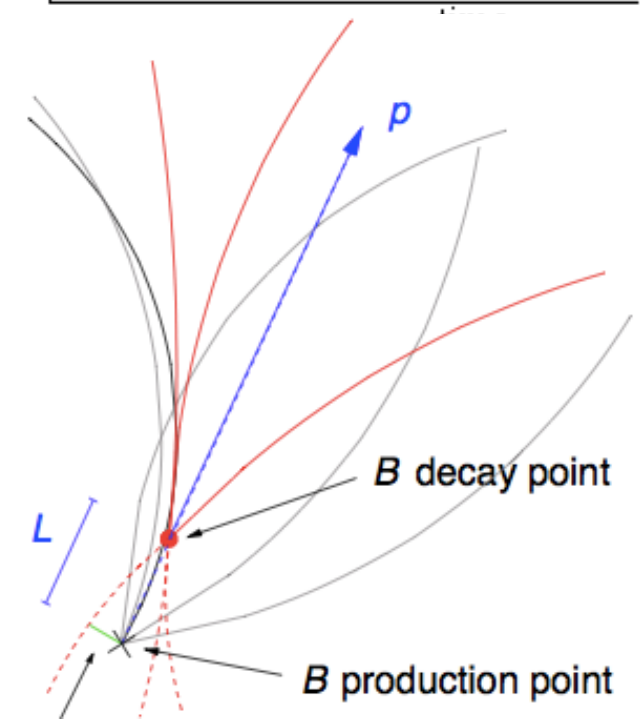
$$\mathcal{P}(t) \sim \frac{1}{\tau} e^{-t/\tau}$$

τ is the lifetime

- t is the proper decay time, experimentally it is measured from the decay length L and momentum p (or their projections on the transverse plane)

$$t = \frac{L}{\beta\gamma} = L\frac{M}{p} = L_{xy}\frac{M}{p_T}$$

Lorentz boost factor



lifetime modeling

signal model → $$L(t|\sigma_t,\tau) = \frac{1}{\mathcal{N}} \cdot \left[\, \frac{1}{\tau} e^{-\frac{t}{\tau}} \theta(t) \otimes G(t;\sigma_t) \,\right] \cdot \mathcal{E}(t)$$

PDF normalization

theory model

t-resolution function

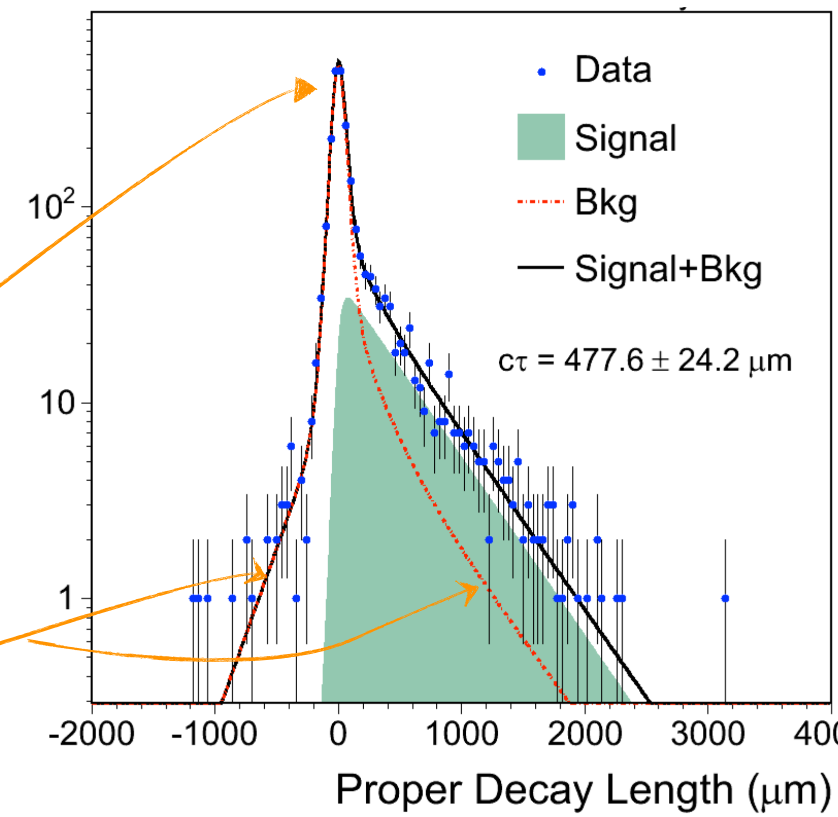
t-acceptance function

- t-resolution
  - use per-event uncertainties σ_t (more precisely reco'd B's get larger weight)
  - calibrate using data (high stat. modes)

$$\sigma_t \mapsto S_t \cdot \sigma_t$$

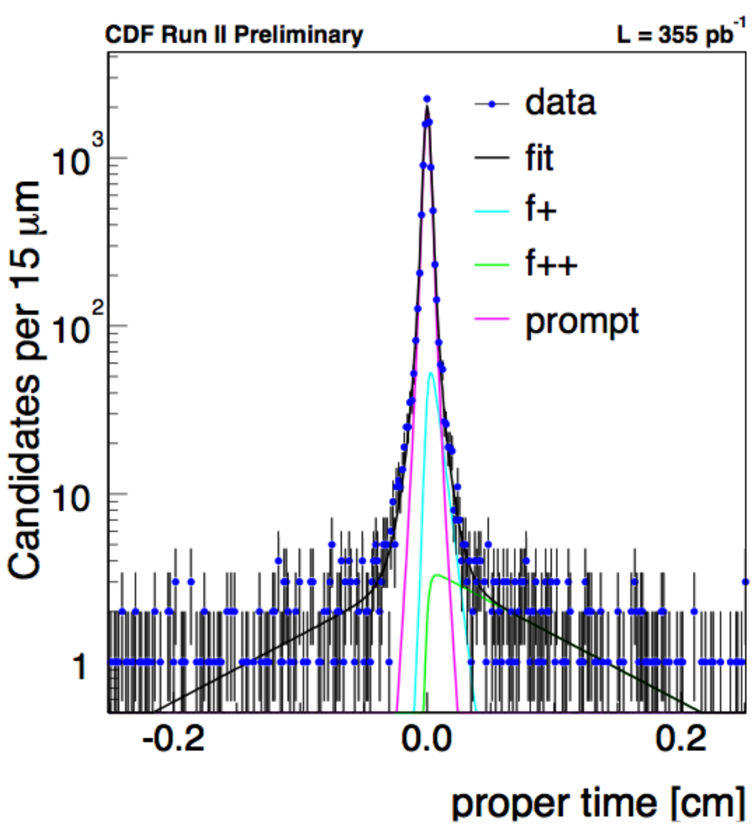
- trigger/selection bias
  - vertex detachment requirements used in selection bias t-distribution, requires acceptance correction, $\varepsilon(t)$
- backgrounds
  - prompt, $\delta(t)$ (→ resolution)
  - long-lived (from decay products of other b-hadrons)

example ⇓ lifetime fit



[t-resolution, σ_t]

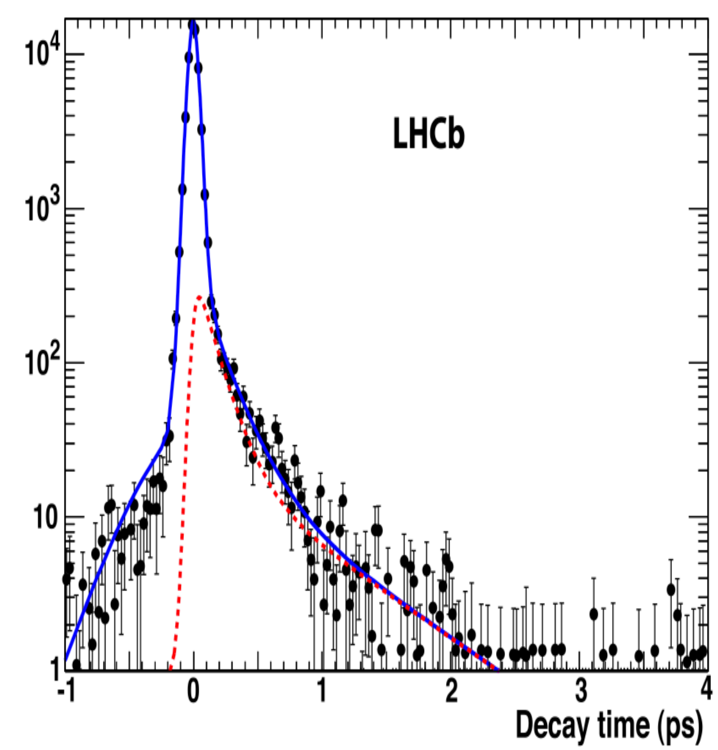
- σ_t may be taken per-event from the vertex kinematic fit
- should be calibrated, using data
- a possible strategy (CDF, also used for example by LHCb)
  - *if* dataset is t-*un*biased: fit prompt peak with scale factor, $e^{-\Gamma \cdot t} \otimes R(t, S_t . \sigma_t)$; *else*:
  - construct a prompt sample of B-like vertices, closely mimicking kinematics and topology of the signal; fit this sample as above, allowing for scale factor
  - to further facilitate transfer to signal sample, parameterize $S_t(\Delta R, I, \eta, z, X^2)$



CDF

signals:
B→Dπ,DlX
triggered by SVT

calibration sample:
SVT-displaced D + prompt tracks



(PLB 713 (2012) 378)

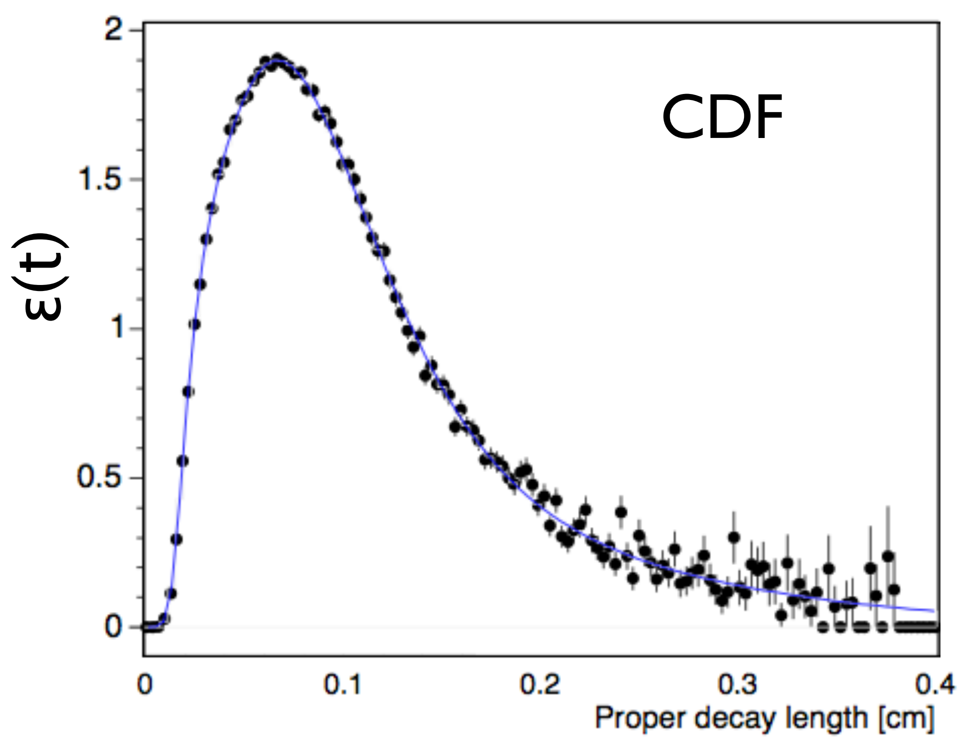
LHCb

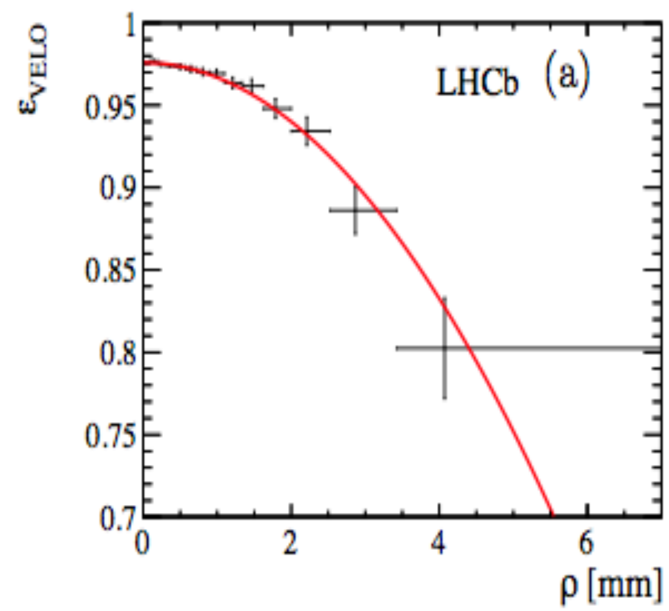
signal:
B→J/ψππ
displaced

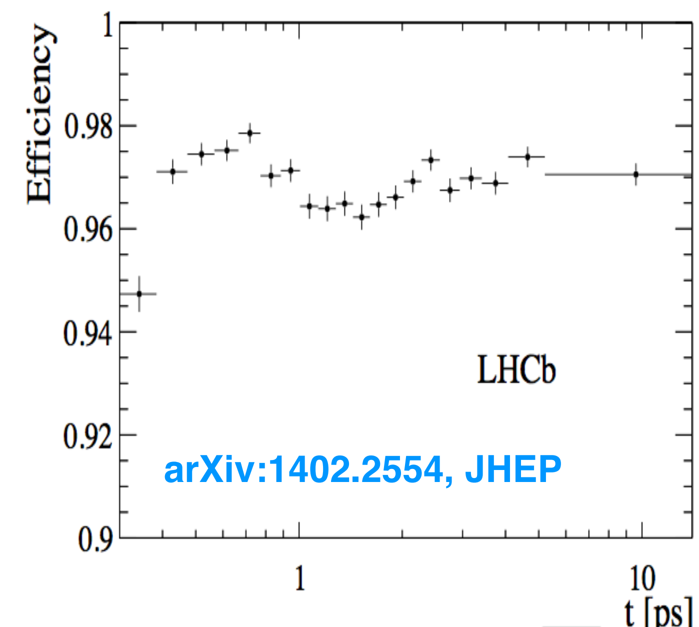
calibration sample:
prompt J/ψ + prompt pion pair

[t-acceptance, ε(t)]

- *if* dataset is not biased, $\varepsilon(t)=1$
- *if* bias corresponds to a threshold (global or per-event) on L_{xy} or t, then the efficiency is given by a threshold function $\varepsilon(t)=\theta(t-t_0)$
- *if* a more general bias, ε(t) can be estimated from MC or data



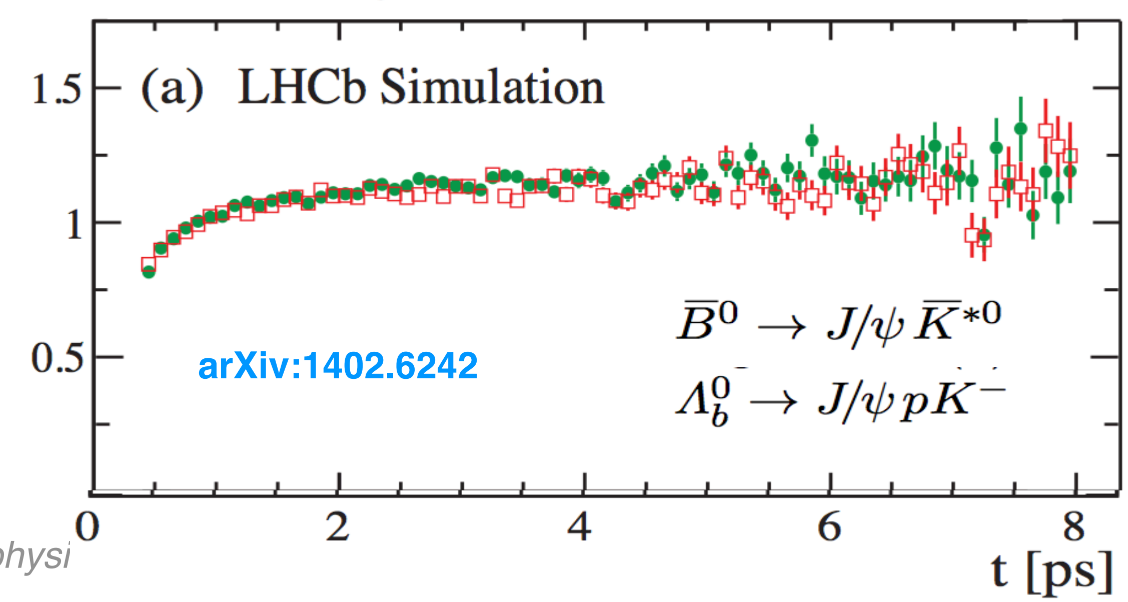




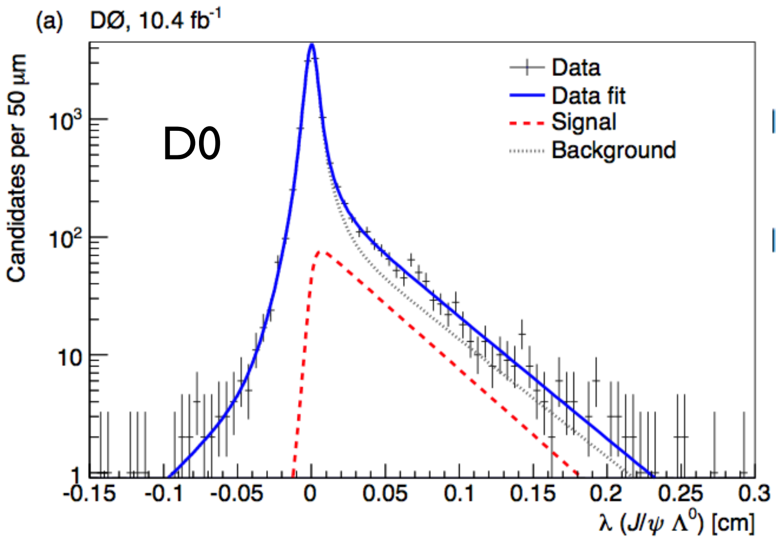
MC driven:

$$\mathcal{E}(t) = \frac{t\text{—distribution after selection}}{\sum_{\{\sigma_t\}} \frac{1}{\tau} e^{-t/\tau} \otimes G(t;\sigma_t)}$$

Data driven: PRD 83 (2011) 032008



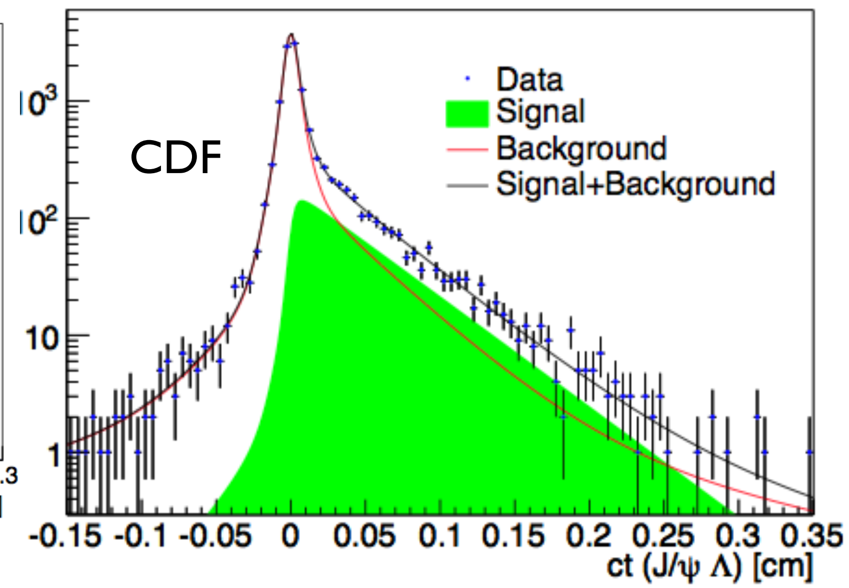
b-hadron lifetimes



PRD 85 (2012) 112003

$\tau(B^0)$ = 1.508 ± 0.025 ± 0.043 ps

$\tau(\Lambda_b)$ = 1.303 ± 0.075 ± 0.035 ps



PRL 106 (2011) 121804

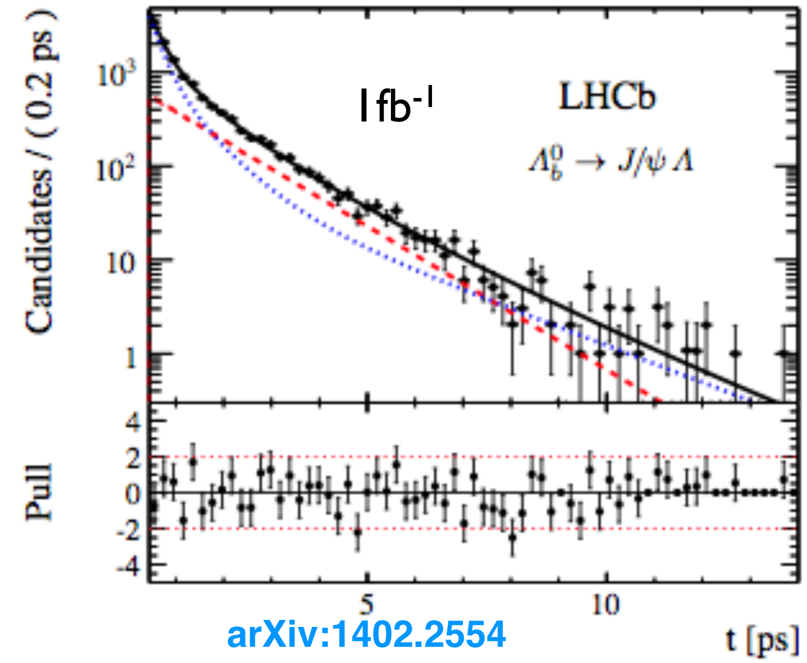
$\tau(B^+)$=1.639 ± 0.009 ± 0.009 ps

$\tau(B^0)$=1.507 ± 0.010 ± 0.008 ps

$\tau(\Lambda_b)$=1.537 ± 0.045 ± 0.014 ps

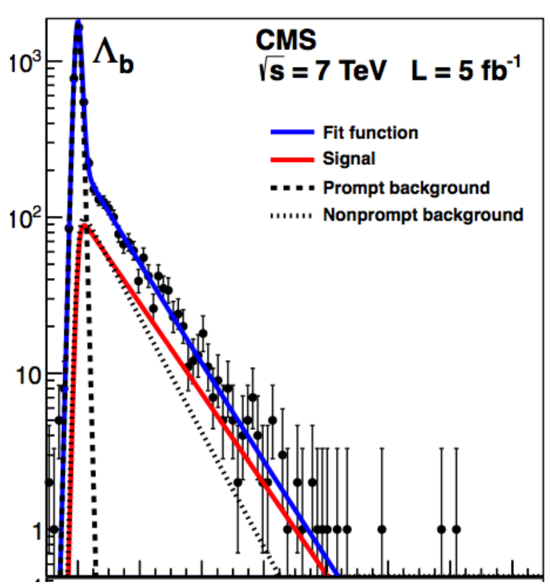
$\tau(B^+)/\tau(B^0)$=1.088 ± 0.009 ± 0.004

$\tau(\Lambda_b)/\tau(B^0)$=1.020 ± 0.030 ± 0.008



arXiv:1402.2554

$$\tau_{B^+\to J/\psi K^+} = 1.637 \pm 0.004 \pm 0.003 \text{ ps}$$
$$\tau_{B^0\to J/\psi K^{*0}} = 1.524 \pm 0.006 \pm 0.004 \text{ ps}$$
$$\tau_{B^0\to J/\psi K^0_S} = 1.499 \pm 0.013 \pm 0.005 \text{ ps}$$
$$\tau_{\Lambda_b^0\to J/\psi \Lambda} = 1.415 \pm 0.027 \pm 0.006 \text{ ps}$$
$$\tau_{B_s^0\to J/\psi \phi} = 1.480 \pm 0.011 \pm 0.005 \text{ ps}$$

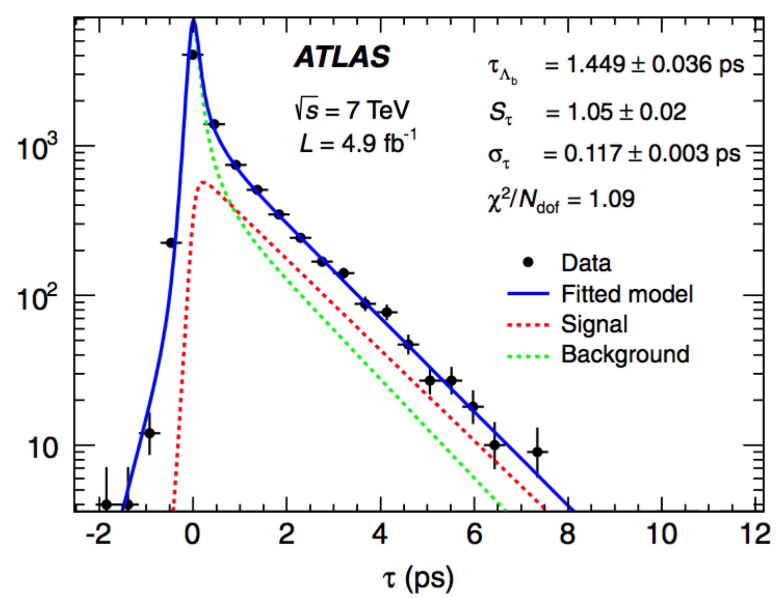


JHEP 07 (2013) 163

$\tau(\Lambda_b)$ =
1.503 ± 0.052 ± 0.031 ps

PRD 87 (2013) 032002

$\tau(\Lambda_b)$ =
1.449 ± 0.036 ± 0.017 ps



PDG 2013

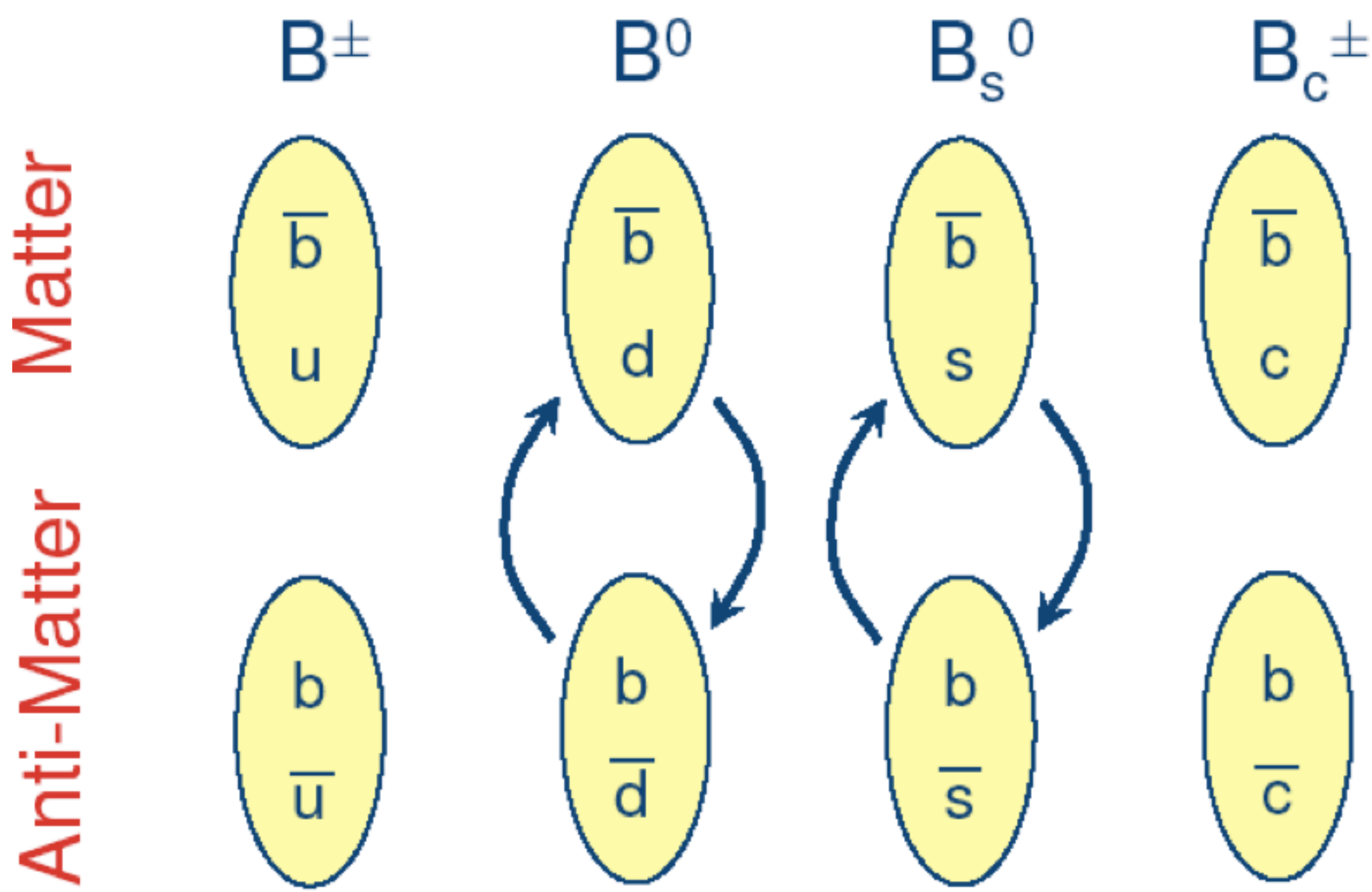
B^0 1.519 ± 0.007 ps
B^+ 1.641 ± 0.008 ps
B_s 1.516 ± 0.011 ps
B_c 0.452 ± 0.033 ps
Λ_b 1.429 ± 0.024 ps

flavor oscillations

&

flavor tagging

B meson mixing



➡ neutral B mesons undergo spontaneous flavor oscillations between particle and antiparticle!

quantum mechanics (ii)

- allowing for a flavor-changing perturbation (ΔF) in the hamiltonian

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{\Delta F} \qquad i\,\frac{d}{dt}\psi = \mathcal{H}\,\psi \qquad i\,\frac{d}{dt}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} m - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & m - \frac{i}{2}\Gamma \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix}$$

$$|\psi\rangle = a\,|P^0\rangle + b\,|\bar{P}^0\rangle$$

- a pure flavor eigenstate at t=0 will evolve to an admixture
    - non-diagonal elements in H ⇒ flavor eigenstates differ from mass eigenstates
- flavor eigenstates

$$\begin{aligned} |P_L\rangle &= p\,|P^0\rangle + q\,|\bar{P}^0\rangle \\ |P_H\rangle &= p\,|P^0\rangle - q\,|\bar{P}^0\rangle \end{aligned} \qquad \text{with} \quad |p|^2 + |q|^2 = 1$$

- time evolution of flavor eigenstates (after finding H eigenvalues $\lambda_{H,L}$)

$$|P_{L,H}\rangle_t \;=\; e^{-i\,\lambda_{L,H}\,t}\,|P_{L,H}\rangle \;=\; e^{-i\,m_{L,H}\,t - \frac{1}{2}\Gamma_{L,H}\,t}\,|P_{L,H}\rangle$$

- probability for particle-antiparticle transition

$$|\langle P^0|\mathcal{H}|\bar{P}^0\rangle|^2 \;=\; \left|\frac{p}{q}\right|^4 |\langle \bar{P}^0|\mathcal{H}|P^0\rangle|^2 \;=\; \left|\frac{p}{q}\right|^2 \frac{1}{2}e^{-\Gamma t}\left[\cosh\left(\frac{\Delta\Gamma}{2}t\right) - \cos\left(\Delta m\,t\right)\right]$$

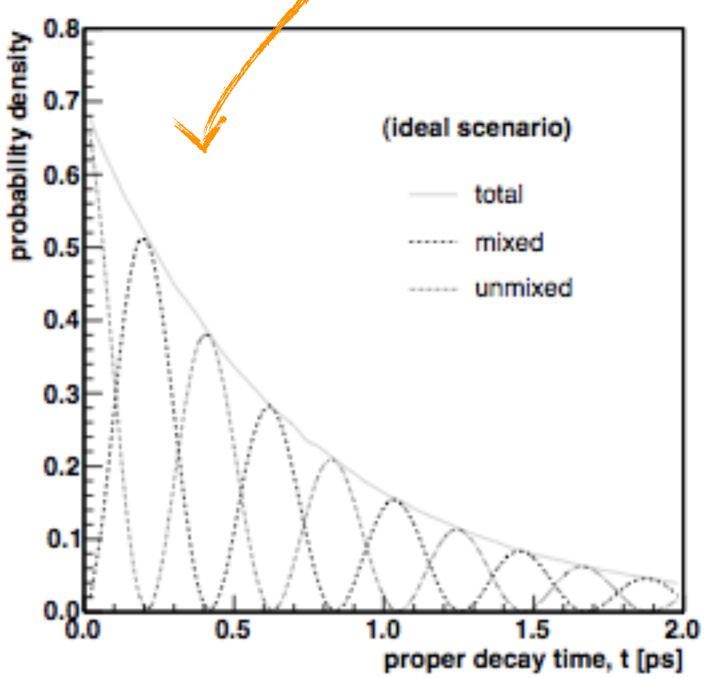
    - with $\Delta\Gamma \equiv \Gamma_L - \Gamma_H$ and $\Delta m \equiv m_H - m_L$
- neglecting CPV in mixing (i.e. p/q=1) and ΔΓ, the mixing probability is:

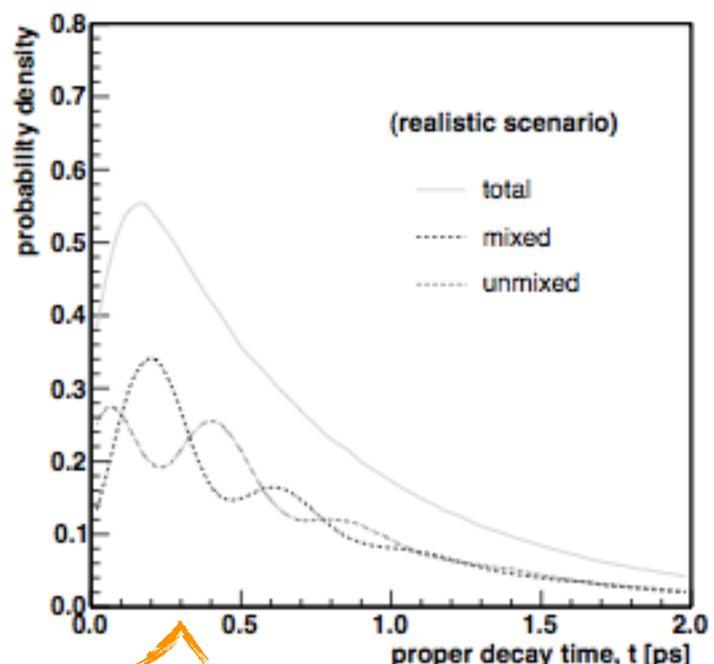
$$\mathcal{P}_{B_q^0 \to \bar{B}_q^0}(t) \;=\; \mathcal{P}_{\bar{B}_q^0 \to B_q^0}(t) \;=\; \frac{\Gamma}{2}e^{-\Gamma\,t}\left[1 - \cos\left(\Delta m\,t\right)\right]$$

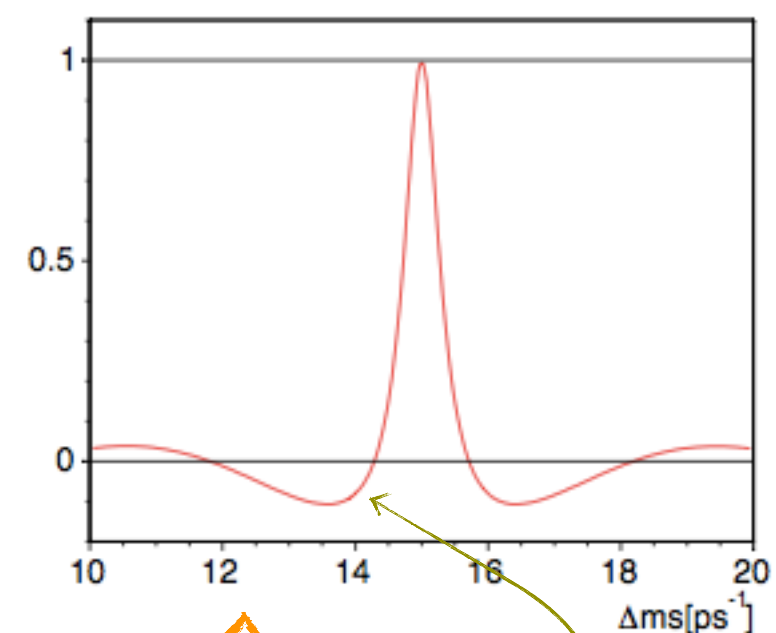
flavor oscillations

$$\left\{ \begin{aligned} p(B \to B) &= \frac{e^{-t/\tau}}{2\tau}(1 + \cos \Delta m t) \\ p(B \to \overline{B}) &= \frac{e^{-t/\tau}}{2\tau}(1 - \cos \Delta m t) \end{aligned} \right.$$

oscillation frequency given by mass difference between heavy and light H eigenstates







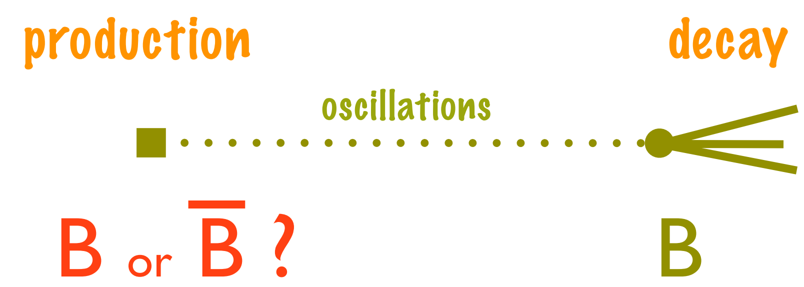
t-resolution, t-bias, dilution

Fourier transform

Exercise: show that a proper time cut $t>t_0$ induces undershootings besides the peak in the Fourier transform of the oscillation signal

- but... one critical ingredient still missing: need to known whether or not a given B candidate in the data has mixed ➠ flavor tagging

particle or antiparticle



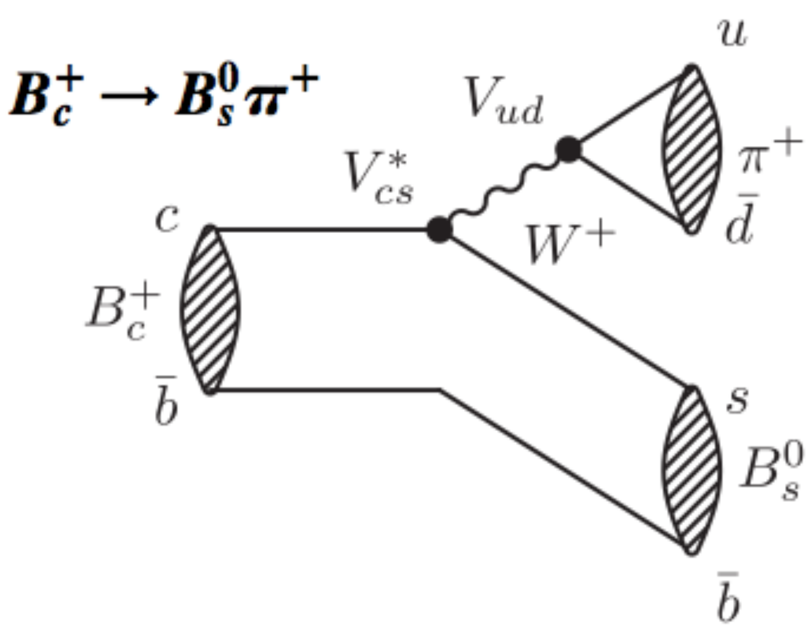
- (let ‘flavor’ here refer to the particle and antiparticle state)
- flavor at decay time:
  - trivially given by the charge of the decay products, if using flavor specific final states
  - (e.g. final flavor given by pion charge in $B_s \to D_s^- \pi^+$ *vs* $\underline{B_s} \to D_s^+ \pi^-$
- flavor at production time: ...

how may it be determined ??

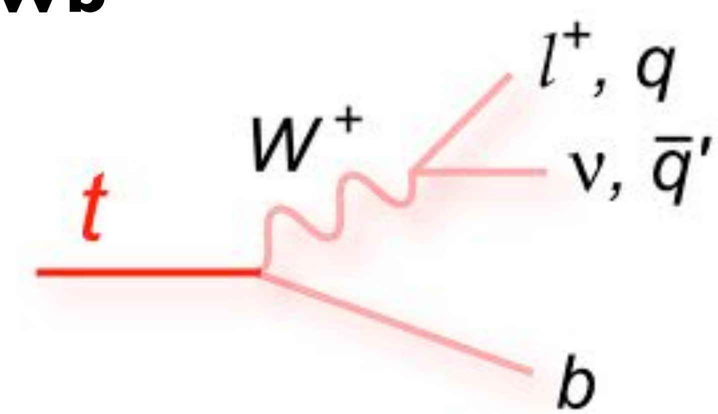
Exercise: think about it before resuming discussion in next 2 slides

how to tag?

- attempt #1: use B_s mesons from the decay of heavier particles



t→Wb

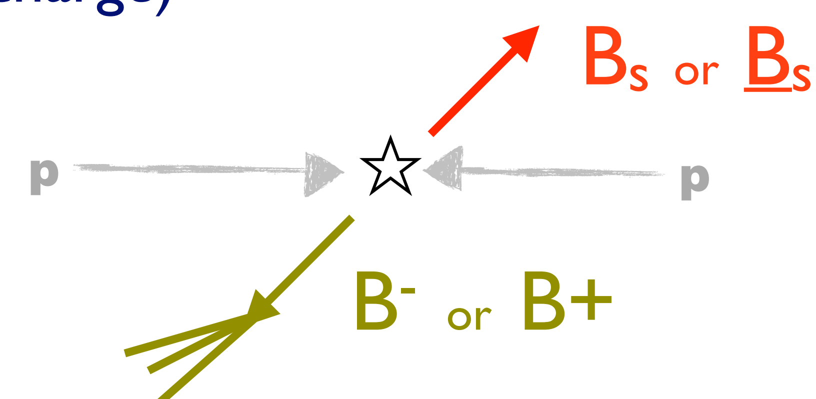


  - the initial B flavor (b or $\bar{b}$) could be inferred from the decay products of the heavier, parent state, eg from the charge of the pion in the examples

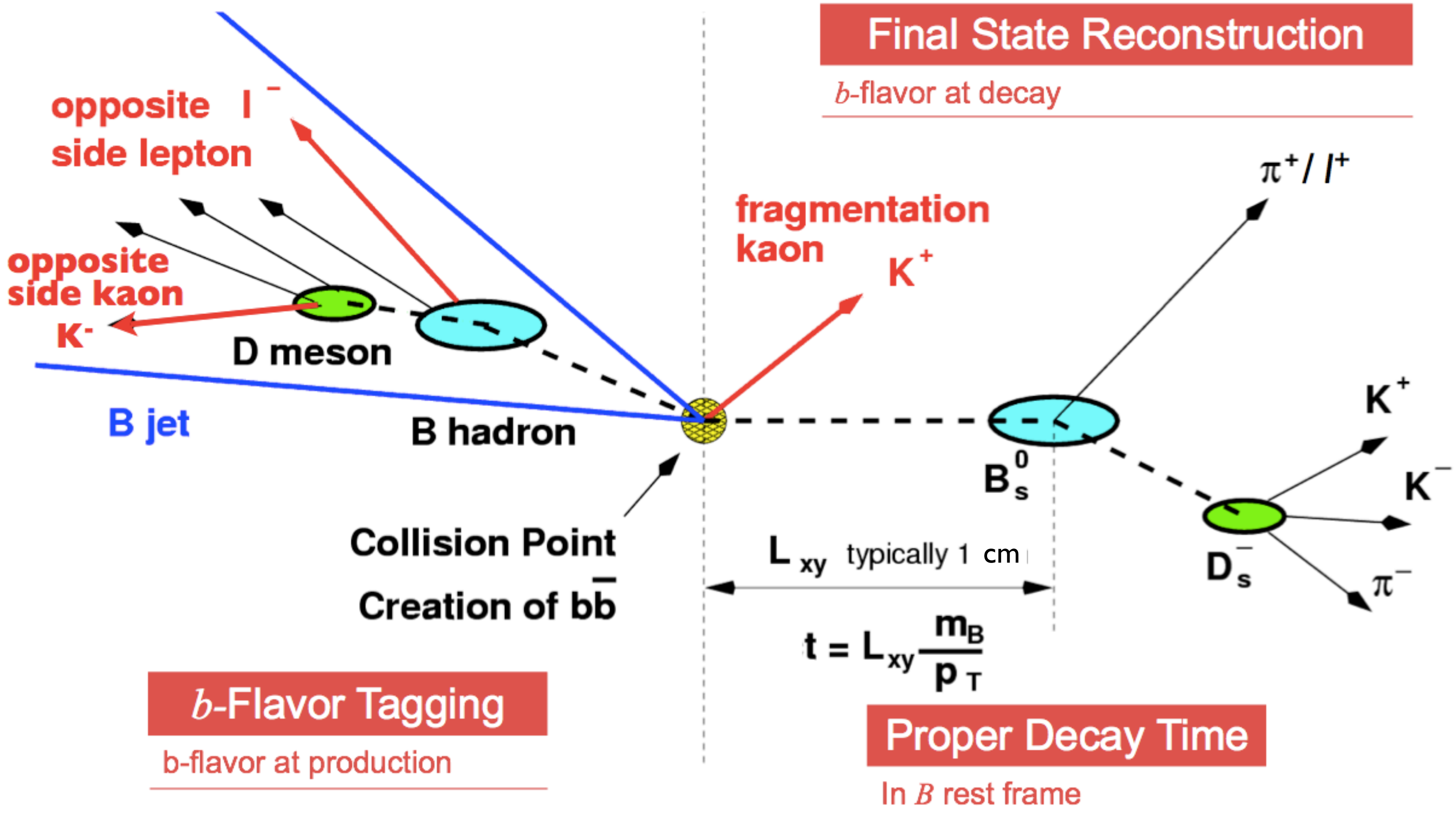
- attempt #2 : make use of the other b quark (from the originally produced b$\bar{b}$ pair), by reconstructing the other b-hadron in the event, say $B^\pm \to J/\psi K^\pm$ (flavor given by the kaon charge)

  - these possibilities are quite interesting! but given reconstruction inefficiencies (of parent or other B), very high signal statistics would/will be required...

➡ catch: infer flavor without full decay reconstruction



flavor tagging methods

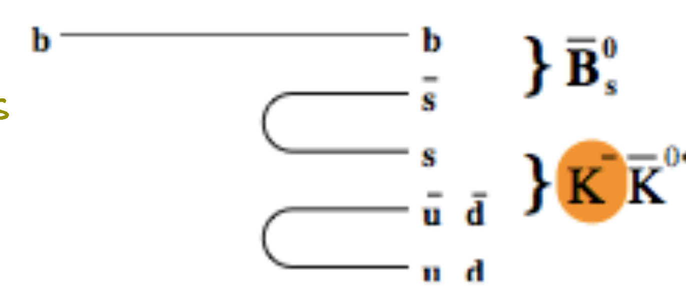


opposite-side tagging

- lepton (e,μ)
- jet-charge
- kaon

Exercise:
1.explain how B flavor oscillations cause an intrinsic dilution of the OST methods performance
2. show how the lepton tagger, based on semileptonic B decays (B→l) is affected by sequential decays such as B→D→l

same-side tagging



Exercise: explain why the performance of SST (OST) should (not) depend on the species of B meson being tagged

dilution factors

- various effects decrease the amplitude of an oscillation signal

$$\left(\begin{array}{c}\text{mixing}\\\text{significance}\end{array}\right)^2 \sim \frac{\epsilon\mathcal{D}^2 S}{2} \cdot \frac{S}{S+B} \cdot e^{-\sigma_t^2 w^2}$$

oscillation frequency w

flavor tagging | signal yield | signal purity | proper time resolution

Exercise: explain why the t-resolution is even more determining for B_s than B_d mixing

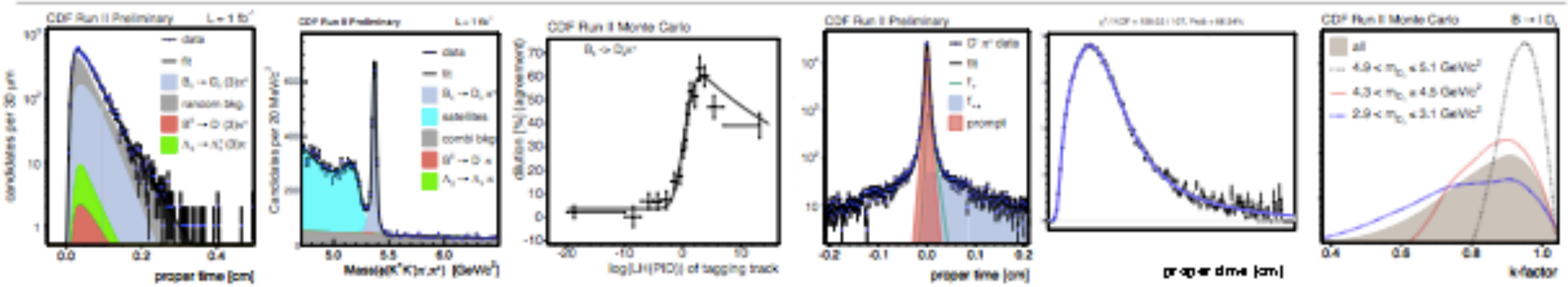
- tagging power εD^2 is given by the algorithm efficiency ε and dilution $D=(1-2w)^2$ where w is the wrong-tag fraction (i.e. probability algorithm gives wrong decision)
- it determines the effective statistical reduction of the sample size: S ➠ S . $\varepsilon_{tag}D^2$

tagger \ εD^2	CDF	D0	ATLAS	CMS	LHCb
for decay $B_s \to J/\psi\Phi$	1.39±0.05% [OST] 3.5±1.4% [SST] **~4.9%**	[OST+SST] 4.68±0.54% **~4.7%**	[OST] 1.45±0.05% **~1.5%**	[OST] **~1%**	2.43±0.08±0.26% [OST] 0.89±0.06% [SST] **~3.3%**

mixing model

$$\mathcal{L} = \mathcal{L}_{mass} \cdot \mathcal{L}_t \cdot \mathcal{L}_{\sigma_t} \cdot \mathcal{L}_D$$

[for each sample component & event]



$$\mathcal{L}_t = \frac{1}{N}\,\kappa\,\frac{e^{-\frac{\kappa\, t'}{\tau}}}{\tau}\;\frac{1 \pm \mathrm{A}\, S_D\, D \cos(\Delta m_s\, \kappa\, t')}{2} \otimes R(t-t'; S_{\sigma_t}\sigma_t)\cdot\mathcal{E}(t) \otimes F(\kappa)$$

analytical computation

(if need be)

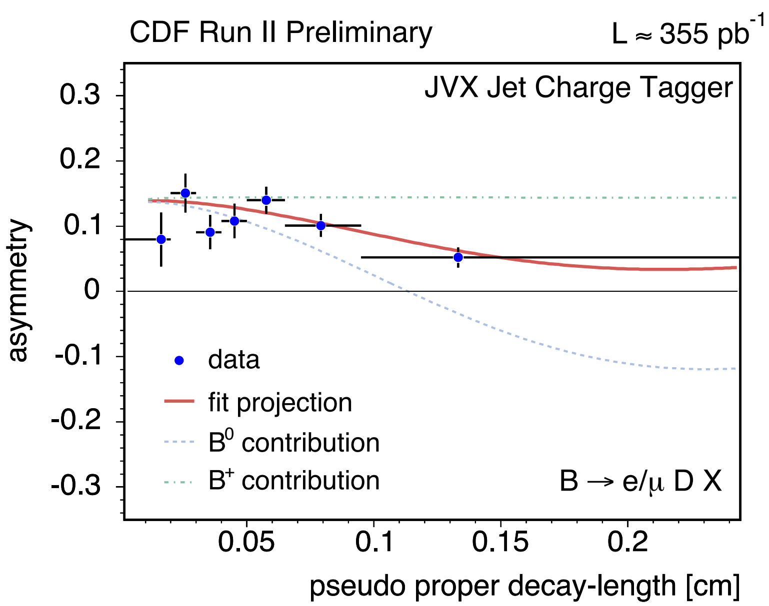
Frequency Δm_s

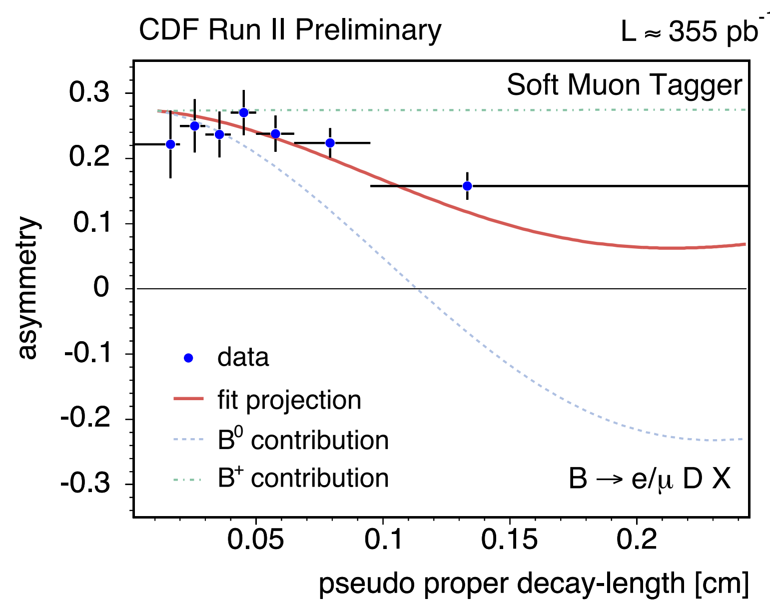
or

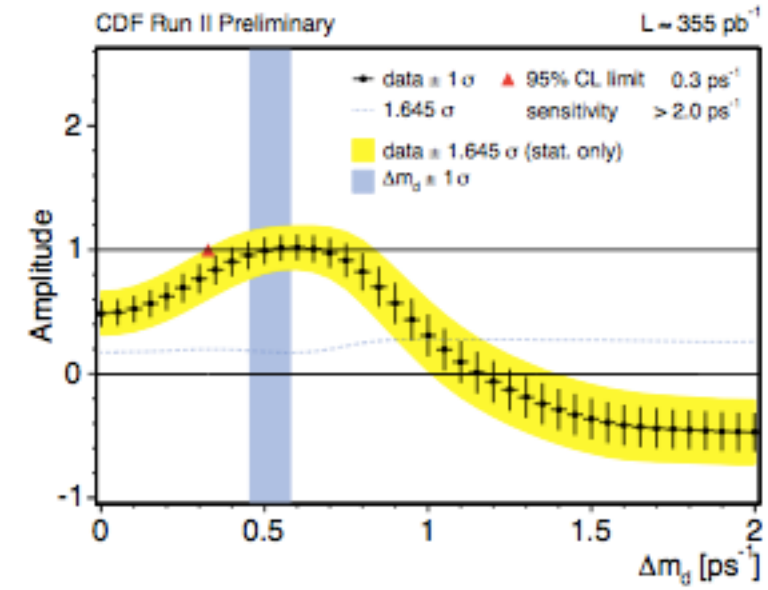
Amplitude (A,σ_A) for fixed Δm_s

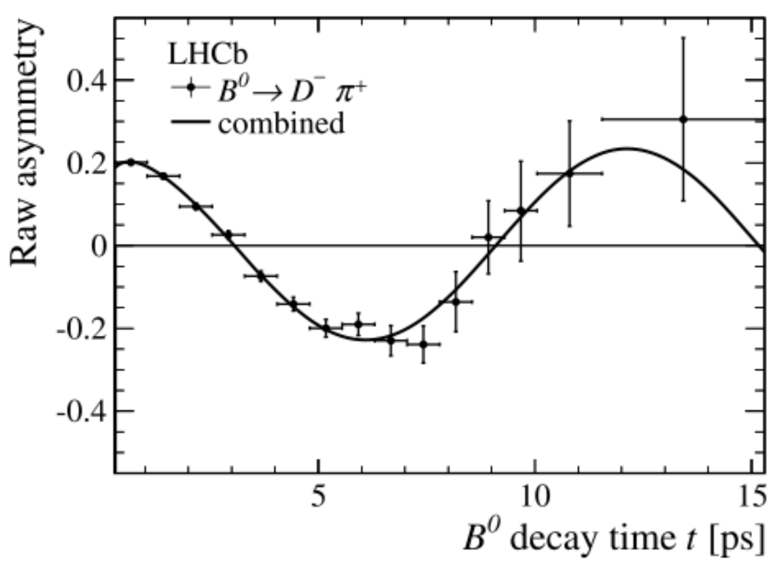
- ingredients: mass, proper time, proper time resolution, t-acceptance function, kinematic factor (for partially reco'd decays), and... flavor tagging

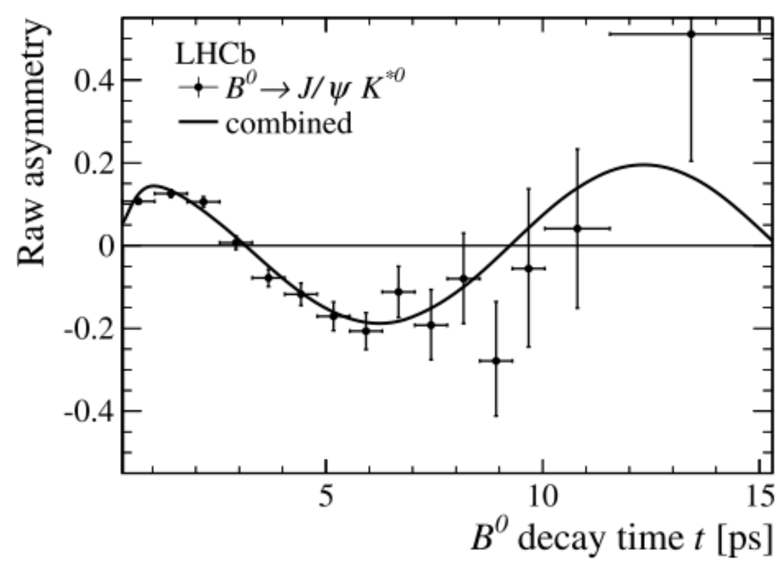
B_d mixing

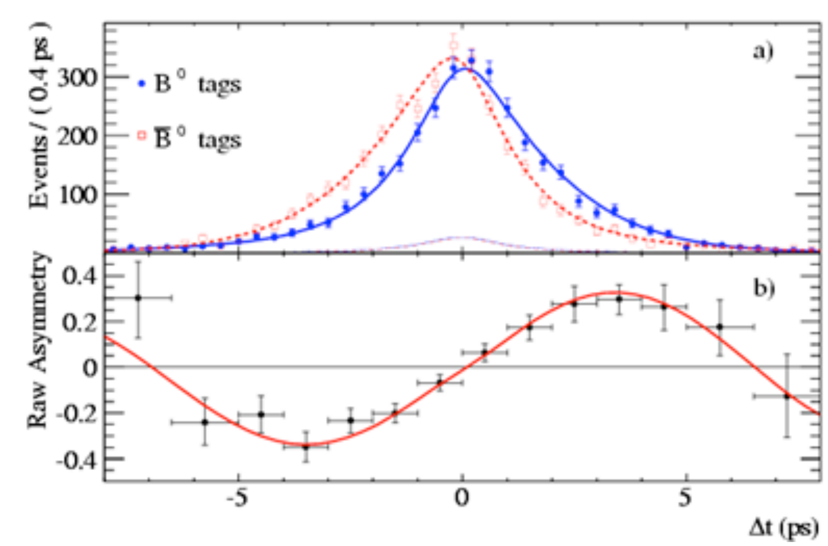






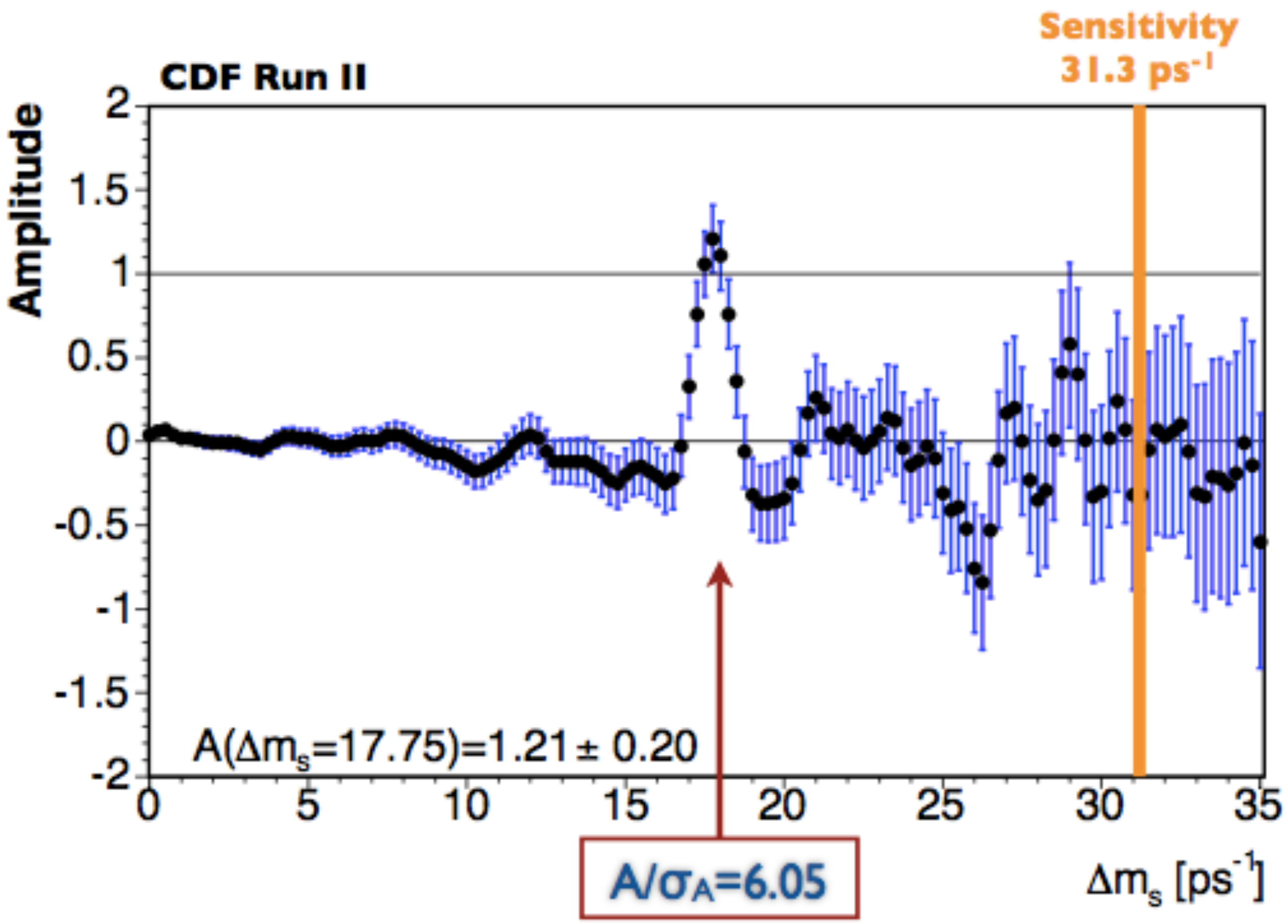


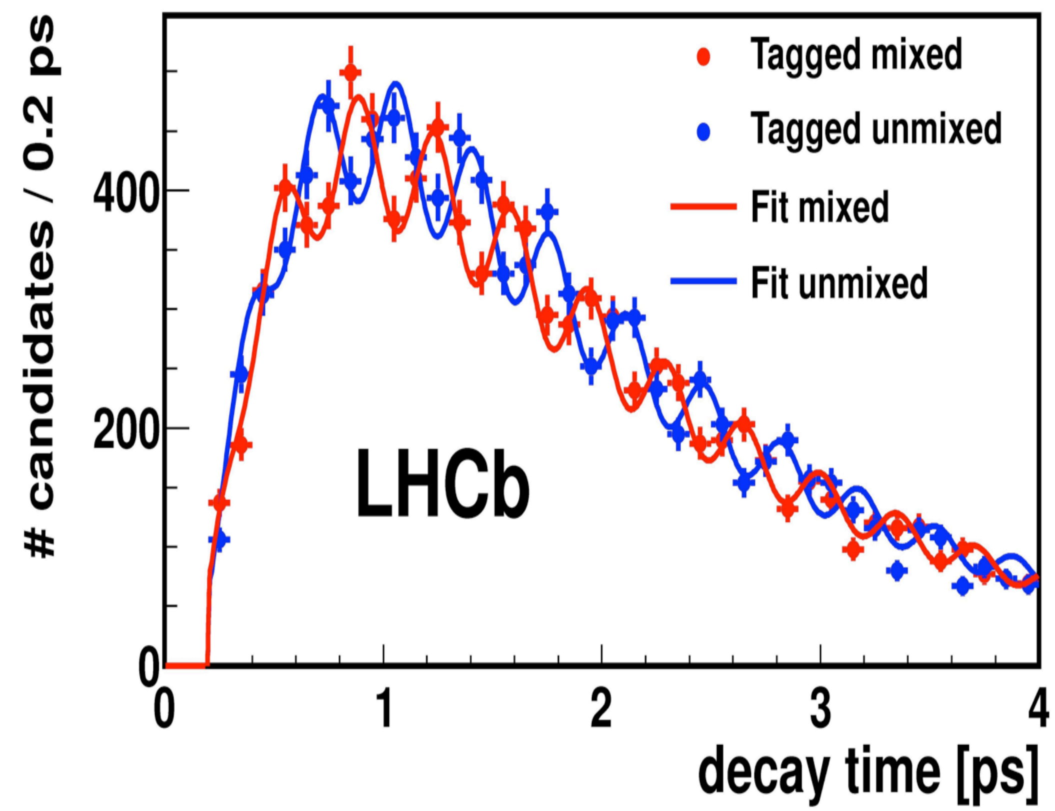




$\Delta m_d = 0.5156 \pm 0.0051\ (\text{stat.}) \pm 0.0033\ (\text{syst.})\ \text{ps}^{-1}$ (most precise measurement)

B_s mixing





frequency space ⇔ time space

observation by CDF (2006)

p-value = 8×10^{-8} corresponding to 5.4σ

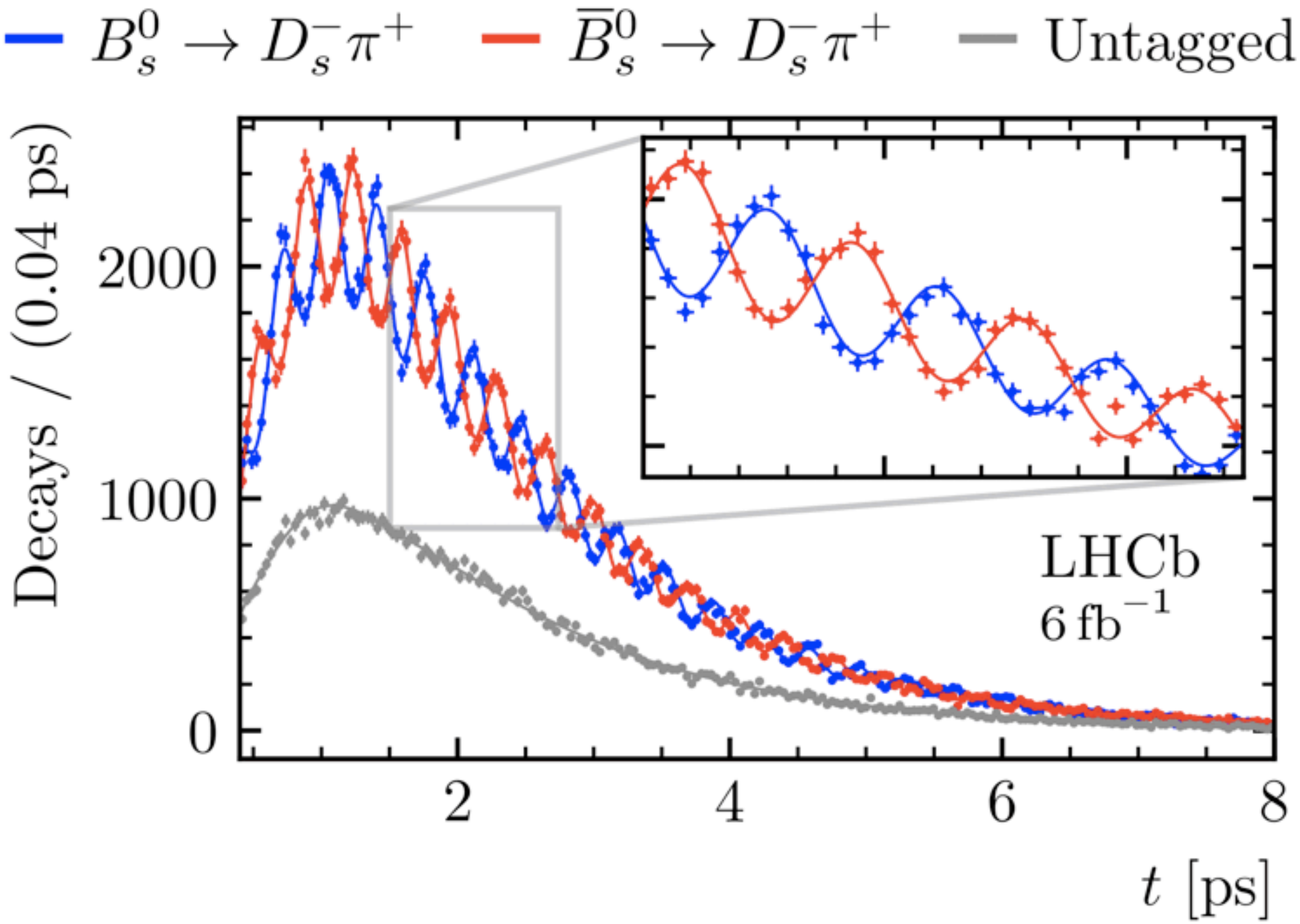
$\Delta m_s = 17.77 \pm 0.10(\text{stat}) \pm 0.07(\text{syst})$ ps-1

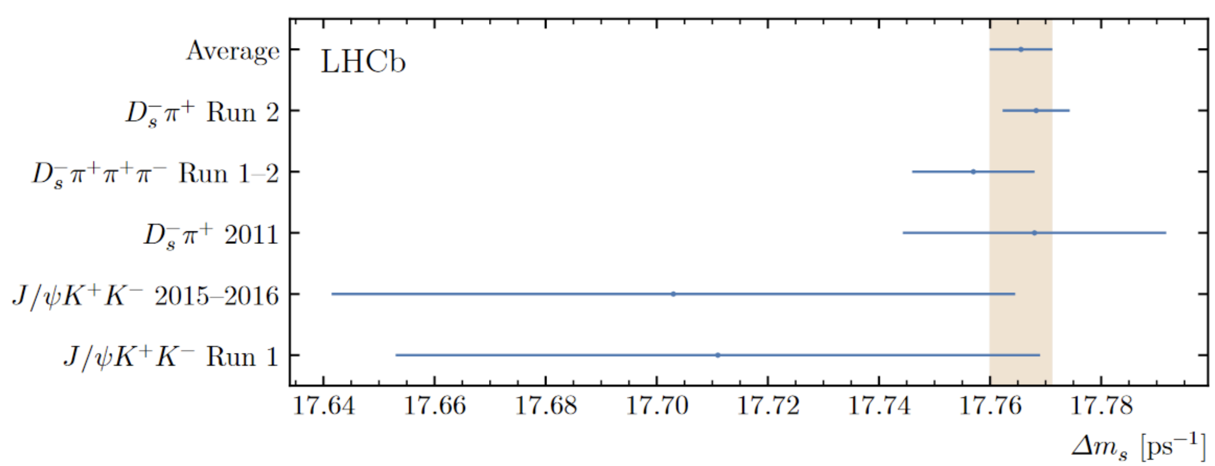
LHCb confirmed (improved precision)

$\Delta m_s = 17.768 \pm 0.023$ (stat) ± 0.006 (syst) ps^{-1}

In agreement with SM expectation $\Delta m_s = 17.3 \pm 2.6$ ps^{-1} [arXiv: 1102.4274]

note: experimental precision $O(10^2)$ times better than theory calculation





CERN LHCb 6/fb 2021

$\Delta m_s = 17.7656 \pm 0.0057 \text{ps}^{-1}$

Fermilab CDF 1/fb 2006

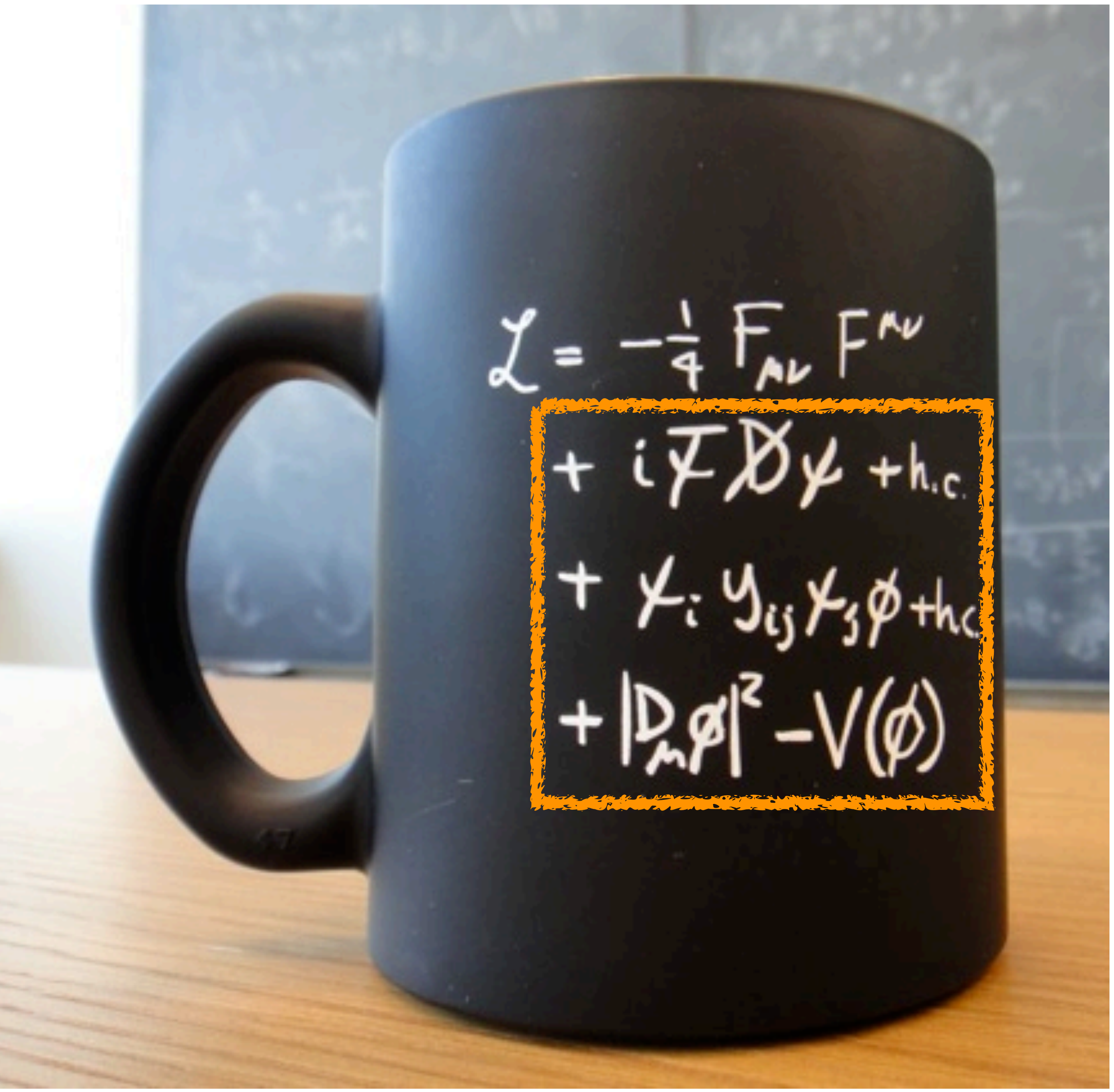
$\Delta m_s = 17.77 \pm 0.10 \pm 0.07 \text{ ps}^{-1}$

Bs mixing provides one of the strongest constraints to NP models that try to explain the observed flavour anomalies. E.g. a Z' could easily alter the B_s^0 oscillation frequency.

CKM

the standard model

the SM lagrangian



$$
\begin{gathered}
\mathcal{L}_{SM} = -\tfrac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \tfrac{1}{4}g^2_s f^{abc}f^{ade}g^b_\mu g^c_\nu g^d_\mu g^e_\nu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - \\
M^2 W^+_\mu W^-_\mu - \tfrac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \tfrac{1}{2c^2_w}M^2 Z^0_\mu Z^0_\mu - \tfrac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - igc_w(\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)) - \\
igs_w(\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + A_\mu(W^+_\nu \partial_\nu W^-_\mu - \\
W^-_\nu \partial_\nu W^+_\mu)) - \tfrac{1}{2}g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \tfrac{1}{2}g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2 c^2_w(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - \\
Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2 s^2_w(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w(A_\mu Z^0_\nu(W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu) - \tfrac{1}{2}\partial_\mu H \partial_\mu H - 2M^2\alpha_h H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - \tfrac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \\
\beta_h\left(\tfrac{2M^2}{g^2} + \tfrac{2M}{g}H + \tfrac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-)\right) + \tfrac{2M^4}{g^2}\alpha_h - \\
g\alpha_h M\left(H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-\right) - \\
\tfrac{1}{8}g^2\alpha_h\left(H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2H^2\right) - \\
gMW^+_\mu W^-_\mu H - \tfrac{1}{2}g\tfrac{M}{c^2_w}Z^0_\mu Z^0_\mu H - \\
\tfrac{1}{2}ig\left(W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)\right) + \\
\tfrac{1}{2}g\left(W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) + W^-_\mu(H\partial_\mu\phi^+ - \phi^+\partial_\mu H)\right) + \tfrac{1}{2}g\tfrac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) + \\
M\left(\tfrac{1}{c_w}Z^0_\mu\partial_\mu\phi^0 + W^+_\mu\partial_\mu\phi^- + W^-_\mu\partial_\mu\phi^+\right) - ig\tfrac{s^2_w}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) + igs_w MA_\mu(W^+_\mu\phi^- - \\
W^-_\mu\phi^+) - ig\tfrac{1-2c^2_w}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + igs_w A_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \\
\tfrac{1}{4}g^2 W^+_\mu W^-_\mu\left(H^2 + (\phi^0)^2 + 2\phi^+\phi^-\right) - \tfrac{1}{8}g^2\tfrac{1}{c^2_w}Z^0_\mu Z^0_\mu\left(H^2 + (\phi^0)^2 + 2(2s^2_w - 1)^2\phi^+\phi^-\right) - \\
\tfrac{1}{2}g^2\tfrac{s^2_w}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) - \tfrac{1}{2}ig^2\tfrac{s^2_w}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \tfrac{1}{2}g^2 s_w A_\mu\phi^0(W^+_\mu\phi^- + \\
W^-_\mu\phi^+) + \tfrac{1}{2}ig^2 s_w A_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\tfrac{s_w}{c_w}(2c^2_w - 1)Z^0_\mu A_\mu\phi^+\phi^- - \\
g^2 s^2_w A_\mu A_\mu\phi^+\phi^- + \tfrac{1}{2}ig_s\,\lambda^a_{ij}(\bar{q}^\sigma_i\gamma^\mu q^\sigma_j)g^a_\mu - \bar{e}^\lambda(\gamma\partial + m^\lambda_e)e^\lambda - \bar{\nu}^\lambda(\gamma\partial + m^\lambda_\nu)\nu^\lambda - \bar{u}^\lambda_j(\gamma\partial + \\
m^\lambda_u)u^\lambda_j - \bar{d}^\lambda_j(\gamma\partial + m^\lambda_d)d^\lambda_j + igs_w A_\mu\left(-(\bar{e}^\lambda\gamma^\mu e^\lambda) + \tfrac{2}{3}(\bar{u}^\lambda_j\gamma^\mu u^\lambda_j) - \tfrac{1}{3}(\bar{d}^\lambda_j\gamma^\mu d^\lambda_j)\right) + \\
\tfrac{ig}{4c_w}Z^0_\mu\{(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{e}^\lambda\gamma^\mu(4s^2_w - 1 - \gamma^5)e^\lambda) + (\bar{d}^\lambda_j\gamma^\mu(\tfrac{4}{3}s^2_w - 1 - \gamma^5)d^\lambda_j) + \\
(\bar{u}^\lambda_j\gamma^\mu(1 - \tfrac{8}{3}s^2_w + \gamma^5)u^\lambda_j)\} + \tfrac{ig}{2\sqrt{2}}W^+_\mu\left((\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)U^{lep}{}_{\lambda\kappa}e^\kappa) + (\bar{u}^\lambda_j\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d^\kappa_j)\right) + \\
\tfrac{ig}{2\sqrt{2}}W^-_\mu\left((\bar{e}^\kappa U^{lep\dagger}{}_{\kappa\lambda}\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\kappa\lambda}\gamma^\mu(1+\gamma^5)u^\lambda_j)\right) + \\
\tfrac{ig}{2M\sqrt{2}}\phi^+\left(-m^\kappa_e(\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa}(1-\gamma^5)e^\kappa) + m^\lambda_\nu(\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa}(1+\gamma^5)e^\kappa\right) + \\
\tfrac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_e(\bar{e}^\lambda U^{lep\dagger}{}_{\lambda\kappa}(1+\gamma^5)\nu^\kappa) - m^\kappa_\nu(\bar{e}^\lambda U^{lep\dagger}{}_{\lambda\kappa}(1-\gamma^5)\nu^\kappa\right) - \tfrac{g}{2}\tfrac{m^\lambda_\nu}{M}H(\bar{\nu}^\lambda\nu^\lambda) - \\
\tfrac{g}{2}\tfrac{m^\lambda_e}{M}H(\bar{e}^\lambda e^\lambda) + \tfrac{ig}{2}\tfrac{m^\lambda_\nu}{M}\phi^0(\bar{\nu}^\lambda\gamma^5\nu^\lambda) - \tfrac{ig}{2}\tfrac{m^\lambda_e}{M}\phi^0(\bar{e}^\lambda\gamma^5 e^\lambda) - \tfrac{1}{4}\,\bar{\nu}_\lambda\,M^R_{\lambda\kappa}\,(1-\gamma_5)\hat{\nu}_\kappa - \\
\tfrac{1}{4}\,\overline{\bar{\nu}_\lambda\,M^R_{\lambda\kappa}\,(1-\gamma_5)\hat{\nu}_\kappa} + \tfrac{ig}{2M\sqrt{2}}\phi^+\left(-m^\kappa_d(\bar{u}^\lambda_j C_{\lambda\kappa}(1-\gamma^5)d^\kappa_j) + m^\lambda_u(\bar{u}^\lambda_j C_{\lambda\kappa}(1+\gamma^5)d^\kappa_j\right) + \\
\tfrac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_d(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j) - m^\kappa_u(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1-\gamma^5)u^\kappa_j\right) - \tfrac{g}{2}\tfrac{m^\lambda_u}{M}H(\bar{u}^\lambda_j u^\lambda_j) - \\
\tfrac{g}{2}\tfrac{m^\lambda_d}{M}H(\bar{d}^\lambda_j d^\lambda_j) + \tfrac{ig}{2}\tfrac{m^\lambda_u}{M}\phi^0(\bar{u}^\lambda_j\gamma^5 u^\lambda_j) - \tfrac{ig}{2}\tfrac{m^\lambda_d}{M}\phi^0(\bar{d}^\lambda_j\gamma^5 d^\lambda_j) + \bar{G}^a\partial^2 G^a + g_s f^{abc}\partial_\mu\bar{G}^a G^b g^c_\mu + \\
\bar{X}^+(\partial^2 - M^2)X^+ + \bar{X}^-(\partial^2 - M^2)X^- + \bar{X}^0(\partial^2 - \tfrac{M^2}{c^2_w})X^0 + \bar{Y}\partial^2 Y + igc_w W^+_\mu(\partial_\mu\bar{X}^0 X^- - \\
\partial_\mu\bar{X}^+ X^0) + igs_w W^+_\mu(\partial_\mu\bar{Y}X^- - \partial_\mu\bar{X}^+ Y) + igc_w W^-_\mu(\partial_\mu\bar{X}^- X^0 - \\
\partial_\mu\bar{X}^0 X^+) + igs_w W^-_\mu(\partial_\mu\bar{X}^- Y - \partial_\mu\bar{Y}X^+) + igc_w Z^0_\mu(\partial_\mu\bar{X}^+ X^+ - \\
\partial_\mu\bar{X}^- X^-) + igs_w A_\mu(\partial_\mu\bar{X}^+ X^+ - \\
\partial_\mu\bar{X}^- X^-) - \tfrac{1}{2}gM\left(\bar{X}^+ X^+ H + \bar{X}^- X^- H + \tfrac{1}{c^2_w}\bar{X}^0 X^0 H\right) + \tfrac{1-2c^2_w}{2c_w}igM\left(\bar{X}^+ X^0\phi^+ - \bar{X}^- X^0\phi^-\right) + \\
\tfrac{1}{2c_w}igM\left(\bar{X}^0 X^-\phi^+ - \bar{X}^0 X^+\phi^-\right) + igMs_w\left(\bar{X}^0 X^-\phi^+ - \bar{X}^0 X^+\phi^-\right) + \\
\tfrac{1}{2}igM\left(\bar{X}^+ X^+\phi^0 - \bar{X}^- X^-\phi^0\right).
\end{gathered}
$$

Quark gauge couplings

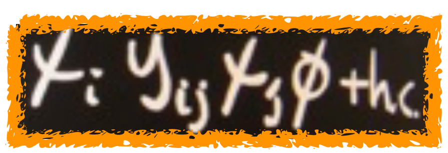
Flavour universality: gauge couplings equal for all generations

$$\mathcal{L}_{\text{fermion}} = \sum_{j=1}^{3} \bar{Q}_j i \not{D}_Q Q_j + \bar{U}_j i \not{D}_U U_j + \bar{D}_J i \not{D}_D D_j$$

$$D_{Q,\mu} \quad = \quad \partial_\mu + i g_s T^a G^a_\mu + i g \tau^a W^a_\mu + i Y_Q g' B_\mu$$

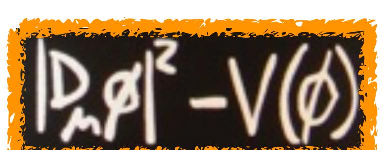
Yukawa couplings

Flavour non-universality: induced by Yukawa couplings between quark and Higgs



$$\mathcal{L}_{\text{Yuk}} = \sum_{i,j=1}^{3} (-Y_{U,ij} \bar{Q}_{Li} \tilde{H} U_{Rj} - Y_{D,ij} \bar{Q}_{Li} H D_{Rj} + h.c.)$$

Higgs sector



$$\mathcal{L}_H = \left[(\partial_\mu - i g W^a_\mu t^a - i g' Y_\phi B_\mu) \, \phi \right]^2 + \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2$$

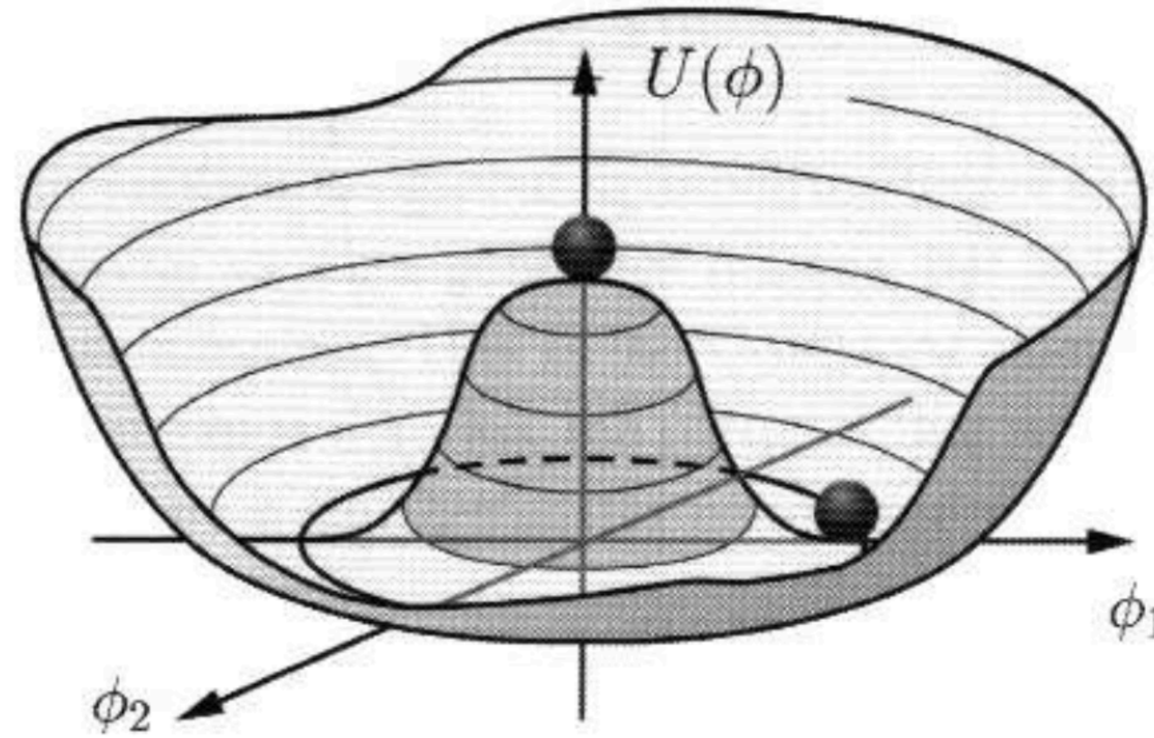
Higgs mechanism

$$V(\phi) = \mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger \phi)^2$$

spontaneous symmetry breaking
by non-zero <ϕ> = v

expand Higgs field around vacuum

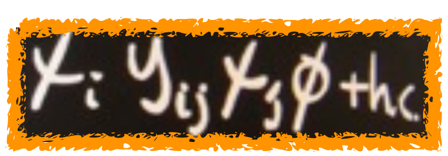
$$\phi(x) = \phi_0 + h(x)$$



$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \Rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

vacuum expectation value v ≠ 0

$$v = \frac{|\mu|}{\sqrt{\lambda}} = \frac{2M_W}{g} = 246\mathrm{GeV}$$



$$\mathcal{L}_f = \frac{g_f}{\sqrt{2}}(\bar{f}_L f_R + \bar{f}_R f_L)v + \frac{g_f}{\sqrt{2}}(\bar{f}_L f_R + \bar{f}_R f_L)h$$

fermions acquire mass: $m_f = \frac{g_f v}{\sqrt{2}} = \frac{\sqrt{2} g_f M_W \sin\theta_W}{e}$

quark masses

Exercise: show this

- a Lagrangian mass term $m\bar{\psi}\psi$ would break chiral gauge symmetry ➠ not allowed
- introducing Yukawa interactions with a scalar field, fermion mass terms get generated

$$-Y\bar{\psi}\psi\phi \quad \xrightarrow[\text{symmetry breaking}]{\text{spontaneous}} \quad -Y\bar{\psi}\psi\,(v+\phi')$$

the Higgs

- the mass terms for up- and down-type quarks have the form

$$\mathcal{L}_M \;=\; -\bar{\mathbf{u}}_R^{\circ T}\,\mathbf{m}_\mathrm{u}\,\mathbf{u}_L^\circ - \bar{\mathbf{d}}_R^{\circ T}\,\mathbf{m}_\mathrm{d}\,\mathbf{d}_L^\circ \;+\; \text{h.c.}$$

- the mass matrices - m_u, m_d - are not diagonal; may be diagonalized (w/ unitary matrices L,R)

$$L_u\mathbf{m}_\mathrm{u}R_u^\dagger = \hat{\mathbf{m}}_\mathrm{u}$$

$$L_d\mathbf{m}_\mathrm{d}R_d^\dagger = \hat{\mathbf{m}}_\mathrm{d}$$

➠ $$\hat{\mathbf{m}}_{\mathrm{u(d)}} = \mathrm{diag}\left(m_{u(d)}, m_{c(s)}, m_{t(b)}\right)$$

- flavor changing interactions in the SM (charged currents) through couplings to $W^\pm$ bosons

$$\mathcal{L}_W \;=\; \frac{g}{\sqrt{2}}\,\bar{\mathbf{u}}_L^{\circ T}\,\gamma^\mu\,\bar{\mathbf{d}}_L^\circ\,W_\mu^+ \;+\; \text{h.c.} \;=\; \frac{g}{2\sqrt{2}}\,\bar{\mathbf{u}}^T\,\gamma^\mu(1-\gamma^5)\,\mathbf{V}\,\mathbf{d}\,W_\mu^+ \;+\; \text{h.c.}$$

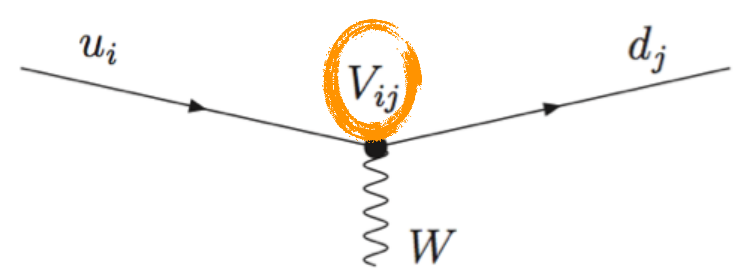
the CKM matrix

- the unitary quark-mixing matrix V is the Cabibbo-Kobayashi-Maskawa matrix

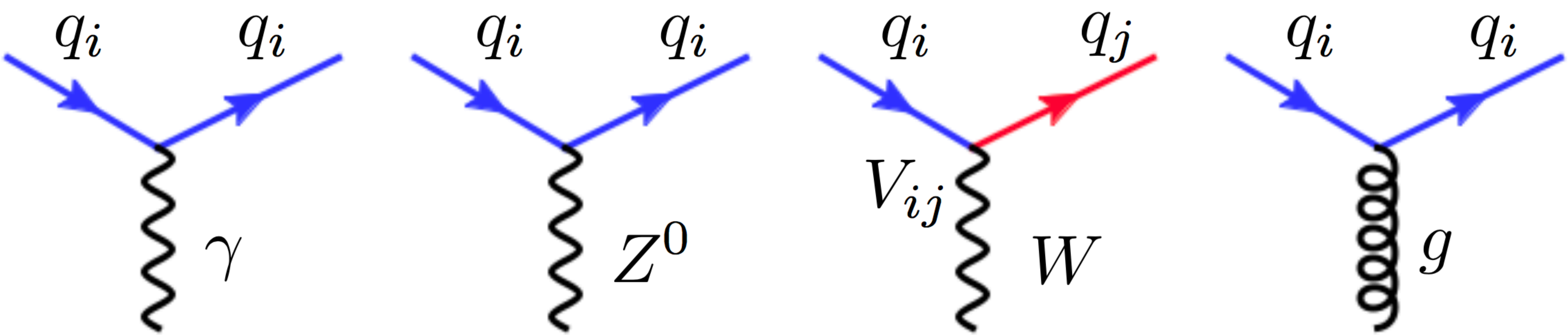
$$\mathbf{V} \equiv L_u L_d^\dagger$$

- describing quark-flavor mixing

$$\mathbf{d}' = \mathbf{V}\,\mathbf{d} \quad \Leftrightarrow \quad \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



recap: no FCNC at tree level in SM

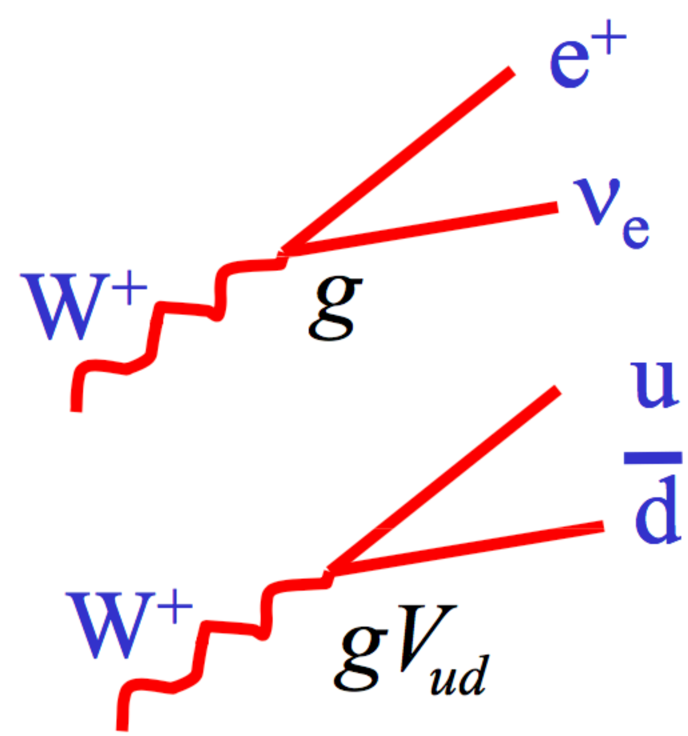


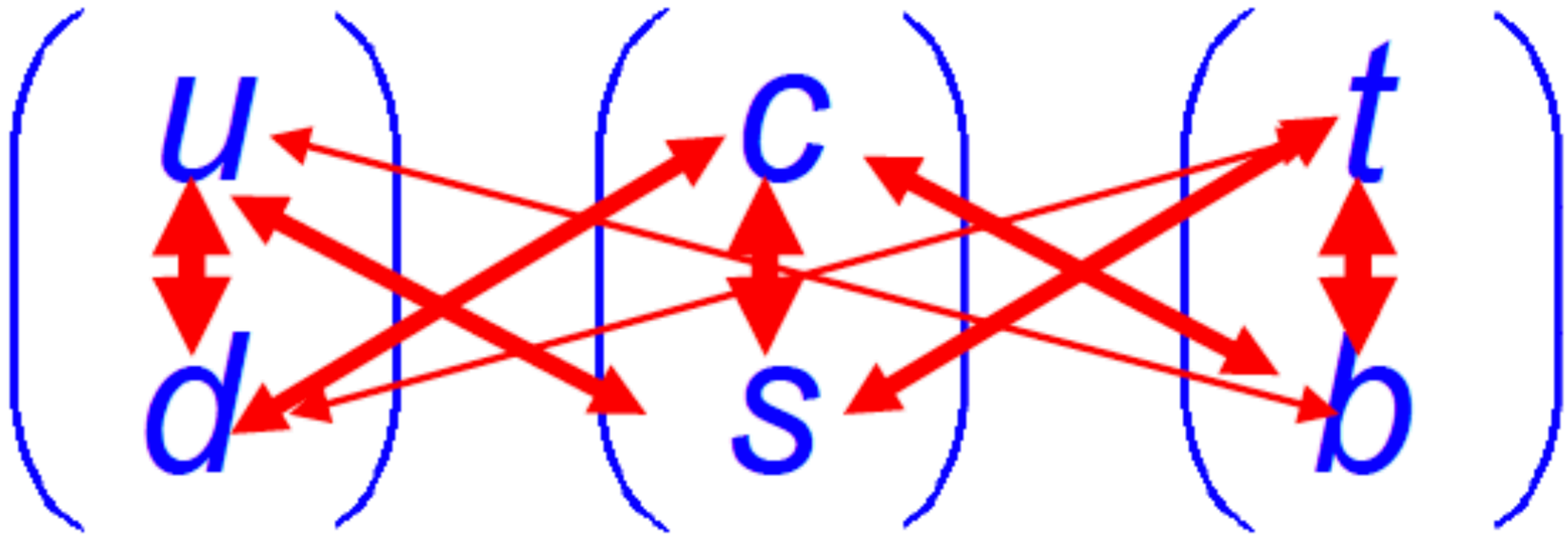
(in SM at tree level)

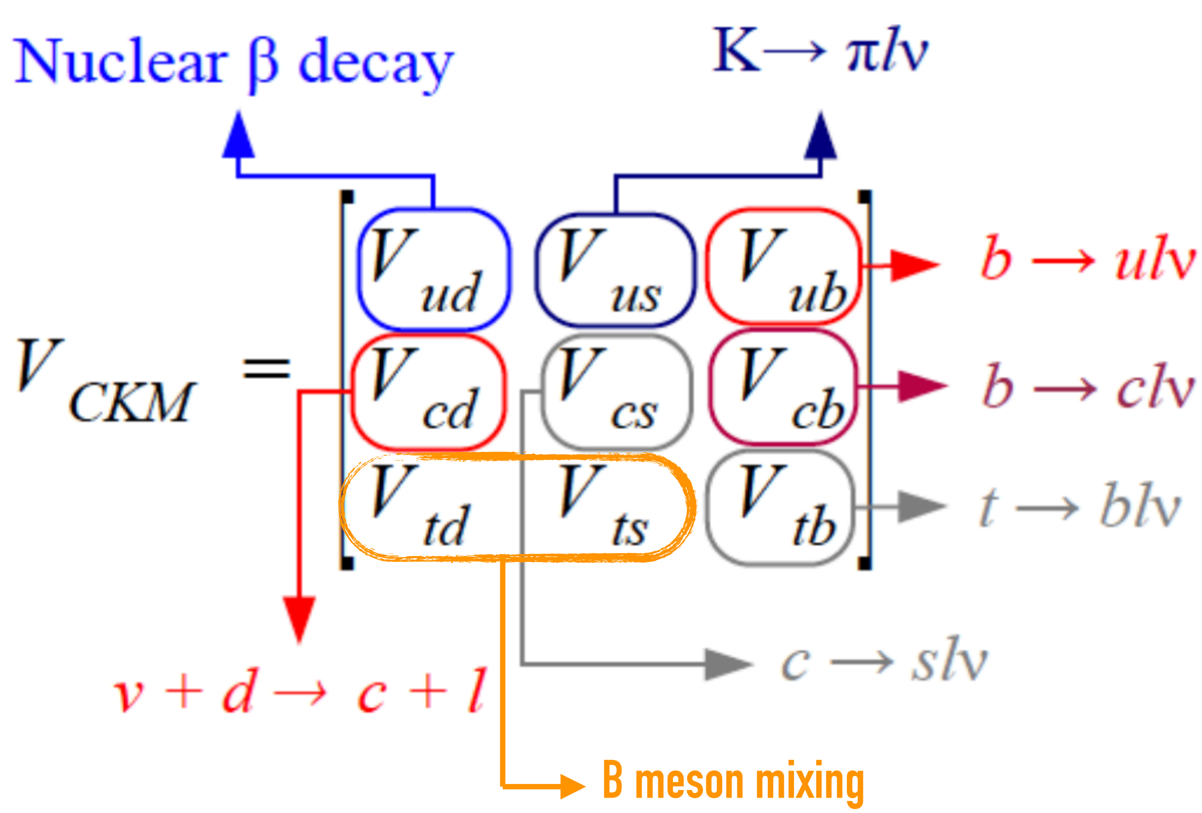
charged currents (W) = flavour changing interactions

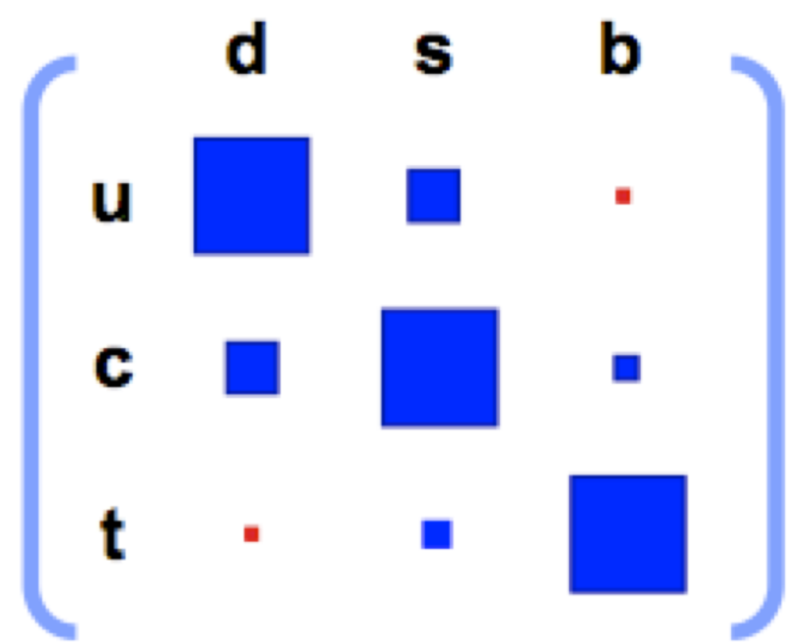
leptons: flavour universality

quarks: flavour non-universality




u
d
c
s
t
b


Nuclear β decay
K→ πlν
$V_{CKM} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$
$b \to ul\nu$
$b \to cl\nu$
$t \to bl\nu$
$c \to sl\nu$
$\nu + d \to c + l$
B meson mixing


d
s
b
u
c
t

quark mixing [CKM]

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \sim \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

- CKM: a unitary 3x3 matrix
  - has 9 parameters: 3 rotation (Euler angles) + 6 phases
  - 5 of these phases can be absorbed by making phase rotations of quark fields
  - we are left with 4 independent parameters: 3 angles & 1 (complex) phase
  - ☛ in a standard parameterization (Wolfenstein) these are: A, λ, ρ & η
- one irreducible phase ➠ the source of CP violation in the SM

Exercise:
* show that in case of N generations, unitarity implies $(N-1)^2$ independent parameters, with $N(N-1)/2$ rotation angles and $(N-1)(N-2)/2$ complex phases
* show that at least three quark generations are required for CP violation

[CKM I parameter counting]

$$\mathbf{V} \equiv L_u L_d^\dagger$$

$$V \rightarrow \begin{pmatrix} e^{-\phi_u} & & \\ & e^{-\phi_c} & \\ & & e^{-\phi_t} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} e^{-\phi_d} & & \\ & e^{-\phi_s} & \\ & & e^{-\phi_b} \end{pmatrix}$$

$$d_{Li} \rightarrow e^{i\phi_{di}} d_{Li}$$

$$u_{Li} \rightarrow e^{i\phi_{ui}} u_{Li}$$

$$V_{\alpha j} \rightarrow \exp\left(i\left(\phi_j - \phi_\alpha\right)\right) V_{\alpha j}$$

- a complex NxN matrix has $2N^2$ parameters
- unitary ➡ $VV^\dagger=1$ ➡ N^2 parameters
- can absorb 2N-1 redefining 2N-1 phases of 2N quarks
- accounting $2N^2 - N^2 - (2N-1) = (N-1)^2$
- this correspond to N(N-1)/2 angles and (N-1)(N-2)/2 complex phases

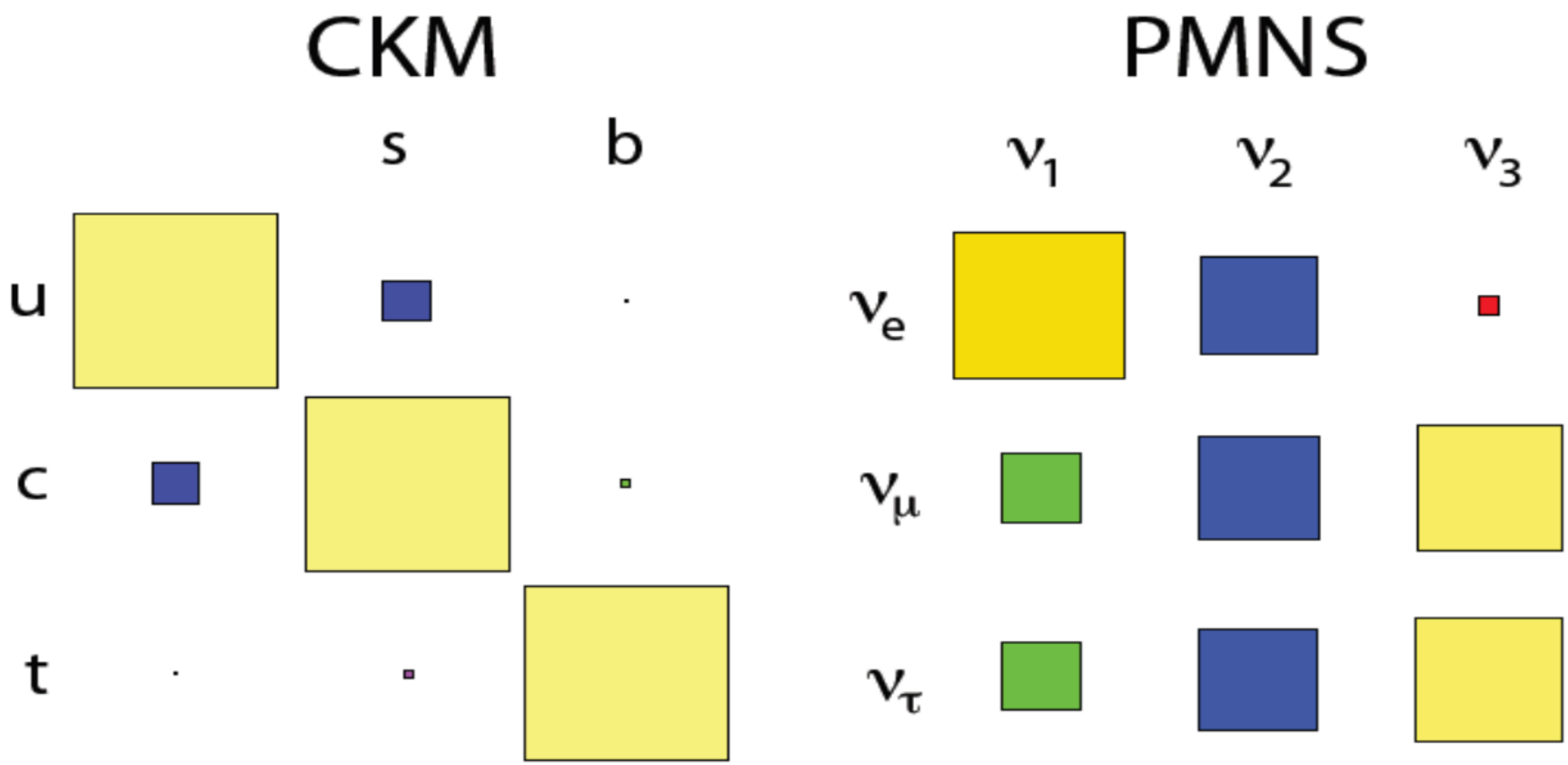
Degrees of freedom in V_{CKM} in	3	N generations
Number of real parameters:	9	$+ N^2$
Number of imaginary parameters:	9	$+ N^2$
Number of constraints ($VV^\dagger = 1$):	-9	$- N^2$
Number of relative quark phases:	-5	$- (2N-1)$
	-----------	-----------
Total degrees of freedom:	4	$(N-1)^2$
Number of Euler angles:	3	$N(N-1)/2$
Number of CP phases:	1	$(N-1)(N-2)/2$

2 generations:

$$V_{CKM} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

No CP violation in SM!
This is the reason Kobayashi and Maskawa *first* suggested a 3rd family of fermions!

[pause: quarks vs leptons]



- in SM, lepton Yukawa matrices can be diagonalised independently ➡ no FCNC …
- however, ν oscillate ➡ lepton flavour not conserved
- there is also a corresponding mixing matrix (PMNS) in the lepton sector
- CLFV depends on mechanism to generate neutrino masses
- CPV in lepton sector
- *are CKM & PMNS related, can explain different structures in quarks vs leptons, …*

CKM unitarity constraints

$$V^\dagger V = 1$$

6 ➡ unitarity triangles

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \text{ (db)}$$

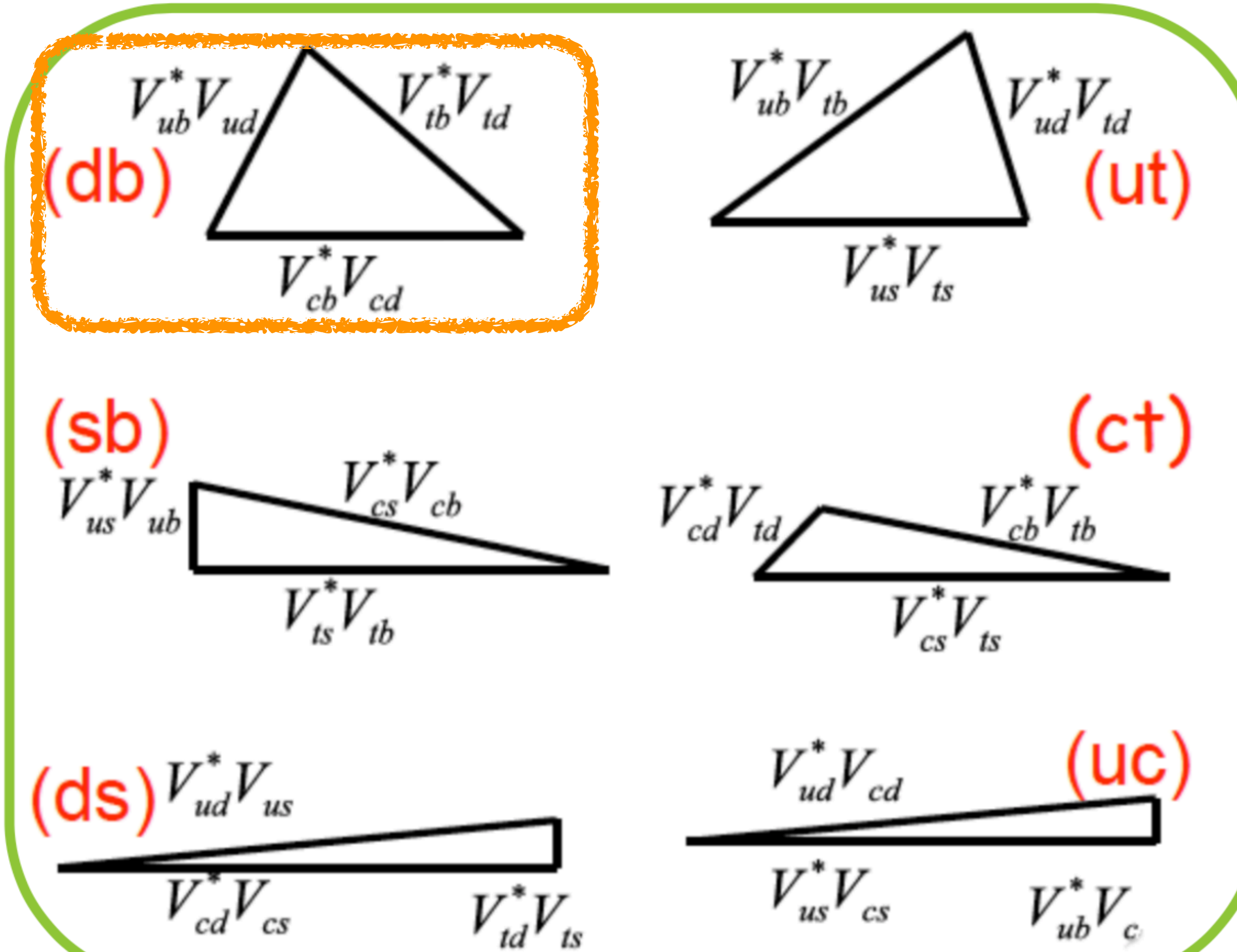
$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0 \text{ (sb)}$$

$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0 \text{ (ds)}$$

$$V_{ud}V_{td}^* + V_{us}V_{ts}^* + V_{ub}V_{tb}^* = 0 \text{ (ut)}$$

$$V_{cd}V_{td}^* + V_{cs}V_{ts}^* + V_{cb}V_{tb}^* = 0 \text{ (ct)}$$

$$V_{ud}V_{cd}^* + V_{us}V_{cs}^* + V_{ub}V_{cb}^* = 0 \text{ (uc)}$$



All 6 triangles have the same area, a measure of CPV in the SM

“the” unitarity triangle

$$V_{ub}^*V_{ud} + V_{cb}^*V_{cd} + V_{tb}^*V_{td} = 0$$

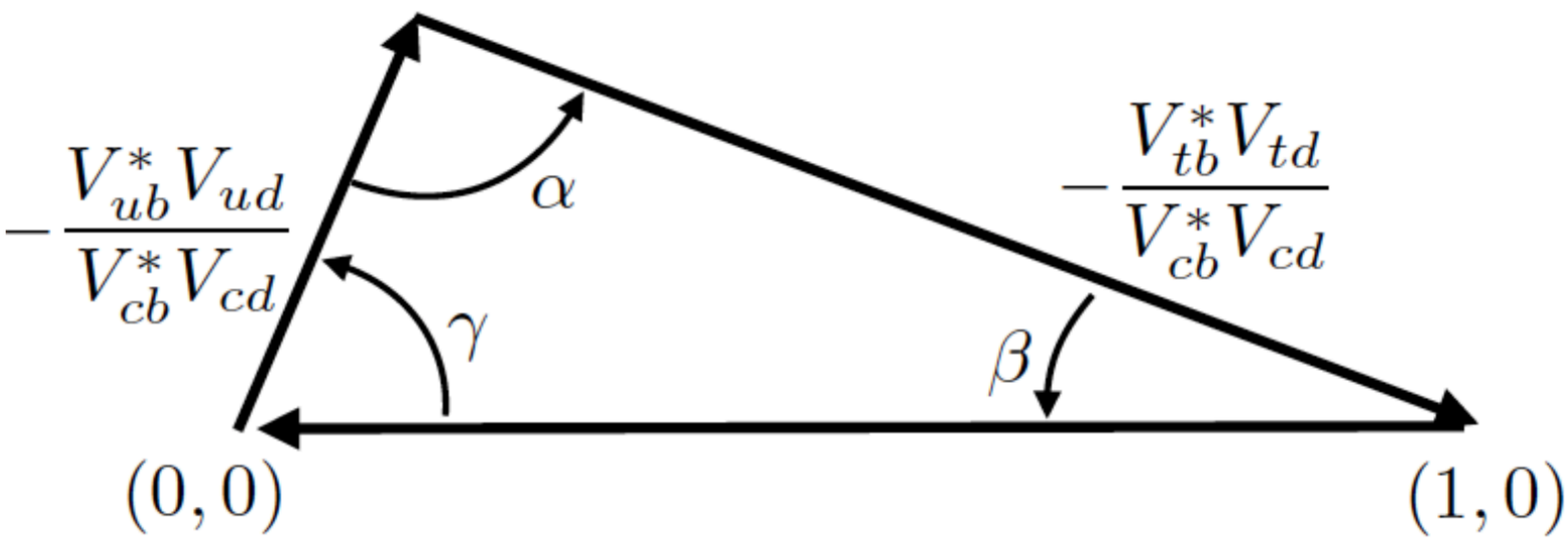
$$\alpha = \arg\left(-\frac{V_{tb}^*V_{td}}{V_{ub}^*V_{ud}}\right)$$

$$\gamma = \arg\left(-\frac{V_{ub}^*V_{ud}}{V_{cb}^*V_{cd}}\right)$$

$$\beta = \arg\left(-\frac{V_{cb}^*V_{cd}}{V_{tb}^*V_{td}}\right)$$

Pick a quark phase convention such that $V_{cb}^*V_{cd}$ is real

Normalize all sides by $-V_{cb}^*V_{cd}$



over-constraining the unitarity triangle

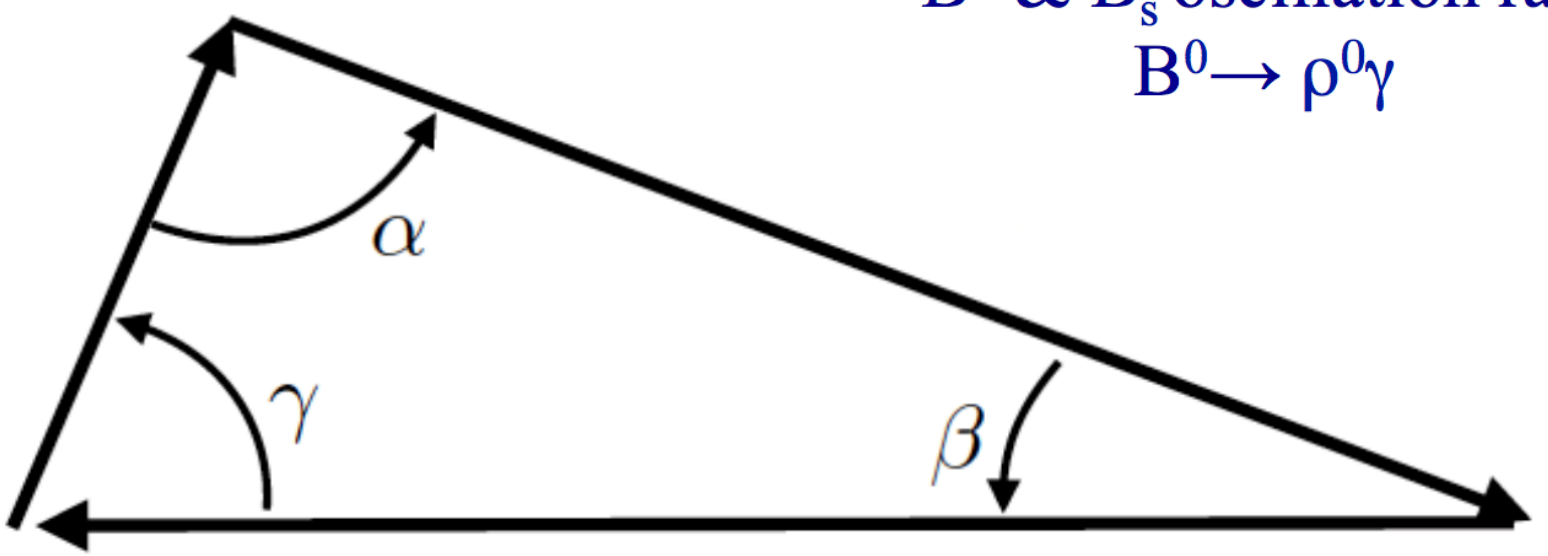
$B^0 \rightarrow \rho^+\rho^-, \rho\pi, \pi^+\pi^-$

B^0 & B_s oscillation rate

$B^0 \rightarrow \rho^0\gamma$

$B \rightarrow X_u\, l\, \nu$

$B \rightarrow \pi\, l\, \nu$



$B^\pm \rightarrow D^0 / D^0 K^\pm$

$B \rightarrow X_c\, l\, \nu$

$B \rightarrow D^*\, l\, \nu$

τ_B

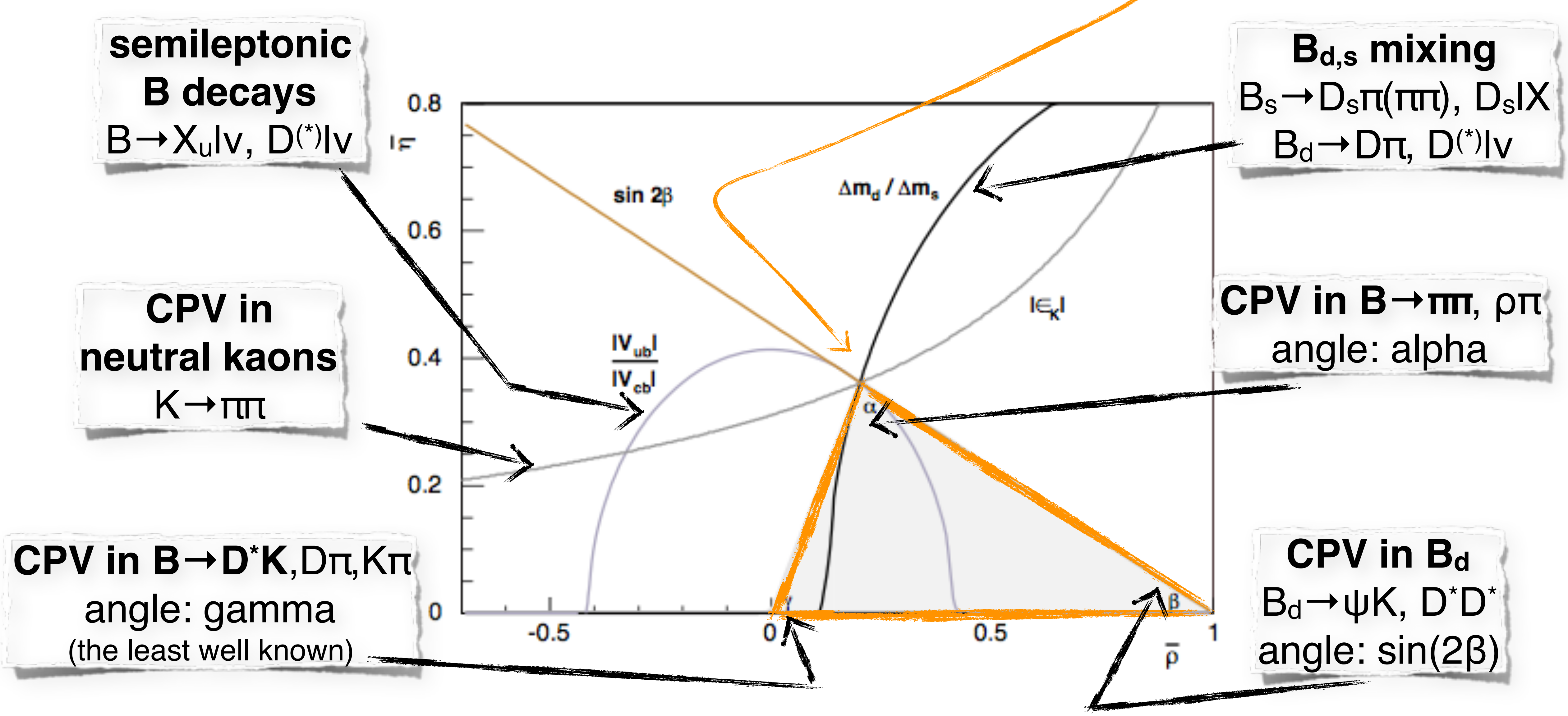
$B^0\ (B^0) \rightarrow cc\ K_s^0$

$B^0\ (B^0) \rightarrow ss\ K_s^0$

$B^0\ (B^0) \rightarrow cc\ dd$

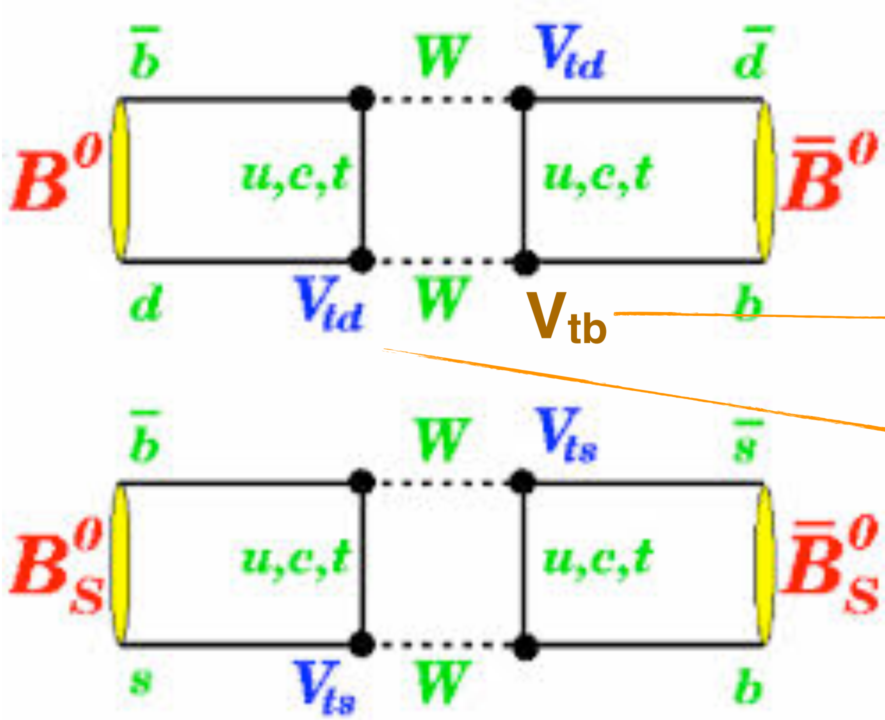
constraining the unitarity triangle

- is the CKM matrix unitary (as expected in the SM)?
  - 4th generation of quarks? New forces? E.g. SUSY?
- over-constrain the UT: measure each side and each angle
  - do all measurements cross at one single point?



meson mixing

B meson mixing in the SM



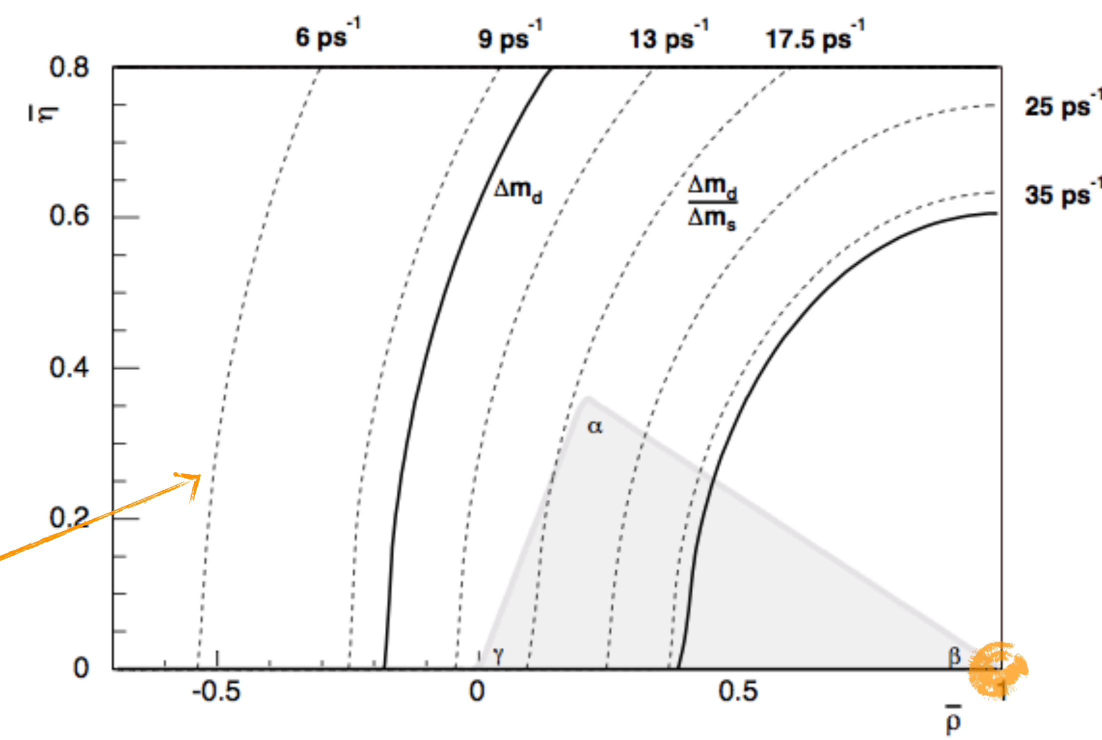
- the mixing process (and oscillation frequency, Δm_q) is proportional to the involved CKM matrix elements

$$\Delta m_q = C_q \left|V_{tb}^* V_{tq}\right|^2 , \qquad (q = d, s)$$

$$\frac{\Delta m_d}{\Delta m_s} = \frac{C_d}{C_s}\frac{|V_{td}|^2}{|V_{ts}|^2} = \frac{m_{B^0}}{m_{B_s}}\xi_\Delta^{-2}\frac{|V_{td}|^2}{|V_{ts}|^2}$$

$$\frac{\Delta m_s}{\Delta m_d} = \xi_\Delta^2 \frac{m_{B_s}}{m_{B^0}}\left(\frac{1-\frac{1}{2}\lambda^2}{\lambda}\right)^2 \frac{1}{(1-\bar\rho)^2+\bar\eta^2}$$

$$\boxed{(1-\bar\rho)^2+\bar\eta^2 = c}$$



- i.e., the ratio of B_d and B_s oscillation frequencies yields a circle centered at the point (ρ=1, η=0)

➡ a measured value of Δm_s away from ~17.5ps^{-1} would have been incompatible with the SM

UT fit

$$\boxed{(1-\bar\rho)^2+\bar\eta^2=c}$$

$$c = \frac{\Delta m_d}{\Delta m_s}\ \xi_\Delta^2\ \frac{m_{B_s}}{m_{B^0}}\left(\frac{1-\frac{1}{2}\lambda^2}{\lambda}\right)^2$$

λ=0.224±0.012

$\xi = 1.210\ ^{+0.047}_{-0.035}$ from lattice QCD

(hep/lat-0510113)

$\Delta m_d = (51.0 \pm 0.4) \times 10^{10}\ \hbar\ s^{-1}$

$\Delta m_s = (17.69 \pm 0.08) \times 10^{12}\ \hbar\ s^{-1}$

(PDG'14)

- if c would be exactly known, the constraint would indeed be a circle

$$f(\bar\rho,\bar\eta|c)=\delta((1-\bar\rho)^2+\bar\eta^2-c)$$

- but… there are uncertainties, both theoretical and experimental

- thus c is described by a probability density function (PDF): f(c)
- upon employing Bayes' theorem

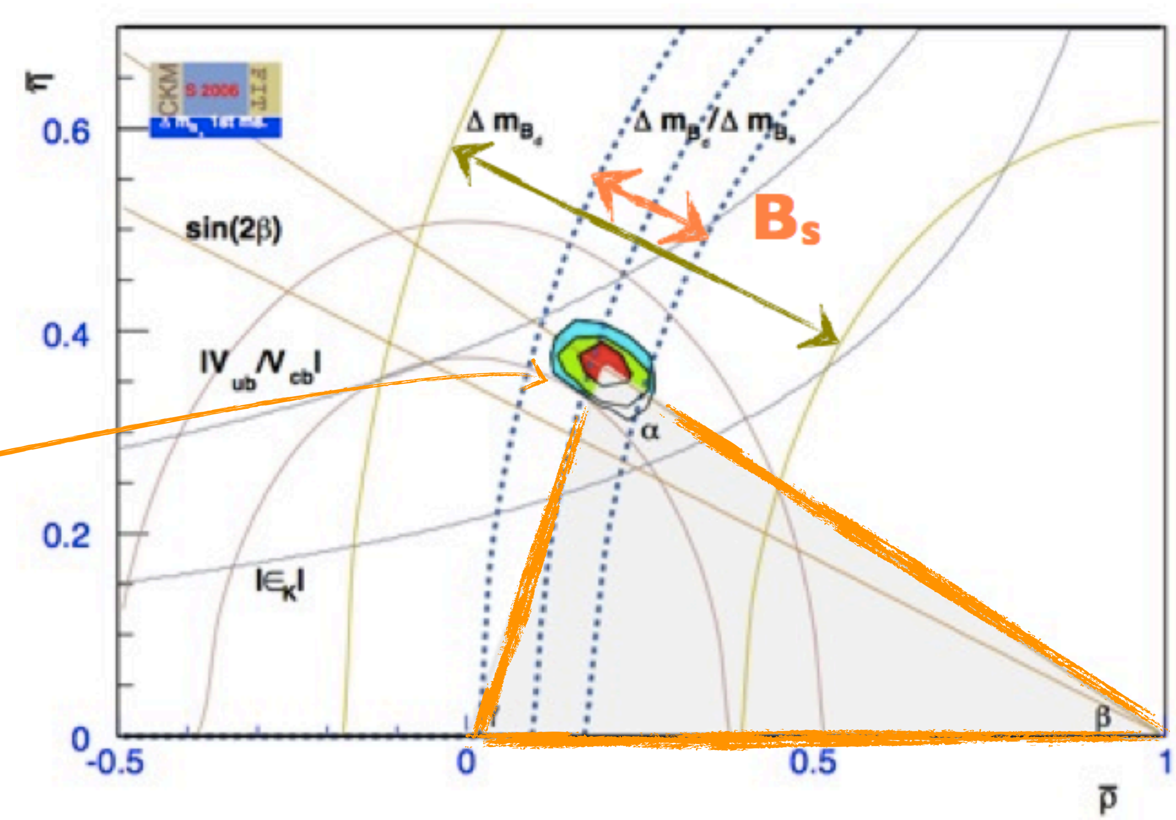
$$\mathcal{L}(\bar\rho,\bar\eta,\mathbf{c},\mathbf{x}|\hat{\mathbf{c}}) \ \propto\ f(\hat{\mathbf{c}}|\bar\rho,\bar\eta,\mathbf{c},\mathbf{x})\cdot f(\mathbf{c},\mathbf{x},\bar\rho,\bar\eta)$$

- we obtain the PDF for ρ,η as

$$\underbrace{\mathcal{L}(\bar\rho,\bar\eta,\mathbf{x})}_{\text{posterior PDF}} \ \propto\ \underbrace{\prod_{j=1,M} f(\hat{c}_j|c_j(\bar\rho,\bar\eta,\mathbf{x}))}_{\text{constraints}} \times \underbrace{\prod_{i=1,N} f_i(x_i)}_{\text{prior PDF}}$$

- integration requires use of numerical and statistical sampling techniques, e.g. Monte Carlo

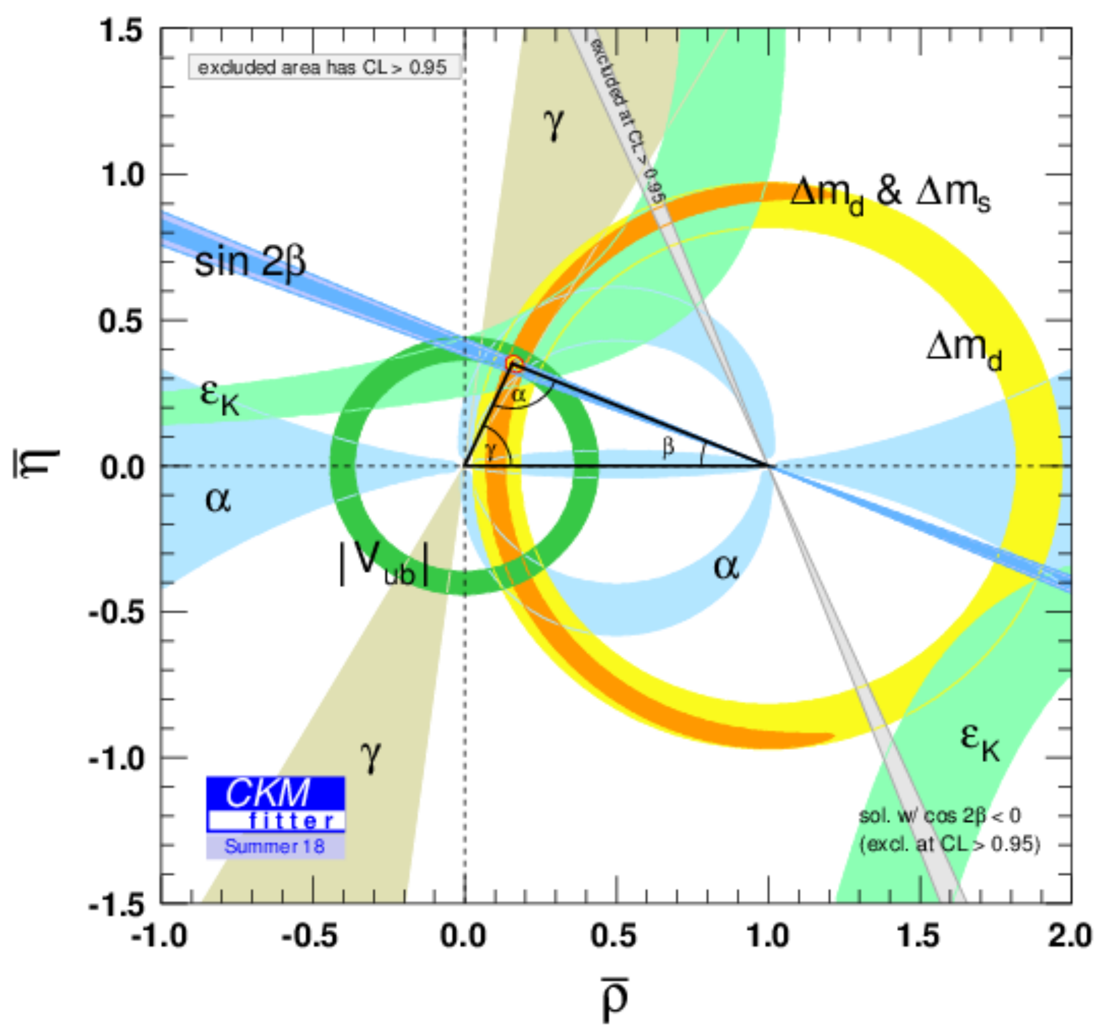
Exercise: which factor limits the CKM-constraining power of B mixing; may it be constrained experimentally

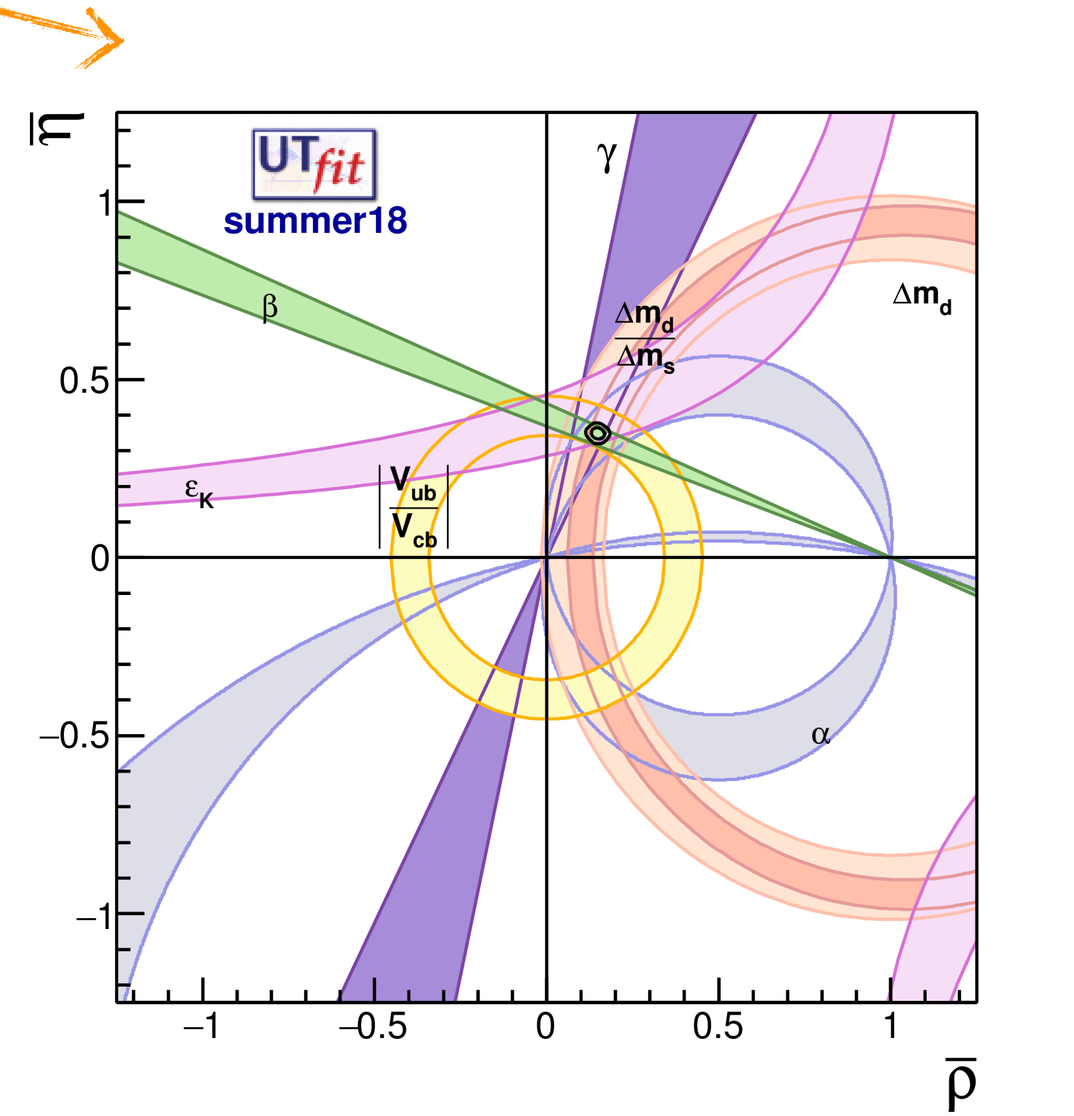


NL, L.Sevilla, https://arxiv.org/abs/hep-ph/0603115

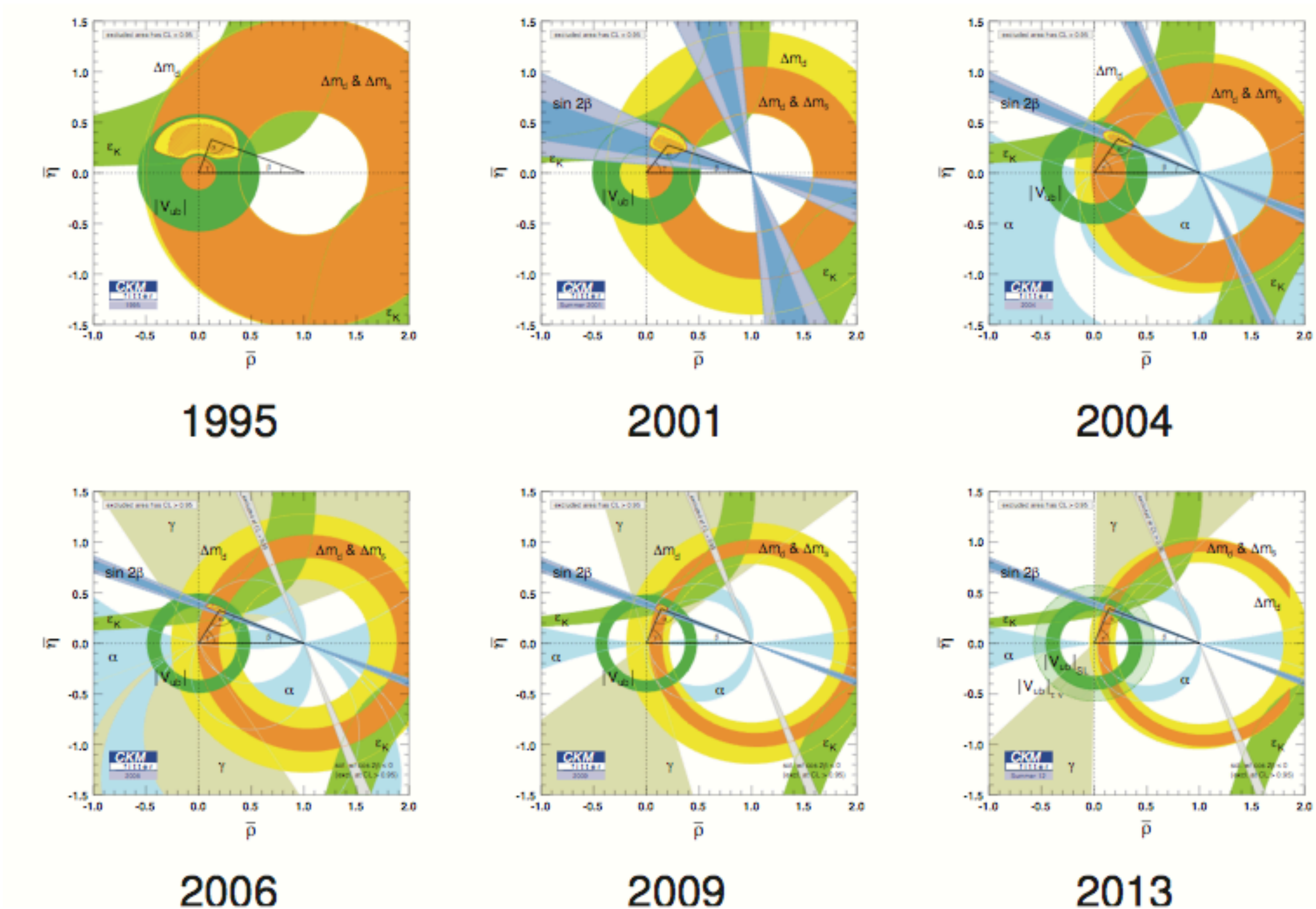
UT fit

- as seen, experimental and theoretical inputs with corresponding uncertainties are combined in global inference frameworks
  - imposing SM relations -- or testing alternative BSM flavor scenarios
  - using frequentist or Bayesian statistical fit approaches, e.g.:

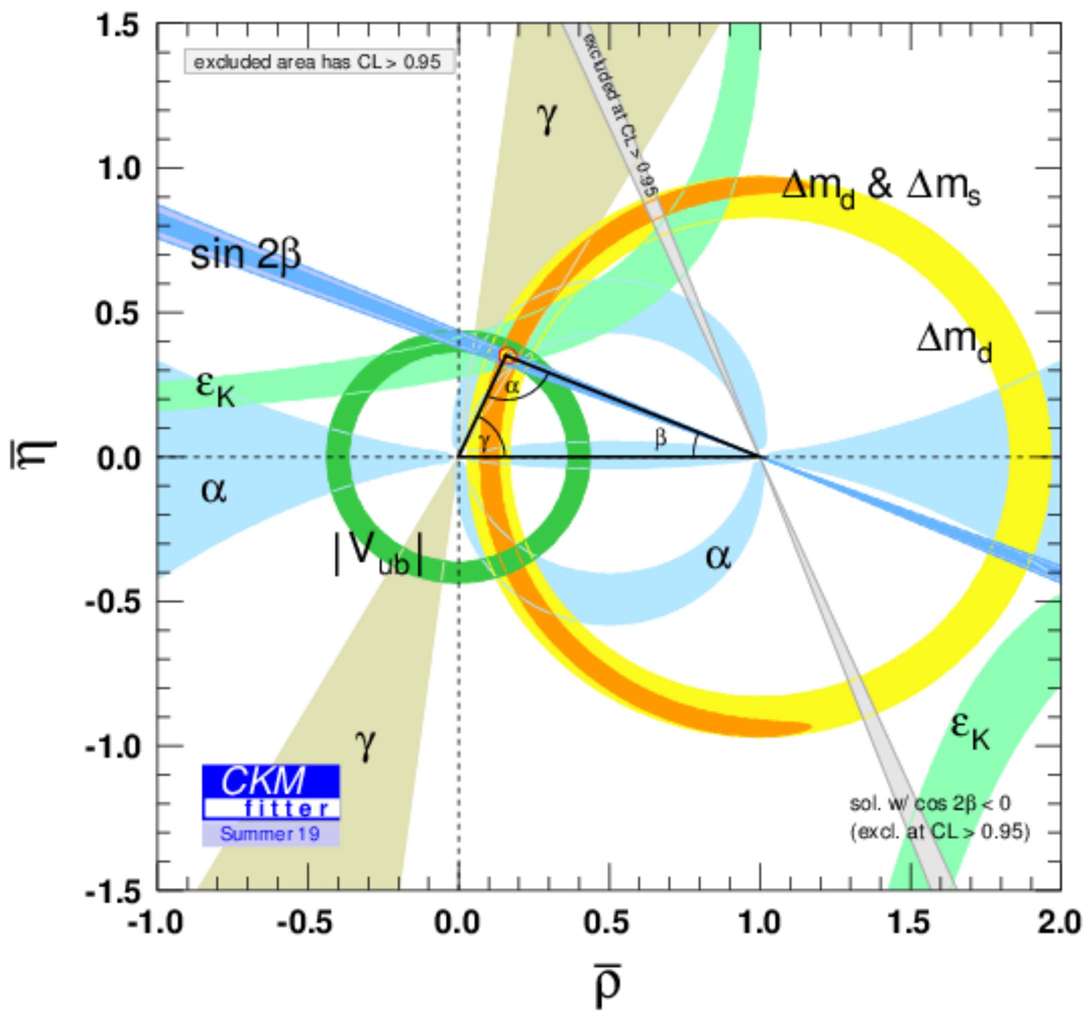


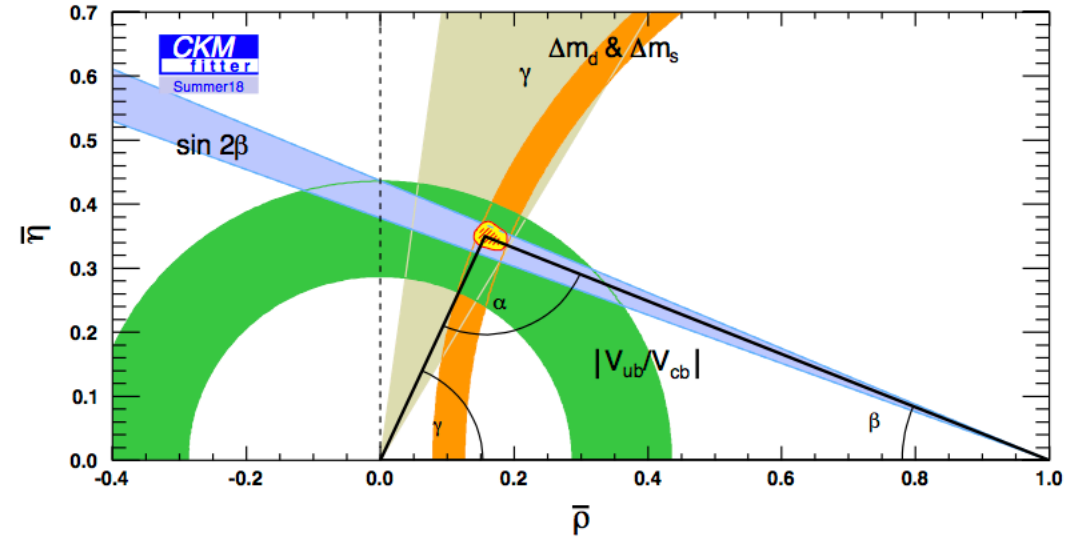


UT fit evolution over 20 years

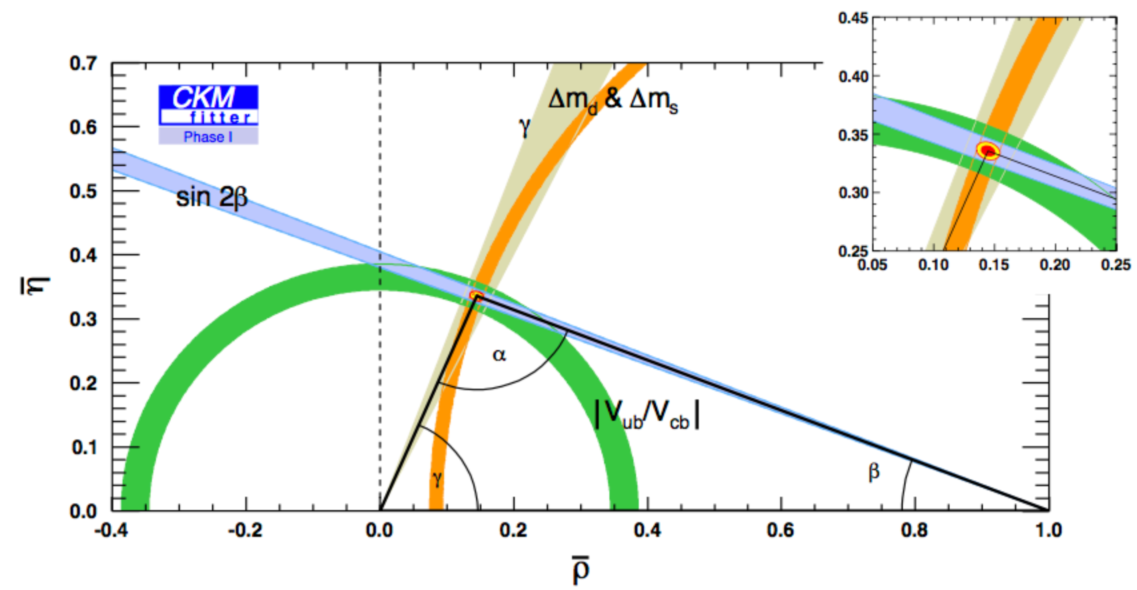


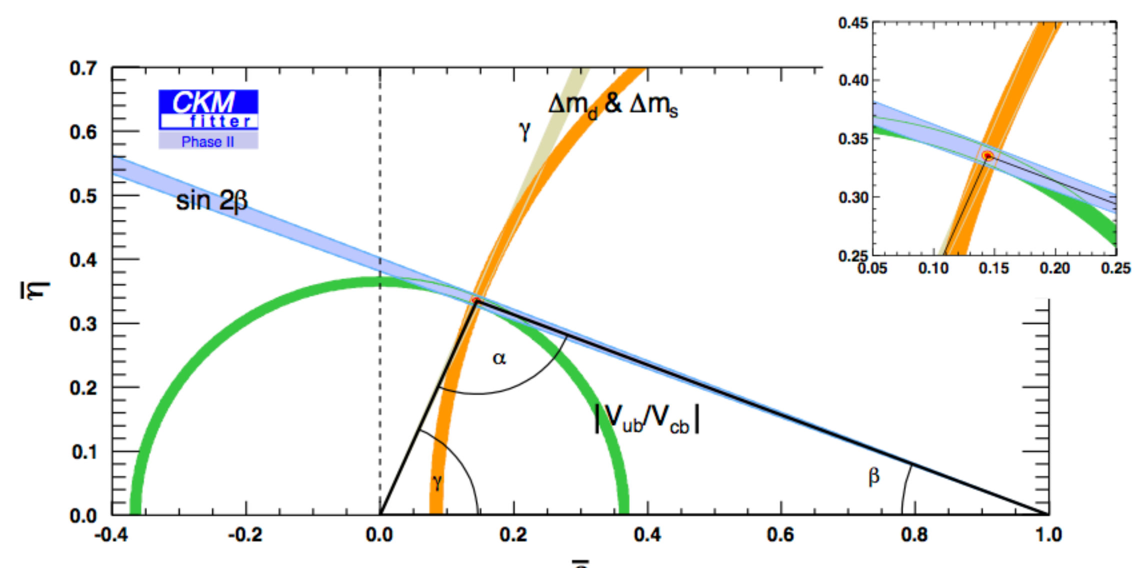
CKM fit today & tomorrow





PBB





constraining NP

- allowing for New Physics contributions, via generic parameterizations
- e.g. NP contribution to off-diagonal B mass mixing matrix M_{12} [see mixing section]
  - $M_{12}^{SM,q} = M_{12}^{SM,q}.\ \Delta_q$, with $\Delta_q = |\Delta_q|.\exp(i\Phi^{\Delta q})$ and q=s,d
  - SM point corresponds to: $\Delta_s = 1 = \Delta_d$
  - NP phases, Φ^{Δ}, shift CP phases from mixing-induced CP asymmetries
  - $2\beta_s \mapsto 2\beta_s - \Phi^{\Delta s}$ ($B_s \to J/\psi\varphi$) and $2\beta_d \mapsto 2\beta_d + \Phi^{\Delta d}$ ($B_d \to J/\psi K$)