Recent developments in GENEVA

Diboson processes at NNLOPS and $p_T$ resummation

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Taming the accuracy of event generators, 24 August 2021

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• Much progress in matching NNLO QCD calculations recently – three main methods available: **UNNLOPS**, **MiNNLOPS**, **GENEVA**

• Initial applications were for $2 \rightarrow 1$ processes e.g. Drell-Yan, Higgs in gluon fusion

• Recently - more complex processes available, such as diphoton, $ZZ$, $t\bar{t}$

• These can improve the description of data significantly and also be vital tools for experimentalists.
Overview of talk

• Introduction to GENEVA as a MC event generator

• Recent applications to diboson processes

• Changing resolution variables and GENEVA $p_T$ with RadISH
The GENEVA method
The GENEVA method

The Three Jewels:

- GENEVA produces **fully differential fixed order** calculations at **NNLO**;
- By **resumming large logarithms at NNLL’**, it provides precise predictions over the whole phase space;
- These are matched to a **parton shower** to produce realistic events (which can further be hadronised, MPI effects included).

The method is fully general.
Work with IR-finite events to which a finite cross section can be assigned:

- Introduce a resolution parameter $\mathcal{T}_N$, $\mathcal{T}_N \to 0$ in the IR region. Emissions below $\mathcal{T}_N^{\text{cut}}$ are unresolved (i.e. integrated over) and the kinematic configuration considered is the one of the event before the emission.
- An $M$-parton event is thus translated to an $N$-jet event, $N \leq M$, fully differential in $\Phi_N$ (no jet-algorithm needed).
  - Price to pay: power corrections in $\mathcal{T}_N^{\text{cut}}$ due to PS projection.
  - Advantage: vanish for IR-safe observables as $\mathcal{T}_N^{\text{cut}} \to 0$
- Iterating the procedure, the phase space is sliced into jet-bins.
Constructing IR-finite events

**Exclusive \(N\)-jet bin**

\[
\frac{d\sigma_{N}^{MC}}{d\Phi_{N}}(\tau_{N}^{\text{cut}})
\]

\(\tau_{0} < \tau_{0}^{\text{cut}}\)

\(\tau_{0} ^{\text{cut}}\)

\(\tau_{1} < \tau_{1}^{\text{cut}}\)

\(\tau_{1}^{\text{cut}}\)

\(\Phi_{0}\)

\(\Phi_{1}\)

\(\Phi_{2}\)

\(\Phi_{2}\)

\[+ \cdots\]

**Inclusive \((N+1)\)-jet bin**

\[
\frac{d\sigma_{\geq N+1}^{MC}}{d\Phi_{N+1}}(\tau_{N} > \tau_{N}^{\text{cut}})
\]

\(\tau_{0} > \tau_{0}^{\text{cut}}\)

\(\tau_{1} > \tau_{1}^{\text{cut}}\)
Constructing IR-finite events

Excl. $N$-jet bin

$$\frac{d\sigma_{N}^{MC}}{d\Phi_{N}}(\mathcal{T}_{N}^{\text{cut}})$$

Excl. $(N + 1)$-jet

$$\frac{d\sigma_{N+1}^{MC}}{d\Phi_{N+1}}(\mathcal{T}_{N} > \mathcal{T}_{N}^{\text{cut}}; \mathcal{T}_{N}^{\text{cut}}, \mathcal{T}_{N+1}^{\text{cut}})$$

Inclusive $(N + 2)$-jet bin

$$\frac{d\sigma_{\geq N+2}^{MC}}{d\Phi_{N+2}}(\mathcal{T}_{N} > \mathcal{T}_{N}^{\text{cut}}, \mathcal{T}_{N+1} > \mathcal{T}_{N+1}^{\text{cut}})$$
Combining resummed and fixed order calculations

Consider colour singlet production at NNLO. We need events with 0, 1 and 2 additional QCD partons in the final state.

Exclusive 0-jet cross section:

\[
\frac{d\sigma_{0\text{MC}}}{d\Phi_0}(\mathcal{T}_0^{\text{cut}}) = \frac{d\sigma_{0\text{NNLL'}}}{d\Phi_0}(\mathcal{T}_0^{\text{cut}}) + \frac{d\sigma_{0\text{sing match}}}{d\Phi_0}(\mathcal{T}_0^{\text{cut}}) + \frac{d\sigma_{0\text{nons}}}{d\Phi_0}(\mathcal{T}_0^{\text{cut}})
\]

- At NNLL', all singular terms included to $\mathcal{O}(\alpha_s^2)$ by definition – singular matching term vanishes.
- Nonsingular matching term determined by requirement of FO NNLO accuracy:

\[
\frac{d\sigma_{0\text{nons}}}{d\Phi_0}(\mathcal{T}_0^{\text{cut}}) = \frac{d\sigma_{0\text{NNLO}}}{d\Phi_0}(\mathcal{T}_0^{\text{cut}}) - \left[ \frac{d\sigma_{0\text{NNLL'}}}{d\Phi_0}(\mathcal{T}_0^{\text{cut}}) \right]_{\text{NNLO}_0}
\]
Choice of the jet resolution variable

- We use $N$-jettiness as resolution parameter. Global physical observable with straightforward definitions for hadronic colliders, in terms of beams $q_{a,b}$ and jet-directions $q_j$

\[
\tau_N = \frac{2}{Q} \sum_k \min \{ q_a \cdot p_k, q_b \cdot p_k, q_1 \cdot p_k, \ldots, q_N \cdot p_k \}
\]

- $N$-jettiness has good factorisation properties, IR safe and resummable at all orders. Resummation known at NNLL for any $N$ in Soft-Collinear Effective Theory

- $\tau_N \to 0$ for $N$ pencil-like jets, $\tau_N \gg 0$ spherical limit.
NNLL’ resummation from SCET

The spectrum in $\mathcal{T}_0$ can be factorised at all-orders as

$$
\frac{d\sigma^{\text{NNLL}’}}{d\Phi_0 d\mathcal{T}_0} (\mathcal{T}_0 > \mathcal{T}_0^{\text{cut}}) = \sum_{ij} \frac{d\sigma_{ij}^B}{d\Phi_0} H_{ij}(Q^2, \mu_H) U_H(\mu_H, \mu) \int dt_a dt_b
$$

$$
\times [B_i(t_a, x_a, \mu_B) \otimes U_B(\mu_B, \mu)]
$$

$$
\times [B_j(t_b, x_b, \mu_B) \otimes U_B(\mu_B, \mu)]
$$

$$
\otimes [S(\mathcal{T}_0 - \frac{t_a + t_b}{Q}, \mu_S) \otimes U_S(\mu_S, \mu)].
$$

- **Hard**, **Beam** and **Soft** functions are each evaluated at their own scale $\Rightarrow$ no large logarithms,

$$
\mu_H = Q, \quad \mu_B = \sqrt{Q \mathcal{T}_0}, \quad \mu_S = \mathcal{T}_0
$$

- RGE kernels $U_X$ evolve functions to a common scale $\mu$ and in so doing resum logarithms.
Matching to a parton shower

- Parton shower makes calculation differential in higher multiplicities by adding radiation.
- Fills the 0- and 1-jet bins with radiation, adds more to the inclusive 2-jet bin.

- Not allowed to affect the accuracy of the cross section reached at partonic level.
- $\tau_i^{\text{cut}}$ constraints must be respected.
Applications to diboson processes
• Diphoton production is an important process at a hadron collider.

• Conceptually, introduces a new problem – process is divergent at Born level, need to introduce isolation criteria to select prompt photons, prevent QED/QCD divergences with cuts.

• We use Frixione isolation for comparison with FO calculations, hybrid procedure to compare with data.

• Compared predictions to ATLAS and CMS data collected at $\sqrt{s} = 7$ TeV.
Comparing standard resummation with event generation

- Standard resummed calculations improve a single observable, and do not generally have information about full events (no recoil).
- Event generators take a full event and calculate the value of the observable on that event. This may involve a projection from a higher to a lower multiplicity PS point.
- These are identical in the limit $\mathcal{T}_0 \to 0$. Away from this limit, same result if we cut only on quantities preserved by $\Phi_1 \to \Phi_0$.
- Cuts on e.g. photon $p_T$ are not preserved – contributions are removed in GENEVA compared to standard resummation.
Comparing standard resummation with event generation

- Resummed only differs from standard resummation at high values – this is cured by matching to FO, where good agreement is found across $\tau_0$ range.
• Compared to NNLO from MATRIX, missing kinematic dependence of power corrections is reduced as $T_0^{\text{cut}} \rightarrow 0$.
• Residual differences visible in e.g. $p_T$ spectra due to fiducial power corrections treated correctly by differential subtraction and higher order terms included by resummation.
- Partonic result is NNLL’+NNLO accurate for $\mathcal{T}_0$ distribution.
- Accuracy is numerically well-preserved after showering.
Comparison with dedicated $p_T$ resummation

- Can compare predictions for $p_T^\gamma\gamma$ with resummed results matched to FO from MATRIX+RadISH.
- Partonic result replicates exact resummation down to small values. After showering, dipole recoil scheme preserves agreement well.
Isolation dependence

- Check dependence on generation cuts used – compare tight generation with loose generation and tight analysis cuts.
- Parton-level results are not strongly dependent on exact choice.
- Shower can reshuffle momenta and affect rate of events passing analysis cuts – reasonable agreement, but bigger effects here.
Diphoton production – ATLAS 7 TeV

- Good agreement across most of range - worsens at low $p_T^{\gamma\gamma}$ where dedicated $p_T$ resummation is important. Better with dipole recoil in shower
- EW effects important at high $M_{\gamma\gamma}$. 

![Graphs showing diphoton production distribution](image)
Diphoton production – CMS 7 TeV

• $\Delta \phi$ distribution in good agreement with CMS measurements across whole range.
ZZ production

- Massive **diboson processes** are important probes of the non-Abelian EW couplings at the LHC; four lepton final states give very clean signatures in a QCD background.
- Simplifications with respect to diphoton in the sense that no isolation/process-defining cuts are needed.
- More complicated in the sense that many resonance structures **contribute**, phase space is of higher dimension.
- Phase space sampling provided by a tunnel between GENEVA and MUNICH, the multi-channel integrator (as used in e.g. MATRIX).
- We consider \( pp \to \ell^+\ell^-\ell'^+\ell'^- \), i.e. ZZ production with decays to distinct flavours.
• Comparison with MATRIX only including $q\bar{q}$ channel ($gg$ added at LO/relative NNLO later)
• Good agreement with data (last bins contain overflow)
• For $p_T, \ell^+\ell^-$ $> 150$ GeV, EW effects are important.
\(W\gamma\) production

- Process has features of both ZZ (multiple resonance structures) and diphoton (process-defining cuts and photon isolation).
- NLO corrections artificially large because of radiation zero at LO – motivates inclusion of NNLO.
- Comparison of predictions with full CKM to 13 TeV LHC data underway.
Changing the resolution variable and GENEVA-RadISH
• The GENEVA approach is not specific to a particular resolution variable.

• In particular, as long as a suitable resummed calculation is available, any appropriate variable can be used.

• An obvious candidate is the $q_T$ of the colour singlet system – in this case, we can obtain resummed predictions at $N^3$LL from RadISH.

• First application to Drell-Yan, but in principle other colour-singlet processes would be straightforward.
Parton-level predictions agree with MATRIX+RadISH only, both resummed and matched.
• Accuracy is numerically well-preserved even after showering.
GENEVA+RadISH for Drell-Yan

- Can compare predictions for one resolution variable using resummed calculation in another
- Agreement in both cases within scale bands
- Differences appear below $\sim 30$ GeV in both cases.
GENEVA+RadISH for Drell-Yan

- Comparison with RadISH+NNLOJET gives good agreement up to large values of $p_T$, where NNLO$_1$ calculation becomes important.
- Excellent agreement with ATLAS data $< 30$ GeV, first two bins sensitive to hadronisation and non-perturbative effects.
Conclusions

- **GENEVA** allows resummed, fixed order and parton shower calculations to be combined in order to provide an event generator which makes accurate predictions over the full range of relevant energy scales.
- Flexibility in terms of resolution variable and in how the resummation is accomplished.
- Several applications to LHC processes already achieved, more forthcoming.
- Double differential resummation also possible in principle ($\overline{T}_0$ and $p_T$), future avenue to explore.
Backup slides
Combining resummed and fixed order calculations

Inclusive 1-jet cross section:

\[
\frac{d\sigma_{\geq 1}^{\text{MC}}}{d\Phi_1}(T_0 > T_0^{\text{cut}}) = \frac{d\sigma_{\geq 1}^{\text{resum}}}{d\Phi_1} \theta(T_0 > T_0^{\text{cut}}) + \frac{d\sigma_{\geq 1}^{\text{sing match}}}{d\Phi_1}(T_0 > T_0^{\text{cut}})
+ \frac{d\sigma_{\geq 1}^{\text{nons}}}{d\Phi_1}(T_0 > T_0^{\text{cut}})
\]

\[
\frac{d\sigma_{\geq 1}^{\text{resum}}}{d\Phi_1} = \frac{d\sigma_{\text{NNLL}'}^{\text{resum}}}{d\Phi_0 dT_0} P(\Phi_1)
\]

- Resummed formula only differential in $\Phi_0$, $T_0$. Need to make it differential in 2 more variables, e.g. energy ratio $z = E_M/E_S$ and azimuthal angle $\phi$.
- We use a normalised splitting probability to make the resummation differential in $\Phi_1$. 
Combining resummed and fixed order calculations

Inclusive 1-jet cross section:

\[
\frac{d\sigma_{\geq 1}^{\text{MC}}(\mathcal{T}_0 > \mathcal{T}_0^{\text{cut}})}{d\Phi_1} = \frac{d\sigma_{\geq 1}^{\text{resum}}}{d\Phi_1} \theta(\mathcal{T}_0 > \mathcal{T}_0^{\text{cut}}) + \frac{d\sigma_{\geq 1}^{\text{sing match}}}{d\Phi_1} (\mathcal{T}_0 > \mathcal{T}_0^{\text{cut}})
\]

\[
+ \frac{d\sigma_{\geq 1}^{\text{nons}}}{d\Phi_1} (\mathcal{T}_0 > \mathcal{T}_0^{\text{cut}})
\]

\[
\frac{d\sigma_{\geq 1}^{\text{resum}}}{d\Phi_1} = \frac{d\sigma^{\text{NNLL}'}_0}{d\Phi_0 d\mathcal{T}_0} \mathcal{P}(\Phi_1)
\]

\[
\mathcal{P}(\Phi_1) = \frac{\rho_{sp}(z, \phi)}{\sum_{sp} \int_{z_{\text{min}}(\mathcal{T}_0)}^{z_{\text{max}}(\mathcal{T}_0)} dzd\phi \rho_{sp}(z, \phi)} \frac{d\Phi_0 d\mathcal{T}_0 dzd\phi}{d\Phi_1}, \quad \int \frac{d\Phi_1}{d\Phi_0 d\mathcal{T}_0} \mathcal{P}(\Phi_1) = 1
\]

\[
\cdot p_{sp} \text{ are based on AP splittings for FSR, weighted by PDF ratio for ISR.}
\]
Combining resummed and fixed order calculations

Inclusive 1-jet cross section:

\[
\frac{d\sigma_{\geq 1}^{MC}}{d\Phi_1} (T_0 > T_0^{cut}) = \frac{d\sigma^{NNLL'}}{d\Phi_0 dT_0} \mathcal{P}(\Phi_1) + \frac{d\sigma_{\geq 1}^{nons}}{d\Phi_1} (T_0 > T_0^{cut})
\]

\[
\frac{d\sigma_{\geq 1}^{nons}}{d\Phi_1} (T_0 > T_0^{cut}) = \frac{d\sigma^{NLO_1}}{d\Phi_1} (T_0 > T_0^{cut}) - \left[ \frac{d\sigma^{NNLL'}}{d\Phi_0 dT_0} \mathcal{P}(\Phi_1) \right]_{NLO_1} \theta(T_0 > T_0^{cut})
\]

• Singular matching vanishes again at NNLL'.
• Nonsingular matching fixed by NLO$_1$ requirement.
Combining resummed and fixed order calculations

- We also split the inclusive 1-jet cross section into exclusive 1-jet and inclusive 2-jet cross sections, using $T_1$ as the resolution variable.
- Resummation of $T_1$ is performed at NLL accuracy.

\[
\frac{d\sigma^{MC}_{1}}{d\Phi_1} (T_0 > T_0^{cut}, T_1^{cut}) = \frac{d\sigma^{resum}_{1}}{d\Phi_1} (T_0 > T_0^{cut}, T_1^{cut}) \]
\[+ \frac{d\sigma^{match}_{1}}{d\Phi_1} (T_0 > T_0^{cut}, T_1^{cut}) \]
\[
\frac{d\sigma^{MC}_{\geq 2}}{d\Phi_2} (T_0 > T_0^{cut}, T_1 > T_1^{cut}) = \frac{d\sigma^{resum}_{\geq 2}}{d\Phi_2} (T_0 > T_0^{cut}) \theta(T_1 > T_1^{cut}) \]
\[+ \frac{d\sigma^{match}_{\geq 2}}{d\Phi_2} (T_0 > T_0^{cut}, T_1 > T_1^{cut}) \]
Combining resummed and fixed order calculations

\[
\frac{d\sigma_{\text{resum}}^1}{d\Phi_1}(\mathcal{T}_0 > T_0^{\text{cut}}, T_1^{\text{cut}}) = \frac{d\sigma_{\geq 1}^c}{d\Phi_1} U_1(\Phi_1, T_1^{\text{cut}}) \theta(T_0 > T_0^{\text{cut}})
\]

\[
\frac{d\sigma_{\geq 2}^{\text{resum}}}{d\Phi_2}(T_0 > T_0^{\text{cut}}) = \frac{d\sigma_{\geq 1}^c}{d\Phi_1} U_1'(\Phi_1, T_1) \theta(T_0 > T_0^{\text{cut}}) \bigg|_{\Phi_1 = \Phi_1^T(\Phi_2)} \times \mathcal{P}(\Phi_2) \theta(T_1 > T_1^{\text{cut}})
\]

\[
\frac{d\sigma_{\geq 1}^c}{d\Phi_1} = \frac{d\sigma_{\geq 1}^{\text{resum}}}{d\Phi_1} + (B_1 + V_1^c)(\Phi_1) - \left[ \frac{d\sigma_{\geq 1}^{\text{resum}}}{d\Phi_1} \right]_{\text{NLO}_1}
\]

• The fully differential $\mathcal{T}_0$ resummation is contained within $\frac{d\sigma_{\geq 1}^{\text{resum}}}{d\Phi_1}$. 
• Resummation is switched off via profile scales – when hard, beam and soft scales become equal, RGE evolution stops.

• Scales are continuous functions of the resolution variable.

• Transition points determined by examination of size of singular vs nonsingular contribution as a function of $\tau$. 
Power-suppressed contributions to the nonsingular cumulant

- The definition of the $\Phi_0$ events depends on a projective map from higher multiplicity partonic events.
- This means observables dependent on the $\Phi_0$ kinematics are correct at $\mathcal{O}(\alpha_s^2)$ only up to power corrections in $T_0^{\text{cut}}$.
- We can use this limitation to simplify the expression for the 0-jet formula and write:

\[
\frac{d\sigma_{0\text{MC}}}{d\Phi_0}(T_0^{\text{cut}}) = \frac{d\sigma_{\text{NNLL}}'}{d\Phi_0}(T_0^{\text{cut}}) - \left[ \frac{d\sigma_{\text{NNLL}}'}{d\Phi_0}(T_0^{\text{cut}}) \right]_{\text{NLO}_0} \\
+ B_0(\Phi_0) + V_0(\Phi_0) \\
+ \int \frac{d\Phi_1}{d\Phi_0} B_1(\Phi_1) \theta (T_0(\Phi_1) < T_0^{\text{cut}}) ,
\]

- The double virtual and real virtual contributions have been dropped, resulting in a missing nonsingular contribution which is also a power correction in $T_0^{\text{cut}}$. 
Power-suppressed contributions to the nonsingular cumulant

The missing nonsingular contribution is:

\[
\frac{d\sigma_{0}^{\text{nons}}}{d\Phi_0}(T_0^{\text{cut}}) = [\alpha_s f_1(T_0^{\text{cut}}, \Phi_0) + \alpha_s^2 f_2(T_0^{\text{cut}}, \Phi_0)] T_0^{\text{cut}}
\]

We include the first term fully but neglect the \(f_2\) piece. How big is this effect?

For \(W_\gamma\), this is very small for \(T_0^{\text{cut}} = 0.01\) GeV – about 0.01% of the total cross section.
Power-suppressed contributions to the nonsingular cumulant

- We include the effects of the integral of the $f_2$ term by reweighting the $\Phi_0$ events such that the correct total cross section is obtained.
- Full NNLO cross section provided by MATRIX in this case.
- Missing $\mathcal{O}(\alpha_s^2)$ dependence on $\Phi_0$ variables is of the same order as that missing due to the projective map, even if a full NNLO fixed order calculation were included.
Matching to a parton shower

We want to ensure preservation of NNLO+NNLL’ accuracy as far as possible. Take each class of event in turn:

- For $\Phi_0$, all events start with $T_0 = 0$. Shower should restore emissions which were integrated over - shape given by PYTHIA, constraint is only on normalisation.
- Starting scale is $\sim \sqrt{Q T_0^{\text{cut}}}$, events are re-showered until $T_0^{\text{PY}} < T_0^{\text{cut}}$. Small spillover allowed to avoid hard border.
- $\Phi_2$ events are the bulk, with nonzero $T_0$, $T_1$. Starting scale is set to $k_{T,2nd} \sim \sqrt{Q T_1}$, reshower until $T_2^{\text{PY}} < T_1$.
- What about $\Phi_1$ events?
Matching to a parton shower

- Jet constraint from $\mathcal{T}_1(\Phi_N) < \mathcal{T}_1^{\text{cut}}$ must be applied on hardest radiation, not necessarily first (real showers are not ordered in $N$-jettiness).
- Force this by using an NLL Sudakov and the $\mathcal{T}_0$-preserving map.

\[
\frac{d\sigma_{N \rightarrow N}^{\text{MC}}}{d\phi_N} (\mathcal{T}_N^{\text{cut}}; \Lambda_N) = \frac{d\sigma_{N}^{\text{MC}}}{d\phi_N} (\mathcal{T}_N^{\text{cut}}) U_N(\mathcal{T}_N^{\text{cut}}, \Lambda_N)
\]

\[
\frac{d\sigma_{N \rightarrow N+1}^{\text{MC}}}{d\phi_{N+1}} (\mathcal{T}_N > \Lambda_N, \mathcal{T}_N^{\text{cut}}) = \frac{d}{d\mathcal{T}_N} \left[ \frac{d\sigma_{N \rightarrow N}^{\text{MC}}}{d\phi_N} (\mathcal{T}_N^{\text{cut}}; \mathcal{T}_N) \right] P(\Phi_{N+1})
\]

\[
\times \theta(\mathcal{T}_N^{\text{cut}} > \mathcal{T}_N > \Lambda_N)
\]

- $\Lambda_N$ is a shower cutoff, much lower than $\mathcal{T}_N^{\text{cut}}$.
- Shower starting from $k_{T,\text{max}} \sim \sqrt{Q\mathcal{T}_1^{\text{cut}}}$ and reshower until $\mathcal{T}_1^{\text{PY}} < \mathcal{T}_1^{\text{cut}}$
- Choosing $\Lambda_1 \sim \Lambda_{\text{QCD}}$, contribution reduced to $\sim 0.1\%$ of the total cross section.
What about the cumulant?

- Above discussion holds for the $\mathcal{T}_0$ spectrum $d\sigma^{NNLL'}/d\mathcal{T}_0$, but not necessarily the cumulant $d\sigma^{NNLL'}(\mathcal{T}_0^{\text{cut}})$.

- Since profile scales have a functional dependence on $\mathcal{T}_0$, choosing scales and integrating over $\mathcal{T}_0$ do not commute – difference is $\mathcal{O}(N^{3\text{LL}})$. Inclusive FO cross section not recovered exactly!

- Solution: add term to spectrum so that
  1. The integral of the modified spectrum gives the correct FO cross section;
  2. Term only contributes in region of $\mathcal{T}_0$ where missing $N^{3\text{LL}}$ terms are large;
  3. Term is itself $\mathcal{O}(N^{3\text{LL}})$ to prevent spoiling NNLL' accuracy.
What about the cumulant?

Add the term:

$$\kappa(T_0) \left[ \frac{d}{dT_0} \frac{d\sigma^{NNLL'}}{d\Phi_0}(T_0, \mu_h(T_0)) - \frac{d\sigma^{NNLL'}}{d\Phi_0 dT_0}(\mu_h(T_0)) \right]$$

- Of higher order (by construction);
- In FO region, $\mu_h = Q$ and difference between terms is zero (scales are constant) – term vanishes;
- Tune $\kappa(T_0 \to 0)$ so that correct inclusive cross section is obtained on integration.
Photon isolation procedures

- We are interested only in **prompt photon production**, where the photons are produced in the hard scattering interaction.
- Need to **remove contribution from fragmentation process**, where photons are radiated off final-state jets.
- A **fixed-cone algorithm** restricts the amount of hadronic energy allowed to lie within a cone around the jet, **BUT**
- **This is not IR-safe** – forbids soft emissions inside the cone.
- Still sensitive to fragmentation, since collinear configurations are still allowed.
Frixione isolation

- An IR-safe method to isolate photons has been provided by Frixione.
- Consider a series of sub-cones with radius $r < R_{iso}$ where $R_{iso}$ is the outer cone radius. We then require

$$E_T^{had}(r) \leq E_T^{max} \chi(r; R_{iso})$$

where the isolation function $\chi$ is smooth and monotonic.
- This reduces hadronic activity in a smooth way when approaching the photon direction.
- Standard choice is

$$\chi(r; R_{iso}) = \left( \frac{1 - \cos r}{1 - \cos R_{iso}} \right)^n$$
Hybrid isolation

• Frixione isolation complicates comparison with experimental analyses, which always use a fixed-cone approach.
• A hybrid-cone procedure uses Frixione isolation with a very small $R_{\text{iso}}$ to remove a tiny slice of phase space around the photon.
• A fixed-cone procedure with a larger radius $R \gg R_{\text{iso}}$ is then applied to events passing the first isolation step.