NNLO event generation for top pair production at the LHC

[arXiv: 2012.14267 + ongoing work]
Why top quarks: Higgs sector

- Bridge between QCD and Higgs sectors of SM Lagrangian: study of $y_t$ plays a central role in Higgs physics
- Hierarchy problem
- Sensitivity to top partners in tails of distributions (large momentum transfer)
- background in many Higgs measurements

**tth observation (consistent with SM $y_t$ within unc.)**

<table>
<thead>
<tr>
<th>Source</th>
<th>$\sqrt{s}$</th>
<th>$79.8$ fb$^{-1}$</th>
<th>$125.09$ GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>ATLAS</td>
<td>$13$ TeV</td>
<td>$7.1$ TeV</td>
<td>$125.09$ GeV</td>
</tr>
<tr>
<td>CMS</td>
<td>$7+8$ TeV</td>
<td>$19.7$ fb$^{-1}$</td>
<td>$125.09$ GeV</td>
</tr>
<tr>
<td>CMS</td>
<td>$13$ TeV</td>
<td>$35.9$ fb$^{-1}$</td>
<td>$125.09$ GeV</td>
</tr>
</tbody>
</table>

$m_H = 125.09$ GeV

All categories

$\ln(1+S/B)$ weighted sum

$\mu$-dependence

Combined

$\Sigma_{Tb} = 13$ TeV, $79.8$ fb$^{-1}$

$\mu_t$ observation (consistent with SM $y_t$ within unc.)
Why top quarks: new physics

- In several NP scenarios, extra states couple dominantly to top quarks
- rich sensitivity to SMEFT dim. 6 op. Different observables within the same process probe different operators
e.g. in $t\bar{t}$
- contact int. in high energy tails, e.g.
  $$(\bar{Q}\gamma_\mu T^A Q)(\bar{q}_i\gamma^\mu T^A q_i) + \ldots$$
- dipole op. in total rates, e.g.
  $$(\bar{Q}\sigma^{\mu\nu} T^A t) \tilde{\phi} G^A_{\mu\nu} + \text{h.c.}$$

[Brivio, Bruggisser, Maltoni, Moutafis, Plehn, Vryonidou, Westhoff, Zhang '19]

Global fit of dimension 6 ops. with Run II top measurements

Theory uncertainties already a limiting factor at Run II!
Why top quarks: precision SM

- Precision measurements/theory in top physics: (outstanding performance of LHC)

- Fast decay allows one to “probe” its pole mass (though linear renormalon ambiguities of $O(\Lambda_{\text{QCD}})$ remain)

  [Beneke, Marquard, Nason, Steinhauser (2017)]
  [Hoang, Lepenik, Preisser (2017)]

- Top mass relevant for running of Higgs trilinear coupling (and e.g. stability of the vacuum)

- Sensitivity of $t\bar{t}$ to $\alpha_s$ and parton densities

  see e.g. [Klijnsma, Bethke, Dissertori, Salam (2017)]

- Spin correlations between top quarks

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**e.g. top pole mass from template fits**

<table>
<thead>
<tr>
<th>ATLAS</th>
<th>$m_{\text{top}} \pm \text{stat.} \pm \text{syst. (total)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_{\text{top}}^{\text{dilepton}} (8 \text{ TeV})$</td>
<td>172.99 ± 0.41 ± 0.74 (0.85 ± 0.04)</td>
</tr>
<tr>
<td>$+ m_{\text{top}}^{+\text{jets}} (8 \text{ TeV})$</td>
<td>172.56 ± 0.28 ± 0.48 (0.56 ± 0.04)</td>
</tr>
<tr>
<td>$+ m_{\text{top}}^{+\text{jets}} (7 \text{ TeV})$</td>
<td>172.51 ± 0.27 ± 0.42 (0.50 ± 0.04)</td>
</tr>
<tr>
<td>$+ m_{\text{top}}^{\text{all jets}} (8 \text{ TeV})$</td>
<td>172.61 ± 0.25 ± 0.42 (0.49 ± 0.03)</td>
</tr>
<tr>
<td>$+ m_{\text{top}}^{\text{all jets}} (7 \text{ TeV})$</td>
<td>172.70 ± 0.24 ± 0.42 (0.48 ± 0.03)</td>
</tr>
<tr>
<td>$+ m_{\text{top}}^{\text{dilepton}} (7 \text{ TeV})$</td>
<td>172.69 ± 0.25 ± 0.41 (0.48 ± 0.03)</td>
</tr>
</tbody>
</table>

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**Comb. according to importance**

- $n_{\text{top}}^{\text{dilepton}} (8 \text{ TeV})$
- $+ m_{\text{top}}^{+\text{jets}} (8 \text{ TeV})$
- $+ m_{\text{top}}^{+\text{jets}} (7 \text{ TeV})$
- $+ m_{\text{top}}^{\text{all jets}} (8 \text{ TeV})$
- $+ m_{\text{top}}^{\text{all jets}} (7 \text{ TeV})$
- $+ m_{\text{top}}^{\text{dilepton}} (7 \text{ TeV})$

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**Combination**

- stat. uncertainty
- total uncertainty

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**Diagram details:**

- $m_{\text{top}}$ range from 170 to 180 GeV
- $m_{\text{top}}$ values with uncertainties
- Graphical representation of combined measurements

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**References:**

- [ATLAS EPJC (2019) 290]
- [Beneke, Marquard, Nason, Steinhauser (2017)]
- [Hoang, Lepenik, Preisser (2017)]
- [Klijnsma, Bethke, Dissertori, Salam (2017)]
Top-pair production: state of the art theory

- Great advances in perturbative calculations (fixed/all orders) led to remarkable theory accuracy for \( \text{tt} \) observables
- NNLO QCD (production & decay in NWA, + spin correlations)
- Full off-shell effects @ NLO
- NLO EW
- Resummations (\( q_\perp \), threshold, Coulomb corrections)
- Bottom quark fragmentation @ NNLO

Yet, most experimental studies rely on MC generators, often with considerable TH uncertainties
Pending issues with MC

- e.g. Unfolding to inclusive phase space may hide subtle issues w/ underlying MC accuracy
- e.g. Significant dependence of the extracted pole top mass on MC used in template fits

[Behring, Czakon, Mitov, Papanastasiou, Poncelet (2019)]

\[ m_t \text{ from moments of leptonic obs.} \]

- Py8.2: \( m_t = 172.500^{+0.794}_{-0.772} \) GeV
- Py6.4: \( m_t = 173.673^{+0.810}_{-0.781} \) GeV
- Hw7.1: \( m_t = 175.354^{+0.821}_{-0.785} \) GeV
- Hw6.5: \( m_t = 177.031^{+0.816}_{-0.778} \) GeV

[Behring, Czakon, Mitov, Papanastasiou, Poncelet (2019)]

\[ \sigma - d \sigma / d \Delta \phi / \pi \text{ vs } \Delta \phi / \pi \]

- LHC 13 TeV \( m_t = 172.5 \) GeV
- Scale: \( H_T/4 \) PDF: NNPDF31nnlo

[Behring, Czakon, Mitov, Papanastasiou, Poncelet (2019)]

\[ \sigma - d \sigma / d \Delta \phi / \pi \text{ vs } \Delta \phi / \pi \]

- LHC 13 TeV \( m_t = 172.5 \) GeV
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Matching to Parton Shower, a simpler example: Z+jet@NLO

Resolved (e.g. $p_T^{\text{jet}} > 30$ GeV) QCD jet
Matching to Parton Shower, a simpler example: Z+jet@NLO

NLO QCD: fixed order exp.\(^n\)

\[
d\sigma = d\sigma^{(0)} \left( 1 + \alpha_s(\mu_R)d\sigma^{(1)} + O(\alpha_s^2(\mu_R)) \right)
\]

- Fixed coupling \(\alpha_s(\mu_R)\)
- Series truncated at FO

Resolved (e.g. \(p_{T\text{jet}} > 30\) GeV) QCD jet
Matching to Parton Shower, a simpler example: Z+jet@NLO

- Running coupling $\alpha_S(k_{t,\text{rad}})$
- Virtual corrections encoded in Sudakov FFs
- Iterated at all orders: resummation of radiative corrections with some accuracy

\[
d\sigma_{n+1} = d\sigma_n \left( \Delta(v_n) + d\Phi_{\text{rad}} \frac{\Delta(v_n)}{\Delta(v_{n+1})} P(\alpha_s(k_{\perp,\text{rad}}, \Phi_{\text{rad}})) \right)
\]

\[
\Delta(v_n) \equiv \exp \left\{ - \int_{v_n > v_{\text{rad}} > \Lambda} d\Phi_{\text{rad}} P(\alpha_s(k_{\perp,\text{rad}}, \Phi_{\text{rad}})) \right\}
\]

Resolved (e.g. $p_{T,\text{jet}} > 30$ GeV) QCD jet
Matching to Parton Shower, a simpler example: $Z+\text{jet}@\text{NLO}$

NLO QCD: fixed order $\exp^n$

$$d\sigma = d\sigma^{(0)} \left( 1 + \alpha_s(\mu_R) d\sigma^{(1)} + O(\alpha_s^2(\mu_R)) \right)$$

Parton Shower: iterate

$$d\sigma_{n+1} = d\sigma_n \left( \Delta(v_n) + d\Phi_{\text{rad}} \frac{\Delta(v_n)}{\Delta(v_{n+1})} P(\alpha_s(k_{\perp, \text{rad}}, \Phi_{\text{rad}})) \right)$$

Double counting of radiative corrections near the singular limits
What do we want from N(N)LO + PS simulations?

- Simple goal … many pitfalls: avoid double counting while
  - i) achieving N(N)LO accuracy of hard scattering process
  - ii) preserving the logarithmic accuracy of the parton shower

- Inevitable price to pay/margin of manoeuvre: inclusion of higher order corrections beyond N(N)LO

- In the following the PS is assumed to have LL accuracy (in the leading colour approximation), i.e. the multi-parton squared amplitude is reproduced correctly in the limit of strongly ordered emissions and $N_c >> 1$

  - A very relevant, completely open question is how to match consistently to more accurate PS algorithms while preserving logarithmic accuracy

  - I will only address MiNNLO$_{PS}$ - other NNLO matching methods discussed today

[Alioli et al. (2013 - 2021)]
[Hoeche, Li, Prestel (2014)] [Hoeche, Kuttimalai, Li (2018)]
[Campbell, Hoeche, Li, Preuss, Skands (2021)]
Problem well understood at NLO, general solutions applicable to virtually any process

[Frixione, Webber (2002); Nason (2004); Frixione, Nason, Oleari (2007); Jadach et al. (2015)]

e.g. one option is to recast the hard scattering as if the radiation were generated by a modified PS …
NLO + PS & merging jet multiplicities: MiNLO


• 1) dress the inclusive NLO with Sudakov FFs, and set the coupling scales to the $k_T$ of the corresponding emission (in a $k_T$-clustering sense - inspired by CKKW procedure)

[Catani, Krauss, Kuhn, Webber (2001)]

e.g. consider a NLO calculation for $Z+$jet differential in $\Phi_{FJ}$

$$\bar{B}_{NLO}^{(FJ)} = \frac{\alpha_s(\mu_R)}{2\pi} \left[ B^{(FJ)} + \frac{\alpha_s(\mu_R)}{2\pi} V^{(FJ)} + \frac{\alpha_s(\mu_R)}{2\pi} \int d\Phi_{rad} R^{(FJ)} \right]$$
1) dress the inclusive NLO with Sudakov FFs, and set the coupling scales to the $k_T$ of the corresponding emission (in a $k_T$-clustering sense - inspired by CKKW procedure)

\[
\Delta_f^2(Q) / \Delta_f^2(q_\perp) = \left(1 - \frac{\alpha_s(q_\perp)}{2\pi} S_f^{(1)}(q_\perp) + \mathcal{O}(\alpha_s^2(q_\perp))\right)
\]

Squared = 2 radiating legs
in the unresolved limit
2) generate real radiation à la PS, namely (POWHEG) 

\[ d\sigma^{(FJ)} = \overline{B}^{(FJ)}_{\text{MiNLO}} d\Phi_{FJ} \left[ \Delta_{\text{pwg}}(\Delta_{\text{gen}}) + d\Phi_{\text{rad}} \Delta_{\text{pwg}}(k_{\perp}) \frac{R(\Phi_{FJ}, \Phi_{\text{rad}})}{B(\Phi_{FJ})} \right] \]

\[ \Delta_{\text{pwg}}(q) \equiv \exp \left\{ - \int_{k_{\perp}^2 > q^2} d\Phi_{\text{rad}} \frac{R(\Phi_{FJ}, \Phi_{\text{rad}})}{B(\Phi_{FJ})} \right\} \]

Mimics a shower step ordered in \( k_T \), with the full real matrix element (virtuals in \( \overline{B}^{(FJ)}_{\text{MiNLO}} \))

[Nason (2004)]
3) NLO calculation now mimics the first two steps of a PS, so it is sufficient to let the actual shower (e.g. Pythia8) generate extra radiation requiring it has a transverse momentum smaller than the POWHEG radiation (PS starting scale)*

* crucial for the PS ordering to match transverse momentum near singular limit, otherwise extra fixes become necessary (e.g. truncated shower for angular ordering)
An important byproduct is that now the jets can go unresolved (i.e. $q_\perp \to 0$)

Merging of 1 and 0 jet multiplicities: can one get NLO accuracy for both?

Unresolved (0-jet) limit approached as the leading jet has $p_{T,jet} \to 0$

\[
\tilde{B}_{\text{MiNLO}}^{(FJ)} = \frac{\alpha_s(q_\perp)}{2\pi} \left\{ \frac{\Delta_f^2(Q)}{\Delta_f^2(q_\perp)} B^{(FJ)} \left( 1 + \frac{\alpha_s(q_\perp)}{2\pi} S_f^{(1)}(q_\perp) \right) + \frac{\alpha_s(q_\perp)}{2\pi} V^{(FJ)} \right\} + \int d\Phi_{\text{rad}} \frac{\alpha_s(q_\perp)}{2\pi} \frac{\Delta_f^2(Q)}{\Delta_f^2(q_\perp)} R^{(FJ)}
\]

With LL accuracy, approximate the $p_{T,jet}$ with the $q_\perp$ of the Z: Sudakov FF must account for the full singularity structure in the limit $q_\perp \to 0 \Rightarrow$ Extract it from $q_\perp$ resummation
Small $q_{\perp}$ limit for colour singlet systems

In the limit $q_{\perp} \to 0$ the differential cross section obeys a simple factorisation theorem*

\[
\frac{d\sigma}{dq_{\perp}d\Phi_F} \sim \sum_f |M_{ff\to F}^{(0)}|^2 \int \frac{d^2b}{(2\pi)^2} e^{ib\cdot q_{\perp}} e^{-R_f(b)} H_f \sum_{i,j} (C_{fi} \otimes h^{[i]})(C_{fj} \otimes h^{[j]})
\]

*Connection with MC can be made manifest in momentum-space formulation (RadISH), not discussed in the following

[PM, Re, Torrielli (2016); Bizon, PM, Re, Rottoli, Torrielli (2017)]
Simple form when averaged over azimuth of $\vec{q}_\perp$ and LL accuracy

$$\left[ \frac{d\sigma}{d\vec{q}_\perp d\Phi_F} \right]_\phi \sim \frac{d}{dq_\perp} \left[ \sum_f e^{-S_f(q_\perp)} \mathcal{L}_f(q_\perp) \right] + \mathcal{O}(\alpha_s^3(q_\perp))$$

Allows us to identify the missing Sudakov FF

$$\tilde{B}_{\text{MiNLO}}^{(FJ)} = \frac{\alpha_s(q_\perp)}{2\pi} \left\{ \frac{\Delta^2_f(Q)}{\Delta^2_f(q_\perp)} B^{(FJ)} \left( 1 + \frac{\alpha_s(q_\perp)}{2\pi} S^{(1)}_f(q_\perp) \right) + \frac{\alpha_s(q_\perp)}{2\pi} V^{(FJ)} + \int d\Phi_{\text{rad}} \frac{\alpha_s(q_\perp)}{2\pi} \frac{\Delta^2_f(Q)}{\Delta^2_f(q_\perp)} R^{(FJ)} \right\}$$

Mind the power counting

$$\int\limits_Q^{Q} \frac{dq_\perp}{q_\perp} \ln^n \frac{Q}{q_\perp} \alpha_s^m(q_\perp) e^{-S(q_\perp)} \sim \alpha_s^{m - \frac{n+1}{2}}(Q) \quad \Rightarrow$$

Full $\alpha_s^2$ resummation structure needed to have NLO in both 0 and 1 jet events
NNLO for 0-jet events could be achieved by a local reweighing in the phase space of the Z boson by \( \frac{d\sigma_{\text{NNLO}}}{d\sigma_{\text{MINLO}}} \): simple procedure but

- discrete grids, hard to access remote regions
- CPU intensive
- tough high-dim. reweighing for many particles

\[ \text{e.g. } W^+W^- \text{ invariant mass} \]

\[ \text{e.g. } W^+W^- \text{ production would require a 9-dim. diff. XS} \]

\[ \text{recast as 81 grids using Collins-Soper decomposition} \]
The MiNNLO\textsubscript{PS} procedure

- **MiNNLO\textsubscript{PS}:** compute full NNLO corrections directly in the weight, i.e.

\[
\tilde{B}_{\text{MiNNLO}_{\text{PS}}}^{(FJ)} = \frac{\alpha_s(q_\perp)}{2\pi} \left\{ \frac{\Delta_f^2(Q)}{\Delta_f^2(q_\perp)} \left[ B^{(FJ)} \left( 1 + \frac{\alpha_s(q_\perp)}{2\pi} S^{(1)}_f(q_\perp) \right) + \frac{\alpha_s(q_\perp)}{2\pi} V^{(FJ)} \right] + \int d\Phi_{\text{rad}} \frac{\alpha_s(q_\perp)}{2\pi} \frac{\Delta_f^2(Q)}{\Delta_f^2(q_\perp)} R^{(FJ)} \right. \\
\left. + \frac{\Delta_f^2(Q)}{\Delta_f^2(q_\perp)} D^{(\geq 3)}(\Phi_F, q_\perp) F_{\ell}^{\text{corr}}(\Phi_{FJ}) \right\}
\]

New term derived from $q_\perp$ resum., contains all terms required to achieve NNLO according to our power counting ($\alpha_s^3 \text{corr.}^{\text{ns}}$ needed)

\[
D^{(\geq 3)}(\Phi_F, q_\perp) = -\frac{dS_f(q_\perp)}{dq_\perp} L_f(q_\perp) + \frac{dL_f}{dq_\perp} - \frac{\alpha_s(q_\perp)}{2\pi} [D(q_\perp)]^{(1)} - \frac{\alpha_s^2(q_\perp)}{(2\pi)^2} [D(q_\perp)]^{(2)}
\]

Spreading of new corr.\textsuperscript{n} across $\Phi_{FJ}$ at fixed $\Phi_F$ (FKS)

\[
F_{\ell}^{\text{corr}}(\Phi_{FJ}) = \int d\Phi'_{FJ} J(\Phi_{FJ}) \delta(q_\perp - q'_\perp) \delta(\Phi_F - \Phi_F') \\
J(\Phi_{FJ}) = P(\Phi_{\text{rad}})(h^{[i]} h^{[j]})
\]
The MiNNLOPS procedure

- \textbf{MiNNLOPS:} compute full NNLO corrections directly in the weight, i.e.

\[ B_{\text{MiNNLOPS}}^{(FJ)} = \frac{\alpha_s(q_\perp)}{2\pi} \left\{ \frac{\Delta_f^2(Q)}{\Delta_f^2(q_\perp)} \left[ B^{(FJ)} \left( 1 + \frac{\alpha_s(q_\perp)}{2\pi} S_f^{(1)}(q_\perp) \right) + \frac{\alpha_s(q_\perp)}{2\pi} V^{(FJ)} \right] + \int d\Phi_{\text{rad}} \frac{\alpha_s(q_\perp)}{2\pi} \frac{\Delta_f^2(Q)}{\Delta_f^2(q_\perp)} R^{(FJ)} \right. 
\]
\[ + \left. \frac{\Delta_f^2(Q)}{\Delta_f^2(q_\perp)} D^{(\geq 3)}(\Phi_F, q_\perp) F_{\ell}^{\text{corr}}(\Phi_{FJ}) \right\} \]

\textbf{New term derived from} \( q_\perp \) resum., contains all terms required to achieve NNLO according to our power counting (\( \alpha_s^3 \text{ corr.}^{\text{ns}} \) needed)

\[ D^{(\geq 3)}(\Phi_F, q_\perp) = \underbrace{-\frac{dS_f(q_\perp)}{dq_\perp} L_f(q_\perp) + \frac{dL_f}{dq_\perp} - \frac{\alpha_s(q_\perp)}{2\pi} [D(q_\perp)]^{(1)}}_{D(q_\perp)} - \frac{\alpha_s^2(q_\perp)}{(2\pi)^2} [D(q_\perp)]^{(2)}} \]

\textbf{Spreading of new corr.\textsuperscript{n} across} \( \Phi_{FJ} \) \textbf{at fixed} \( \Phi_F \) (FKS)

- Fully differential NNLO upon integration over \( q_\perp \)
- Marginal loss in speed w.r.t. NLO calculation
- Possible to tackle complex processes

[PM, Nason, Re, Wiesemann, Zanderighi (2019)]
[PM, Re, Wiesemann (2020)]
MiNNLO\textsubscript{PS} for colour singlet prod.\textsuperscript{n}

\[ \text{do/dp}_{\ell\ell\gamma} \text{ [fb/GeV]} \]

$pp \to \ell^+\ell^-\gamma@LHC \text{ 13 TeV}$

\[ \text{do/dz\textsubscript{bin}} \text{ [pb]} \]

\[ \text{MINNLO\textsubscript{PS}} \]
\[ \text{NNLO (MATRIX)} \]

\[ \text{do/d\textsubscript{dMINNLO\textsubscript{PS}}} \]

\[ \text{do/d\textsubscript{dMINNLO\textsubscript{PS}} (PY8)} \]

\[ \text{do/d\textsubscript{dMINNLO\textsubscript{PS}} (PY8)} \]

$pp \to \ell^+\ell^-\nu@LHC \text{ 13 TeV}$

\[ \text{d\sigma/dm_{\text{\ell\ell\gamma}}} \text{ [fb/GeV]} \]

\[ \text{MINNLO\textsubscript{PS} (PY8)} \]
\[ \text{MINNLO\textsubscript{PS} (PY8)} \]
\[ \text{NNLO\textsubscript{PS} (PY8)} \]

\[ \text{min\textsubscript{\ell\ell\gamma}} \text{ [GeV]} \]

\[ \text{do/d\textsubscript{dMINNLO\textsubscript{PS}}} \]

\[ \text{do/d\textsubscript{dMINNLO\textsubscript{PS}}} \]

\[ \text{pp \to \ell^+\ell^-\nu@LHC \text{ 13 TeV}} \]

[PM, Nason, Re, Wiesemann, Zanderighi (2019)]
[PM, Re, Wiesemann (2020)]
[Lombardi, Wiesemann, Zanderighi (2020-2021)]
[Buonocore, Koole, Lombardi, Rottoli, Wiesemann, Zanderighi (2021)]

[...]
Colour charges in the final states: top pair production

Reminder:

$$\bar{B}^{(FJ)}_{\text{MiNNLO}_{PS}} \sim \frac{\Delta^2_f(Q)}{\Delta^2_f(q_{\perp})} \ldots$$

Squared $\Rightarrow$ 2 radiating legs.

Doesn’t account for radiation off tops, notably initial-final & final-final soft interference
Colour charges in the final states: top pair production

\[
\frac{d\sigma}{d\vec{q}_\perp d\Phi_F} \sim \sum_f |M_{f\bar{f} \rightarrow F}^{(0)}|^2 \int \frac{d^2\vec{b}}{(2\pi)^2} e^{i\vec{b} \cdot \vec{q}_\perp} e^{-R_f(b)} H_f \sum_{i,j} (C_{fi} \otimes h^{[i]})(C_{\bar{f}j} \otimes h^{[j]})
\]

\[
\frac{d\sigma}{d\vec{q}_\perp d\Phi_F} \sim \sum_f |M_{f\bar{f} \rightarrow t\bar{t}}^{(0)}|^2 \int \frac{d^2\vec{b}}{(2\pi)^2} e^{i\vec{b} \cdot \vec{q}_\perp} e^{-R_f(b)} \text{Tr} (H_f \Delta_{\text{soft}}) \sum_{i,j} (C_{fi} \otimes h^{[i]})(C_{\bar{f}j} \otimes h^{[j]})
\]

\[
\text{Tr} (H_f \Delta_{\text{soft}}) = \frac{\langle M_{f\bar{f}} | \Delta | M_{f\bar{f}} \rangle}{|M_{f\bar{f}}^{(0)}|^2}, \quad \Delta = V^\dagger DV
\]

\[
V = \mathcal{P} \exp \left\{ -\int_{b_0^2}^{M_{t\bar{t}}^2} \frac{dq^2}{q^2} \Gamma_t(\Phi_{t\bar{t}}, \alpha_s(q)) \right\}
\]

V and D encode soft interference up to two loops
[new sources of colour & azimuthal correlations in \(\Delta CC\)]

[Zhu, Li, Li, Shao, Yang (2013)]
[Catani, Grazzini, Torre (2014)]
Colour charges in the final states: top pair production

- With LL and NNLO accuracy, the azimuthally averaged distribution takes a simpler form

\[
\frac{d\sigma}{dq\perp d\Phi_F} \phi \sim \frac{d}{dq\perp} \left[ \sum_f e^{-S_f(q\perp)} \langle M_{ff}^{(0)} | (V_{NLL})^\dagger V_{NLL} | M_{ff}^{(0)} \rangle [\text{Tr} (H_f D_{\text{soft}}) \sum_{i,j} (C_{fi} \otimes h^{[i]})(C_{fj} \otimes h^{[j]})] \phi + O(\alpha_s^5(q\perp)) \right]
\]

\[
S_f(q\perp) = \int_{q\perp}^{Q^2} \frac{dq^2}{q^2} \left( A(\alpha_s(q)) \ln \frac{Q^2}{q^2} + B(\alpha_s(q)) \right)
\]

Ingredients used in slicing NNLO calculations and derived in:
[Baernreuther, Czakon, Fiedler (2013)] [Czakon (2008)]
[Catani, Grazzini, Torre (2014)]
[Catani, Grazzini, Sargsyan (2018)]
[Catani, Devoto, Grazzini, Kallweit, Mazzitelli (2019) + Sargsyan (2019)]

[Amoroso, PM, Nason, Re, Wiesemann, Zanderighi (2020)]

**Soft interference pattern is split into 3 contributions that can be matched to the MiNNLOPS weight**

\[
A(\alpha_s) = \frac{\alpha_s}{2\pi} A^{(1)} + \frac{\alpha_s^2}{(2\pi)^2} A^{(2)}
\]

\[
B(\alpha_s) = \frac{\alpha_s}{2\pi} B^{(1)} + \frac{\alpha_s^2}{(2\pi)^2} B^{(2)}
\]

⇒ Upon diagonalisation of \(V_{NLL}\) this result can be recast as a sum of colour–single-like factorisation theorems with complex exponentials.
Numerical results at LHC

- Scales setup:
  - 2 Born powers of the coupling @ \( \mu_R = K_R \mu_0 \)
  - Everywhere else (\( Q = \frac{m_{tt}}{2} \)): \( \mu_R = K_R \bar{\mu} \ e^{-L}, \mu_F = K_F \bar{\mu} \ e^{-L} \)
  - Vary scales by a factor of 2 (7 points)
  - Smooth freezing of PDFs at \( Q_0 = 2 \text{ GeV} \)
  - Pythia8 with stable top quarks [decays included later]

\( L = \begin{cases} \ln \frac{Q}{q} & \text{for } q_\perp \lesssim \frac{Q}{2} \\ 0 & \text{for } q_\perp \geq Q \end{cases} \)
smooth interpolation to zero in \([Q/2, Q]\)

\( \Rightarrow \text{MiNLO vs. MiNNLO}_\text{PS vs. NNLO (MATRIX with } \mu_R = K_R \bar{\mu} \text{ & } \mu_F = K_F \bar{\mu} \)\)*

\[ \text{Sub-percent agreement with NNLO} - \text{more sources of scale uncertainty (e.g. Sudakov) in MiNNLO}_\text{PS} \]

| Total cross sections for \( \bar{\mu} = \mu_0 = \frac{m_{tt}}{2} \) |
|-----------------|-----------------|-----------------|
| M1NLO'          | NNLO            | MiNNLO\text{PS} |
| 695.6(3)\(+22\)% pb | 769.8(9)\(+5.0\)% pb | 775.5(2)\(+9.8\)% pb |

| Total cross sections for \( \bar{\mu} = \mu_0 = m_{tt} \) |
|-----------------|-----------------|-----------------|
| M1NLO'          | NNLO            | MiNNLO\text{PS} |
| 572.9(2)\(+21\)% pb | 719.1(8)\(+7.0\)% pb | 719.8(2)\(+7.6\)% pb |

* Results in published version (arXiv not yet updated)
Top-pair rapidity

Excellent agreement with NNLO for inclusive distributions, slightly larger uncertainty in MiNNLO\textsubscript{PS} for certain central scales.

CMS data [arXiv:1803.08856]

NNLO calculation in: [Baernreuther, Czakon, Mitov (2012); Czakon, Fiedler, Mitov (2013); Czakon, Heymes, Mitov (2015); Behring, Czakon, Mitov, Papanastasiou, Poncelet (2019); Czakon, Mitov, Poncelet (2020), …]

\[ \bar{\mu} = \mu_0 = \frac{m_{tt}}{2} \]

NB: scale settings not equivalent to NNLO!
Observables involving jets

Obs. which resolve QCD radiation expected to be NLO accurate, good agreement with MiNLO for sufficiently resolved jets
Perturbative hard scales

Dynamical hard ($\mu_0$) scales different from $m_{tt}$ preferable when comparing to experimental data. $H_T^{tt}$, $H_T^{tt+jets}$ scales now available, both for $\bar{\mu}$ and $\mu_0$ (overall coupling)

E.g. sensible scales across topologies

[Caola, Dreyer, McDonald, Salam (2020)]
Top decays

- Inclusion of top decays paramount for realistic experimental cuts. Full NLO (off-shell+spin corr. and non-resonant channels) available @ NLO+PS
  
  e.g. bb4l [Jezo, Lindert, Nason, Oleari, Pozzorini (2016)]

- Possible avenue: inclusion of N(N)LO decays in NWA, significant work is required to retain spin correlations (as well as matching)

- Alternative: Generate decays at LO in NWA according to (only double-resonant diagrams)

\[
dP(\Phi_{\text{dec}}|\Phi_{\text{undec.}}) = \frac{1}{\text{BR}(t \to b\bar{v}) \text{BR}(\bar{t} \to \bar{b}\bar{v})} \frac{M_{\text{dec.}}(\Phi_{\text{undec.}}, \Phi_{t \to b\bar{v}}, \Phi_{\bar{t} \to \bar{b}\bar{v}})}{M_{\text{undec.}}(\Phi_{\text{undec.}})} d\Phi_{t \to b\bar{v}} d\Phi_{\bar{t} \to \bar{b}\bar{v}}
\]

[Alioli, Moch, Uwer (2011)]

Impact of radiative corrections on spin correlations expected to be moderate

[Behring, Czakon, Mitov, Papanastasiou, Poncelet (2019)]
Top decays: leptonic channel

$$\mu_0 = H_T^{tt}/4, \text{ excl. } \tau, 120 < m_{e\mu}/\text{GeV} < 200$$

ATLAS data [arXiv:1910.08819]

$$\bar{\mu} = m_{tt}, \mu_0 = H_T^{tt}/4$$

Many more observables under study…

$$\mu_0 = H_T^{tt}/4, \text{ excl. } \tau, 80 < m_{e\mu}/\text{GeV} < 120$$

Leptons azimuthal separation (sensitive to spin correlations)
Top decays: semi-leptonic channel

\[ \mu_0 = \frac{H_T^{tt}}{4}, \text{ particle level} \]

CMS data [arXiv:1803.08856]

\[ \bar{\mu} = m_{tt}, \mu_0 = \frac{H_T^{tt}}{4} \]

\[ \mu_0 = \frac{H_T^{tt}}{4}, \text{ particle level} \]

[much more observables under study…]
Summary & Outlook

- Matching of NNLO to PS for $t\bar{t}$ production (+LO decay): first NNLO event generator for a reaction with colour charges in the final state

- Very good description of data - ongoing phenomenological studies across multiple observables, scale settings, etc.

- Avenue towards NNLO+PS for jet processes, many challenges ahead:
  - Matching with higher order corrections to decays?
  - Resolution variables for light-jet processes [mind the PS accuracy]?
  - Matching to higher-order (e.g. NLL) showers? Several aspects relevant to log accuracy [resolution variable vs. PS ordering, kinematic maps & constraint on the shower, …]
  - NLL showers for heavy quarks