Lecture on

(heavy) flavor physics

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the role of flavor physics

• in searching for **New Physics**
  ‣ discovery potential beyond energy frontier e.g. via searches for rare processes

• in understanding why the **SM** appears so fundamental
  ‣ in that no phenomena beyond the SM has (yet) been detected at LHC

• in learning about standing mysteries of the **flavor structure** of SM (and BSM)

• in connecting **CP violation** to the matter-antimatter asymmetry in the observable universe

• in understanding **QCD**, and probing the properties of **deconfined** matter at high temperature and density

• extra: as an experimental tool & probe
  ‣ serve as probe or a **dominant background** in SM measurements and BSM searches
  ‣ used for **detector calibration** (e.g. material budget, magnetic field, detector performance)
Direct search for NP

- searching for the decay products of NP particles produced in collision

Indirect search for NP

- searching for effects of NP particles running in quantum loops (virtual)

Flavour!
indirect discovery via precision

- new physics can show up at precision frontier before energy frontier
  - kaon (1947), $\Lambda^0$ (1950) led to discovery of strangeness
  - neutral current (1973) before discovery of $Z$ (1983)
  - precision W and t mass meas. constrained Higgs mass, etc!

1970
GIM mechanism
(Glashow–Iliopoulos–Maiani)
required the fourth quark: charm
e.g. $K^+ \rightarrow \mu^+\nu_\mu$ so why not $K^0 \rightarrow \mu^+\mu^-$?

1973
3rd quark generation predicted
bottom and top
by Kobayachi and Maskawa
to explain observed CP violation
(discovered in 1964 in the kaon system)

(bottom line: historically, precision measurements at lower energies predicted the existence of new, heavier states)

1974
$J/\psi$ meson
discovery
(SPEAR+AGS)

1977
$\Upsilon$ meson
discovery
(E288)

1995
top quark
discovery
(CDF+D0)

2012
Higgs boson
discovery
(ATLAS+CMS)

1983
$Z$ boson
discovery
(UA1+UA2)

(neat: quarks postulated 1964 [Gellman&Zweig], based on hadron classification ['eightfold way'], directly confirmed experimentally 1968 [DIS])
**Indirect searches**: fuelled by Quantum Mechanics

Flavour observables provide access to NP energy scales well beyond \( \sqrt{s} \)!

Quark Mixing & CP  Lepton Flavour  EDM  Higgs-LFV  top-FCNC  EWK
Precision tests of the SM

The “nightmare”

The “dream”

https://arxiv.org/abs/0710.3799
Anomalies!

**Intriguing new result from the LHCb experiment at CERN**

The LHCb results strengthen hints of a violation of lepton flavour universality

23 March, 2021

Very rare decay of a beauty meson involving an electron and positron observed at LHCb (Image: CERN)

**First results from Fermilab’s Muon g-2 experiment strengthen evidence of new physics**

April 7, 2021

Fermilab result

Experiment Average

Standard Model Prediction

Brookhaven result

$R_K$

- **BaBar**
  - $0.1 < q^2 < 8.12 \text{ GeV}^2$
- **Belle**
  - $1.0 < q^2 < 6.0 \text{ GeV}^2$
- **LHCb 3 fb$^{-1}$**
  - $1.0 < q^2 < 6.0 \text{ GeV}^2$
- **LHCb 5 fb$^{-1}$**
  - $1.1 < q^2 < 6.0 \text{ GeV}^2$
- **LHCb 9 fb$^{-1}$**
  - $1.1 < q^2 < 6.0 \text{ GeV}^2$

$\mu \times 10^9 - 1165900$
the standard model (of particle physics)

A great triumph of 20th century science.
The standard model (of particle physics)

The SM Lagrangian

New Physics

Going Beyond the SM — New Physics!
(heavy) flavor?
a tabela ‘periódica’ das partículas elementares

- sabores pesados
- sabores leves
- leptões carregados
- neutrinos

sector de sabor
sector gauge
sector escalar
a tabela ‘periódica’ das partículas elementares — testes de precisão e buscas de novas partículas

sabores pesados
LHC, BelleII, BESIII, (Belle, BaBar), …

sabores leves
Compass, NA62, KOTO, (Amber, KLEVER), …

leptões carregados
MEG, Mu2e, Mu3e, BelleII, (TauFV), …

neutrinos
SNO, Minos, Nova, MicroBoone, DayBay, T2K, SK, Opera, ANTARES, BOREXINO, (DUNE, HK), SND@LHC

Ions: RHIC, (FAIR,EIC), …
Forward: Totem / PPS, Alfa / LHCf, (AFP), …
FIPs: MoEDAL, FASER, (Codex-b, MATHUSLA, MAPP, SHiP), …
Cosmic: AMS, Auger, IceCube, Axion, …
DM: LUX, ZEPLIN, DAMMA, CDMS, ADMX, CRESST, EDELWEISS, EURECA, XENON, DEAP, ArDM, WARP, DarkSide, PandaX, … …
a tabela ‘períódica’ das partículas elementares: flavour @ LHC

Just as ice cream has both color and flavour, so do quarks

**top**: too heavy | does not hadronize | see in detain in previous lectures

**neutrinos**: interact too feebly | studied at LHC by dedicated experiments (e.g. SND)
a rich «flavor» phenomenology

- the SM flavor sector arises from interplay of fermion-weak–gauge and fermion-Higgs couplings

Out of the 19 parameters of the SM (excluding neutrino masses/mixing), 14 arise from the flavor sector.

Rich flavor phenomenology ...

The parameters of the SM

- 3 gauge couplings
- 2 Higgs parameters
- strong CPV parameter, $\Theta$
- 6 quark masses
- 3 quark mixing angles + 1 phase (CKM)
- 3 (+3) lepton masses
- (3 lepton mixing angles + 1 phase (PMNS))

( ) = with Dirac neutrino masses
flavor «puzzle»

- there are standing mysteries intrinsic to the SM flavor sector
  - why are there so many free parameters
    - why do these parameters exhibit strong hierarchical structure spanning several orders of magnitude
  - why are there so many fermions
  - what is responsible for their organization into generations
    - and why are there 3 such generations each of leptons and quarks
  - why wide range of fermion couplings and masses
    - for example: $O(10^{-5}) \cdot m_c \sim m_u \sim m_\nu \cdot O(10^{+6})$, $|V_{ub}| \sim O(10^{-3}) \cdot |V_{td}|$
  - why are there flavor symmetries
    - and what breaks them
  - why is $\theta_{QCD} < 10^{-9}$
  - what is the origin of CP violation

- various solutions to this puzzle have been proposed (but not established), inevitably leading to beyond-the-SM scenarios
  - for within the SM these parameters can only be accommodated, not explained
another, related «puzzle»: BAU
(baryon asymmetry in the universe)

- Sakharov conditions (1967), necessary for dynamical evolution of matter dominated universe from symmetric initial state
  1. baryon number violation
  2. C & CP violation
  3. thermal inequilibrium
- no significant amounts of antimatter observed
  - \( \Delta N_B/N_Y \equiv [N(\text{baryon})-N(\text{antibaryon})] / N_Y \sim 10^{-10} \)
- amount of CP violation in SM not sufficient to explain BAU
  - CPV in quark sector (CKM) would yield an asymmetry of \( O(10^{-17}) \ll 10^{-10} \)
- more CPV is needed!
  - to create a larger asymmetry, require: new sources of CP violation ... that occur at higher energies
- where might it be found?
  - lepton sector: CPV in neutrino oscillations
  - quark sector: discrepancies with KM predictions
  - gauge/higgs sector; extra dimensions or other new physics?
  - precision measurements of flavor observables sensitive to additions to SM
«heavy» flavor, aka B Physics

Light quarks: \( m \approx \Lambda_{\text{QCD}} \)
- u, d: realm of nuclear physics
- s: rare kaon decays test SM

<table>
<thead>
<tr>
<th>Quark</th>
<th>Mass (MeV)</th>
</tr>
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<tbody>
<tr>
<td>( m_u )</td>
<td>( \approx 3 )</td>
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<tr>
<td>( m_d )</td>
<td>( \approx 5 )</td>
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<tr>
<td>( m_s )</td>
<td>( \approx 100 )</td>
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<tr>
<td>( m_c )</td>
<td>( \approx 1300 )</td>
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<tr>
<td>( m_b )</td>
<td>( \approx 4200 )</td>
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<tr>
<td>( m_t )</td>
<td>( \approx 170000 )</td>
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</tbody>
</table>

Top (not that heavy!)
- The top quark has its own phenomenology (since it does not hadronize)
- \( m_t \approx 170000 \text{ MeV} \)

Neutrinos
- Have their own phenomenology, not detected (directly) at LHC
- Electric and magnetic dipole moments test SM
- \( m_{\nu_1} \leq 10^{-6} \text{ MeV} \)
- \( m_{\nu_2} \leq 10^{-6} \text{ MeV} \)
- \( m_{\nu_3} \leq 10^{-6} \text{ MeV} \)
- \( m_e \approx 0.5 \text{ MeV} \)
- \( m_\mu \approx 100 \text{ MeV} \)
- \( m_\tau \approx 1800 \text{ MeV} \)

Study Beauty and Charm quarks
- Hidden flavor aka quarkonia: \( \psi (cc), \Upsilon (bb), \chi_{c,b} \)
- Open charm: D mesons
- Open beauty, B mesons (\( B_u, B_d, B_s, B_c \)) and b-baryons (\( \Lambda_b, \Xi_b, \Omega_b, \ldots \))
- Exotic hadrons: X, Y, Z states

Note:
- «B physics» refers to study of flavor-changing interactions of b-quark mesons
  - Heavier states accessible only at the LHC — \( \Upsilon, \chi_b, B_s, B_c, b\)-baryons
the SM discovery

1974

J/ψ, charm

Ting&Richter, Nobel prize 1976
the discovery of the b quark
the SM re-discovery @ LHC

1977

the discovery of the b quark

2010

decades worth of particle physics discovery … in a single plot!
the SM re-discovery @ LHC

... with 100’000 times more data  (⇒ precision + new particles?)
not-so-rare ➔ precision!
medium rare ⇐ EWK Penguins
very rare ⇐ FCNC/GIM+helicity
ultra rare ⇐ LFV
production
HF production

- high HF production rates at the LHC
  - very large production cross section ($\sigma$)
  - large accumulated luminosity (L)
- LHC: HF ‘factory’ (N=L.$\sigma$)
  - allow to perform precision measurements, as well as to search for very rare processes
- HF production is ubiquitous
  - forming backgrounds for many physics processes explored at the LHC
  - need to be thoroughly understood
hadron production

• different mechanisms contribute to HF production

• produced quarks evolve into hadrons: known as fragmentation
  ‣ involving short-distance/perturbative vs long-distance processes

• heavy quarkonia $QQ=(bb, cc)$ are an ideal laboratory in which to study the strong force and the mechanisms of hadron formation
  ‣ non-perturbative evolution of $QQ$ pair into a quarkonium state
  ‣ employ effective theories: e.g. non-relativistic QCD (NRQCD; CSM, CEM...)

need to carry out detailed of HF production, including cross sections, polarizations, etc
quarkonia production

- precision measurements of quarkonium production
- LHC allows to probe higher \( p_T \) region for the first time
1. Trigger (online selection)
2. Muon identification and reconstruction
3. Y meson candidate reconstruction
4. Extract signal (offline selection) $N$
5. Determine detector acceptance $A$ and selection efficiency $\varepsilon$
6. Luminosity $L$ and branching fraction $B$
7. Systematic uncertainties (on $N$, $A$, $L$, $B$)
8. Compare to theory predictions (NRQCD)
Trigger

CMS Preliminary

13.1 fb⁻¹ (13 TeV, 2016)

Trigger paths:
- φ
- J/ψ
- ψ'
- B_s
- Y
- low mass double muon + track
- double muon inclusive

Events / GeV

μ⁺μ⁻ invariant mass [GeV]
(Di)Muon signal in detector
di-muon Invariant Mass

- Statistical procedure: extended unbinned maximum likelihood (EUML)

\[ \mathcal{L}(\vec{\lambda}|m_i) = \left( \prod_{i=1}^{N_{\text{obs}}} \sum_{\alpha} N_{\alpha} \mathcal{P}_{\alpha}(m_i|\vec{\lambda}) \right) \times \frac{e^{-N} N^{N_{\text{obs}}}}{N_{\text{obs}}!} \]

- Fit PDFs: Signal (3 x Crystal Ball) and background (polynomial) models

  - yield \( N \pm \sigma_{\text{stat}} \), mass \( m \pm \sigma_m \)
Differential yield measurement

- Perform measurement as function of given observable, e.g. pT
- Split the dataset in ranges
- Perform analysis produce in each range
- note backgrounds levels, resolutions vary!
production cross section measurement, $\sigma$

\[ N = L \times \sigma \]

\[
\frac{d^2\sigma(Q\bar{Q})}{dp_Tdy} B\left(Q\bar{Q} \to \mu^+\mu^-\right) = \frac{N_{\text{fit}}(Q\bar{Q})}{L \cdot A \cdot \epsilon \cdot \Delta p_T \cdot \Delta y}
\]

1. Trigger (online selection)
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8. Compare to theory predictions (NRQCD)
**tag & probe**

explore $J/\psi, \Upsilon, Z \rightarrow \mu\mu$

- **example:**
  - $\mu$-identification efficiency

- **data:**
  - single-muon **trigger** dataset

- **tag**
  - good global muon
  - matched to single-muon path to remove trigger bias

- **probe**
  - inner track
  - passing criteria: identified as muon
explore $J/\psi, \Upsilon, Z \rightarrow \mu\mu$
cross section

"N=L.σ"

d^2σ(Q\bar{Q}) / dp_T dy \cdot B(Q\bar{Q} → μ^+ μ^-) = \frac{N_{fit}(Q\bar{Q})}{L \cdot A \cdot ε \cdot Δp_T \cdot Δy}

Acceptance simulation [data-driven]

Efficiency

N = L \cdot σ

CMS, \sqrt{s} = 7 TeV, L = 3 pb⁻¹, |y| < 2

Signal yields

| n^μ | < 0.4

Events/(0.1 GeV/c²)

μ^+ μ^- mass (GeV/c²)
Systematics

- quantify all possible sources of systematic uncertainty
  - signal yield, $N$
    - signal & background PDF
    - resolution, FSR, …
    - fit procedure, possible bias
  - acceptance, $A$
    - polarization, limited MC statistics
  - efficiency, $\varepsilon$
    - data vs MC, T&P stat and syst errors, limited size of T&P calibration sample
  - luminosity, $L$
    - stat & syst err on $L$ (measured separately) give syst error on $x$-section
  - branching fraction
    - world average (PDG) uncertainty
- finally, can then quote $Y$ measured production cross section:
  - $N \pm \sigma_{\text{stat}} \pm \sigma_{\text{syst}}$
b cross sections

Cross sections vs momentum -> vs rapidity -> vs $\sqrt{s}$ ->

Data (13 TeV, $p_T^\Lambda<10$ GeV) vs $p_T^\Lambda$ [GeV] vs $|y|$ [10^{-1}] (13 TeV)

CMS (48.1 pb^{-1} (13 TeV))

Data (13 TeV, $p_T^\Lambda>17$ GeV) vs $p_T^\Lambda$ [GeV] vs $|y|$ [10^{-2}] (13 TeV)

Data (7 TeV, scaled to $p_T^\Lambda<10$ GeV) vs $p_T^\Lambda$ [GeV] vs $|y|$ [10^{-2}] (7 TeV)

Data (7 TeV, scaled to $p_T^\Lambda>17$ GeV) vs $p_T^\Lambda$ [GeV] vs $|y|$ [10^{-2}] (7 TeV)

48.1 pb^{-1} (13 TeV)

CMS data vs. theoretical predictions.

Open flavour ->

<- Hidden flavour

$\gamma(1S)$ vs $p_T^\gamma$ (GeV/c) as a function of $B(\mu\mu)$.

 Entries / 20 MeV

CMS $\sqrt{s} = 7$ TeV, $L = 36$ pb^{-1}

$|\gamma| < 2$

CMS data, NLO NRQCD, NLO CSM, NNLO* CSM, PYTHIA (normalized), CEM, CASCADE

CMS data vs. theoretical predictions.

$|\gamma| < 2$

CMS $\sqrt{s} = 7$ TeV, $L = 36$ pb^{-1}

$|\gamma| < 2$

CMS data vs. theoretical predictions.

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CMS data vs. theoretical predictions.

$|\gamma| < 2$
spectroscopy
bottomonium
Review of bottomonium measurements from CMS

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We review the results on the bottomonium system from the CMS experiment at the Large Hadron Collider. Measurements have been carried out at different center-of-mass energies in proton collisions and in collisions involving heavy ions. These include precision measurements of cross sections and polarizations, shedding light on hadroproduction mechanisms, and the observation of quarkonium sequential suppression, a notable indication of quark-gluon plasma formation. The observation of the production of bottomonium pairs is also reported along with searches for new states. We close with a brief outlook of the future physics program.

Keywords: Quarkonia; bottomonia; cross section; polarization; suppression; QGP; LHC.

PACS numbers: 14.40.Pq, 25.75.Nq, 13.85.Ni

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Upsilon production cross section in pp collisions at $\sqrt{s} = 7$ TeV

V. Khachatryan et al.*
(CMS Collaboration)
(Received 27 December 2010; published 15 June 2011)

The $Y(1S)$, $Y(2S)$, and $Y(3S)$ production cross sections in proton-proton collisions at $\sqrt{s} = 7$ TeV are measured using a data sample collected with the CMS detector at the LHC, corresponding to an integrated luminosity of $3.1 \pm 0.3$ pb$^{-1}$. Integrated over the rapidity range $|y| < 2$, we find the product of the $Y(1S)$ production cross section and branching fraction to dimuons to be $\sigma(pp \rightarrow Y(1S)X) \cdot B(Y(1S) \rightarrow \mu^+\mu^-) = 7.37 \pm 0.13^{+0.55}_{-0.42} \pm 0.81$ nb, where the first uncertainty is statistical, the second is systematic, and the third is associated with the estimation of the integrated luminosity of the data sample. This cross section is obtained assuming unpolarized $Y(1S)$ production. With the assumption of fully transverse or fully longitudinal production polarization, the measured cross section changes by about 20%. We also report the measurement of the $Y(1S)$, $Y(2S)$, and $Y(3S)$ differential cross sections as a function of transverse momentum and rapidity.

arXiv:1708.02913

arXiv:1012.5545
charmonium

\[ m_{\mu^+\mu^-} \in [2.5, 4.5] \text{ GeV/c}^2 \]

Events / (0.02 GeV/c^2)

\( \chi^2 / \text{ndf} = 94.4 / 99 \)

CMS - \( \sqrt{s} = 7 \text{ TeV} \)
\( L = 37 \text{ pb}^{-1} \)

\( 8 < p_T < 9 \text{ GeV/c} \)
|\( \eta | < 1.2 \)

Data
- Total fit
- Background

\( \chi_c(1P) \)
The first new particles found at LHC

- First new particle discovered by ATLAS
- Reconstruct the radiative bottomonium decay by exploring photon conversions in tracker material

\[ \chi_b(3P) \rightarrow \gamma \gamma \rightarrow e^+ e^- \rightarrow \mu^+ \mu^- \]

Note: These orthogonal capabilities further illustrate the ability of general purpose detectors to make flavor discoveries

- First new particle discovered by CMS
- First new b baryon observed at LHC
- Complex cascade decay topology
  - 4 displaced vertices
  - 6 final state tracks
the latest new particles found @LHC

First LHC result with full Run2 dataset!
(2015+2016+2017+2018 ~140fb⁻¹)

CMS
L = 140 fb⁻¹
\( \sqrt{s} = 13 \text{ TeV} \)

Two new bottom-charm states

B_c (2S)
**B_c**

- meson with different heavy flavors -- unique in SM
  - sometimes also referred to as ‘quarkonium’: similar non-relativist potential techniques used to predict properties
  - formed of $b+c$ quarks: the heaviest quark flavors expected to form mesons
  - $b$ and $c$ may both decay weakly
    - much shorter lifetime than other B mesons
- state by now observed in several modes
  - no excited states observed yet (many expected)
Before 2006, only one b baryon had been seen: $\Lambda_b$

CDF and D0 contributed several such discoveries:
- $\Sigma_b^-$ (2006)
- $\Xi_b^-$ (2007)
- $\Omega_b$ (2008)

LHC:
- $\chi_b(3P)$ (ATLAS' 2011)
- $\Xi_b^{*0}$ (CMS'2012)
- $B_c(2S)$ (ATLAS'2014)
- $\Xi_b^{*-} \Xi_b^{-*}$ (LHCb'2014)

Several other composite particles awaiting to be discovered!
exotic spectroscopy

• while not all of the predicted states have been observed yet... many unexpected ones already have
• referred to as XYZ states
• all started with the discovery of the X(3872) state by Belle in 2003
  ‣ quickly confirmed by Babar, CDF, D0
  ‣ other unconventional states popped up

Many theoretical interpretations in discussion:

- conventional quarkonia;
- tetra-quarks states;
- meson–molecules;
- hybrid mesons;
- threshold effects;

⇒ properties do not well fit the quarkonia picture

[EUR. PHYS. J. C 71 (2011) 1534]
**XYZ states**

**X(3872) → J/ψππ**

**Y(4140) in B_u → X[J/ψφ]K decays**
pentaquarks
New particles discovered@LHC?
New particles discovered @ LHC? 59+1

LHCb observes four new tetraquarks

3 March 2021

FLAVOUR PHYSICS | NEWS

LHCb observes four new tetraquarks

3 March 2021

March’2021 @CERN
nature of hadrons, a tricky business

\[ \Sigma_i (q\bar{q})_i \Sigma_j g_j \]

Nature of several particles unknown... exotic hadrons
heavy ion collisions
heavy flavor studies at the LHC are opening up new research lines in nuclear physics, benefitting from the exquisite capability of the detectors and unprecedented collision energies at the LHC

several ground-breaking results already delivered, many more to come

large HF production cross section + precision HF detection capability
at large energy densities, QCD predicts the existence of a deconfined state of quarks and gluons -- the quark gluon plasma (QGP)
- studied in heavy ion collisions
- the goal is to characterize and quantify the properties of the dense and hot medium produced at the unprecedented LHC energies

• heavy-flavor states are ideal “hard probes” for studying the properties of the created medium
probing matter at extreme conditions

Particles melt in the QGP! (sequentially)

Particles loose energy in the QGP! (sequentially)

1.5 nb⁻¹ (PbPb) 5.02 TeV

CMS Experiment at the LHC, CERN
Data recorded: 2018 No.747Pb, in the GLEAM GEMT
Run / Event / LS: 312329 / 280207 / 3

57
quarkonium suppression

- first (quantitative) measurements of the $Y(nS)$ states in HI collisions
- unprecedented resolutions, allowing to separate the three states
  - experimentally and theoretically robust
- excited states observed ($>5\sigma$) to be more suppressed than ground state
- spectacular indication of formation of Quark Gluon Plasma in heavy ion coll.

“the LHC heavy-ion text book result”

Exercise: the excited states being suppressed, what may be expected also of the observed ground state (hint: $nS\rightarrow 1S$ feed-down)
quarkonium sequential suppression

quarkonia suppression pattern experimentally established: less tightly bound states are more suppressed in the medium
in medium hadron suppression

- measure of suppression, $R_{AA}$ (nuclear modification factor)
  - cross section ration in PbPb vs pp, scaled by number of binary collisions
- different particle species undergo different energy loss in the medium
  - colorless probes ($W, Z, \gamma$) are not suppressed ($R_{AA} \sim 1$)
- study flavor dependence of energy loss
b-hadron detection

• prior to LHC, b-hadron detection was pursued mostly through inclusive-lepton ($B \rightarrow lX$) and inclusive-charmonia ($B \rightarrow J/\psi X$) studies

• with LHC, moved to a new class of more reliable and precise new measurements
  ‣ through non-prompt charmonia: remove prompt contribution through lifetime analysis [see next section]
  ‣ through exclusive state reconstruction [see next slide]
    ➡ both achieved for the first time at the LHC
**first! B mesons in ion collisions**

- first B meson decays (fully) reconstructed in collisions involving heavy ions
- novel probes of the QGP!

These systems constitute precise handles that will facilitate a much improved understanding of the mechanisms of energy loss of hadrons in the deconfined (‘hot’) and nuclear (‘cold’) media -- and of its flavor dependence

Currently actively searching for Bs and Bc, using dataset collected November 2018
top quark in ion collisions

- top observed already in p+Pb collisions at 8 TeV
  - used signature: e or muon plus &ge;4 jets
- yet another novel probe of the QGP
- may be used to resolve time dependence of jet quenching effects

- next step: search for the top in PbPb at 5 TeV collected in 2018
particle enhancement !?
lifetime
a distinctive experimental signature

- bottom and charm hadrons live longer than the other unstable particles
  - $\tau(D) \sim 0.5-1\text{ ps}$, $\tau(B) \sim 1.5\text{ ps}$
  - they travel macroscopic (i.e. measurable) distances in the detector before decaying, producing a displaced vertex topology

- extensively explored
  - in heavy-flavor analyses themselves
  - b-jet tagging: discriminate b-jets from the lighter quark jets
  - in SM measurements and BSM searches: to detect signal HF components (e.g. $t \rightarrow Wb$, $H \rightarrow b\bar{b}$,...) or control HF backgrounds (e.g. $bb$ dijets,...)
quantum mechanics (i)

• an unstable particle may be described by an effective Hamiltonian

• through the non-relativistic Schrödinger equation

• the solution reproduces the law of radioactive decay

$$\mathcal{P}(t) \sim \frac{1}{\tau} e^{-t/\tau} \quad \tau \text{ is the lifetime}$$

• $t$ is the proper decay time, experimentally it is measured from the decay length $L$ and momentum $p$ (or their projections on the transverse plane)

$$t = \frac{L}{\beta \gamma} = \frac{M}{p} = L_{xy} \frac{M}{p_T}$$

\[
\mathcal{H} = m - \frac{i}{2} \Gamma,
\]

\[
i \partial_t \psi = \mathcal{H} \psi
\]

\[
|\psi_t \rangle = e^{-imt} e^{-\frac{1}{2} \Gamma t} |\psi_0 \rangle
\]

\[
|\langle \psi_0 | \psi_t \rangle|^2 = e^{-\Gamma t},
\]

\[
\tau \equiv 1/\Gamma
\]
lifetime modeling

\[
L(t|\sigma_t, \tau) = \frac{1}{N} \cdot \left[ \frac{1}{\tau} e^{-\frac{t}{\tau}} \theta(t) \otimes G(t; \sigma_t) \right] \cdot \mathcal{E}(t)
\]

- **t-resolution**
  - use per-event uncertainties \(\sigma_t\)
    (more precisely reco'd B's get larger weight)
  - calibrate using data (high stat. modes)
    \[ \sigma_t \rightarrow S_t \cdot \sigma_t \]

- **trigger/selection bias**
  - vertex detachment requirements
    used in selection bias \(t\)-distribution,
    requires acceptance correction, \(\mathcal{E}(t)\)

- **backgrounds**
  - prompt, \(\delta(t)\) (\(\rightarrow\) resolution)
  - long-lived (from decay products of other b-hadrons)

**PDF normalization**

**theory model**

**t-resolution function**

**t-acceptance function**

**example \rightarrow lifetime fit**

- Data
- Signal
- Bkg
- Signal+Bkg

\[ c_t = 477.6 \pm 24.2 \text{ \(\mu\text{m}\)} \]
\[ \text{[t-resolution, } \sigma_t] \]

- \( \sigma_t \) may be taken per-event from the vertex kinematic fit
- should be calibrated, using data
- a possible strategy (CDF, also used for example by LHCb)
  - if dataset is \( t \)-unbiased: fit prompt peak with scale factor, \( e^{-\Gamma \cdot t} \otimes \mathcal{R}(t, S_t, \sigma_t) \); else:
  - construct a prompt sample of B-like vertices, closely mimicking kinematics and topology of the signal; fit this sample as above, allowing for scale factor
  - to further facilitate transfer to signal sample, parameterize \( S_t(\Delta R, l, \eta, z, \chi^2) \)

**CDF**

signals: 
\[ B \to D\pi, D\pi\pi \]
triggered by SVT

**LHCb**

signal: 
\[ B \to J/\psi \pi\pi \]
displaced

calibration sample: 

prompt \( J/\psi \) + prompt pion pair

PLB 713 (2012) 378
[t-acceptance, $\varepsilon(t)$]

- *if* dataset is not biased, $\varepsilon(t)=1$
- *if* bias corresponds to a threshold (global or per-event) on $L_{xy}$ or $t$, then the efficiency is given by a threshold function $\varepsilon(t)=\theta(t-t_0)$
- *if* a more general bias, $\varepsilon(t)$ can be estimated from MC or data

**MC driven:**
$$\varepsilon(t) = \frac{t-\text{distribution after selection}}{\sum \{ \sigma_t \} \frac{1}{\tau} e^{-t/\tau} \otimes G(t; \sigma_t)}$$

**Data driven:** [PRD 83 (2011) 032008]
b-hadron lifetimes

**PRD 85 (2012) 112003**
\[
\tau(B^0) = 1.508 \pm 0.025 \pm 0.043 \text{ ps} \\
\tau(\Lambda_b) = 1.303 \pm 0.075 \pm 0.035 \text{ ps}
\]

**PRL 106 (2011) 121804**
\[
\tau(B^+) = 1.639 \pm 0.009 \pm 0.009 \text{ ps} \\
\tau(B^0) = 1.507 \pm 0.010 \pm 0.008 \text{ ps} \\
\tau(\Lambda_b) = 1.537 \pm 0.045 \pm 0.014 \text{ ps} \\
\tau(B^+)/\tau(B^0) = 1.088 \pm 0.009 \pm 0.004 \text{ ps} \\
\tau(\Lambda_b)/\tau(B^0) = 1.020 \pm 0.030 \pm 0.008 \text{ ps}
\]

**JHEP 07 (2013) 163**
\[
\tau(\Lambda_b) = 1.503 \pm 0.052 \pm 0.031 \text{ ps}
\]

**PRD 87 (2013) 032002**
\[
\tau(\Lambda_b) = 1.449 \pm 0.036 \pm 0.017 \text{ ps}
\]

**arXiv:1402.2554**
\[
\tau(\Lambda_b) = 1.503 \pm 0.052 \pm 0.031 \text{ ps}
\]

**PDG 2013**
\[
B^0 = 1.519 \pm 0.007 \text{ ps} \\
B^+ = 1.641 \pm 0.008 \text{ ps} \\
B_s = 1.516 \pm 0.011 \text{ ps} \\
B_c = 0.452 \pm 0.033 \text{ ps} \\
\Lambda_b = 1.429 \pm 0.024 \text{ ps}
\]
flavor oscillations
&
flavor tagging
neutral B mesons undergo spontaneous flavor oscillations between particle and antiparticle!
quantum mechanics (ii)

- allowing for a flavor-changing perturbation ($\Delta F$) in the hamiltonian
  \[ \mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{\Delta F} \]
  \[ i \frac{d}{dt} \psi = \mathcal{H} \psi \]
  \[ i \frac{d}{dt} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} m - \frac{i}{2} \Gamma & M_{12} - \frac{i}{2} \Gamma_{12} \\ M_{12}^* - \frac{i}{2} \Gamma_{12}^* & m + \frac{i}{2} \Gamma \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \]

- a pure flavor eigenstate at $t=0$ will evolve to an admixture
  \[ |\psi\rangle = a |P^0\rangle + b |\bar{P}^0\rangle \]
  ‣ non-diagonal elements in $\mathcal{H} \Rightarrow$ flavor eigenstates differ from mass eigenstates

- flavor eigenstates
  \[ |P_L\rangle = p |P^0\rangle + q |\bar{P}^0\rangle \]
  \[ |P_H\rangle = p |P^0\rangle - q |\bar{P}^0\rangle \]
  with $|p|^2 + |q|^2 = 1$

- time evolution of flavor eigenstates (after finding $\mathcal{H}$ eigenvalues $\lambda_{H,L}$)
  \[ |P_{L,H}\rangle_t = e^{-i\lambda_{L,H} t} |P_{L,H}\rangle = e^{-im_{L,H} t - \frac{1}{2} \Gamma_{L,H} t} |P_{L,H}\rangle \]

- probability for particle-antiparticle transition
  \[ |\langle P^0 | \mathcal{H} | \bar{P}^0 \rangle|^2 = \left| \frac{p}{q} \right|^4 |\langle \bar{P}^0 | \mathcal{H} | P^0 \rangle|^2 = \left| \frac{p}{q} \right|^2 \frac{1}{2} e^{-\Gamma t} \left[ \cosh \left( \frac{\Delta \Gamma}{2} t \right) - \cos (\Delta m t) \right] \]
  ‣ with $\Delta \Gamma \equiv \Gamma_L - \Gamma_H$ and $\Delta m \equiv m_H - m_L$

- neglecting CPV in mixing (i.e. $p/q=1$) and $\Delta \Gamma$, the mixing probability is:
  \[ \mathcal{P}_{B_q^0 \rightarrow \bar{B}_q^0} (t) = \mathcal{P}_{\bar{B}_q^0 \rightarrow B_q^0} (t) = \frac{\Gamma}{2} e^{-\Gamma t} [1 - \cos (\Delta m t)] \]
flavor oscillations

\[
p(B \rightarrow B) = \frac{e^{-t/\tau}}{2\tau} (1 + \cos \Delta m t)
\]

\[
p(B \rightarrow \bar{B}) = \frac{e^{-t/\tau}}{2\tau} (1 - \cos \Delta m t)
\]

- oscillation frequency given by mass difference between heavy and light H eigenstates
- \( \Delta m \) [ps^-1]
- \( t \)-resolution, \( t \)-bias, dilution
- Fourier transform

Exercise: show that a proper time cut \( t > t_0 \) induces undershootings besides the peak in the Fourier transform of the oscillation signal

- but... one critical ingredient still missing: need to known whether or not a given B candidate in the data has mixed flavor tagging
particle or antiparticle

- (let ‘flavor’ here refer to the particle and antiparticle state)

- flavor at decay time:
  - trivially given by the charge of the decay products, if using flavor specific final states
  - (e.g. final flavor given by pion charge in $B_s \rightarrow D_s^- \pi^+$ vs $B_s \rightarrow D_s^+ \pi^-$)

- flavor at production time: ...

how may it be determined ??

Exercise: think about it before resuming discussion in next 2 slides
how to tag?

• attempt #1: use $B_s$ mesons from the decay of heavier particles

- $B^+_c \rightarrow B^0_s \pi^+$

  - the initial B flavor ($b$ or $\bar{b}$) could be inferred from the decay products of the heavier, parent state, eg from the charge of the pion in the examples

• attempt #2: make use of the other $b$ quark (from the originally produced $bb$ pair), by reconstructing the other $b$-hadron in the event, say $B^± \rightarrow J/\Psi K^±$ (flavor given by the kaon charge)

- these possibilities are quite interesting! but given reconstruction inefficiencies (of parent or other $B$), very high signal statistics would/will be required...

⇒ catch: infer flavor without full decay reconstruction

$B^+_s$ or $B^-_s$
flavor tagging methods

**Exercise:**
1. Explain how $B$ flavor oscillations cause an intrinsic dilution of the OST methods performance.
2. Show how the lepton tagger, based on semileptonic $B$ decays ($B \rightarrow l$), is affected by sequential decays such as $B \rightarrow D \rightarrow l$.

**Exercise:** Explain why the performance of SST (OST) should (not) depend on the species of $B$ meson being tagged.

---

opposite-side tagging

- lepton ($e, \mu$)
- jet-charge
- kaon

same-side tagging

- $b$-flavor tagging

---

**Final State Reconstruction**

$b$-flavor at decay

**Proper Decay Time**

In $B$ rest frame

**b-Flavor Tagging**

$b$-flavor at production

---

**Collision Point**

Creation of $b\bar{b}$

---

**D meson**

$K^-$

---

**B jet**

---

**B hadron**

---

**Kaon**

---

**Lepton ($e, \mu$)**

---

**Jet-charge**

---

**Kaon**

---
dilution factors

• various effects decrease the amplitude of an oscillation signal
  
  \[
  \left( \frac{\text{mixing significance}}{\text{signal}} \right)^2 \sim \frac{\epsilon D^2 S}{2} \cdot \frac{S}{S + B} \cdot e^{-\sigma^2 w^2}
  \]

  - flavor tagging
  - signal yield
  - signal purity
  - proper time resolution

  Exercise: Explain why the t-resolution is even more determining for B_s than B_d mixing

• tagging power \( \epsilon D^2 \) is given by the algorithm efficiency \( \epsilon \) and dilution \( D = (1 - 2w)^2 \) where \( w \) is the wrong-tag fraction (i.e. probability algorithm gives wrong decision)
• it determines the effective statistical reduction of the sample size: \( S \Rightarrow S \cdot \epsilon_{\text{tag}} D^2 \)

<table>
<thead>
<tr>
<th>tagger ( \epsilon D^2 )</th>
<th>CDF</th>
<th>D0</th>
<th>ATLAS</th>
<th>CMS</th>
<th>LHCb</th>
</tr>
</thead>
<tbody>
<tr>
<td>for decay ( B_s \rightarrow J/\psi \Phi )</td>
<td>( 1.39 \pm 0.05 % ) [OST] ( 3.5 \pm 1.4 % ) [SST] ~4.9%</td>
<td>[OST+SST] ( 4.68 \pm 0.54 % ) ~4.7%</td>
<td>[OST] ( 1.45 \pm 0.05 % ) ~1.5%</td>
<td>[OST] ~1%</td>
<td>( 2.43 \pm 0.08 \pm 0.26 % ) [OST] ( 0.89 \pm 0.06 % ) [SST] ~3.3%</td>
</tr>
</tbody>
</table>
mixing model

\[ \mathcal{L} = \mathcal{L}_{mass} \cdot \mathcal{L}_t \cdot \mathcal{L}_{\sigma_t} \cdot \mathcal{L}_D \]

[for each sample component & event]

- ingredients: mass, proper time, proper time resolution, t-acceptance function, kinematic factor (for partially reco’d decays), and... flavor tagging

\[ \mathcal{L}_t = \frac{1}{N} \kappa \frac{e^{-\frac{\kappa t^\prime}{\tau}}}{\tau} \left( 1 \pm A S_D D \cos(\Delta m_s \kappa t') \right) \times R(t - t'; S_{\sigma_t} \sigma_t) \cdot \mathcal{E}(t) \otimes F(\kappa) \]
$B_d$ mixing

CDF Run II Preliminary  
$L = 355 \text{ pb}^{-1}$

**JVX Jet Charge Tagger**

- data
- fit projection
- $B^0$ contribution
- $B^+$ contribution

**Soft Muon Tagger**

- data
- fit projection
- $B^0$ contribution
- $B^+$ contribution

LHCb

$+ B^0 \rightarrow D^+ \pi^+$

- combined

LHCb

$+ B^0 \rightarrow J/\psi K^+$

- combined

$\Delta m_d = 0.5156 \pm 0.0051 \text{ (stat.)} \pm 0.0033 \text{ (syst.) ps}^{-1}$  
(most precise measurement)
B_s mixing

observation by CDF (2006)

- p-value = 8x10^{-8} corresponding to 5.4σ
- \Delta m_s = 17.77 \pm 0.10\text{(stat)} \pm 0.07\text{(syst)} \text{ps}^{-1}

\[ A/\sigma_A = 6.05 \]

LHCb confirmed (improved precision)

\[ \Delta m_s = 17.768 \pm 0.023 \text{ (stat)} \pm 0.006 \text{ (syst) } \text{ps}^{-1} \]

In agreement with SM expectation \[ \Delta m_s = 17.3 \pm 2.6 \text{ ps}^{-1} \]

[arXiv: 1102.4274]

note: experimental precision O(10^2) times better than theory calculation
Bs mixing provides one of the strongest constraints to NP models that try to explain the observed flavour anomalies. E.g. a Z' could easily alter the $B_s$ oscillation frequency.

$\Delta m_s = 17.7656 \pm 0.0057 \text{ps}^{-1}$

CERN LHCb 6/fb 2021

$\Delta m_s = 17.77 \pm 0.10 \pm 0.07 \text{ ps}^{-1}$

Fermilab CDF 1/fb 2006
the SM lagrangian

\[ \mathcal{L}_{\text{SM}} = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} + i \bar{f} D \gamma_\mu f + (\bar{\chi} \gamma_\mu \chi) + \frac{1}{2} \partial_\mu \partial^\mu \phi - (\phi^2)^2 \]
Quark gauge couplings

**Flavour universality:** gauge couplings equal for all generations

\[
\mathcal{L}_{\text{fermion}} = \sum_{j=1}^{3} \bar{Q}_j i \not{\! D} Q_j + \bar{U}_j i \not{\! D} U_j + \bar{D}_j i \not{\! D} D_j
\]

\[
D_{Q,\mu} = \partial_\mu + ig_s T^a G^{a\mu} + ig_\tau a W^{a\mu} + iY_Q g' B_\mu
\]

Yukawa couplings

**Flavour non-universality:** induced by Yukawa couplings between quark and Higgs

\[
\mathcal{L}_{\text{Yuk}} = \sum_{i,j=1}^{3} \left( -Y_{U,ij} \bar{Q}_{Li} \tilde{H} U_{Rj} - Y_{D,ij} \bar{Q}_{Li} H D_{Rj} + h.c. \right)
\]

Higgs sector

\[
\mathcal{L}_H = \left( \partial_\mu - ig W^a_\mu t^a - ig' Y_\phi B_\mu \right) \phi^2 + \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2
\]
**Higgs mechanism**

\[ V(\phi) = \mu^2 \phi^\dagger \phi - \frac{\lambda}{2} (\phi^\dagger \phi)^2 \]

spontaneous symmetry breaking by non-zero \( \langle \phi \rangle = v \)

expand Higgs field around vacuum

\[ \phi(x) = \phi_0 + h(x) \]

\[ \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \Rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \]

vacuum expectation value \( v \neq 0 \)

\[ v = \frac{|\mu|}{\sqrt{\lambda}} = \frac{2M_W}{g} = 246 \text{GeV} \]

\[ \mathcal{L}_f = \frac{g_f}{\sqrt{2}} (\bar{f}_Lf_R + \bar{f}_Rf_L)v + \frac{g_f}{\sqrt{2}} (\bar{f}_Lf_R + \bar{f}_Rf_L)h \]

fermions acquire mass:

\[ m_f = \frac{g_f v}{\sqrt{2}} = \frac{\sqrt{2} g_f M_W \sin \theta_W}{e} \]
quark masses

- a Lagrangian mass term $m \overline{\psi} \psi$ would break chiral gauge symmetry $\nRightarrow$ not allowed
- introducing Yukawa interactions with a scalar field, fermion mass terms get generated

\[ -Y \overline{\psi} \psi \phi \xrightarrow{\text{spontaneous symmetry breaking}} -Y \overline{\psi} \psi (\nu + \phi') \]

- the mass terms for up- and down-type quarks have the form

\[ \mathcal{L}_M = -\overline{u}_R^{\alpha T} m_u u_L^\alpha - \overline{d}_R^{\alpha T} m_d d_L^\alpha + \text{h.c.} \]

- the mass matrices - $m_u, m_d$ - are not diagonal; may be diagonalized (w/ unitary matrices $L, R$)

\[ L_u m_u R_u^\dagger = \hat{m}_u \quad \Rightarrow \quad \hat{m}_{u(d)} = \text{diag} \left( m_{u(d)}, m_{c(s)}, m_{t(b)} \right) \]

- flavor changing interactions in the SM (charged currents) through couplings to $W^\pm$ bosons

\[ \mathcal{L}_W = \frac{g}{\sqrt{2}} \overline{u}_L^{\alpha T} \gamma^\mu \overline{d}_L^{\alpha} W_{\mu}^+ + \text{h.c.} = \frac{g}{2\sqrt{2}} \overline{u}^T \gamma^\mu (1 - \gamma^5) V_d W_{\mu}^+ + \text{h.c.} \]

- the unitary quark-mixing matrix $V$ is the Cabibbo-Kobayashi-Maskawa matrix

\[ V \equiv L_u L_d^\dagger \]

- describing quark-flavor mixing

\[ d' = V d \quad \Leftrightarrow \quad \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \]
recap: no FCNC at tree level in SM

charged currents (W) = flavour changing interactions

leptons: flavour universality
quarks: flavour non-universality
quark mixing [CKM]

\[ V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \sim \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4) \]

- **CKM**: a unitary 3x3 matrix
  - has 9 parameters: 3 rotation (Euler angles) + 6 phases
  - 5 of these phases can be absorbed by making phase rotations of quark fields
  - we are left with 4 independent parameters: 3 angles & 1 (complex) phase
  - ➡️ in a standard parameterization (Wolfenstein) these are: A, \( \lambda \), \( \rho \) & \( \eta \)
- **one irreducible phase** ➡️ the source of CP violation in the SM

**Exercise:**
* show that in case of \( N \) generations, unitarity implies \((N-1)^2\) independent parameters, with \( N(N-1)/2 \) rotation angles and \((N-1)(N-2)/2\) complex phases
* show that at least three quark generations are required for CP violation
[CKM | parameter counting]

\[ V \equiv L_u L_d^\dagger \]

- a complex \( N \times N \) matrix has \( 2N^2 \) parameters
- unitary \( \Rightarrow VV^\dagger = 1 \Rightarrow N^2 \) parameters
- can absorb \( 2N-1 \) redefining \( 2N-1 \) phases of \( 2N \) quarks
- accounting \( 2N^2 - N^2 - (2N-1) = (N-1)^2 \)
- this correspond to \( N(N-1)/2 \) angles and \( (N-1)(N-2)/2 \) complex phases

**Degrees of freedom in \( V_{\text{CKM}} \)**

<table>
<thead>
<tr>
<th>Generations</th>
<th>Number of real parameters:</th>
<th>Number of imaginary parameters:</th>
<th>Number of constraints ((VV^\dagger = 1)):</th>
<th>Number of relative quark phases:</th>
<th>Total degrees of freedom:</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 generations</td>
<td>9 + N^2</td>
<td>9 + N^2</td>
<td>-9 - N^2</td>
<td>-5 - (2N-1)</td>
<td>4 (N-1)^2</td>
</tr>
<tr>
<td>2 generations</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**No CP violation in SM!**

This is the reason Kobayashi and Maskawa *first* suggested a 3\(^{rd}\) family of fermions!
- in SM, lepton Yukawa matrices can be diagonalised independently ➞ no FCNC ...
- however, \( \nu \) oscillate ➞ lepton flavour not conserved
- there is also a corresponding mixing matrix (PMNS) in the lepton sector
- CLFV depends on mechanism to generate neutrino masses
- CPV in lepton sector
- are CKM & PMNS related, can explain different structures in quarks vs leptons, …
CKM unitarity constraints

\[ V^\dagger V = 1 \]

All 6 triangles have the same area, a measure of CPV in the SM
“the” unitarity triangle

\[ V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0 \]

Pick a quark phase convention such that \( V_{cb}^* V_{cd} \) is real

Normalize all sides by \( -V_{cb}^* V_{cd} \)

\[
\alpha = \arg \left( -\frac{V_{tb}^* V_{td}}{V_{ub}^* V_{ud}} \right)
\]

\[
\gamma = \arg \left( -\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}} \right)
\]

\[
\beta = \arg \left( -\frac{V_{cb}^* V_{cd}}{V_{tb}^* V_{td}} \right)
\]
over-constraining the unitarity triangle

\[ \text{B}^0 \rightarrow \rho^+\rho^-, \rho\pi, \pi^+\pi^- \]

\[ \text{B} \rightarrow X_u \lambdabar \nu \]
\[ \text{B} \rightarrow \pi \lambdabar \nu \]

\[ \text{B}^\pm \rightarrow D^0 / D^0 K^\pm \]

\[ \text{B} \rightarrow X_c \lambdabar \nu \]
\[ \text{B} \rightarrow D^* \lambdabar \nu \]
\[ \tau_B \]

\[ \text{B}^0 (\text{B}^0) \rightarrow \text{cc} K_s^0 \]
\[ \text{B}^0 (\text{B}^0) \rightarrow \text{ss} K_s^0 \]
\[ \text{B}^0 (\text{B}^0) \rightarrow \text{cc} \text{ dd} \]
constraining the unitarity triangle

• is the CKM matrix unitary (as expected in the SM)?
  ‣ 4th generation of quarks? New forces? E.g. SUSY?
• over-constrain the UT: measure each side and each angle
  ‣ do all measurements cross at one single point?

semileptonic B decays
B → Xυν, D(′)υν

CPV in neutral kaons
K → ππ

CPV in B → D′K, Dπ, Kπ
angle: gamma
(the least well known)

B_d,s mixing
B_s → D_sπ(ππ), D_sΧ
B_d → Dπ, D(′)ν

CPV in B → ππ, ρπ
angle: alpha

CPV in B_d
B_d → ψK, D′D′
angle: sin(2β)
meson mixing
B meson mixing in the SM

- the mixing process (and oscillation frequency, $\Delta m_q$) is proportional to the involved CKM matrix elements

$$\Delta m_q = C_q \left| V_{tb}^* V_{tq} \right|^2, \quad (q = d, s)$$

\[
\begin{align*}
\frac{\Delta m_d}{\Delta m_s} &= \frac{C_d}{C_s} \left( \frac{|V_{td}|^2}{|V_{ts}|^2} \right) = \frac{m_{B^0_d}}{m_{B^0_s}} \xi^2_\Delta \left( \frac{|V_{td}|^2}{|V_{ts}|^2} \right) \\
\frac{\Delta m_s}{\Delta m_d} &= \xi^2_\Delta \frac{m_{B^0_s}}{m_{B^0_d}} \left( \frac{1 - \frac{1}{2} \lambda^2}{\lambda} \right)^2 \frac{1}{(1 - \bar{\rho})^2 + \bar{\eta}^2} \\
(1 - \bar{\rho})^2 + \bar{\eta}^2 &= c
\end{align*}
\]

- i.e., the ratio of $B_d$ and $B_s$ oscillation frequencies yields a circle centered at the point $(\rho=1, \eta=0)$

$\Rightarrow$ a measured value of $\Delta m_s$ away from $\sim 17.5\text{ps}^{-1}$ would have been incompatible with the SM
UT fit

\[(1 - \tilde{\rho})^2 + \tilde{\eta}^2 = c\]

- if \(c\) would be exactly known, the constraint would indeed be a circle
  \[f(\tilde{\rho}, \tilde{\eta}|c) = \delta((1 - \tilde{\rho})^2 + \tilde{\eta}^2 - c)\]
- but... there are uncertainties, both theoretical and experimental

- thus \(c\) is described by a probability density function (PDF): \(f(c)\)
- upon employing Bayes’ theorem
  \[\mathcal{L}(\tilde{\rho}, \tilde{\eta}, c, x|\hat{c}) \propto f(\hat{c}|\tilde{\rho}, \tilde{\eta}, c, x) \cdot f(c, x, \tilde{\rho}, \tilde{\eta})\]
- we obtain the PDF for \(\rho, \eta\) as
  \[\mathcal{L}(\tilde{\rho}, \tilde{\eta}, x) \propto \prod_{j=1,M} f(\hat{c}_j|c_j(\tilde{\rho}, \tilde{\eta}, x)) \times \prod_{i=1,N} f_i(x_i)\]

- integration requires use of numerical and statistical sampling techniques, e.g. Monte Carlo

Exercise: which factor limits the CKM-constraining power of \(B\) mixing; may it be constrained experimentally
UT fit

• as seen, experimental and theoretical inputs with corresponding uncertainties are combined in global inference frameworks
  ‣ imposing SM relations -- or testing alternative BSM flavor scenarios
  ‣ using frequentist or Bayesian statistical fit approaches, e.g.
UT fit evolution over 20 years
CKM fit today & tomorrow
constraining NP

• allowing for New Physics contributions, via generic parameterizations
• e.g. NP contribution to off-diagonal B mass mixing matrix $M_{12}$
  
  - $M_{12}^{SM,q} = M_{12}^{SM,q} \cdot \Delta_q, \text{ with } \Delta_q = |\Delta_q| \cdot \exp(i\Phi^\Delta) \text{ and } q=s,d$
  
  - SM point corresponds to: $\Delta_s=1=\Delta_d$

  - NP phases, $\Phi^\Delta$, shift CP phases from mixing-induced CP asymmetries
  
  - $2\beta_s \rightarrow 2\beta_s-\Phi^\Delta_s (B_s \rightarrow J/\psi \phi)$ and $2\beta_d \rightarrow 2\beta_d+\Phi^\Delta_d (B_d \rightarrow J/\psi K)$