# Higher orders and showers

Paolo Nason

Università di Milano Bicocca

Taming the accuracy of event generators, 23/8/2021

UNIVERSITÀ DEGLI STUDI DI MILANO BICOCCA

INFN Istituto Nazionale di Fisica Nucleare

ACCADEMIA NAZIONALE DEI LINCEI

# Outline

- Several methods have been proposed for improving parton shower generators to NLO accuracy.
- NLO+PS methods are routinely used by theorists and experamentalists for everyday LHC physics
- NNLO+PS methods are growing in importance.
- Challenges:
  - Going to NNLO order in a systematic way. At the moment we are climbing up from easy processes to more complex ones ...
  - Improving resummation accuracy. Given that proposals for NLL showers are appearing, how do we adapt (or formulate new) NLO+PS methods that can comply with them?
  - Multi-jet merging strategies: still not quite satisfactory (in my opinion). Can they become an alternative to better showers?

# Outline

From a theoretical viewpoint, there is still no clear consensus on what are the theoretical requirements an $N^nLO+PS$ generator should have, besides of (the obvious one) being $N^nLO$ accurate for inclusive observables

This situation goes in pair with the general lack of a precise qualification of theoretical requirements that Shower generators should have, a problem that has been directly addressed only in recent times.

In order to make progress, it is useful to understand where we are, analize the various methods, the basic ideas underlying them, and their differences.

# Outline

In this talk I will review several $\text{N}^n\text{LO}+\text{PS}$ methods
with the following aims:

- Formulate the essence of the methods, in a language that is as much as possible common to all of them.
- Pinpoint "features" that can help discussing the differences among the methods.
- I will mostly discuss $n = 1$, i.e. NLO. For methods that have only been developed at NNLO: I will downgrade them to NLO to discuss them, for two reasons:
  - It is likely that features that are present or absent at NLO will be present or absent also at NNLO.
  - NNLO methods don't fit well in slides.

**Caveats:**

- The "common language" that I choose, is POWHEG-centric for obvious reasons.
- I only focus upon selected aspects (the most elementary ones). (A full review of all methods would be highly desirable, but would require a wide collaboration, much more work and a lot of patience.)
- Not all methods are discussed. For example, DEDUCTOR is not included (Nagy+Soper, 2014) (maybe I should have tried harder ...)

# Common Language

I define a mapping from the phase space with radiation to the Born phase space as:

$$\Phi = \{\Phi_0, \Phi_{\rm rad}\}, \quad \Phi' = \{\Phi_0, \Phi'_{\rm rad}\}$$

The (unspecified) "hardness", or ordering variable defined in terms of the phase space with radiation is dubbed $t_\Phi$.
When I write

$$B(\Phi_0)\mathrm{d}\Phi_0 + R(\Phi)\theta(t_\Phi - t_{\rm cut})\mathrm{d}\Phi$$

or

$$(B(\Phi_0) + R(\Phi)\theta(t_\Phi - t_{\rm cut})\mathrm{d}\Phi_{\rm rad})\mathrm{d}\Phi_0$$

I *usually* mean:

- Events generated with weight $B(\Phi_0)$, to be showered with hardness less than $t_{\rm cut}$ (if $t_{\rm cut}$ is small they are not showered).
- plus events generated with weight $R(\Phi)\mathrm{d}\Phi$, having $t_\Phi$ above $t_{\rm cut}$, to be shower with hardness less than $t_\Phi$

Exception (i.e. not *usual* behaviour) will be further specified.

# NLO+PS: MC@NLO, POWHEG

- MC@NLO: Frixione,Webber,2002; Alwall,Frederix,Frixione,Hirschi,Maltoni,Mattelaer,Shao, Stelzer,Torrielli,Zaro,2014
- POWHEG: P.N.2004;Frixione,Oleari,P.N.2007;Alioli,Oleari,Re,P.N.2011; several contributions to the core code by Zanderighi,Hamilton,Jezo,Ferrario-Ravasio, plus several authors of specific processes.

# NLO+PS: MC@NLO, POWHEG

One defines

$$\bar{B}_s = B_0(\Phi_0) + V(\Phi_0) + \int R_S(\Phi, \Phi_{\rm rad}) {\rm d}\Phi_{\rm rad}.$$

$$S(t_\Phi, \Phi_0) = \exp\left[ - \int_{t_{\Phi'} > t_\Phi} \frac{R_S(\Phi_0, \Phi'_{\rm rad})}{B_0(\Phi_0)} {\rm d}\Phi'_{\rm rad} \right]$$

Basic NLO+PS core formula:

$$\begin{aligned} {\rm d}\sigma = & \underbrace{\bar{B}_s(\Phi_0) S(t_{\rm cut}, \Phi_0) {\rm d}\Phi_0}_{\text{(Small!) Shower} < t_{\rm cut}} \\ & \underbrace{+ \bar{B}_s(\Phi_0) S(t_\Phi, \Phi_0) \times \frac{R_S(\Phi_0, \Phi_{\rm rad})}{B_0(\Phi_0)} \theta(t_\Phi - t_{\rm cut}) {\rm d}\Phi}_{\text{Shower} < t(\Phi)} \\ & + \underbrace{(R - R_s) {\rm d}\Phi}_{\text{Shower} < t(\Phi)} \end{aligned}$$

Why it works:

$$\int S(t_\Phi, \Phi_0) \times \frac{R_S(\Phi_0, \Phi_{\rm rad})}{B_0(\Phi_0)} {\rm d}\Phi_{\rm rad} \theta(t_\Phi - t_{\rm cut}) = 1 - S(t_{\rm cut}, \Phi_0)$$

(same reason why it can be generated by a shower algorithm). So

$$\begin{aligned}\frac{{\rm d}\sigma}{{\rm d}\Phi_0} &= \bar{B}_s(\Phi_0) S(\Phi_0, t_{\rm cut}) + \bar{B}_s(\Phi_0)(1 - S_{\Phi_0}(t_{\rm cut})) + \int (R - R_S) {\rm d}\Phi \\ &= \bar{B}_s(\Phi_0) + \int (R - R_S) {\rm d}\Phi \\ &= B_0(\Phi_0) + V(\Phi_0) + \int R(\Phi, \Phi_{\rm rad}) {\rm d}\Phi_{\rm rad}.\end{aligned}$$

(NLO inclusive cross section at fixed underlying Born kinematics).

- MC@NLO: $R_S$ is the Shower approximation to the real term; $R - R_s$: ($\mathcal{H}$-events).
- POWHEG: $R_S = RF(t_\Phi)$, $F(t_\Phi) < 1$, $F(t_\Phi) \to 1$ in the singular limit (i.e. as $t_\Phi \to 0$).

Notice that:

- This uniform formulation of POWHEG and MC@NLO appeared in the NLO+PS review by Webber,P.N.2012.
- One feature that appears in the MC@NLO method has to do with the IR finiteness of the hard contribution, i.e. $R - R_S$. Often shower MC's are not accurate in the soft region, leading to an incomplete cancellation of IR singularities that has to be handled in some way.
- The other well-known feature is the presence of negative weights in the MC@NLO formulation. Whether this is acceptable is not a theoretical issue but a practical question, depending upon the size of the fraction of negative weights and the availability of computer resources (see also Frederix,Frixione,Prestel,Torrielli,2020).

Problems with the matching to parton showers:

- In transverse momentum ordered shower these methods rely upon the shower for LL accuracy
- For angular ordered showers, the generation of the hardest emission as the first one is in conflict with angular ordering. This affects all POWHEG events, and $\mathcal{H}$-events in MC@NLO.

These problems have been known since the very beginning (most of the POWHEG 2004 paper is about this). They are now dealt with in Herwig7 (that provides appropriate truncated showers).

For $p_T$-ordered shower and POWHEG the current statement is that the resummation accuracy of the NLO+PS implementation is as good as that of the shower.

This may turn out to become insufficient in the near future, and more problems related to merging may come up as studies on shower accuracy proceed.

(Jadach,Płaczek,Sapeta,Siódmok,Skrzypek,2015)

$$\mathrm{d}\sigma = \bar{B}_s(\Phi_0)\left\{S(t_\Phi,\Phi_0)\times\frac{R_S(\Phi)}{B_0(\Phi_0)}\right\}\times\left[\frac{R(\Phi)}{R_S(\Phi)}\right]\mathrm{d}\Phi$$

As in `MC@NLO`, $R_S$ is the MC approximation to $R$, but it reweights by $R/R_S$ rather than adding $R - R_S$.

- No negative weights!
- Cross section accurate at NLO, but not equal to the NLO cross section ... (may not please the "purists" of the NLO...)
- Needs full coverage of phase space by the shower:
  - Needs also a good coverage: if $R/R_S$ becomes large, unweighting efficiency will drop to unacceptable levels.
  - Needs good (perfect?) cancellation of singularities in $R/R_S$. If $R_S$ misses some singular region, it will diverge.

# NLO+PS: KrKNLO

On NLO accuracy:

$$
\begin{aligned}
\mathrm{d}\sigma &= \bar{B}_s(\Phi_0)\left\{S(t_\Phi,\Phi_0)\times\frac{R_S(\Phi)}{B_0(\Phi_0)}\right\}\times\left[\frac{R(\Phi)}{R_S(\Phi)}\right]\mathrm{d}\Phi \\
&= \bar{B}_s(\Phi_0)\left\{S(t_\Phi,\Phi_0)\times\frac{R_S(\Phi)}{B_0(\Phi_0)}\right\}\times\mathrm{d}\Phi \\
&+ \bar{B}_s(\Phi_0)\left\{S(t_\Phi,\Phi_0)\times\frac{R_S(\Phi)}{B_0(\Phi_0)}\right\}\times\left[\frac{R(\Phi)}{R_S(\Phi)}-1\right]\mathrm{d}\Phi. \qquad \text{(F)}
\end{aligned}
$$

In (F) the singularity cancel in the square bracket, so we can set the Sudakov to 1, and get

$$
(F) = \frac{\bar{B}_s(\Phi_0)}{B_0(\Phi_0)}\times[R(\Phi)-R_S(\Phi)]\,\mathrm{d}\Phi
$$

equal to the $\mathcal{H}$-events of `MC@NLO` up to NNLO terms.

# MC@NLO+KrK?

Why not both?

$$d\sigma = \bar{B}_s(\Phi_0)\left\{S(t_\Phi,\Phi_0)\times\frac{R_S(\Phi)}{B_0(\Phi_0)}\right\}\times\left[\frac{R}{R_S}\theta(R_S-R)\right]d\Phi$$
$$+\,\theta(R-R_S)\,[R-R_S]\,d\Phi$$

- Reweighting factor $< 1$, can be done by hit and miss.
- No full coverage needed.
- $H$-events positive.
- Possible variants: replace $R \to RF(t)$ (hdamp factor) and add $(1-F(t))\,R$ to the $H$-events. Can tune the amount of NNLO terms injected in the calculation.

# MC@NLO+KrK?

Why not both?

$$\mathrm{d}\sigma = \bar{B}_s(\Phi_0)\left\{S(t_\Phi,\Phi_0)\times\frac{R_S(\Phi)}{B_0(\Phi_0)}\right\}\times\left[\frac{R}{R_S}\theta(R_S-R)\right]\mathrm{d}\Phi$$
$$+\,\theta(R-R_S)\,[R-R_S]\,\mathrm{d}\Phi$$

- Reweighting factor $< 1$, can be done by hit and miss.
- No full coverage needed.
- *H*-events positive.
- Possible variants: replace $R \to RF(t)$ (hdamp factor) and add $(1-F(t))\,R$ to the *H*-events. Can tune the amount of NNLO terms injected in the calculation.
- Think about it: MadGraph5_aMC_KrK@NLO ...

# UNLOPS

- CKKW-L, Lönnblad,2002; Lavesson,Lönnblad,2005; Lönnblad,Prestel,2012.
- UMEPS, Lönnblad,Prestel,2012
- UNLOPS, Lönnblad,Prestel,2012+, (Lavesson,Lönnblad,2008)
- See also: Plätzer,2012; Bellm,Gieseke,Plätzer,2017.
- $UN^2LOPS$, Höche,Li,Prestel,2014,2014+
- $N^3LO$-PS attempts, Prestel,2021.

I follow mostly Höche,Li,Prestel,2014 (where a downgraded NLO version of the method is also illustrated) for this presentation.

# UNLOPS

What is implemented.

$$\mathrm{d}\sigma = \left\{ \left[ \bar{B}(\Phi_0) - \int_{t_c} S_0(t,Q) R(\Phi_0,\Phi_{\mathrm{rad}}) \mathrm{d}\Phi_{\mathrm{rad}} \right] + S_0(t,Q) R(\Phi_0,\Phi_{\mathrm{rad}}) \theta(t-t_c)\, \mathrm{d}\Phi_{\mathrm{rad}} \right\} \mathrm{d}\Phi_0$$

How it is implemented:

$$\mathrm{d}\sigma = \left\{ \left[ \bar{B}^{(t_c)}(\Phi_0) + \int_{t_c} \left[1 - S_0(t,Q)\right] R(\Phi_0,\Phi_{\mathrm{rad}}) \mathrm{d}\Phi_{\mathrm{rad}} \right] + S_0(t,Q) R(\Phi_0,\Phi_{\mathrm{rad}}) \theta(t-t_c)\, \mathrm{d}\Phi_{\mathrm{rad}} \right\} \mathrm{d}\Phi_0$$

$$\begin{aligned} \bar{B}^{(t_c)}(\Phi_0) &= \bar{B}(\Phi_0) - \int_{t_c} R(\Phi_0,\Phi_{\mathrm{rad}}) \mathrm{d}\Phi_{\mathrm{rad}} \\ &= B(\Phi_0) + V(\Phi_0) + \int^{t_c} R(\Phi_0,\Phi_{\mathrm{rad}}) \mathrm{d}\Phi_{\mathrm{rad}} \end{aligned}$$

So:

- Events with a Born-like kinematics and weight $\bar{B}^{(t_c)}$ are generated, and NOT showered
- Events with weight $R$ are generated with a cutoff $t_c$. They are weighted with the Sudakov form factor for not radiating anything harder $S(t, Q)$, and dressed with further shower radiation. At the same time an event with Born kinematics, weighted with $1 - S(t, Q)$, is generated and not showered.

Notice that $\bar{B}^{(t_c)}$ becomes negative when $\alpha_S \log^2 \frac{Q}{t_c} \gtrapprox 1$, its value being compensated by the real cross section of the $1 - S(t, Q)$ term, at the price of generating negative weighted events.
Should I think of $t_{\rm cut}$ as a Shower cutoff or a merging scale?

# UNLOPS

Consider that if $\bar{B} = B$ and $R$ is the shower approximation to $R$ used to define $S_0$, we have

$$\lim_{t_c \to 0} \left[ \bar{B}(\Phi_0) - \int_{t_c} S_0(t, Q) R(\Phi_0, \Phi_{\mathrm{rad}}) \mathrm{d}\Phi_{\mathrm{rad}} \right] = 0$$

Thus, in the full UNLOPS formula, if $t_{\mathrm{cut}}$ is small (i.e. is a shower cutoff), unshowered events are an NLO fraction wrt showered ones.

# UNLOPS

- Its origin is back to the CKKW: generate matrix elements according to the tree-level formula (with a cutoff) and reweight them with Sudakov form factors.
- In CKKW-L, given a matrix element one reconstract a shower history, and implement Sudakov form factors using the shower itself. (about reconstructing shower histories: more on Sector Showers later in this workshop).
- LO or NLO accuracy is enforced by subtracting the (Sudakov reweighted) cross section for the $n+1$-matrix element to the $n$-matrix element one.

Here I only look at one emission to expose the most elementary features of the method.

# UNLOPS

Already at the NLO level for one emission, a feature of the UNLOPS method shows up: the presence of unsuppressed, unshowered events with cross section $\sigma_U$ ($U$ for unshowered) that, for small $t_{\rm cut}$, have typical NLO size:

$$\mathrm{d}\sigma_U = \left[\bar{B}^{(t_c)}(\Phi_0) + \int_{t_c} \left[1 - S_0(t, Q)\right] R(\Phi_0, \Phi_{\rm rad})\mathrm{d}\Phi_{\rm rad}\right] \mathrm{d}\Phi_0$$

- Is this theoreticall acceptable to have them? (I believe not, the authors seem to think differently)
- What should we do with them?

Remembering that

$$\mathrm{d}\sigma_U = \left[B + V + \int \left[1 - S_0(t, Q)\, R(\Phi_0, \Phi_{\rm rad})\, \theta(t - t_c)\right] \mathrm{d}\Phi_{\rm rad}\right] \mathrm{d}\Phi_0$$

it seems clear that, for small $t_{\rm cut}$ the right thing to do is just to shower them.

# UNLOPS

- The $\sigma_U$ terms originate mostly from virtual and reals having $t$ in the Sudakov region. Showering them will spread them (mostly) in the Sudakov region.
- The high-$t$ region would be corrected by beyond NLO order, since $1 - S(t)$ is of order $\alpha_s(t)$ for large $t$.
- Variants on how large a $t$ one should allow may be possible.
- The direct computation of $\sigma_U$ (without using the give and take algorithm) is no more difficult that typical $\bar{B}$ calculations in POWHEG.

Whether the interesting features of UNLOPS as far as multijet are concerned can be maintained with the above strategy, may requires furhter thinking ...

# GENEVA

- Alioli,Bauer,Berggren,Tackmann,Walsh,2015, DY at NNLO
- Alioli,Bauer,Berggren,Tackmann,Walsh,et al.,2014, GENEVA at NNLO.
- Alioli,Bauer,Berggren,Hornig,Tackmann,2013, NLO merging.

For the present talk: Alioli,Broggio,Kallweit,Lim,Rottoli, 2019, where a quite clear formulation of the method is presented.

At NLO, the GENEVA formula may be written as follows

$$
\begin{aligned}
\mathrm{d}\sigma = {} & B(\Phi_0) S(t_{\mathrm{cut}}, \Phi_0) \mathrm{d}\Phi_0 \\
& + B(\Phi_0) \frac{\mathrm{d}S(t, \Phi_0)}{\mathrm{d}t} \theta(t - t_{\mathrm{cut}}) \mathcal{P}(\Phi) \mathrm{d}\Phi \\
& + \left( R(\Phi) - B(\Phi_0) \left[ \frac{\mathrm{d}S(t, \Phi_0)}{\mathrm{d}t} \right]_1 \mathcal{P}(\Phi) \right) \theta(t - t_{\mathrm{cut}}) \mathrm{d}\Phi
\end{aligned}
$$

- $B(\Phi_0) S(t_{\mathrm{cut}}, \Phi_0)$ is the resummed, inclusive cross section for $t < t_{\mathrm{cut}}$
- $\mathcal{P}$ is a splitting function, such that $\int \mathrm{d}\Phi_{\mathrm{rad}}\, \delta(t - t_0) \mathcal{P}(\Phi) = 1$
- $[\ldots]_1$ takes out the first order term from what's inside.

Integrating it up to a given $t_0$, it yields the resummed, NLO matched cross section for the process, provided $B(\Phi_0)S(t,\Phi_0)$ is NLL' accurate:

$$B(\Phi_0)S(t,\Phi_0) = (B(\Phi_0) + SV(\Phi_0)) \times \exp\left[-L\left(\alpha_s L + \alpha_s^2 L^2 + \ldots\right) + \left(\alpha_s L + \alpha_s^2 L^2 + \ldots\right)\right]$$

In order to have NLO accuracy, the red terms should be included. NLL' accuracy guarantees this, but it is not mandatory.

The hope/aim of GENEVA (I think) is to achieve also NLL accuracy. This is, by construction, would be the case for the $t$, provided

- $S$ is the full NLL Sudakov
- The shower does not change the initial $t$ at order $\alpha_s$.

- Local cancellation of singularities in the last line is not guaranteed in case of complex soft patterns ($\mathcal{P}$ is in the collinear approximation). Cancellation takes place by construction in integrated quantities.
- GENEVA is the only method attempting to improve resummation aspects of NLO$^n$PS generators.
- In what sense resummation is improved?

It seems that there is a conflict in trying to maintain the NLL accuracy of the resummation observable without spoiling global LL accuracy of the generator.
Understanding these issue at the NLO,NLL level would be already very useful.

- Originaly formulated for matching without merging Hamilton,Zanderighi,P.N.2012, Rikkert,Hamilton,2015
- Method for NNLO+PS Hamilton,Oleari,Zanderighi,P.N.2013,Hamilton,Re,Zanderighi,P.N.2013, by reweighting.
- MiNNLO$_{\rm PS}$: NNLO+PS by correcting, Monni,Re,Wiesemann,Zanderighi,P.N.2020.

MiNLO$_{\rm PS}$ (with one N) does not exist, the following is a downgrade of MiNNLO.

# MiNLO$_{\rm PS}$ by Reweighting

MiNLO$_{\rm PS}$ formula:

$$\mathrm{d}\sigma = \frac{\sigma_{\rm NLO}(\Phi_0)}{\sigma_{\rm LO}(\Phi_0)} S(t_\Phi, \Phi_0) R(\Phi) \mathrm{d}\Phi.$$

- $S(t, \Phi_0)$ is a Sudakov form factor at a scale $t$ (typically a transverse momentum)
- $R(\Phi)$ has the couplings and PDFs evaluated at the scake $t_\Phi$.

For this to work, we must guarantee that

$$\int S(t_\Phi, \Phi_0) R(\Phi) \mathrm{d}\Phi_{\rm rad} = \sigma_{\rm LO}(\Phi_0) + \mathcal{O}(\alpha_s).$$

# MiNLO$_{\rm PS}$ by Reweighting

In the MiNLO procedure we identify the Monte Carlo result for a $t$ differential distribution with a resummation formula that is at least LO accuracy when integrated:

$$
\begin{aligned}
\frac{\mathrm{d}\sigma(t,\Phi_0)}{\mathrm{d}t} &= B(\Phi_0)\frac{\mathrm{d}}{\mathrm{d}t}S(t,\Phi_0) + R_F && \text{(F1)}\\
&= B(\Phi_0)\,S(t,\Phi_0)\frac{1}{t}(A\,L\,\alpha_s(t) + B\,\alpha_s(t) + \ldots) + R_F && \text{(F2)}\\
&= \int S(t,\Phi_0)\,R(\Phi)\,\delta(t-t_\Phi)\,\mathrm{d}\Phi_{\rm rad} && \text{(F3)}
\end{aligned}
$$

where $L = \log \mu^2/t^2$. We have the properties:

- (F1) integrates to the LO result up to terms of $\mathcal{O}(\alpha_s)$.
- (F2) is obtained by taking the derivative and expanding up to the order needed to match (F3) (i.e. $\mathcal{O}(\alpha_s)$).

# MiNLO$_{\rm PS}$ by Reweighting

When integrating in $t$, remember that the Sudakov peaks when $\alpha_s(t)L^2 \approx 1$, so each power of $L$ (including $\mathrm{d}t/t$) counts as $\alpha_s^{-\frac{1}{2}}$:

$$\int_{t_{\rm cut}}^{Q} S(t,\Phi_0) L^m \alpha_s^n(t) \frac{\mathrm{d}t}{t} \approx (\alpha_s(Q))^{n-\frac{m+1}{2}} . \qquad (1)$$

What is negleced in (F2) has the form

$$B(\Phi_0)\, S(t,\Phi_0) \frac{1}{t} \left( L\, \alpha_s^2(t) + \alpha_s^2(t) + \ldots \right) \qquad (2)$$

that upon $t$ integration starts at order $\alpha_s(Q)$. So, (F1) and (F2) differ by terms of $\mathcal{O}(\alpha_s)$, and (F2) matches (F3) by construction,

Thus the whole thing is LO accurate up to terms of order $\alpha_s$.

The equality of the second and third line is certainly true if $S(t,\Phi_0)$ is the NLL Sudakov. However, it is enough to require that $A$ and $B$ are the same that one gets with the full NLL Sudakov.

# MiNLO$_{\mathrm{PS}}$ by Correcting

MiNLO$_{\mathrm{PS}}$ formula:

$$\mathrm{d}\sigma = S(t_\Phi, \Phi_0)\left[R(\Phi) + C(t_\Phi, \Phi_0)F(\Phi)\right]\mathrm{d}\Phi.$$

- $C(t_\Phi, \Phi_0)$ is the correction.
- Not much is required on $F$: $\int F(\Phi)\delta(t - t_\Phi)\mathrm{d}\Phi_{\mathrm{rad}} = 1$. In other words, the correction $C$ lives at an underlying Born kinematics, but with a finite $t$. $F$ must spread out the $\Phi_0$ kinematics at a given $t$ over the full $\Phi$ kinematics.

# MiNLO$_{\text{PS}}$ by Correcting

Now we have

$$
\begin{aligned}
\frac{\mathrm{d}\sigma(t,\Phi_0)}{\mathrm{d}t} &= B(\Phi_0)\frac{\mathrm{d}}{\mathrm{d}t}\Big[S(t,\Phi_0)(1+H(\Phi_0)\,\alpha_s(t))\Big] + R_F && \text{(F1)}\\
&= B(\Phi_0)\,S(t,\Phi_0)\frac{1}{t}\Big(A\,L\,\alpha_s(t) + B\,\alpha_s(t) \\
&\qquad + P\,L\,\alpha_s^2(t) + Q\,\alpha_s^2(t) + \ldots\Big) + R_F && \text{(F2)}\\
&= \int S(t,\Phi_0)\,R(\Phi)\,\delta(t-t_\Phi)\,\mathrm{d}\Phi_{\mathrm{rad}} + D(t,\Phi_0) && \text{(F3)}\\
&= \int \left[S(t,\Phi_0)\,R(\Phi) + D(t_\Phi,\Phi_0)F(\Phi)\right]\,\delta(t-t_\Phi)\,\mathrm{d}\Phi_{\mathrm{rad}} && \text{(F4)}
\end{aligned}
$$

where

- In (F2), $P$ integrates to $\mathcal{O}(\alpha_s)$, $Q$ to to $\mathcal{O}(\alpha_s^{1+\frac{1}{2}})$, and must be kepth for $\mathcal{O}(\alpha_s(Q))$ accuracy!
- $P$ and $Q$ are not in $S(t,\Phi_0)\,R(\Phi)$, must be added:

$$D(t,\Phi_0) = P\,L\,\alpha_s^2(t) + Q\,\alpha_s^2(t).$$

- The goal of the method was to formulate a merging approach, and the an NNLO+PS approach without merging scales. In fact, in MiNLO the only scale that appears must be the non-perturbative cutoff, near the Landau pole.
- Is it possible to build a $\texttt{MiNLO}_{\mathrm{PS}}$ system as good as POWHEG or MC@NLO?

# Actual NNLO+PS implementations

Of the methods discussed so far, their application for NNLO+PS follows different paths:

- `GENEVA` follows a uniform method, going one further step to reach NNLO (and NNLL') accuracy.
- $\texttt{MiNNLO}_{\mathrm{PS}}$, and the reweighting approach, uses a hybrid method. In the example of Higgs production, $H + J$ is generated at NLO using `POWHEG`, while the extension to NNLO uses MiNLO.
- UNLOPS also uses a hybrid approach, with the first stage ($H + J$) carried out with an `MC@NLO` like approach, and the NNLO extension uses UNLOPS.

# NNLO+PS

Of the methods discussed so far, their application for NNLO+PS follows different paths:

- GENEVA follows a uniform method, going one further step to reach NNLO (and NNLL') accuracy.
- $\texttt{MiNNLO}_{\text{PS}}$, and the reweighting approach, uses a hybrid method. In the example of Higgs production, $H+J$ is generated at NLO using POWHEG, while the extension to NNLO uses MiNLO.
- UNLOPS also uses a hybrid approach, with the first stage ($H+J$) carried out with an MC@NLO like approach, and the NNLO extension uses UNLOPS.

# Conclusions and General Remarks

- We are now for the NNLO+PS at a similar stage of NLO+PS in the middle 2000: several implemented processes for neutral systems; $t\bar{t}$ just appeared.
  Can we do jets? Can we improve over NLO+PS jets?
- NLO+PS implementations with NLL accuracy.
  By NLL here I mean that includes all the following terms

$$\exp(Lg(\alpha_s L) + g_1(\alpha_s L) + \mathcal{O}(\alpha_s(\alpha_s L)^m)) \tag{3}$$

  Are there available methods that extend well in this direction?
- NLO+PS implementation with NLL accuracy for a single key observable. requiring at least general LL accuracy (does `GENEVA` already fulfill this?)
- Changing the rules of the game: how about asking only for the a fixed number of powers in $g_1$ to be correct? Or any less demanding requirement that can be acceptable in practice?