Recent developments in GENEVA

Diboson processes at NNLOPS and p_T resummation

Matthew Lim

Taming the accuracy of event generators, 24 August 2021

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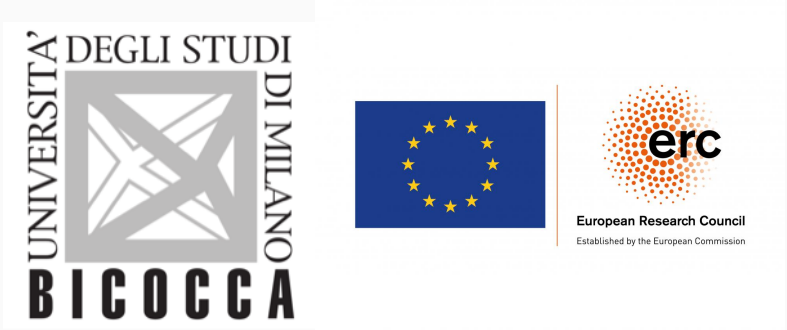
arXiv:2010.10498, 2103.01214, 2105.13214, 2102.08390



Motivation

- Much progress in matching NNLO QCD calculations recently – three main methods available: UNNLOPS, MiNNLO$_{\mathrm{PS}}$, GENEVA
- Initial applications were for $2 \to 1$ processes e.g. Drell-Yan, Higgs in gluon fusion
- Recently - more complex processes available, such as diphoton, ZZ, $t\bar{t}$
- These can improve the description of data significantly and also be vital tools for experimentalists.

Overview of talk

- Introduction to GENEVA as a MC event generator
- Recent applications to diboson processes
- Changing resolution variables and GENEVA p_T with RadISH

The GENEVA method

The GENEVA method

The Three Jewels:

- GENEVA produces fully differential fixed order calculations at NNLO;
- By resumming large logarithms at NNLL′, it provides precise predictions over the whole phase space;
- These are matched to a parton shower to produce realistic events (which can further be hadronised, MPI effects included).

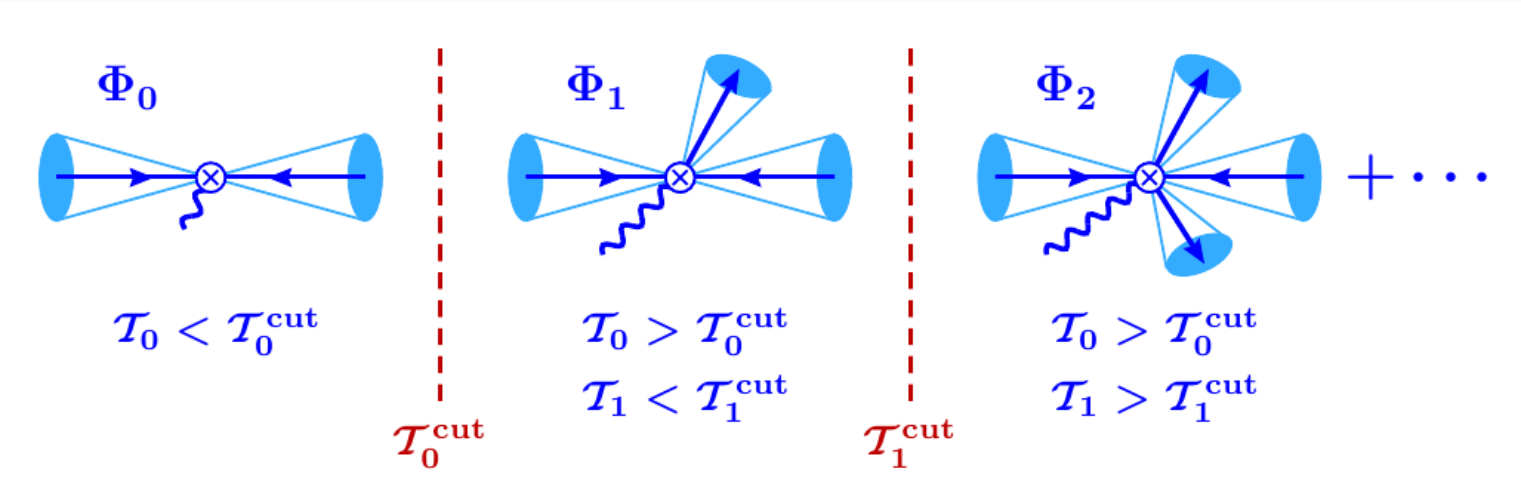
The method is fully general.

Constructing IR-finite events

Work with IR-finite events to which a finite cross section can be assigned:

- Introduce a resolution parameter $\mathcal{T}_N$, $\mathcal{T}_N \to 0$ in the IR region. Emissions below $\mathcal{T}_N^{\text{cut}}$ are unresolved (i.e. integrated over) and the kinematic configuration considered is the one of the event before the emission.
- An M-parton event is thus translated to an N-jet event, $N \leq M$, fully differential in Φ_N (no jet-algorithm needed).
  - Price to pay: power corrections in $\mathcal{T}_N^{\text{cut}}$ due to PS projection.
  - Advantage: vanish for IR-safe observables as $\mathcal{T}_N^{\text{cut}} \to 0$
- Iterating the procedure, the phase space is sliced into jet-bins.

Constructing IR-finite events



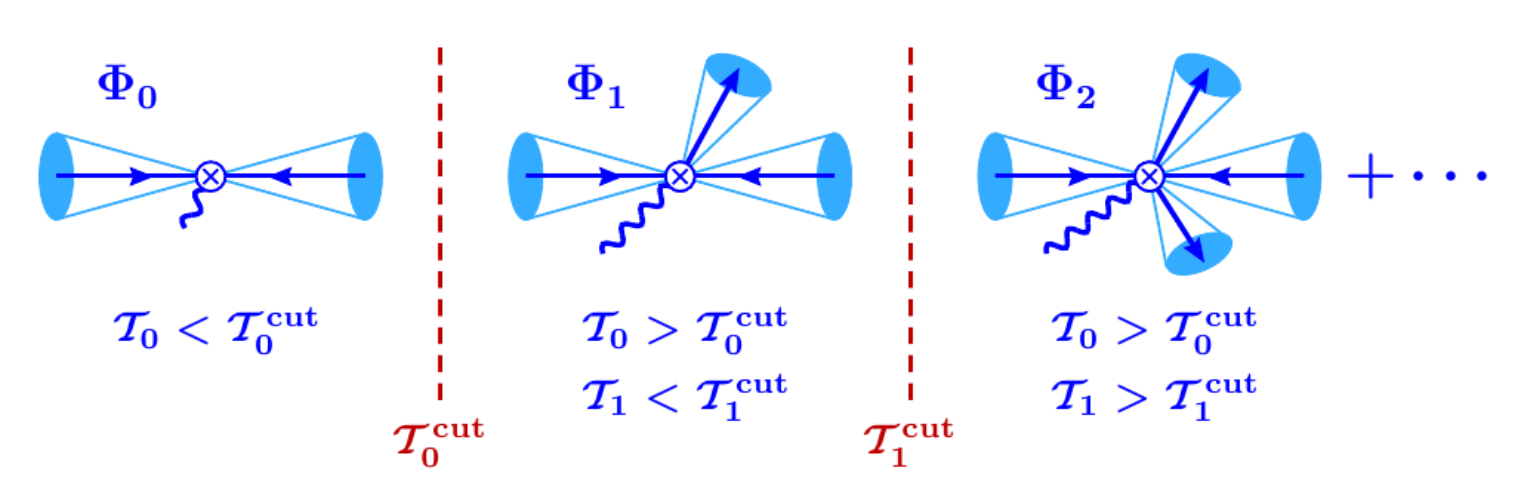
Exclusive N-jet bin

$$\frac{\mathrm{d}\sigma_N^{\mathrm{MC}}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}})$$

Inclusive $(N+1)$-jet bin

$$\frac{\mathrm{d}\sigma_{\geq N+1}^{\mathrm{MC}}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}})$$

Constructing IR-finite events



Excl. N-jet bin

$$\frac{\mathrm{d}\sigma_N^{\text{MC}}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\text{cut}})$$

Excl. $(N+1)$-jet

$$\frac{\mathrm{d}\sigma_{N+1}^{\text{MC}}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\text{cut}};\ \mathcal{T}_{N+1}^{\text{cut}})$$

Inclusive $(N+2)$-jet bin

$$\frac{\mathrm{d}\sigma_{\geq N+2}^{\text{MC}}}{\mathrm{d}\Phi_{N+2}}(\mathcal{T}_N > \mathcal{T}_N^{\text{cut}},\ \mathcal{T}_{N+1} > \mathcal{T}_{N+1}^{\text{cut}})$$

Combining resummed and fixed order calculations

Consider colour singlet production at NNLO. We need events with 0, 1 and 2 additional QCD partons in the final state.
Exclusive 0-jet cross section:

$$\frac{\mathrm{d}\sigma_0^{\mathrm{MC}}}{\mathrm{d}\Phi_0}(\mathcal{T}_0^{\mathrm{cut}}) = \frac{\mathrm{d}\sigma_0^{NNLL'}}{\mathrm{d}\Phi_0}(\mathcal{T}_0^{\mathrm{cut}}) + \overbrace{\frac{\mathrm{d}\sigma_0^{sing\ match}}{\mathrm{d}\Phi_0}(\mathcal{T}_0^{\mathrm{cut}})}^{=0} + \frac{\mathrm{d}\sigma_0^{nons}}{\mathrm{d}\Phi_0}(\mathcal{T}_0^{\mathrm{cut}})$$

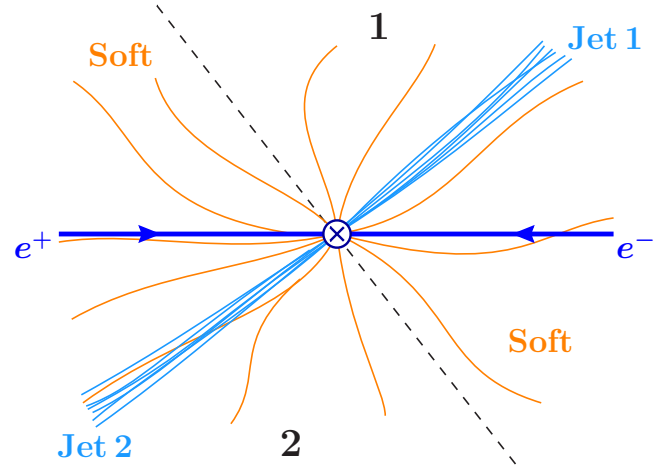
- At NNLL′, all singular terms included to $\mathcal{O}(\alpha_s^2)$ by definition – singular matching term vanishes.
- Nonsingular matching term determined by requirement of FO NNLO accuracy:

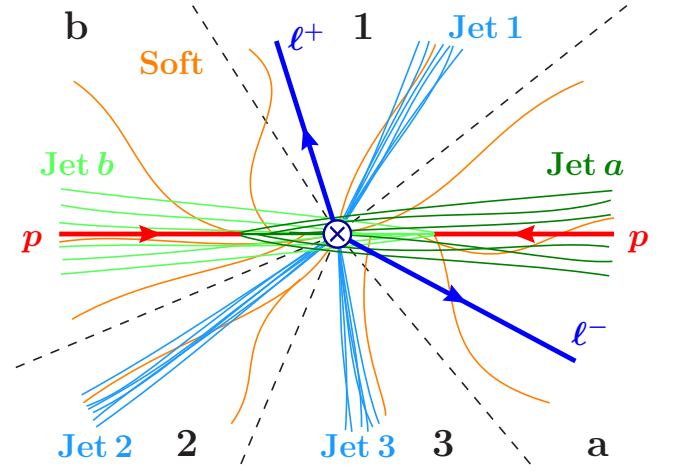
$$\frac{\mathrm{d}\sigma_0^{nons}}{\mathrm{d}\Phi_0}(\mathcal{T}_0^{\mathrm{cut}}) = \frac{\mathrm{d}\sigma_0^{NNLO_0}}{\mathrm{d}\Phi_0}(\mathcal{T}_0^{\mathrm{cut}}) - \left[\frac{\mathrm{d}\sigma^{NNLL'}}{\mathrm{d}\Phi_0}(\mathcal{T}_0^{\mathrm{cut}})\right]_{NNLO_0}$$

Choice of the jet resolution variable

- We use N-jettiness as resolution parameter. Global physical observable with straightforward definitions for hadronic colliders, in terms of beams $q_{a,b}$ and jet-directions q_j

$$\mathcal{T}_N = \frac{2}{Q} \sum_k \min\{q_a \cdot p_k,\ q_b \cdot p_k, q_1 \cdot p_k,\ \ldots,\ q_N \cdot p_k\}$$





- N-jettiness has good factorisation properties, IR safe and resummable at all orders. Resummation known at NNLL for any N in Soft-Collinear Effective Theory
- $\mathcal{T}_N \to 0$ for N pencil-like jets, $\mathcal{T}_N \gg 0$ spherical limit.

NNLL′ resummation from SCET

The spectrum in $\mathcal{T}_0$ can be factorised at all-orders as

$$\frac{\mathrm{d}\sigma^{NNLL'}}{\mathrm{d}\Phi_0 \mathrm{d}\mathcal{T}_0}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}) = \sum_{ij} \frac{\mathrm{d}\sigma_{ij}^B}{\mathrm{d}\Phi_0} H_{ij}(Q^2, \mu_H)\, U_H(\mu_H, \mu) \int \mathrm{d}t_a \mathrm{d}t_b$$
$$\times \left[B_i(t_a, x_a, \mu_B) \otimes U_B(\mu_B, \mu)\right]$$
$$\times \left[B_j(t_b, x_b, \mu_B) \otimes U_B(\mu_B, \mu)\right]$$
$$\otimes \left[S(\mathcal{T}_0 - \frac{t_a + t_b}{Q}, \mu_S) \otimes U_S(\mu_S, \mu)\right],$$

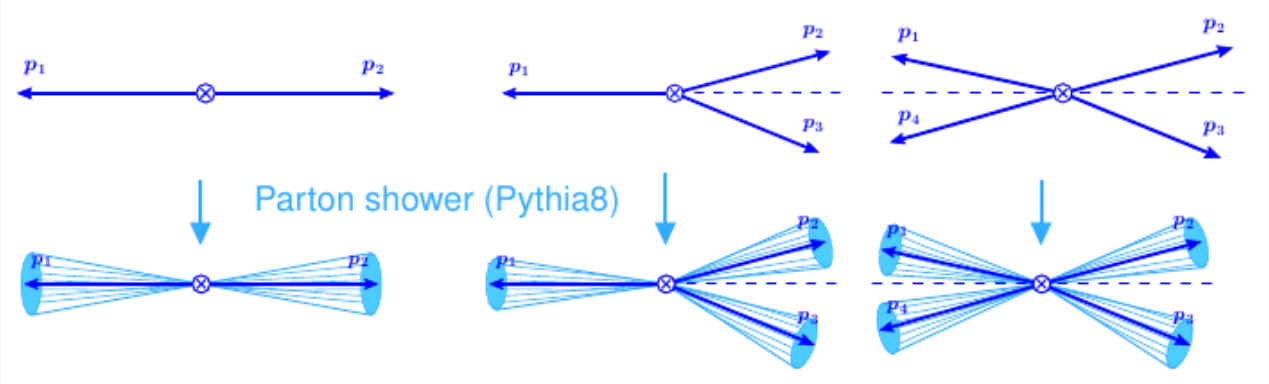
- Hard, Beam and Soft functions are each evaluated at their own scale $\Rightarrow$ no large logarithms,

$$\mu_H = Q, \quad \mu_B = \sqrt{Q\mathcal{T}_0}, \quad \mu_S = \mathcal{T}_0$$

- RGE kernels U_X evolve functions to a common scale μ and in so doing resum logarithms.

Matching to a parton shower

- Parton shower makes calculation differential in higher multiplicities by adding radiation.
- Fills the 0- and 1-jet bins with radiation, adds more to the inclusive 2-jet bin.



- Not allowed to affect the accuracy of the cross section reached at partonic level.
- $\mathcal{T}_i^{\mathrm{cut}}$ constraints must be respected.

Applications to diboson processes

Diphoton production

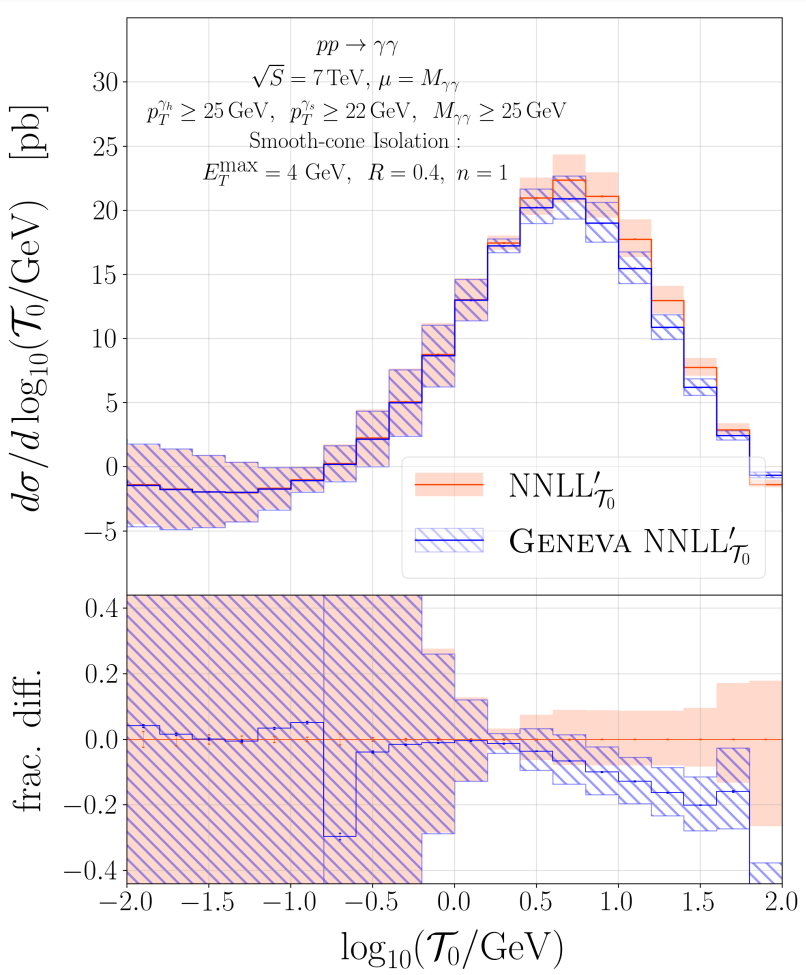
- Diphoton production is an important process at a hadron collider.
- Conceptually, introduces a new problem – process is divergent at Born level, need to introduce isolation criteria to select prompt photons, prevent QED/QCD divergences with cuts.
- We use Frixione isolation for comparison with FO calculations, hybrid procedure to compare with data.
- Compared predictions to ATLAS and CMS data collected at $\sqrt{s} = 7$ TeV.

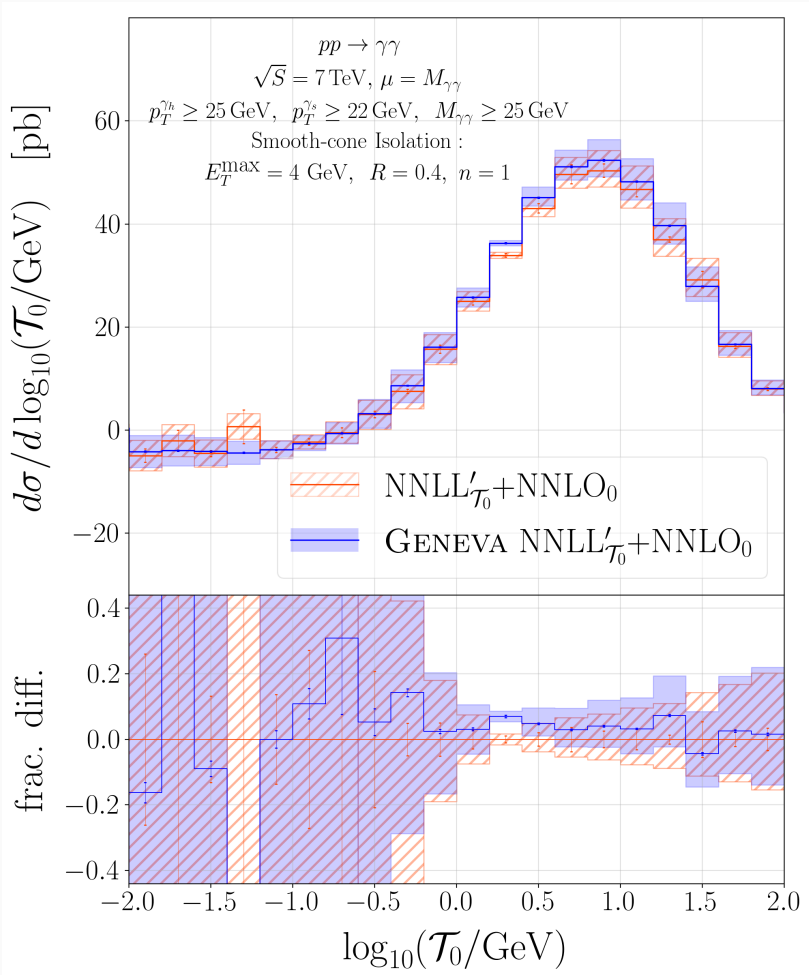
Comparing standard resummation with event generation

- Standard resummed calculations improve a single observable, and do not generally have information about full events (no recoil).
- Event generators take a full event and calculate the value of the observable on that event. This may involve a projection from a higher to a lower multiplicity PS point.
- These are identical in the limit $\mathcal{T}_0 \to 0$. Away from this limit, same result if we cut only on quantities preserved by $\Phi_1 \to \Phi_0$.
- Cuts on e.g. photon p_T are not preserved – contributions are removed in GENEVA compared to standard resummation.

Comparing standard resummation with event generation

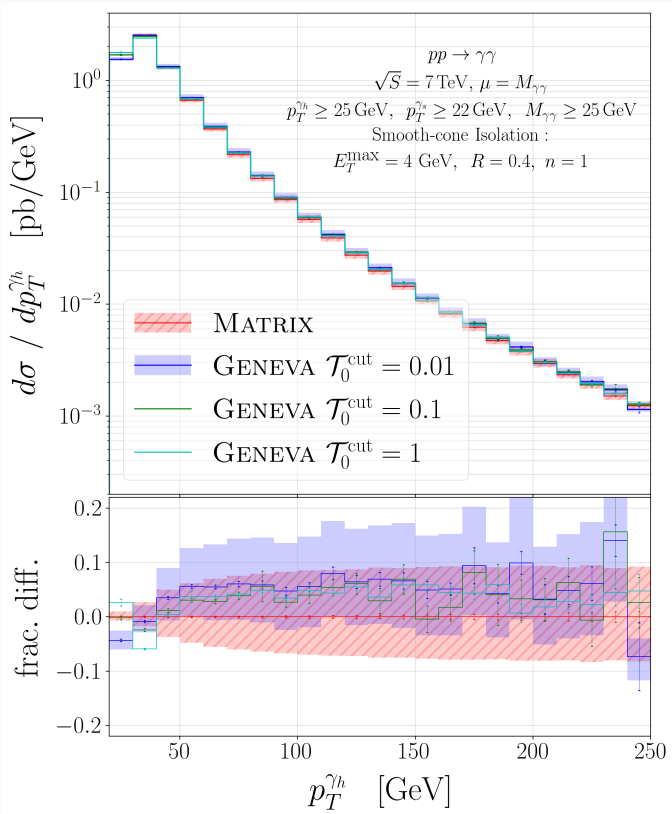
- Resummed only differs from standard resummation at high values – this is cured by matching to FO, where good agreement is found across $\mathcal{T}_0$ range.

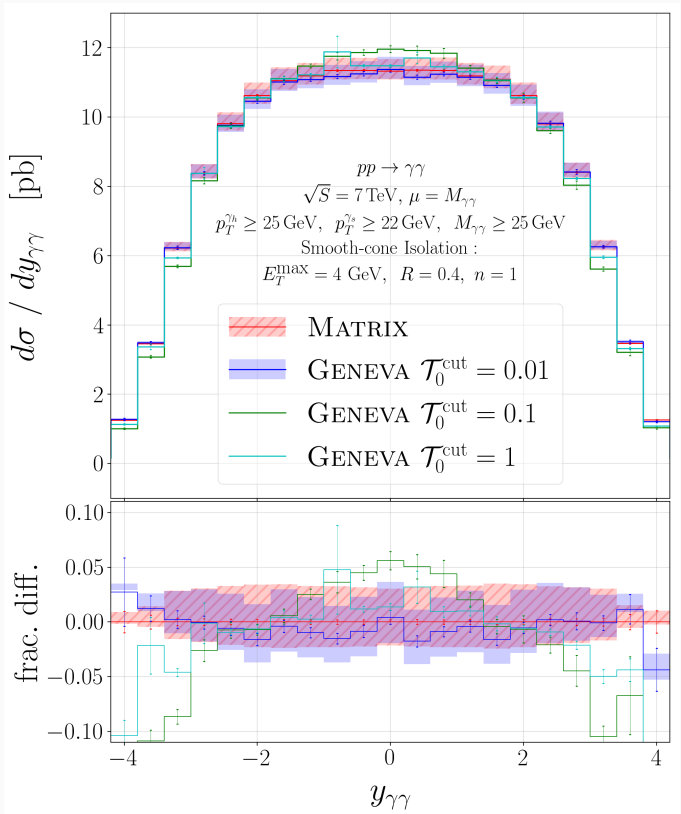




NNLO validation

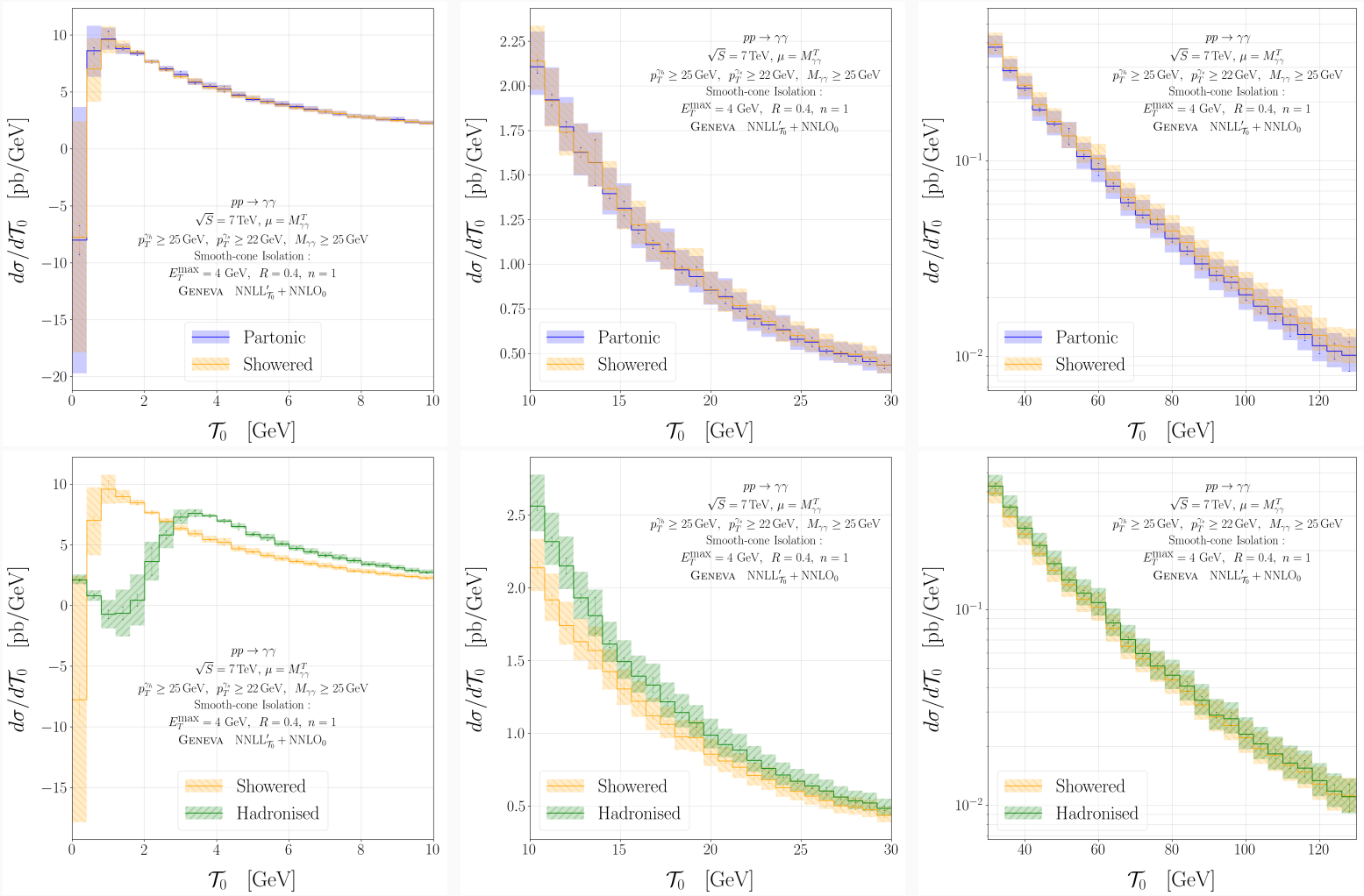
- Compared to NNLO from MATRIX, missing kinematic dependence of power corrections is reduced as $\mathcal{T}_0^{\text{cut}} \to 0$.
- Residual differences visible in e.g. p_T spectra due to fiducial power corrections treated correctly by differential subtraction and higher order terms included by resummation.





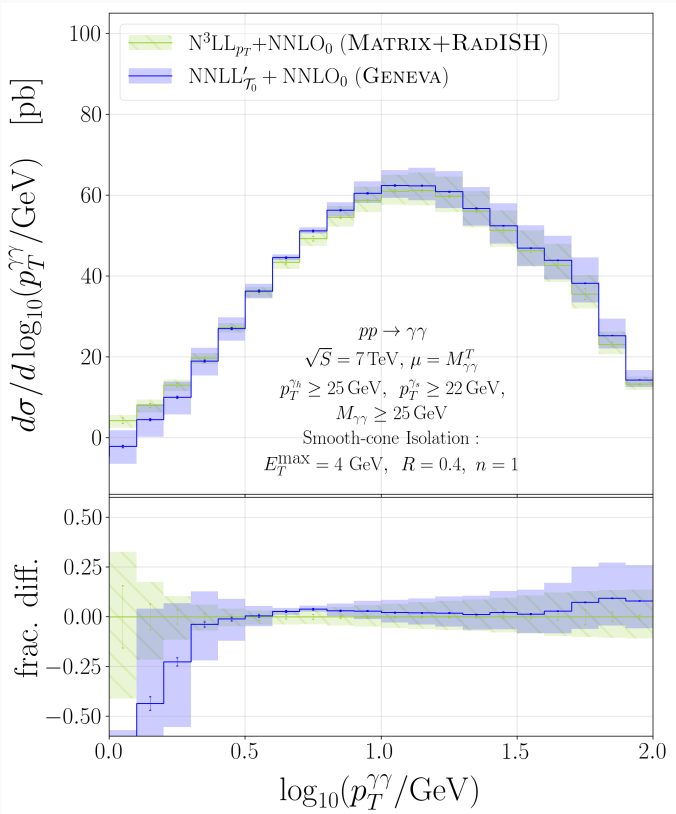
$\mathcal{T}_0$ spectrum

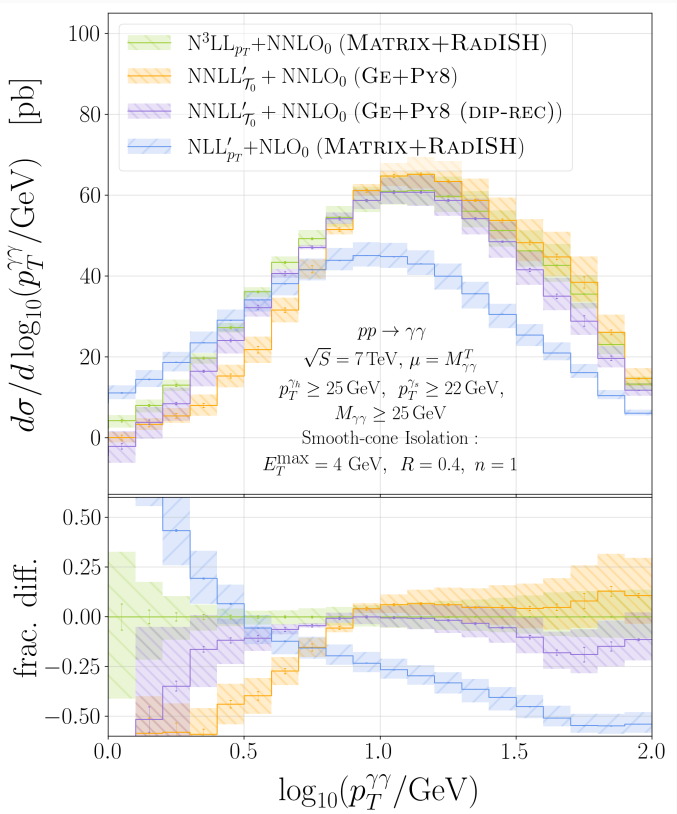
- Partonic result is NNLL′+NNLO accurate for $\mathcal{T}_0$ distribution.
- Accuracy is numerically well-preserved after showering.



Comparison with dedicated p_T resummation

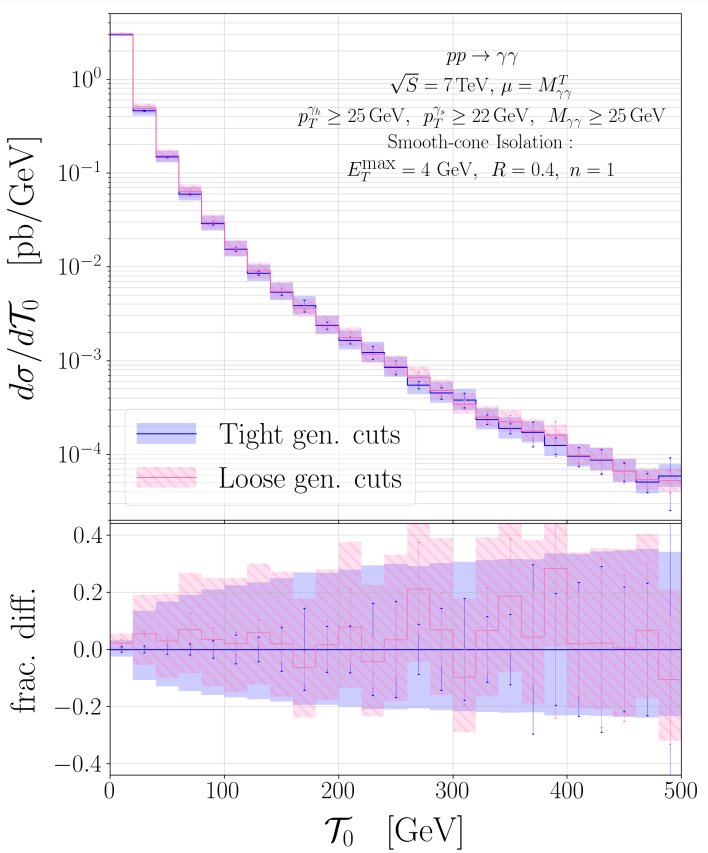
- Can compare predictions for $p_T^{\gamma\gamma}$ with resummed results matched to FO from MATRIX+RadISH.
- Partonic result replicates exact resummation down to small values. After showering, dipole recoil scheme preserves agreement well.

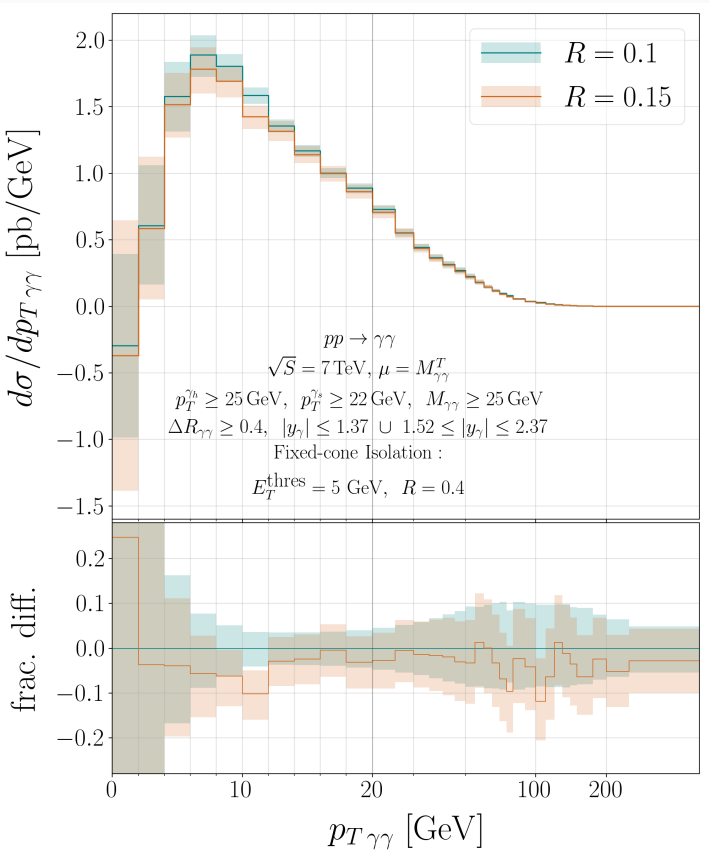




Isolation dependence

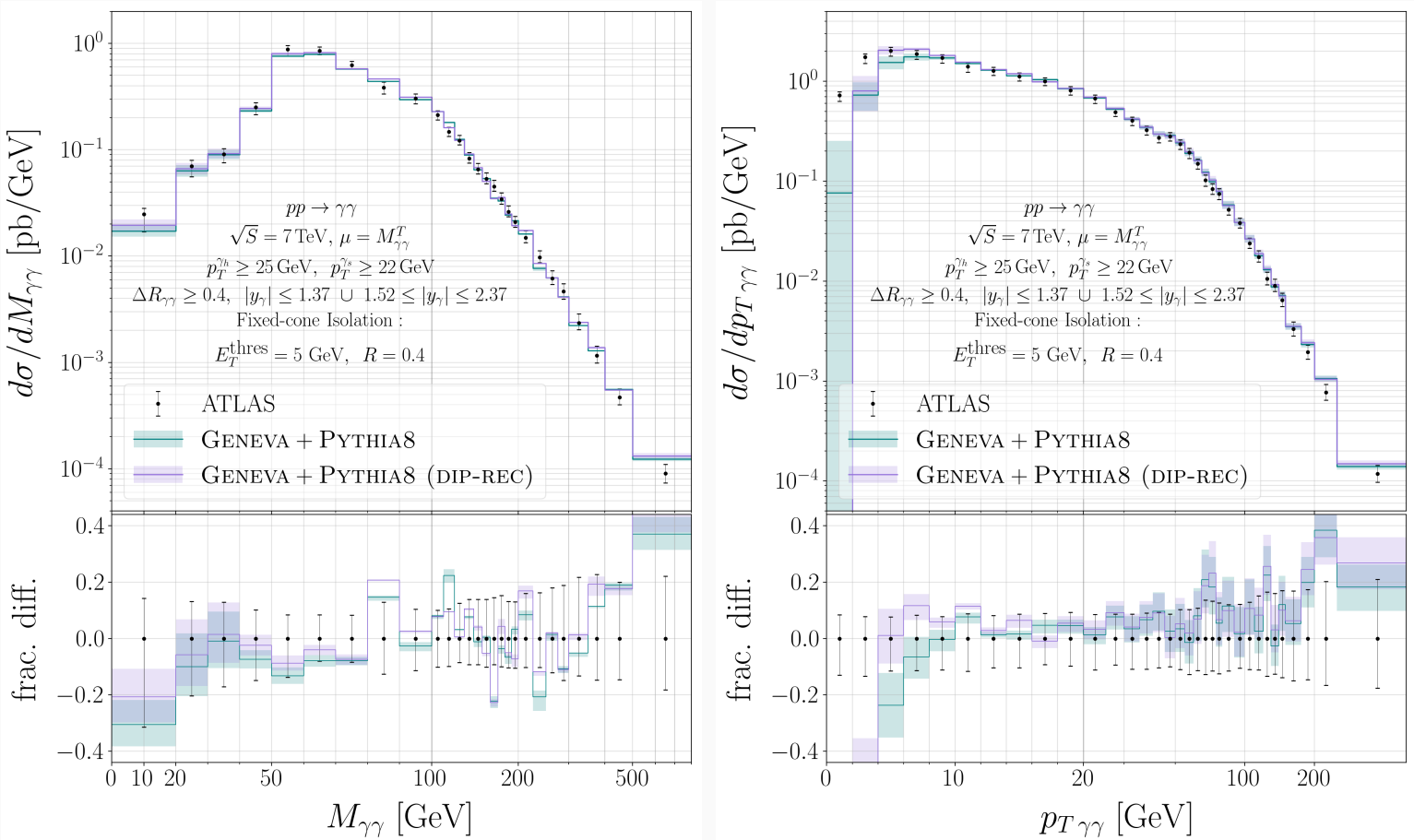
- Check dependence on generation cuts used – compare tight generation with loose generation and tight analysis cuts.
- Parton-level results are not strongly dependent on exact choice.
- Shower can reshuffle momenta and affect rate of events passing analysis cuts – reasonable agreement, but bigger effects here.





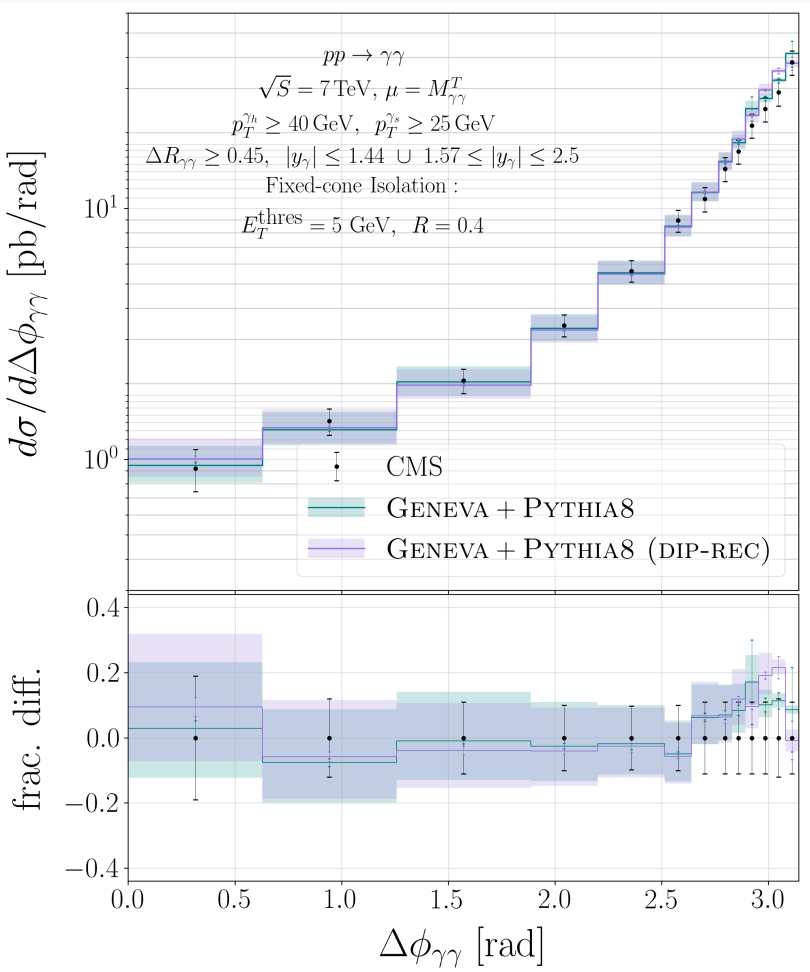
Diphoton production – ATLAS 7 TeV

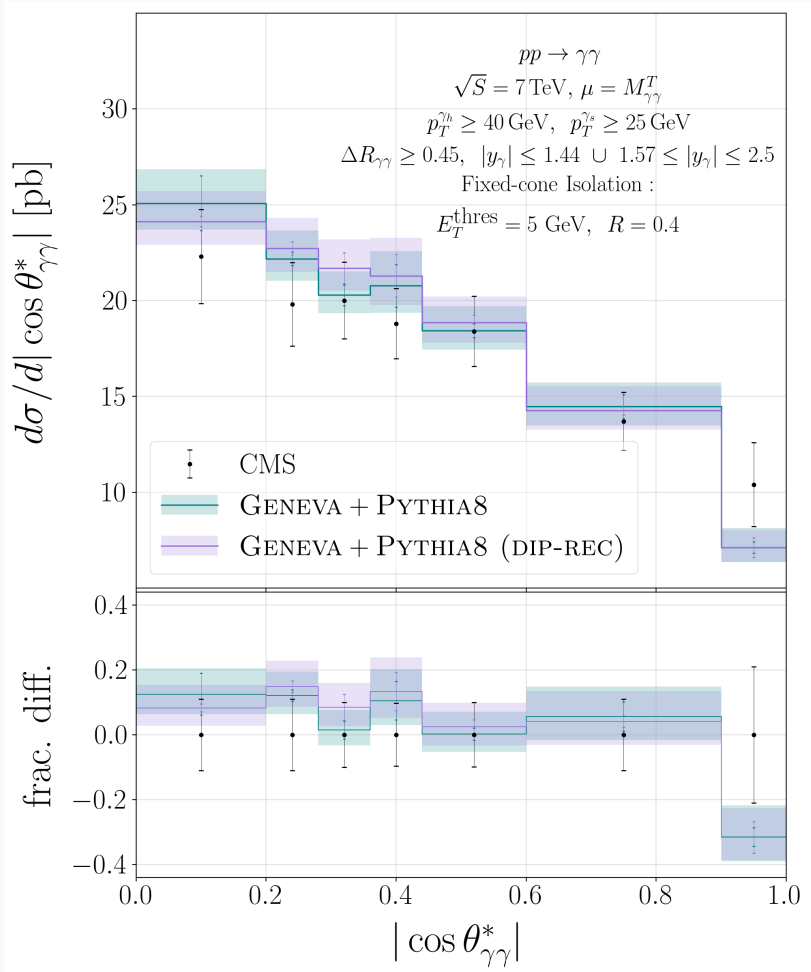
- Good agreement across most of range - worsens at low $p_T^{\gamma\gamma}$ where dedicated p_T resummation is important. Better with dipole recoil in shower
- EW effects important at high $M_{\gamma\gamma}$.



Diphoton production – CMS 7 TeV

- $\Delta\phi$ distribution in good agreement with CMS measurements across whole range.



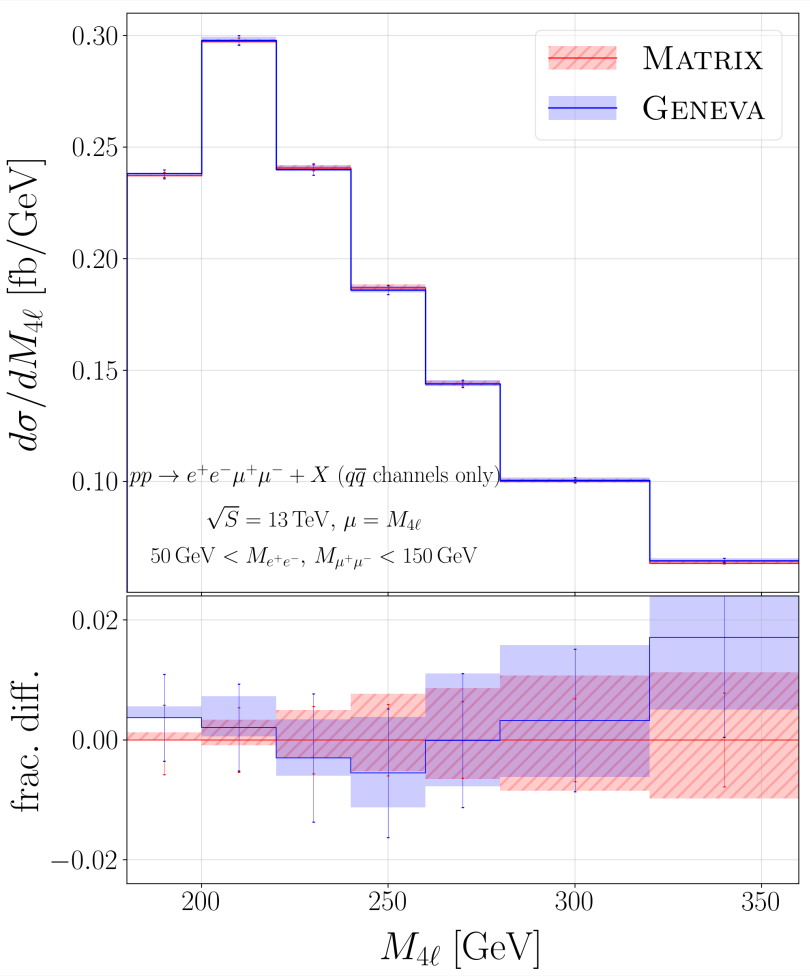


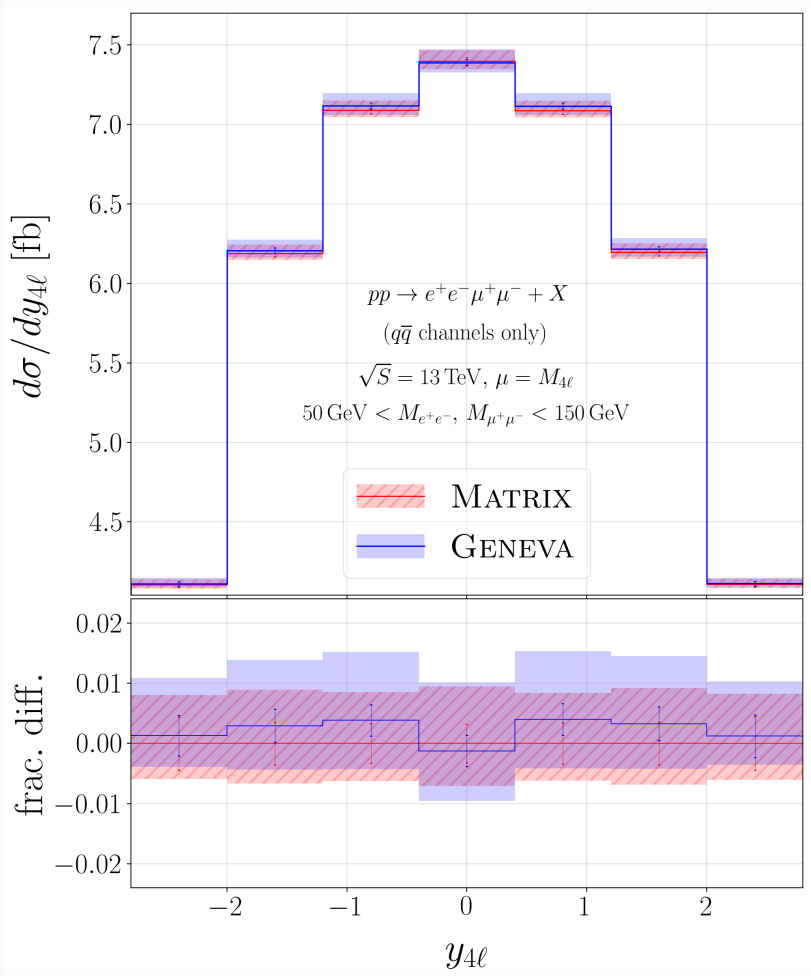
ZZ production

- Massive diboson processes are important probes of the non-Abelian EW couplings at the LHC; four lepton final states give very clean signatures in a QCD background.
- Simplifications with respect to diphoton in the sense that no isolation/process-defining cuts are needed.
- More complicated in the sense that many resonance structures contribute, phase space is of higher dimension.
- Phase space sampling provided by a tunnel between GENEVA and MUNICH, the multi-channel integrator (as used in e.g. MATRIX).
- We consider $pp \to \ell^+\ell^-\ell'^+\ell'^-$, i.e. ZZ production with decays to distinct flavours.

NNLO validation

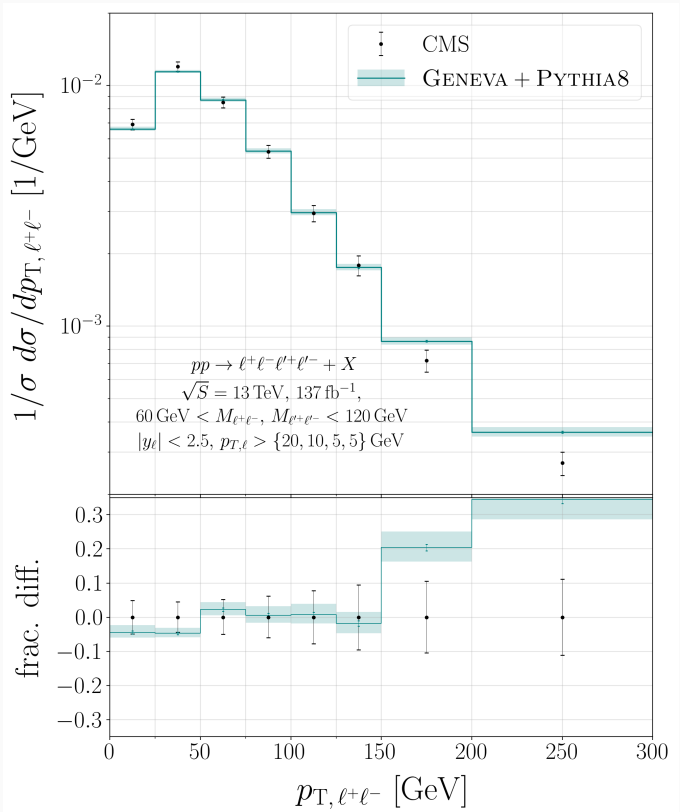
- Comparison with MATRIX only including $q\bar{q}$ channel (gg added at LO/relative NNLO later)

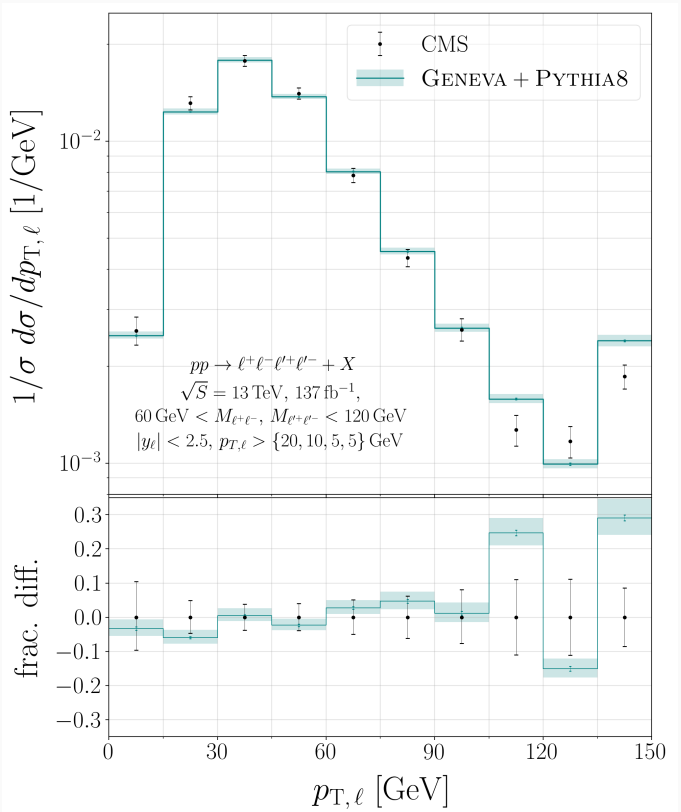




ZZ production – CMS 13 TeV

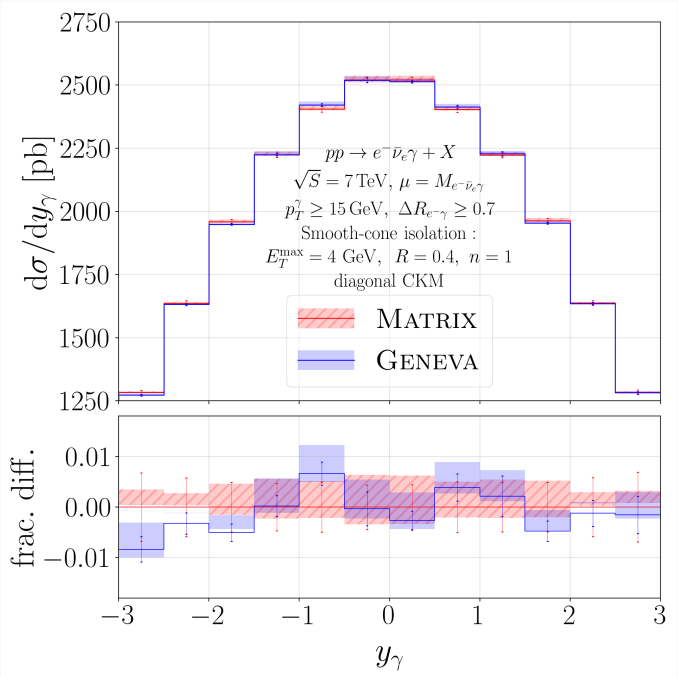
- Good agreement with data (last bins contain overflow)
- For $p_{T,\ell^+\ell^-} > 150$ GeV, EW effects are important.

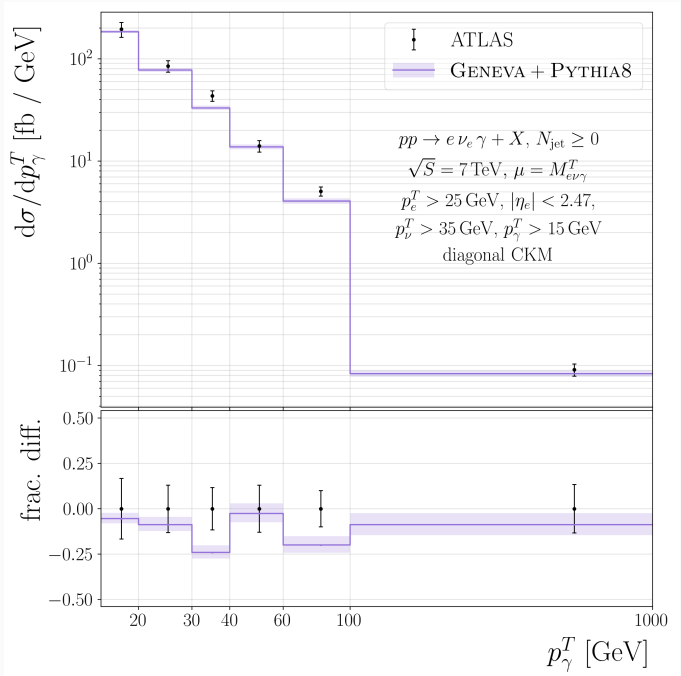




$W\gamma$ production

- Process has features of both ZZ (multiple resonance structures) and diphoton (process-defining cuts and photon isolation).
- NLO corrections artificially large because of radiation zero at LO – motivates inclusion of NNLO.
- Comparison of predictions with full CKM to 13 TeV LHC data underway.





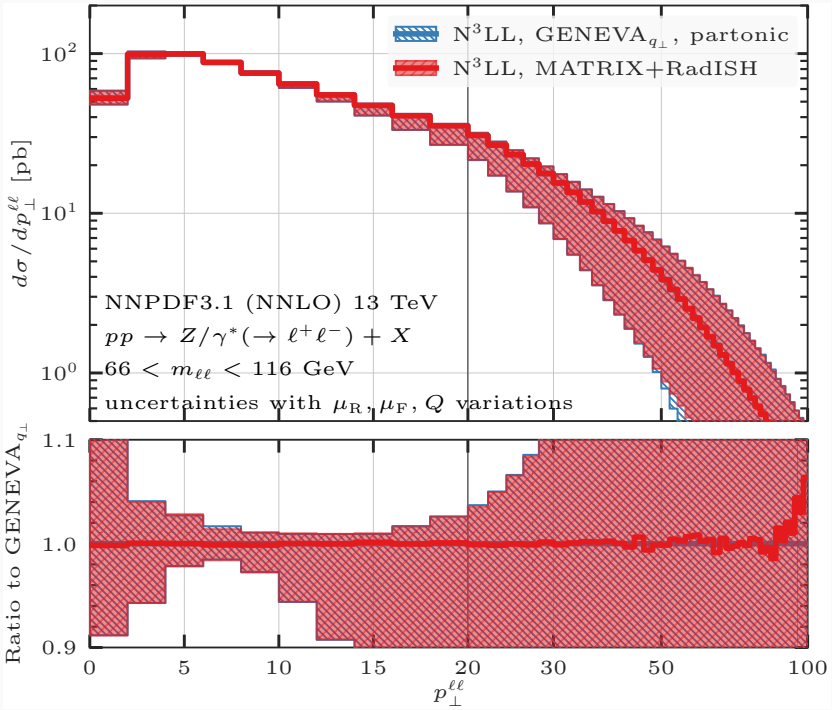
Changing the resolution variable and GENEVA-RadISH

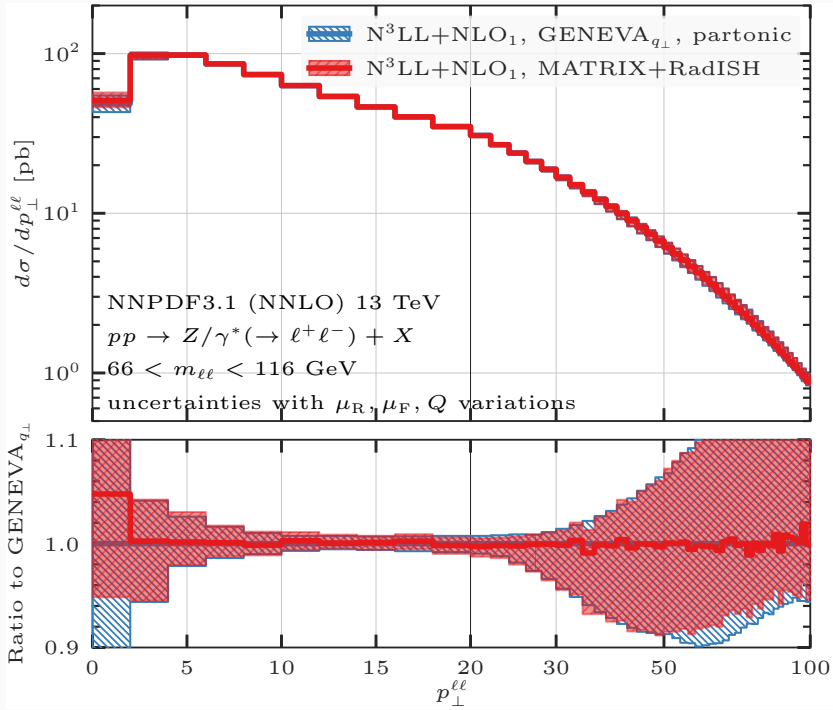
Changing the resolution variable

- The GENEVA approach is not specific to a particular resolution variable.
- In particular, as long as a suitable resummed calculation is available, any appropriate variable can be used.
- An obvious candidate is the q_T of the colour singlet system – in this case, we can obtain resummed predictions at N^3LL from RadISH.
- First application to Drell-Yan, but in principle other colour-singlet processes would be straightforward.

GENEVA+RadISH for Drell-Yan

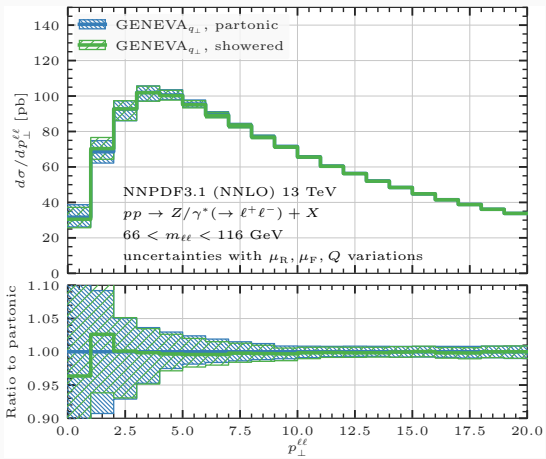
- Parton-level predictions agree with MATRIX+RadISH only, both resummed and matched.

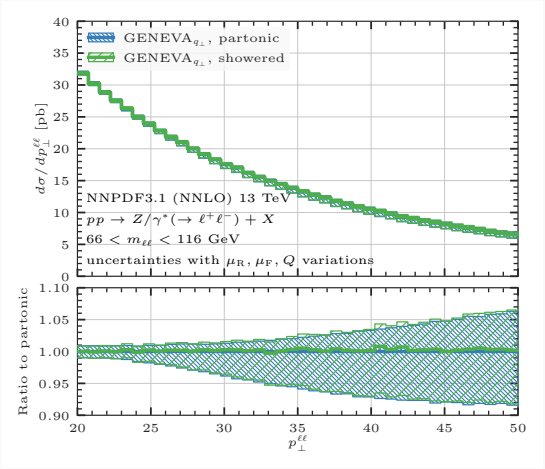


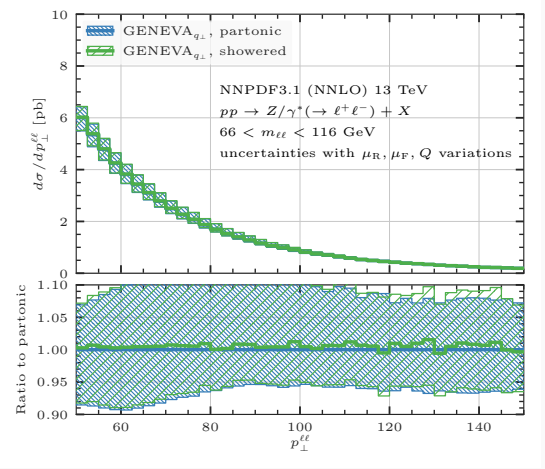


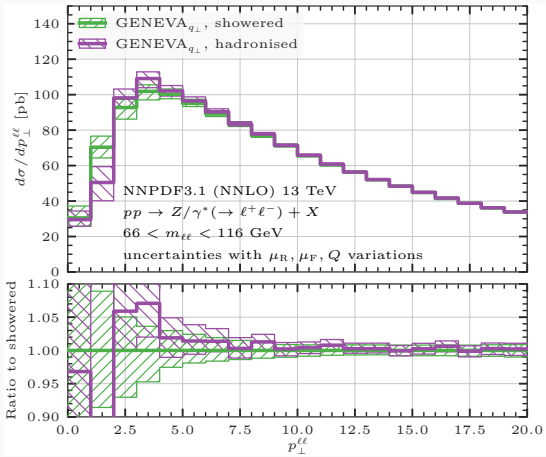
GENEVA+RadISH for Drell-Yan

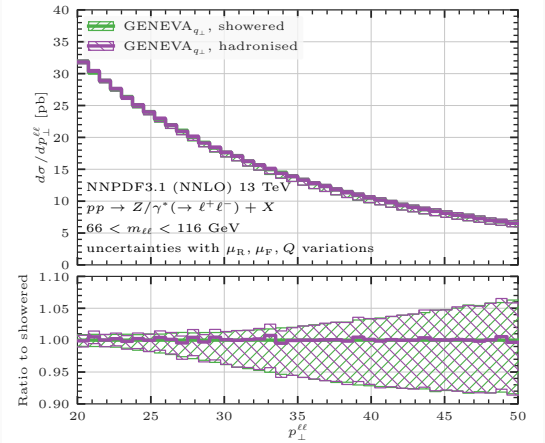
- Accuracy is numerically well-preserved even after showering.

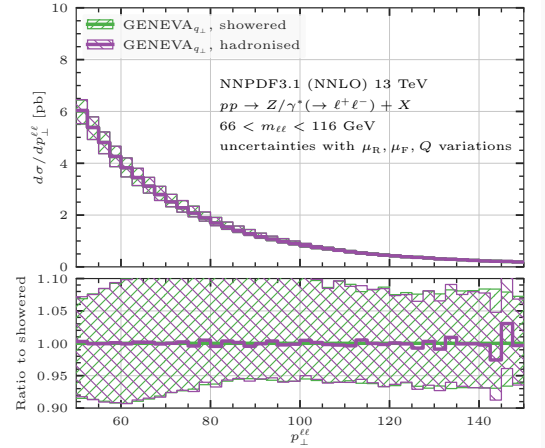






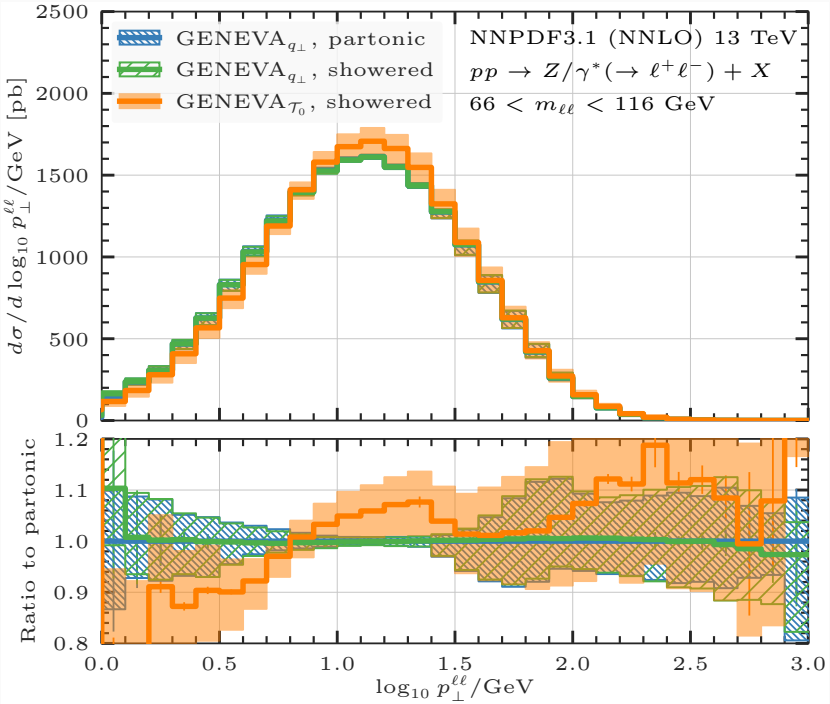


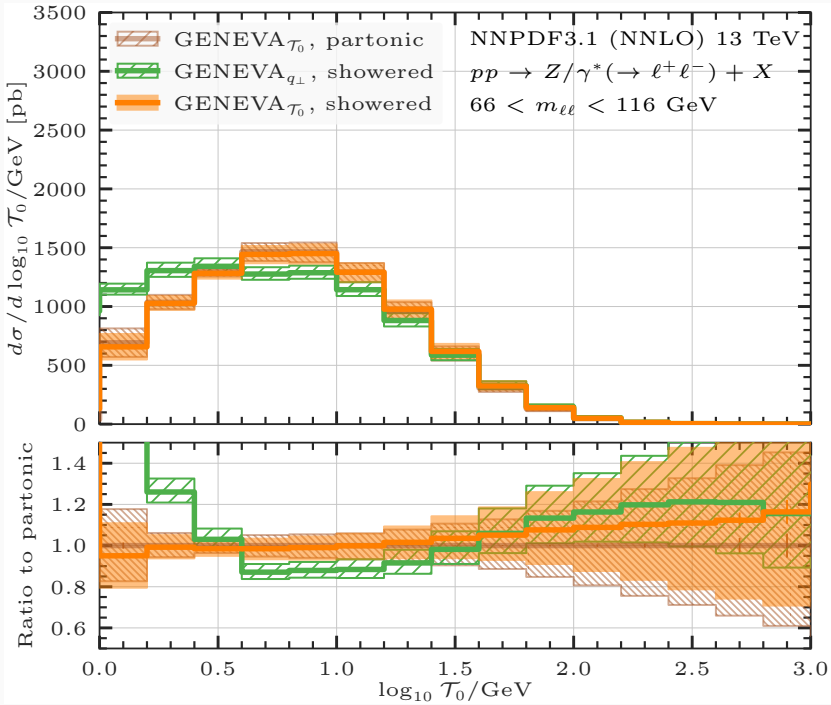




GENEVA+RadISH for Drell-Yan

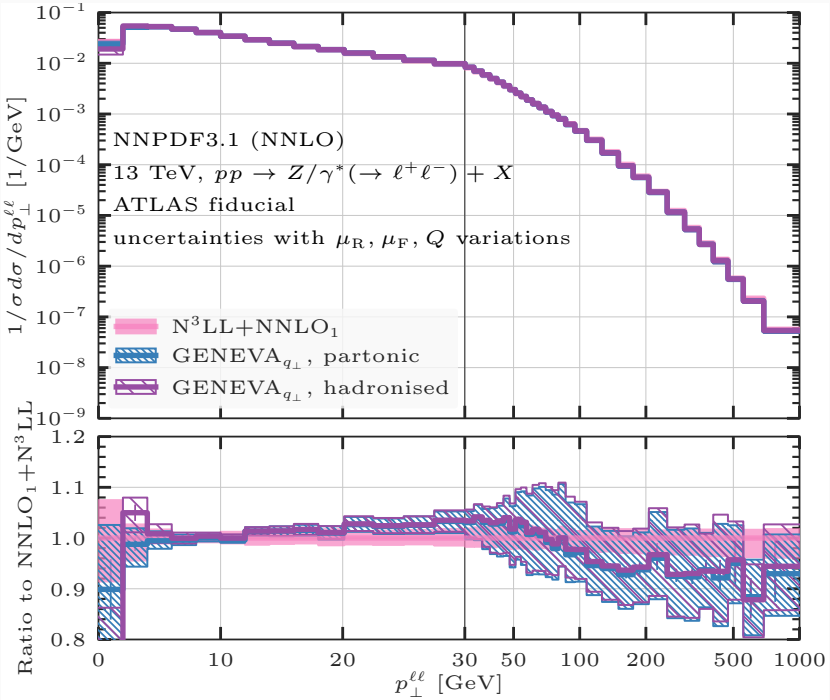
- Can compare predictions for one resolution variable using resummed calculation in another
- Agreement in both cases within scale bands
- Differences appear below ~ 30 GeV in both cases.

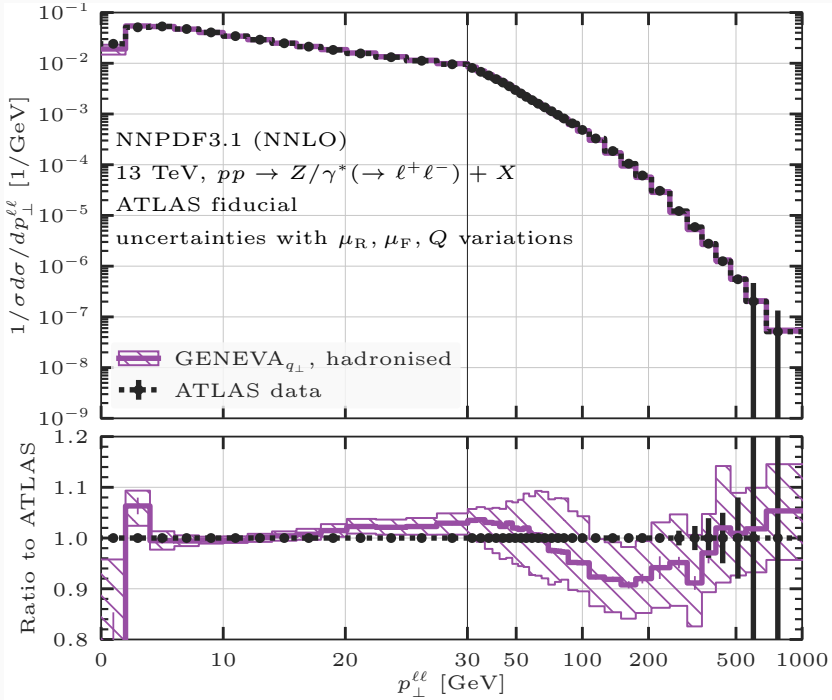




GENEVA+RadISH for Drell-Yan

- Comparison with RadISH+NNLOJET gives good agreement up to large values of p_T, where NNLO_1 calculation becomes important.
- Excellent agreement with ATLAS data < 30 GeV, first two bins sensitive to hadronisation and non-perturbative effects.





Conclusions

- GENEVA allows resummed, fixed order and parton shower calculations to be combined in order to provide an event generator which makes accurate predictions over the full range of relevant energy scales.
- Flexibility in terms of resolution variable and in how the resummation is accomplished.
- Several applications to LHC processes already achieved, more forthcoming.
- Double differential resummation also possible in principle ($\mathcal{T}_0$ and p_T), future avenue to explore.

Backup slides

Combining resummed and fixed order calculations

Inclusive 1-jet cross section:

$$\frac{\mathrm{d}\sigma^{\mathrm{MC}}_{\geq 1}}{\mathrm{d}\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}) = \frac{\mathrm{d}\sigma^{\mathit{resum}}_{\geq 1}}{\mathrm{d}\Phi_1}\,\theta(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}) + \frac{\mathrm{d}\sigma^{\mathit{sing\ match}}_{\geq 1}}{\mathrm{d}\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}) + \frac{\mathrm{d}\sigma^{\mathit{nons}}_{\geq 1}}{\mathrm{d}\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}})$$

$$\frac{\mathrm{d}\sigma^{\mathit{resum}}_{\geq 1}}{\mathrm{d}\Phi_1} = \frac{\mathrm{d}\sigma^{\mathit{NNLL'}}}{\mathrm{d}\Phi_0 \mathrm{d}\mathcal{T}_0}\,\mathcal{P}(\Phi_1)$$

- Resummed formula only differential in $\Phi_0, \mathcal{T}_0$. Need to make it differential in 2 more variables, e.g. energy ratio $z = E_M/E_S$ and azimuthal angle ϕ.
- We use a normalised splitting probability to make the resummation differential in Φ_1.

Combining resummed and fixed order calculations

Inclusive 1-jet cross section:

$$\frac{\mathrm{d}\sigma^{\mathrm{MC}}_{\geq 1}}{\mathrm{d}\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}) = \frac{\mathrm{d}\sigma^{resum}_{\geq 1}}{\mathrm{d}\Phi_1}\,\theta(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}) + \frac{\mathrm{d}\sigma^{sing\ match}_{\geq 1}}{\mathrm{d}\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}) + \frac{\mathrm{d}\sigma^{nons}_{\geq 1}}{\mathrm{d}\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}})$$

$$\frac{\mathrm{d}\sigma^{resum}_{\geq 1}}{\mathrm{d}\Phi_1} = \frac{\mathrm{d}\sigma^{NNLL'}}{\mathrm{d}\Phi_0 \mathrm{d}\mathcal{T}_0}\,\mathcal{P}(\Phi_1)$$

$$\mathcal{P}(\Phi_1) = \frac{p_{sp}(z,\phi)}{\sum_{sp}\int_{z_{min}(\mathcal{T}_0)}^{z_{max}(\mathcal{T}_0)} \mathrm{d}z\mathrm{d}\phi\, p_{sp}(z,\phi)}\,\frac{\mathrm{d}\Phi_0 \mathrm{d}\mathcal{T}_0 \mathrm{d}z\mathrm{d}\phi}{\mathrm{d}\Phi_1}, \qquad \int \frac{\mathrm{d}\Phi_1}{\mathrm{d}\Phi_0 \mathrm{d}\mathcal{T}_0}\,\mathcal{P}(\Phi_1) = 1$$

- p_{sp} are based on AP splittings for FSR, weighted by PDF ratio for ISR.

Combining resummed and fixed order calculations

Inclusive 1-jet cross section:

$$\frac{\mathrm{d}\sigma^{\mathrm{MC}}_{\geq 1}}{\mathrm{d}\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}) = \frac{\mathrm{d}\sigma^{NNLL'}}{\mathrm{d}\Phi_0 \mathrm{d}\mathcal{T}_0}\,\mathcal{P}(\Phi_1) + \frac{\mathrm{d}\sigma^{nons}_{\geq 1}}{\mathrm{d}\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}})$$

$$\frac{\mathrm{d}\sigma^{\mathrm{nons}}_{\geq 1}}{\mathrm{d}\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}) = \frac{\mathrm{d}\sigma^{NLO_1}_{\geq 1}}{\mathrm{d}\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}) - \left[\frac{\mathrm{d}\sigma^{NNLL'}}{\mathrm{d}\Phi_0 \mathrm{d}\mathcal{T}_0}\,\mathcal{P}(\Phi_1)\right]_{NLO_1} \theta(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}})$$

- Singular matching vanishes again at NNLL′.
- Nonsingular matching fixed by NLO_1 requirement.

Combining resummed and fixed order calculations

- We also split the inclusive 1-jet cross section into exclusive 1-jet and inclusive 2-jet cross sections, using $\mathcal{T}_1$ as the resolution variable
- Resummation of $\mathcal{T}_1$ is performed at NLL accuracy.

$$\frac{\mathrm{d}\sigma_1^{\mathrm{MC}}}{\mathrm{d}\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}; \mathcal{T}_1^{\mathrm{cut}}) = \frac{\mathrm{d}\sigma_1^{resum}}{\mathrm{d}\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}; \mathcal{T}_1^{\mathrm{cut}}) + \frac{\mathrm{d}\sigma_1^{match}}{\mathrm{d}\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}; \mathcal{T}_1^{\mathrm{cut}})$$

$$\frac{\mathrm{d}\sigma_{\geq 2}^{\mathrm{MC}}}{\mathrm{d}\Phi_2}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}, \mathcal{T}_1 > \mathcal{T}_1^{\mathrm{cut}}) = \frac{\mathrm{d}\sigma_{\geq 2}^{resum}}{\mathrm{d}\Phi_2}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}})\,\theta(\mathcal{T}_1 > \mathcal{T}_1^{\mathrm{cut}}) + \frac{\mathrm{d}\sigma_{\geq 2}^{match}}{\mathrm{d}\Phi_2}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}, \mathcal{T}_1 > \mathcal{T}_1^{\mathrm{cut}})$$

Combining resummed and fixed order calculations

$$\frac{\mathrm{d}\sigma_1^{resum}}{\mathrm{d}\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}; \mathcal{T}_1^{\mathrm{cut}}) = \frac{\mathrm{d}\sigma_{\geq 1}^{C}}{\mathrm{d}\Phi_1} U_1(\Phi_1, \mathcal{T}_1^{\mathrm{cut}})\, \theta(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}})$$

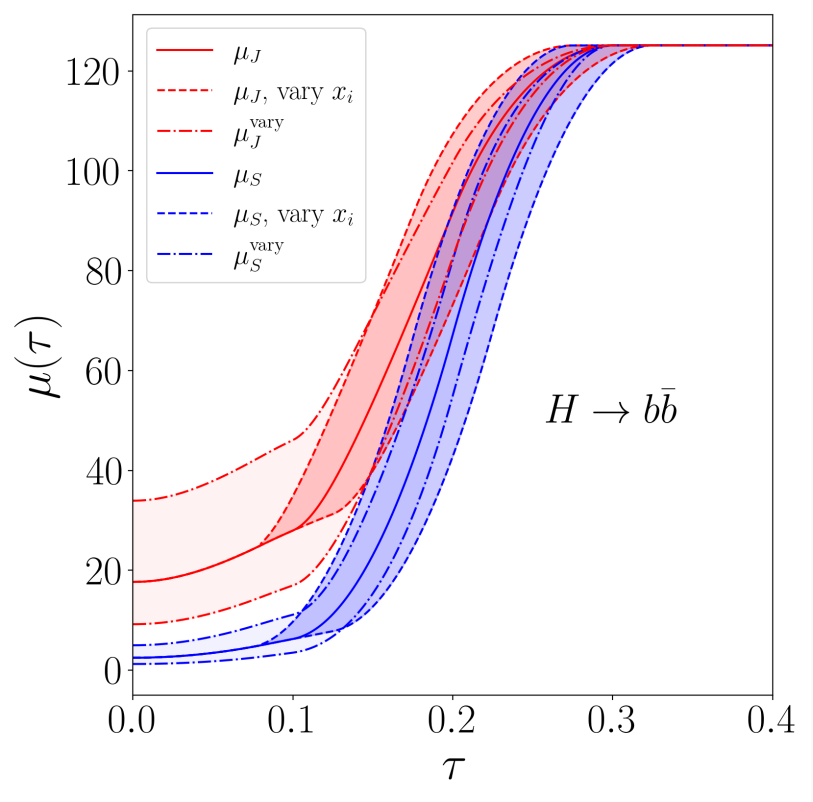
$$\frac{\mathrm{d}\sigma_{\geq 2}^{resum}}{\mathrm{d}\Phi_2}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}) = \frac{\mathrm{d}\sigma_{\geq 1}^{C}}{\mathrm{d}\Phi_1} U_1'(\Phi_1, \mathcal{T}_1)\, \theta(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}})\Big|_{\Phi_1 = \Phi_1^{\mathcal{T}}(\Phi_2)} \times \mathcal{P}(\Phi_2)\, \theta(\mathcal{T}_1 > \mathcal{T}_1^{\mathrm{cut}})$$

$$\frac{\mathrm{d}\sigma_{\geq 1}^{C}}{\mathrm{d}\Phi_1} = \frac{\mathrm{d}\sigma_{\geq 1}^{resum}}{\mathrm{d}\Phi_1} + (B_1 + V_1^{C})(\Phi_1) - \left[\frac{\mathrm{d}\sigma_{\geq 1}^{resum}}{\mathrm{d}\Phi_1}\right]_{NLO_1}$$

- The fully differential $\mathcal{T}_0$ resummation is contained within $\frac{\mathrm{d}\sigma_{\geq 1}^{resum}}{\mathrm{d}\Phi_1}$.

Profile scales

- Resummation is switched off via profile scales – when hard, beam and soft scales become equal, RGE evolution stops.
- Scales are continuous functions of the resolution variable.
- Transition points determined by examination of size of singular vs nonsingular contribution as a function of τ.



Power-suppressed contributions to the nonsingular cumulant

- The definition of the Φ_0 events depends on a projective map from higher multiplicity partonic events.
- This means observables dependent on the Φ_0 kinematics are correct at $\mathcal{O}(\alpha_s^2)$ only up to power corrections in $\mathcal{T}_0^{\mathrm{cut}}$.
- We can use this limitation to simplify the expression for the 0-jet formula and write:

$$\begin{aligned}\widetilde{\frac{\mathrm{d}\sigma_0^{\mathrm{MC}}}{\mathrm{d}\Phi_0}}(\mathcal{T}_0^{\mathrm{cut}}) = \frac{\mathrm{d}\sigma^{NNLL'}}{\mathrm{d}\Phi_0}(\mathcal{T}_0^{\mathrm{cut}}) - \left[\frac{\mathrm{d}\sigma^{NNLL'}}{\mathrm{d}\Phi_0}(\mathcal{T}_0^{\mathrm{cut}})\right]_{NLO_0} \\ + B_0(\Phi_0) + V_0(\Phi_0) \\ + \int \frac{\mathrm{d}\Phi_1}{\mathrm{d}\Phi_0} B_1(\Phi_1)\, \theta\left(\mathcal{T}_0(\Phi_1) < \mathcal{T}_0^{\mathrm{cut}}\right) ,\end{aligned}$$

- The double virtual and real virtual contributions have been dropped, resulting in a missing nonsingular contribution which is also a power correction in $\mathcal{T}_0^{\mathrm{cut}}$.

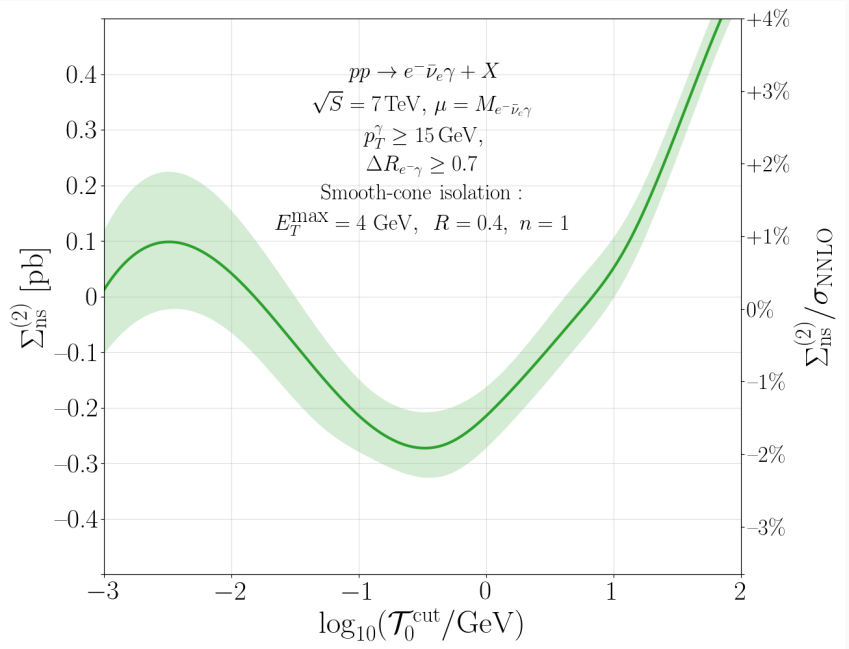
Power-suppressed contributions to the nonsingular cumulant

The missing nonsingular contribution is:

$$\frac{\mathrm{d}\sigma_0^{\mathrm{nons}}}{\mathrm{d}\Phi_0}(\mathcal{T}_0^{\mathrm{cut}}) = \left[\alpha_s f_1(\mathcal{T}_0^{\mathrm{cut}}, \Phi_0) + \alpha_s^2 f_2(\mathcal{T}_0^{\mathrm{cut}}, \Phi_0)\right]\mathcal{T}_0^{\mathrm{cut}}$$

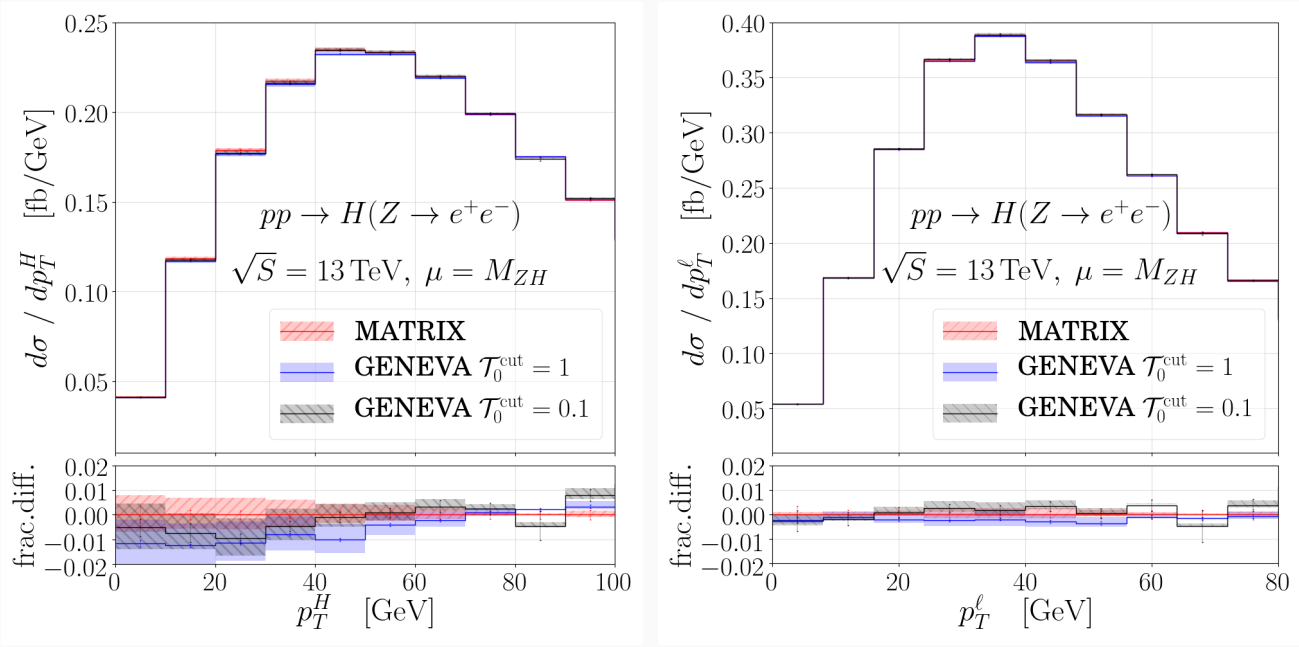
We include the first term fully but neglect the f_2 piece. How big is this effect?

For $W\gamma$, this is very small for $\mathcal{T}_0^{\mathrm{cut}} = 0.01$ GeV – about 0.01% of the total cross section.



Power-suppressed contributions to the nonsingular cumulant

- We include the effects of the integral of the f_2 term by reweighting the Φ_0 events such that the correct total cross section is obtained.
- Full NNLO cross section provided by MATRIX in this case.
- Missing $\mathcal{O}(\alpha_s^2)$ dependence on Φ_0 variables is of the same order as that missing due to the projective map, even if a full NNLO fixed order calculation were included.



Matching to a parton shower

We want to ensure preservation of NNLO+NNLL′ accuracy as far as possible. Take each class of event in turn:

- For Φ_0, all events start with $\mathcal{T}_0 = 0$. Shower should restore emissions which were integrated over - shape given by PYTHIA, constraint is only on normalisation.
- Starting scale is $\sim \sqrt{Q\mathcal{T}_0^{\mathrm{cut}}}$, events are re-showered until $\mathcal{T}_0^{\mathrm{PY}} < \mathcal{T}_0^{\mathrm{cut}}$. Small spillover allowed to avoid hard border.
- Φ_2 events are the bulk, with nonzero $\mathcal{T}_0$, $\mathcal{T}_1$. Starting scale is set to $k_{T,2nd} \sim \sqrt{Q\mathcal{T}_1}$, reshower until $\mathcal{T}_2^{\mathrm{PY}} < \mathcal{T}_1$.
- What about Φ_1 events?

Matching to a parton shower

- Jet constraint from $\mathcal{T}_1(\Phi_N) < \mathcal{T}_1^{\text{cut}}$ must be applied on hardest radiation, not necessarily first (real showers are not ordered in N-jettiness).
- Force this by using an NLL Sudakov and the $\mathcal{T}_0$-preserving map.

$$\frac{\mathrm{d}\sigma^{\text{MC}}_{N\to N}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\text{cut}};\Lambda_N) = \frac{\mathrm{d}\sigma^{\text{MC}}_{N}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\text{cut}})\, U_N(\mathcal{T}_N^{\text{cut}},\Lambda_N)$$

$$\frac{\mathrm{d}\sigma^{\text{MC}}_{N\to N+1}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}_N > \Lambda_N, \mathcal{T}_N^{\text{cut}}) = \frac{\mathrm{d}}{\mathrm{d}\mathcal{T}_N}\left[\frac{\mathrm{d}\sigma^{\text{MC}}_{N\to N}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\text{cut}};\mathcal{T}_N)\right] \mathcal{P}(\Phi_{N+1}) \times \theta(\mathcal{T}_N^{\text{cut}} > \mathcal{T}_N > \Lambda_N)$$

- Λ_N is a shower cutoff, much lower than $\mathcal{T}_N^{\text{cut}}$.
- Shower starting from $k_{T,\max} \sim \sqrt{Q\mathcal{T}_1^{\text{cut}}}$ and reshower until $\mathcal{T}_1^{\text{PY}} < \mathcal{T}_1^{\text{cut}}$
- Choosing $\Lambda_1 \sim \Lambda_{QCD}$, contribution reduced to $\sim 0.1\%$ of the total cross section.

What about the cumulant?

- Above discussion holds for the $\mathcal{T}_0$ spectrum $\mathrm{d}\sigma^{NNLL'}/\mathrm{d}\mathcal{T}_0$, but not necessarily the cumulant $\mathrm{d}\sigma^{NNLL'}(\mathcal{T}_0^{\mathrm{cut}})$.
- Since profile scales have a functional dependence on $\mathcal{T}_0$, choosing scales and integrating over $\mathcal{T}_0$ do not commute – difference is $\mathcal{O}(\mathrm{N^3LL})$. Inclusive FO cross section not recovered exactly!
- Solution: add term to spectrum so that
  1. The integral of the modified spectrum gives the correct FO cross section;
  2. Term only contributes in region of $\mathcal{T}_0$ where missing $\mathrm{N^3LL}$ terms are large;
  3. Term is itself $\mathcal{O}(\mathrm{N^3LL})$ to prevent spoiling NNLL′ accuracy.

What about the cumulant?

Add the term:

$$\kappa(\mathcal{T}_0)\left[\frac{\mathrm{d}}{\mathrm{d}\mathcal{T}_0}\frac{\mathrm{d}\sigma^{NNLL'}}{\mathrm{d}\Phi_0}(\mathcal{T}_0,\mu_h(\mathcal{T}_0)) - \frac{\mathrm{d}\sigma^{NNLL'}}{\mathrm{d}\Phi_0\mathrm{d}\mathcal{T}_0}(\mu_h(\mathcal{T}_0))\right]$$

- Of higher order (by construction);
- In FO region, $\mu_h = Q$ and difference between terms is zero (scales are constant) – term vanishes;
- Tune $\kappa(\mathcal{T}_0 \to 0)$ so that correct inclusive cross section is obtained on integration.

Photon isolation procedures

- We are interested only in prompt photon production, where the photons are produced in the hard scattering interaction
- Need to remove contribution from fragmentation process, where photons are radiated off final-state jets.
- A fixed-cone algorithm restricts the amount of hadronic energy allowed to lie within a cone around the jet, BUT
- This is not IR-safe – forbids soft emissions inside the cone.
- Still sensitive to fragmentation, since collinear configurations are still allowed.

Frixione isolation

- An IR-safe method to isolate photons has been provided by Frixione.
- Consider a series of sub-cones with radius $r < R_{\mathrm{iso}}$ where R_{iso} is the outer cone radius. We then require

$$E_T^{\mathrm{had}}(r) \leq E_T^{\mathrm{max}} \chi(r; R_{\mathrm{iso}})$$

  where the isolation function χ is smooth and monotonic.
- This reduces hadronic activity in a smooth way when approaching the photon direction.
- Standard choice is

$$\chi(r; R_{\mathrm{iso}}) = \left(\frac{1 - \cos r}{1 - \cos R_{\mathrm{iso}}} \right)^n$$

Hybrid isolation

- Frixione isolation complicates comparison with experimental analyses, which always use a fixed-cone approach.
- A hybrid-cone procedure uses Frixione isolation with a very small R_{iso} to remove a tiny slice of phase space around the photon.
- A fixed-cone procedure with a larger radius $R \gg R_{\mathrm{iso}}$ is then applied to events passing the first isolation step.