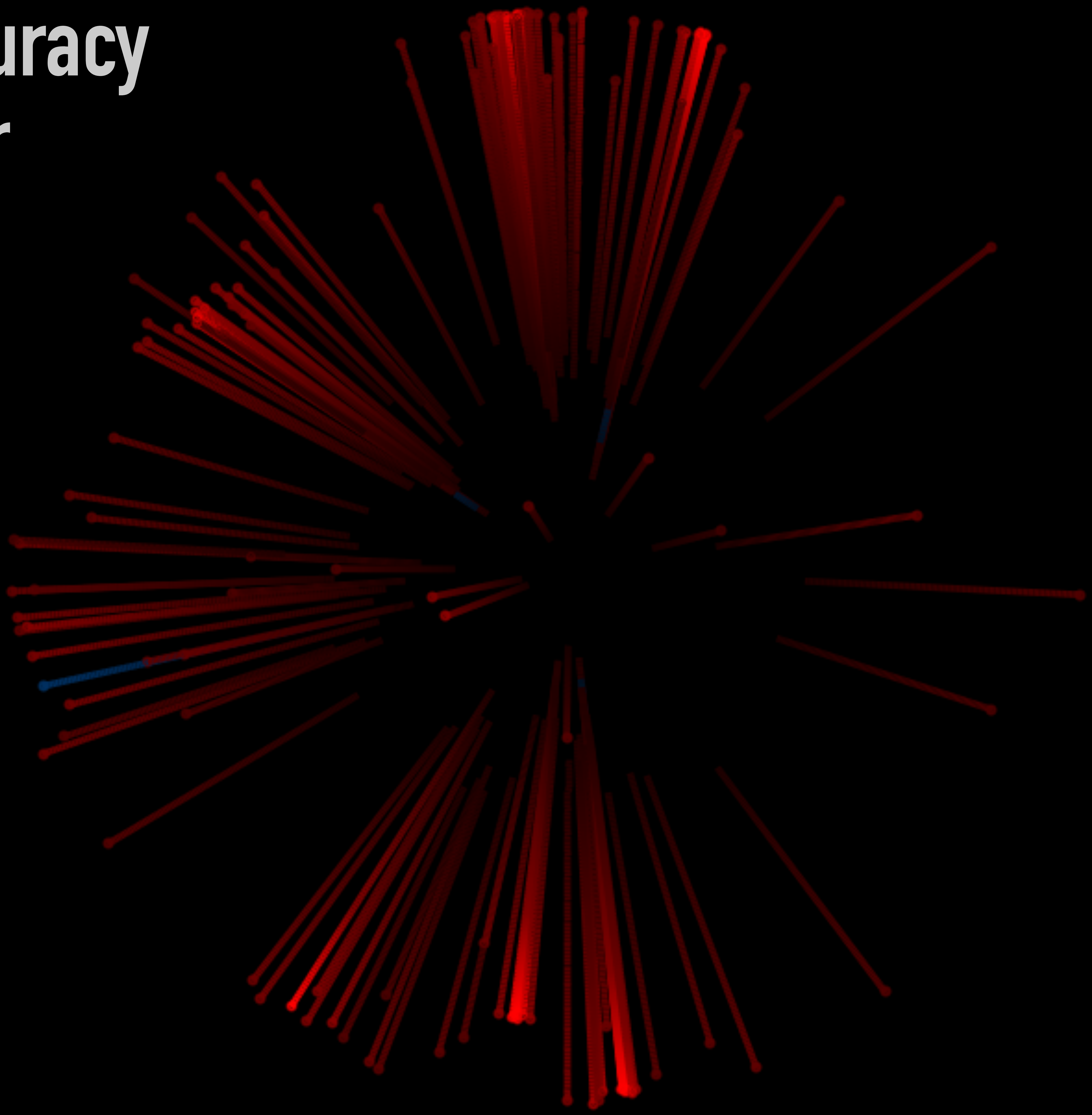
Perturbative accuracy and higher-order parton showers

Taming the accuracy of event generators (Part 2)
CERN, via Zoom
August 2021



Gavin Salam
Rudolf Peierls Centre for Theoretical Physics
& All Souls College, Oxford

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Logarithmic perturbative accuracy and higher-order (NLL) parton showers

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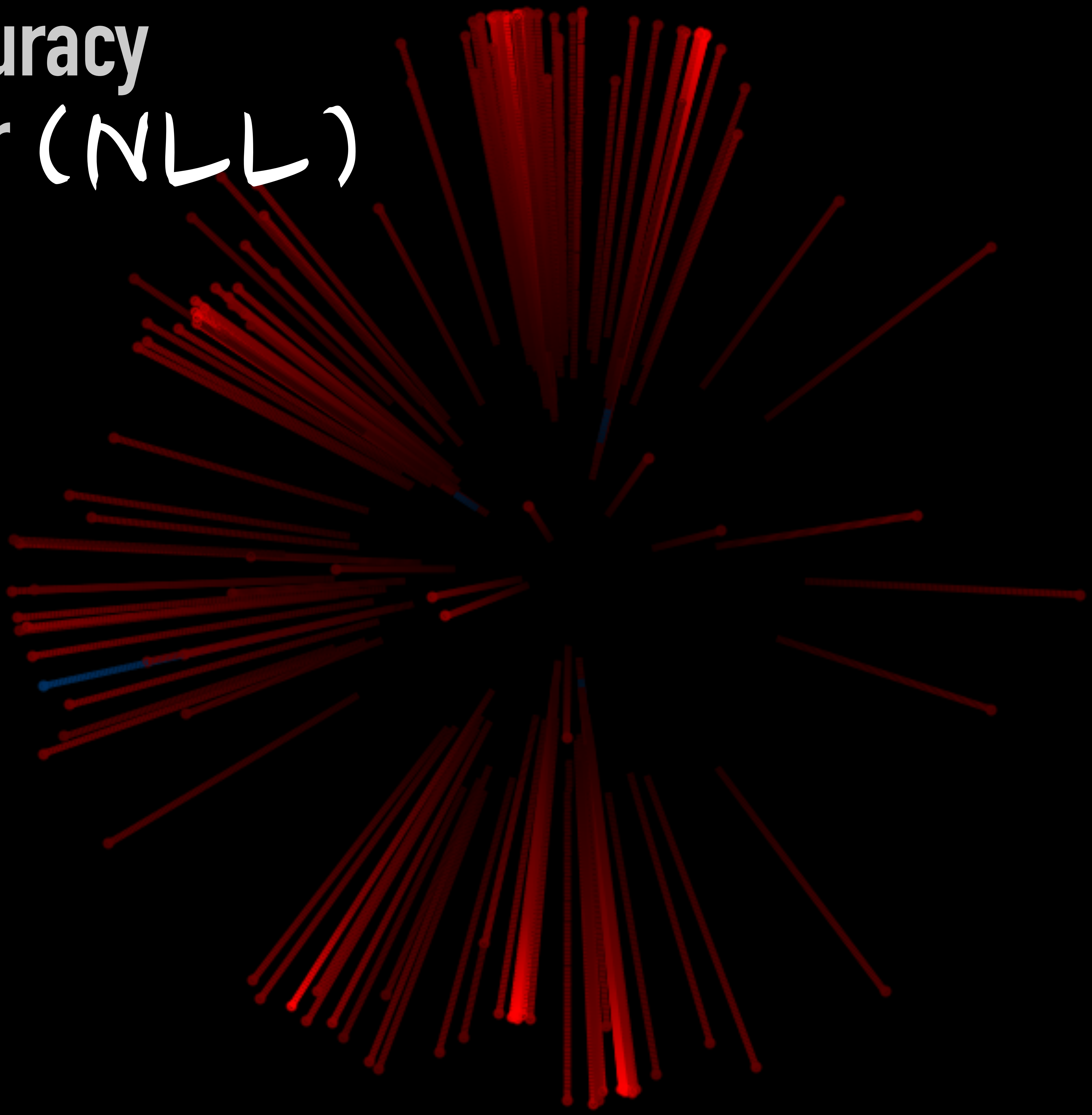

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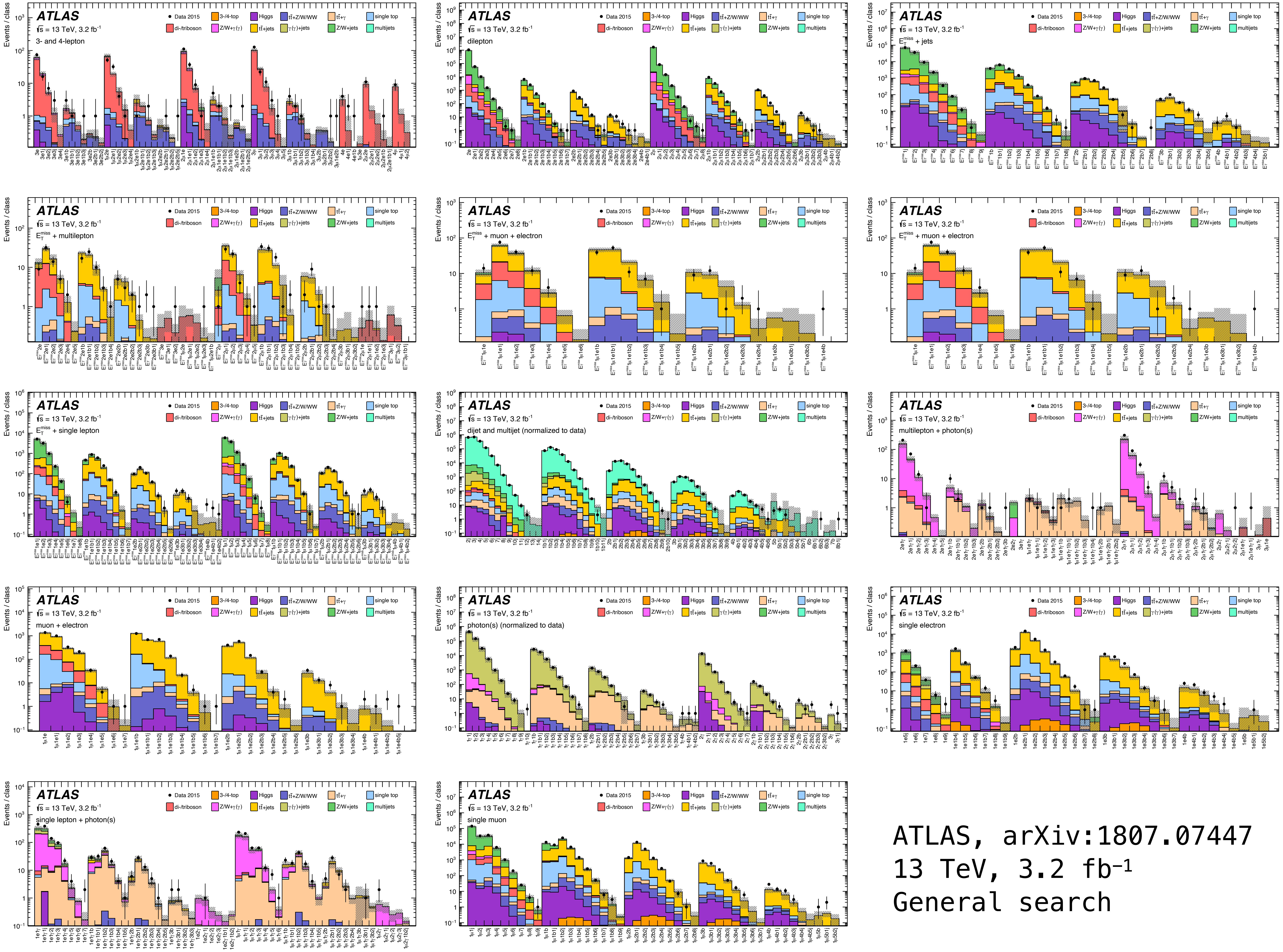
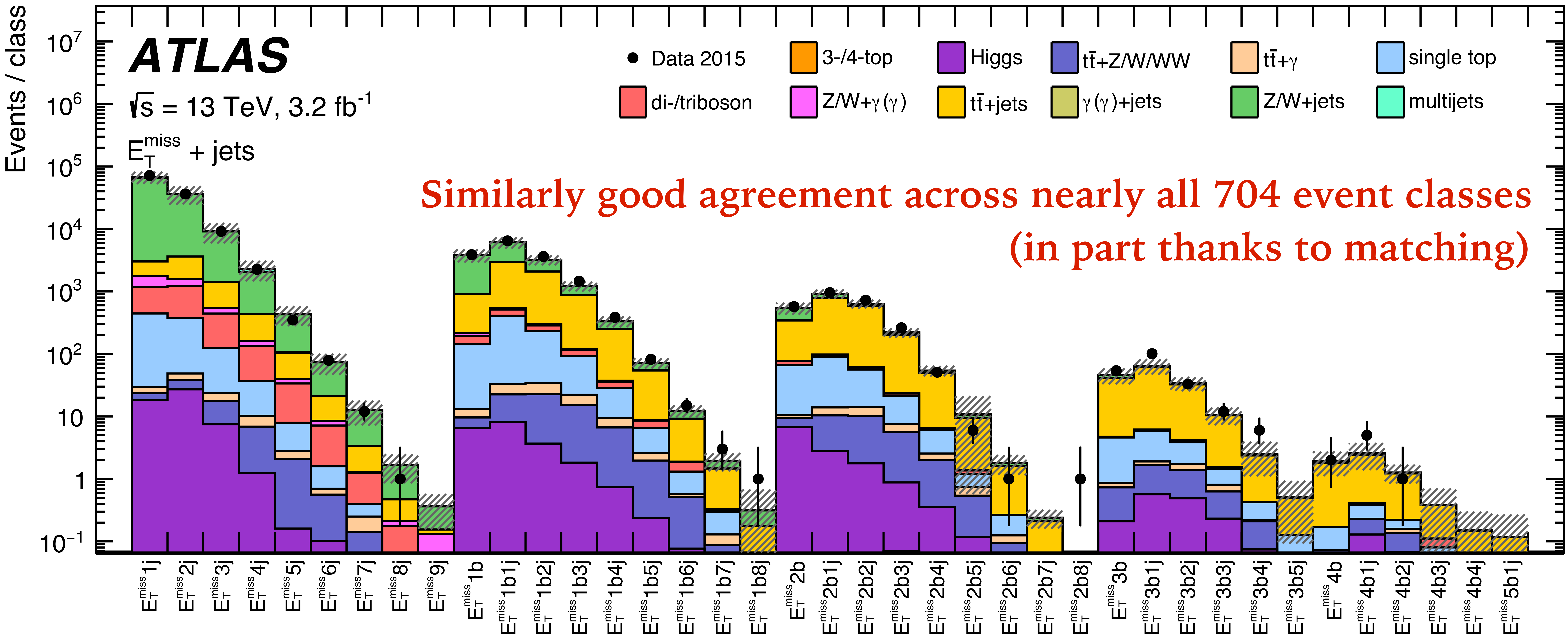
GPMCs and their parton showers are amazingly successful



A broadband search with 704 event classes

ATLAS, arXiv:1807.07447
13 TeV, 3.2 fb-1
General search

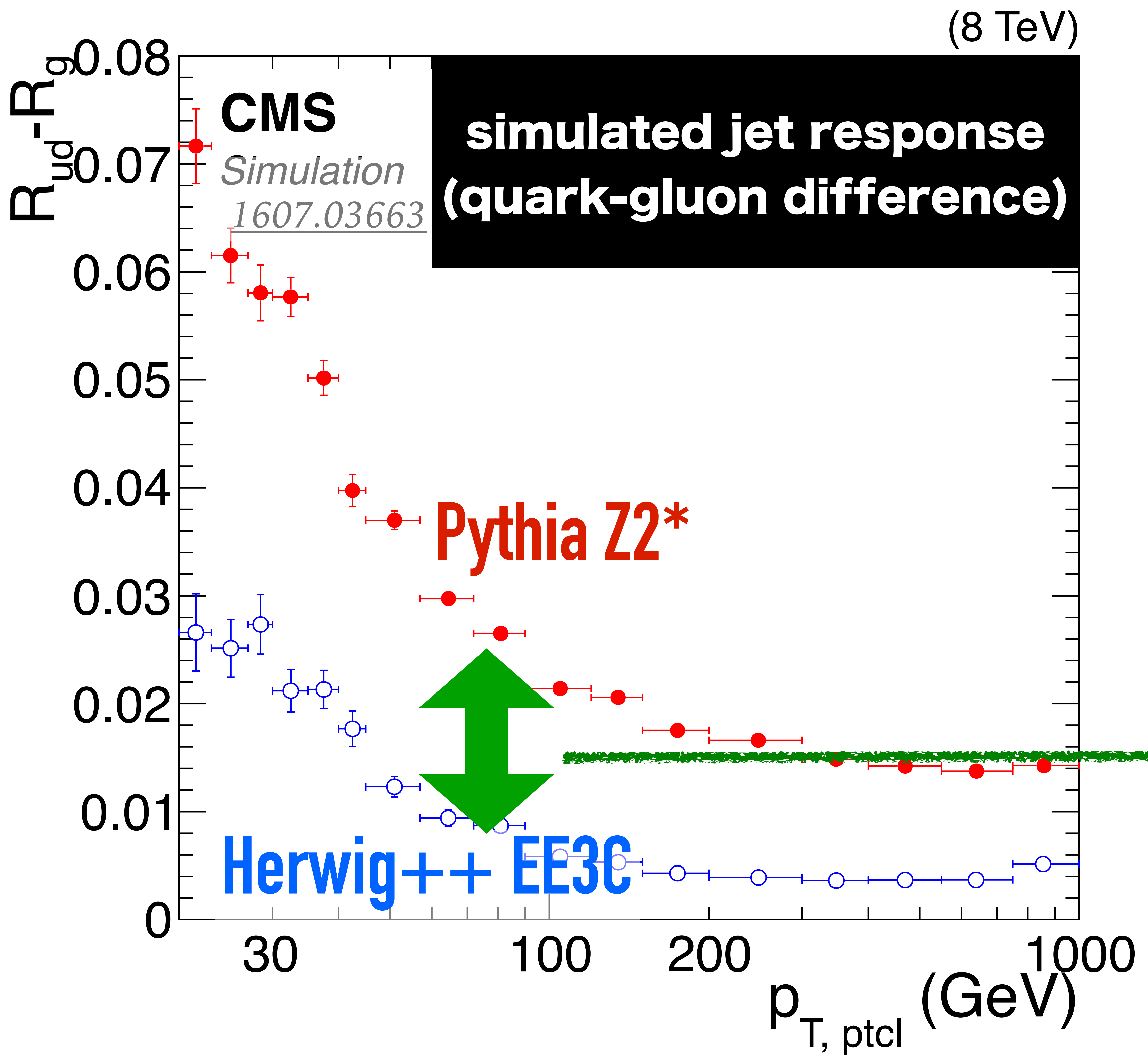
GPMCs and their parton showers are amazingly successful

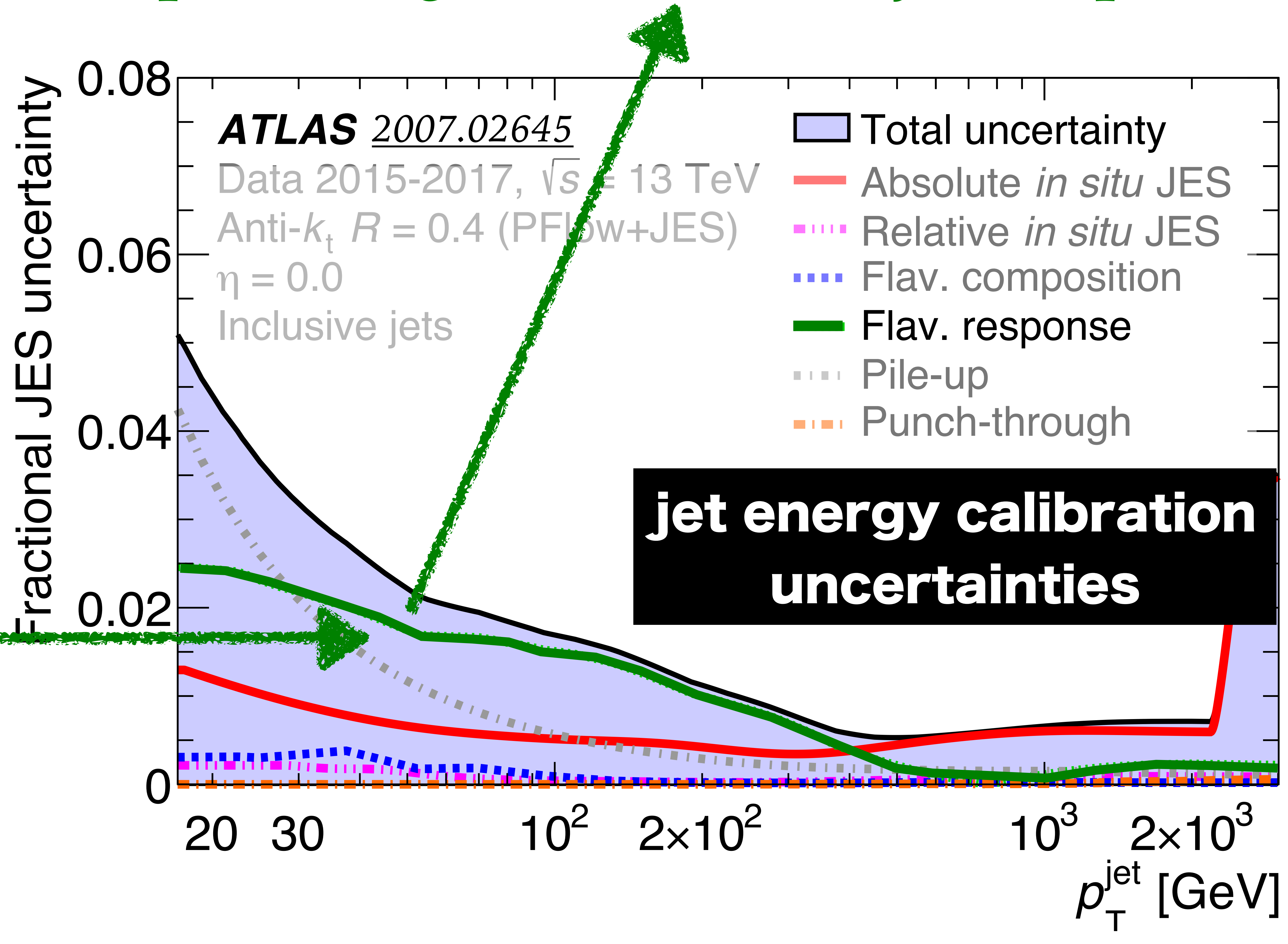


Similarly good agreement across nearly all 704 event classes (in part thanks to matching)

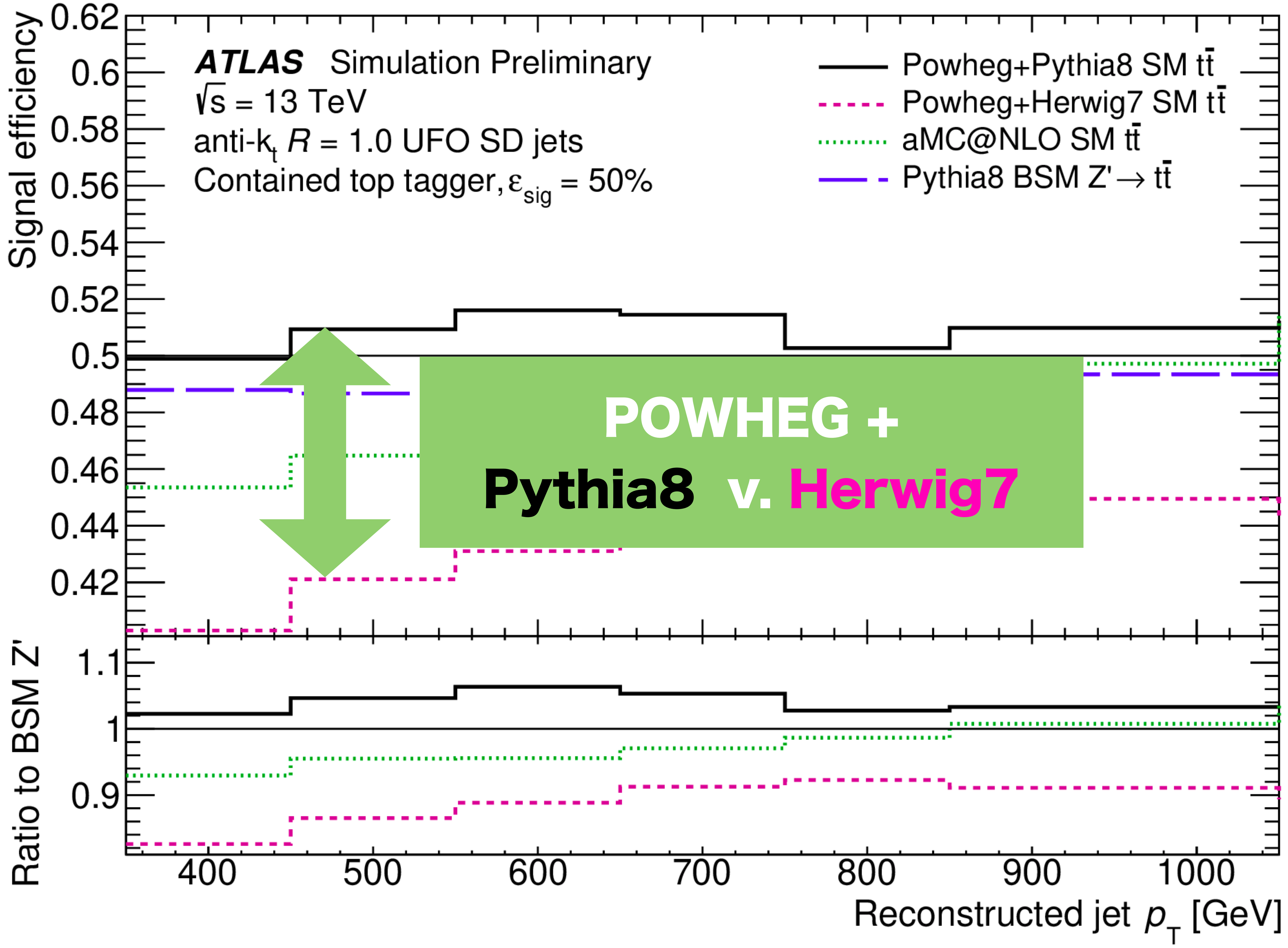
But imperfections matter: e.g. for jet energy calibration (affects ~1500 papers)

Largest uncertainty source is poor understanding of [parton shower simulations of] quark v. gluon-induced jet responses





High-p_t top tagging



signal efficiency

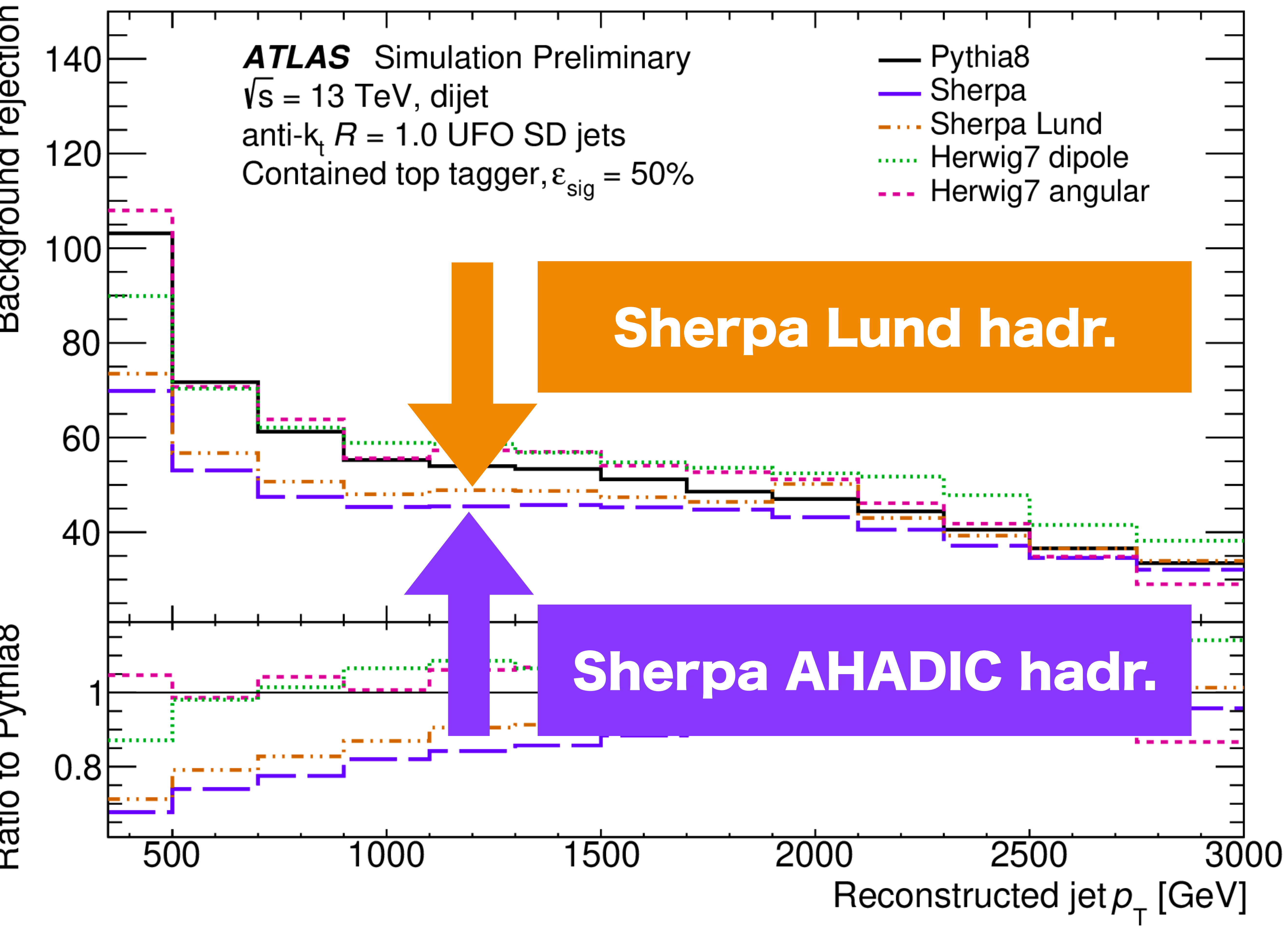
HL-LHC will produce ~10^5 top-pairs with p_t > 1 TeV (i.e. stat accuracy < 1%)

Yet top tagging efficiency has systematics ~ 10-15% today, driven by differences between showers

ATL-PHYS-PUB-2021-028

BOOST2021

High-p_t top tagging



background rejection

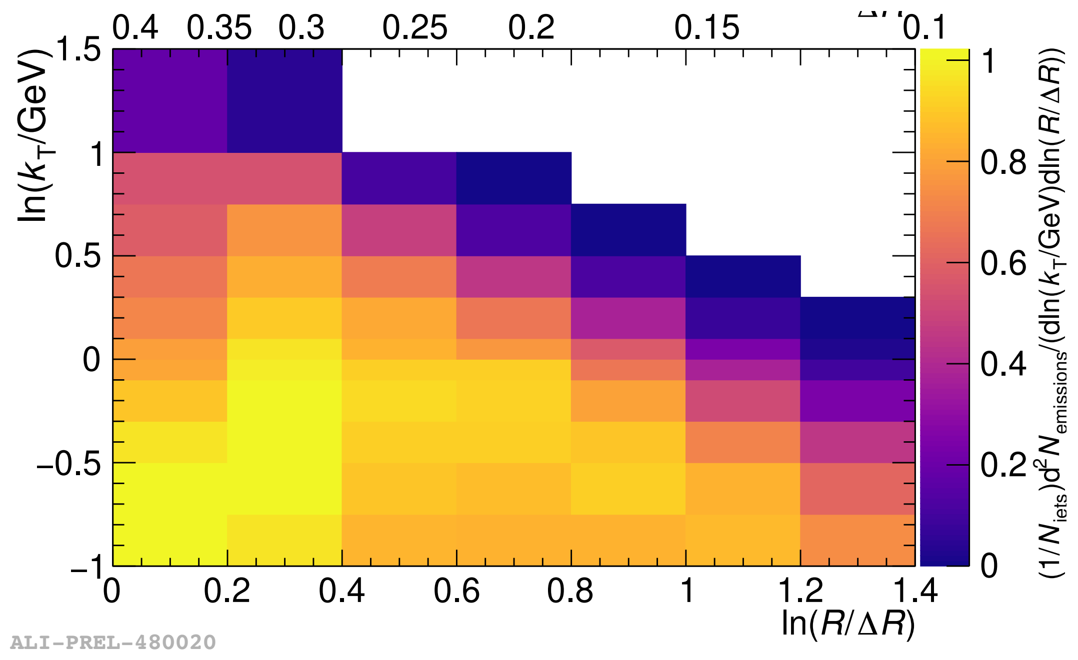
HL-LHC will produce ~10^5 top-pairs with p_t > 1 TeV (i.e. stat accuracy < 1%)

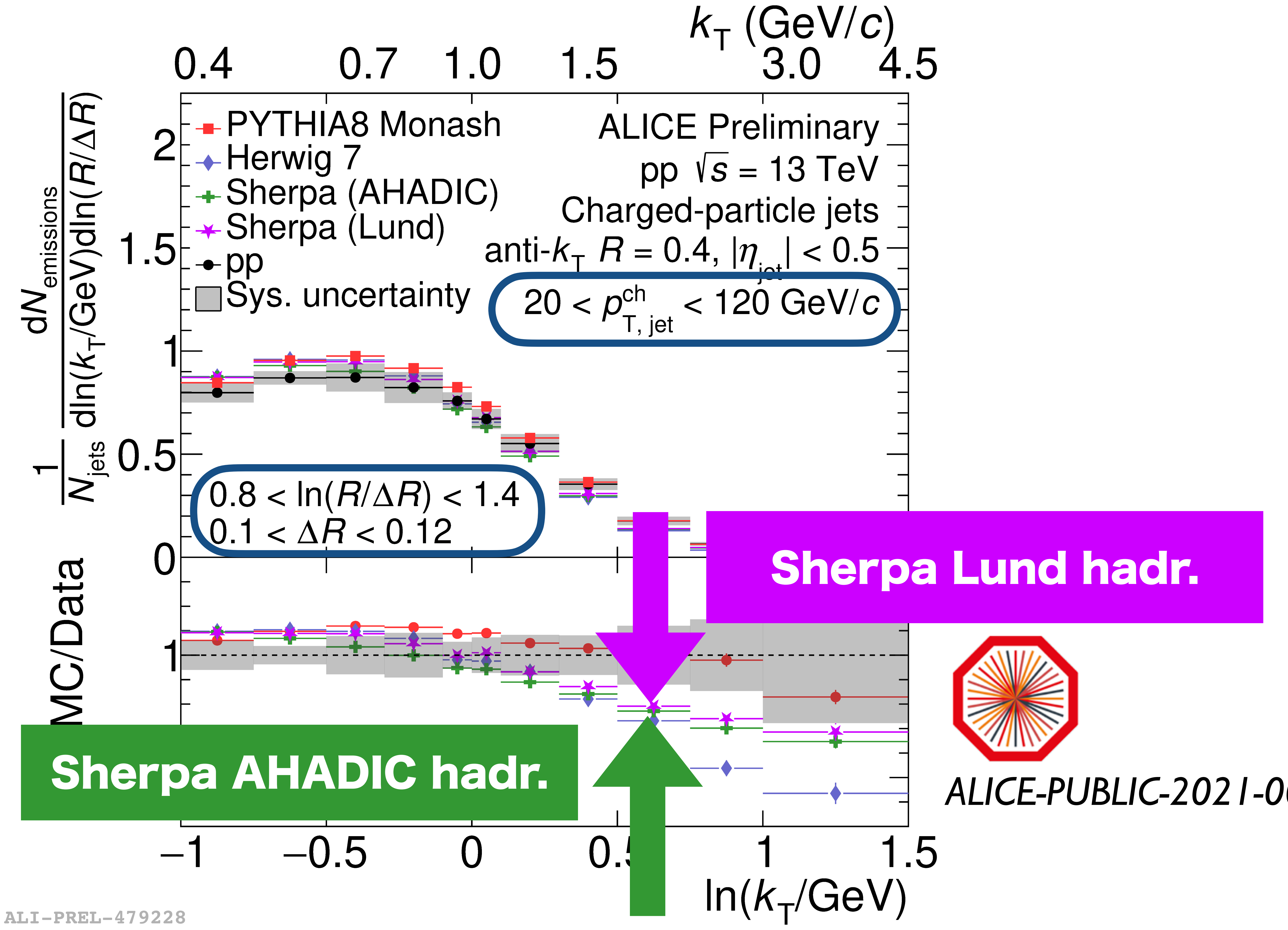
Yet top tagging efficiency has systematics ~ 10% today, driven by differences between showers

Differences are not necessarily affected by non-perturbative hadronisation model

ATL-PHYS-PUB-2021-028

Similar observations hold in low-k_t Lund-plane measurements





ALICE-PUBLIC-2021-002

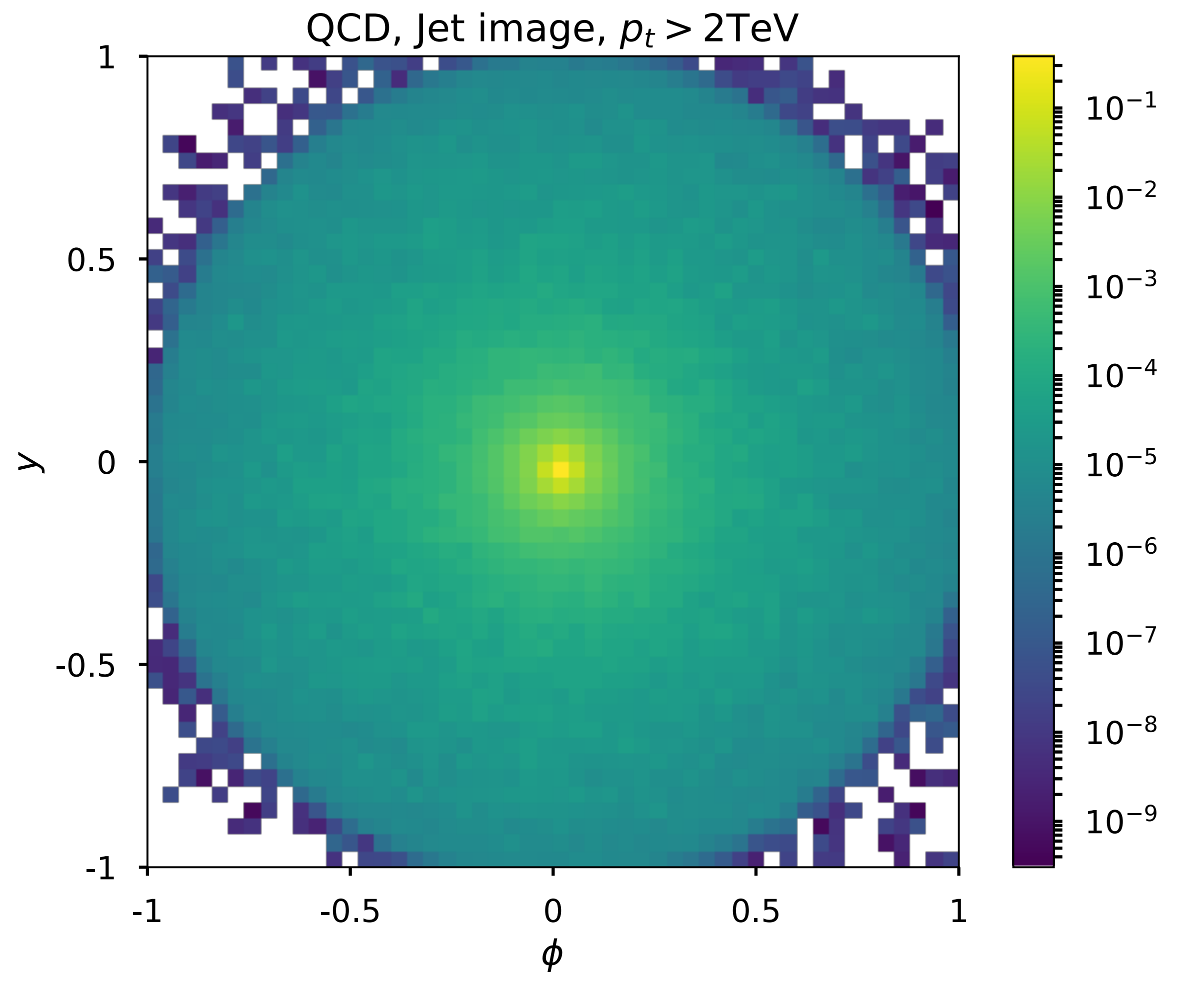
shower differences persist for k_t in range 1-5 GeV

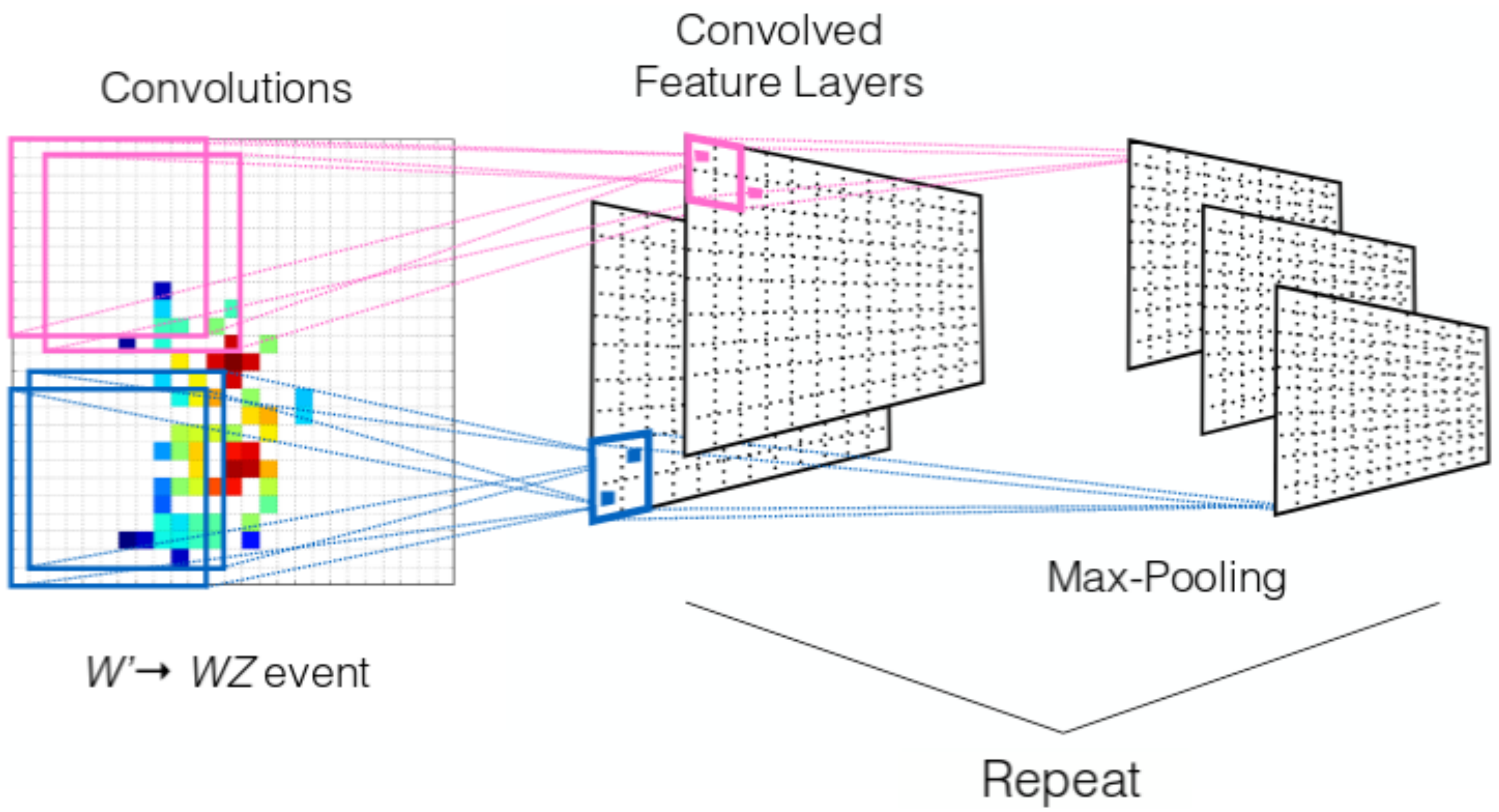
hadronisation has much smaller impact

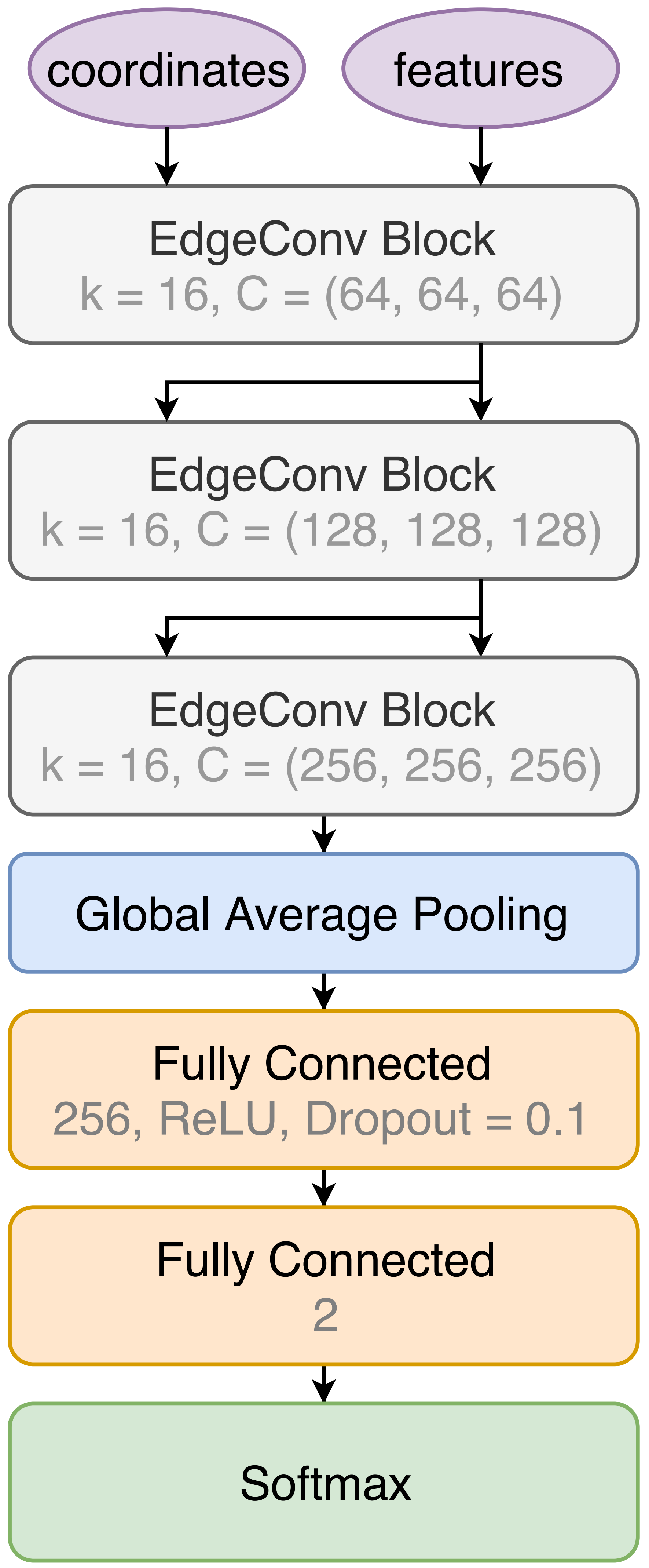
NB: this may not hold for other observables

Parton showers contain immense information accessible via ML

- Project a jet onto a fixed $n \times n$ pixel image in rapidity-azimuth, where each pixel intensity corresponds to the momentum of particles in that cell.
- Can be used as input for classification methods used in computer vision, such as deep convolutional neural networks.







(a) ParticleNet

[Cogan, Kagan, Strauss, Schwartzman JHEP 1502 (2015) 118]

[de Oliveira, Kagan, Mackey, Nachman, Schwartzman JHEP 1607 (2016) 069]

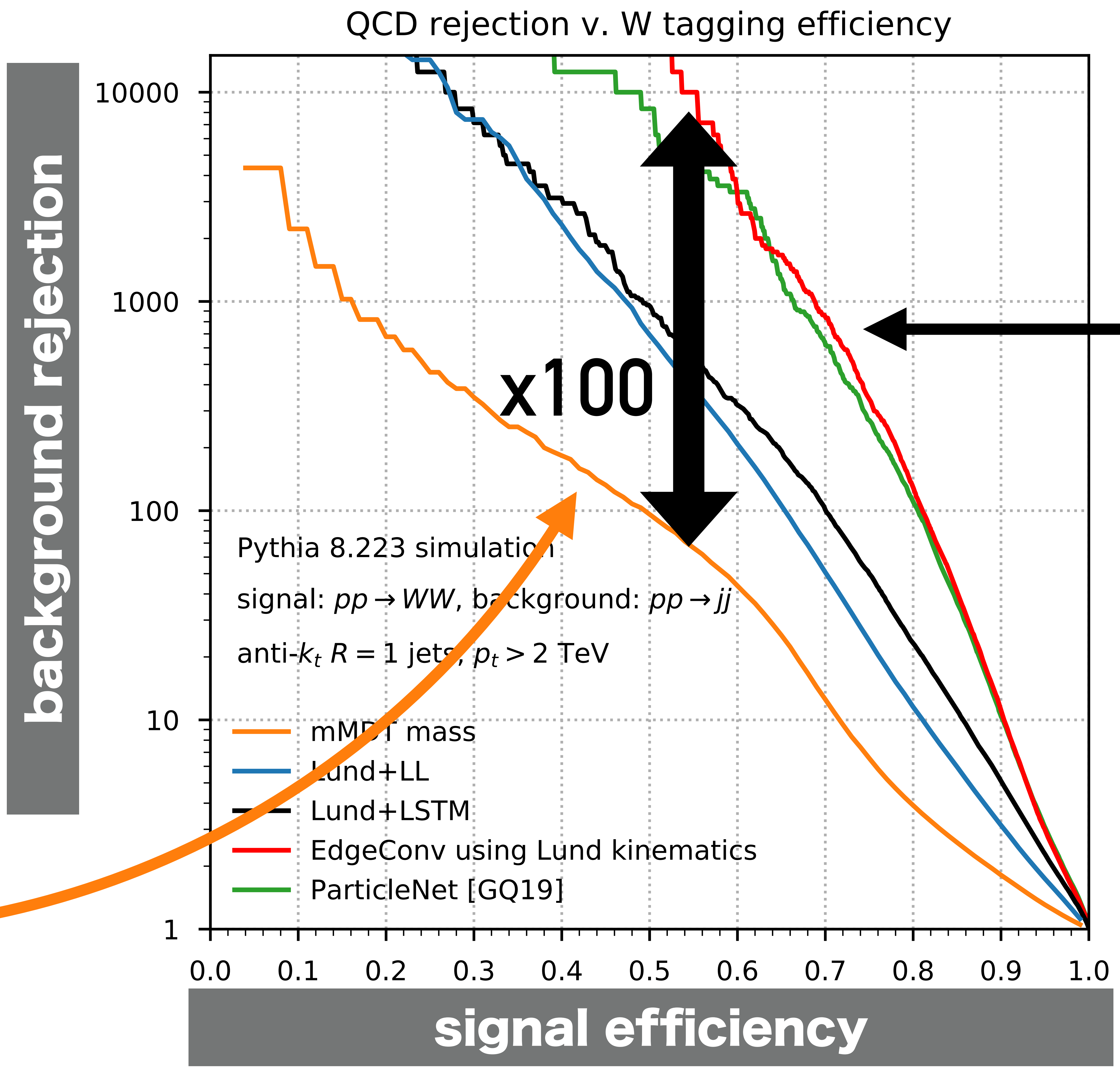
↓

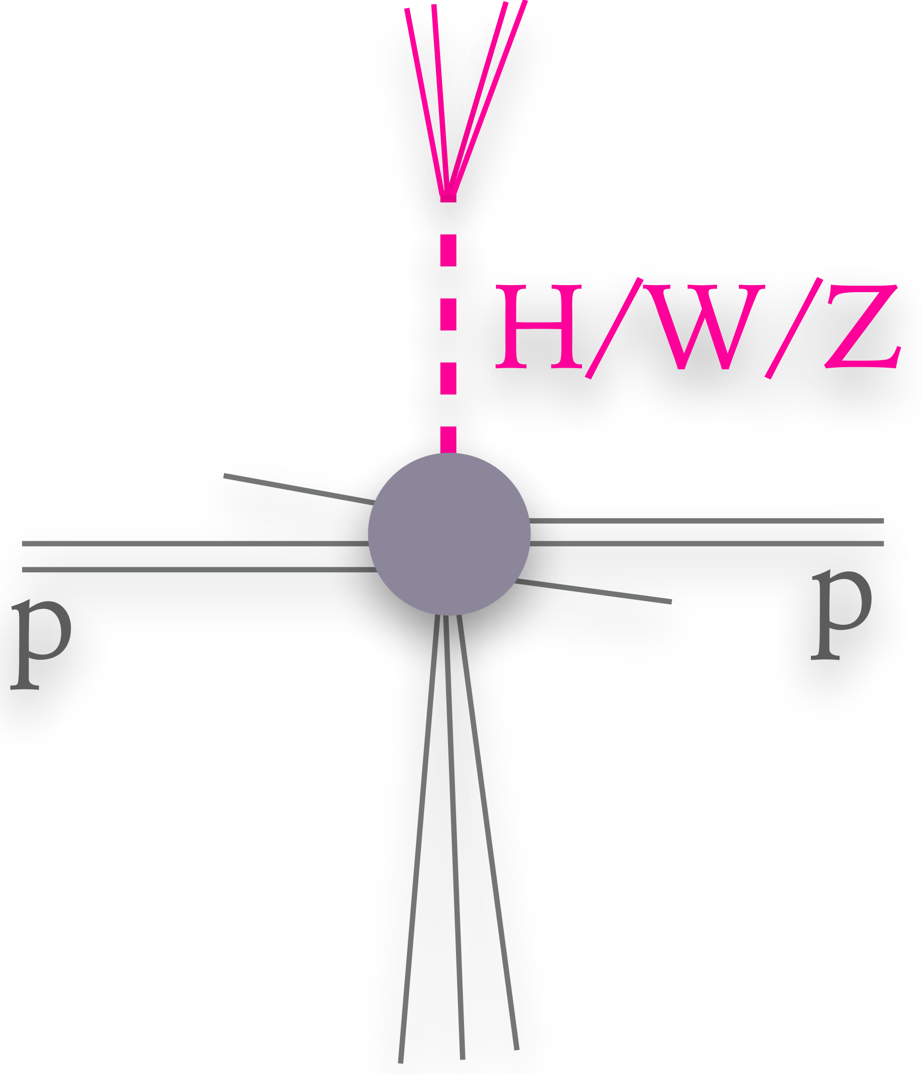
2021 Young Experimental Physicist Prize EPS HEPP prize

Qu & Guskos, arXiv:1902.08570

using full jet/event information for H/W/Z-boson tagging

adapted from Dreyer & Qu [2012.08526]





QCD rejection with just jet mass (SD/mMDT) i.e. 2008 tools & their 2013/14 descendants

QCD rejection with use of full jet substructure (2021 tools)

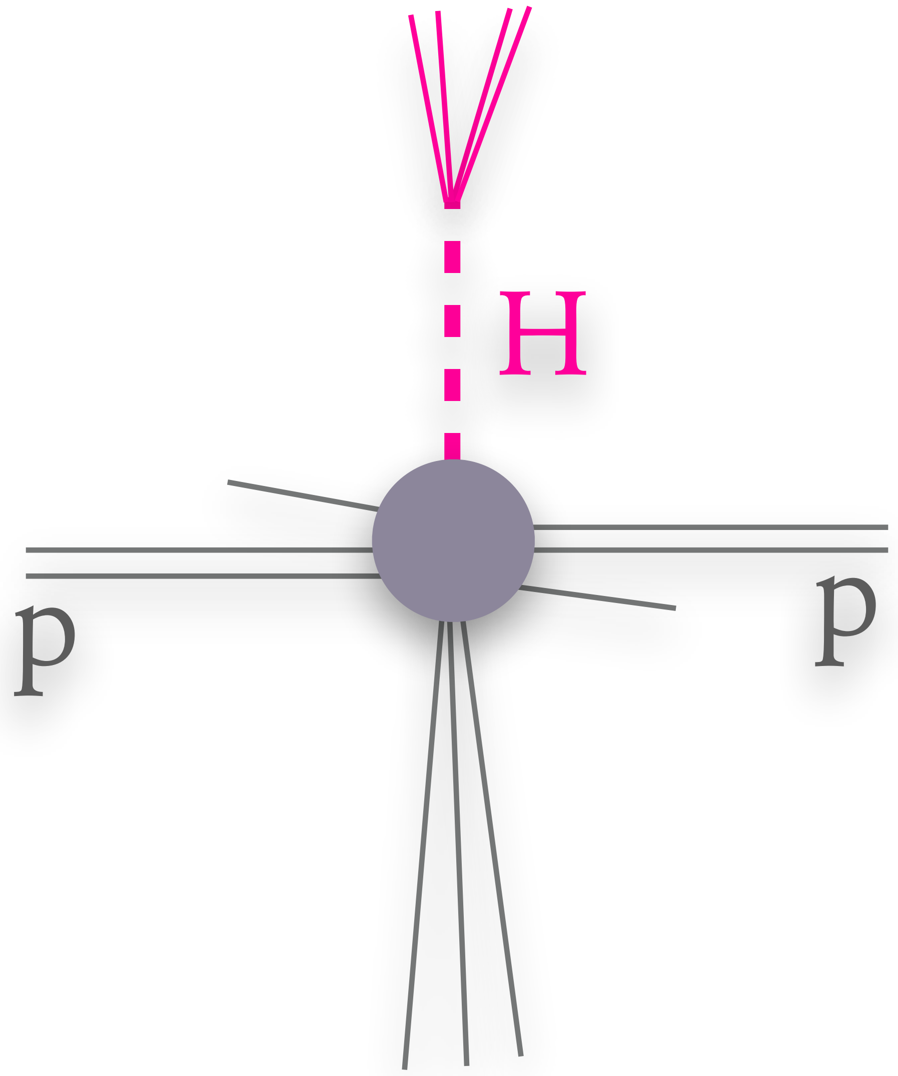
100x better

First started to be exploited by Thaler & Van Tilburg with "N-subjettiness" (2010/11)

high p_T Higgs & [SD] jet mass

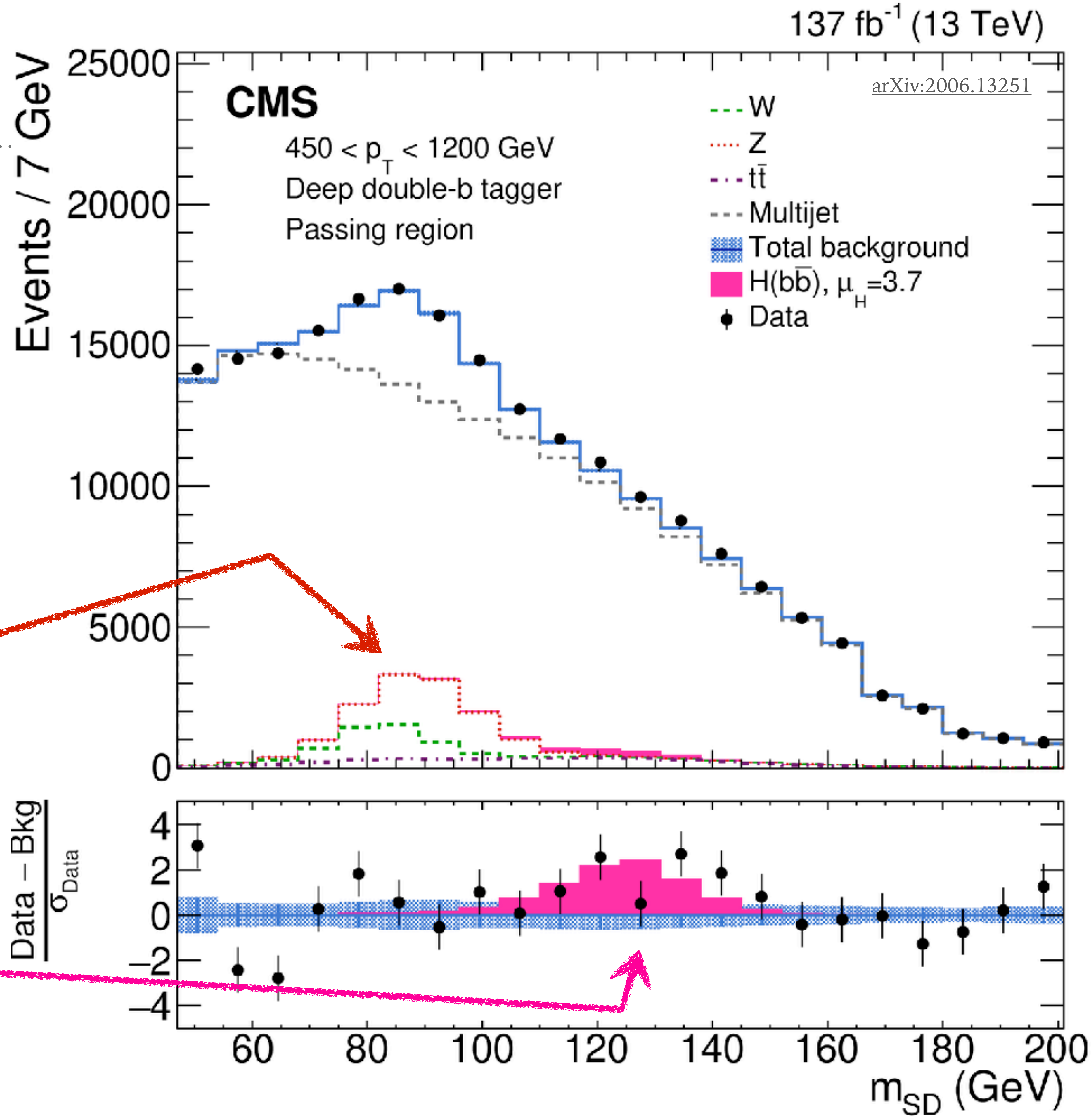
We wouldn't trust electromagnetism if we'd only tested it at one length/ momentum scale.

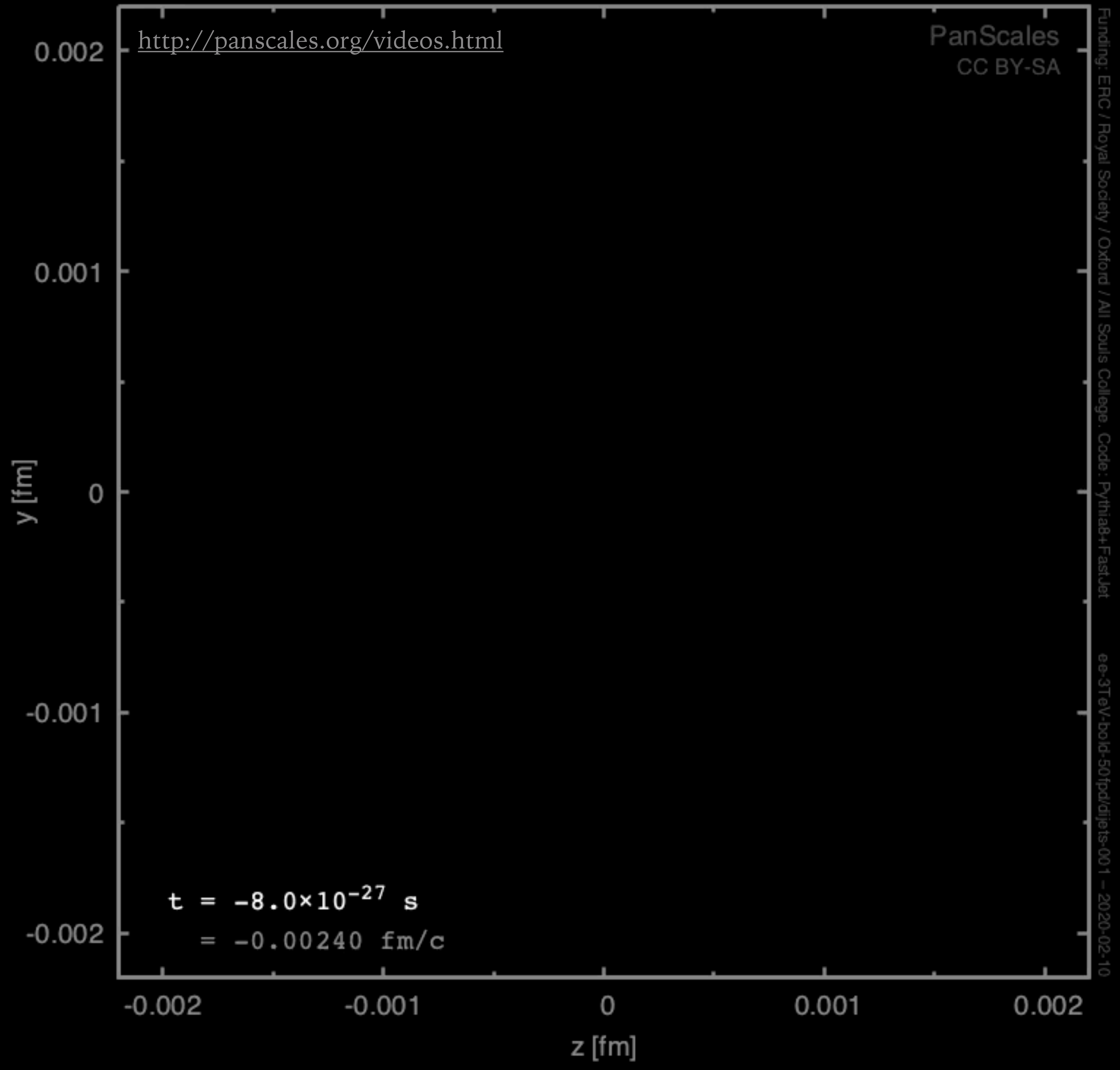
New Higgs interactions need testing at both low and (here) **high momenta.**



high-p_T
$Z \to bb$

high-p_T
$H \to bb$
(2.5 σ)





$$e^+e^- \to q\bar{q}, \sqrt{s} = 3\,\text{TeV}$$

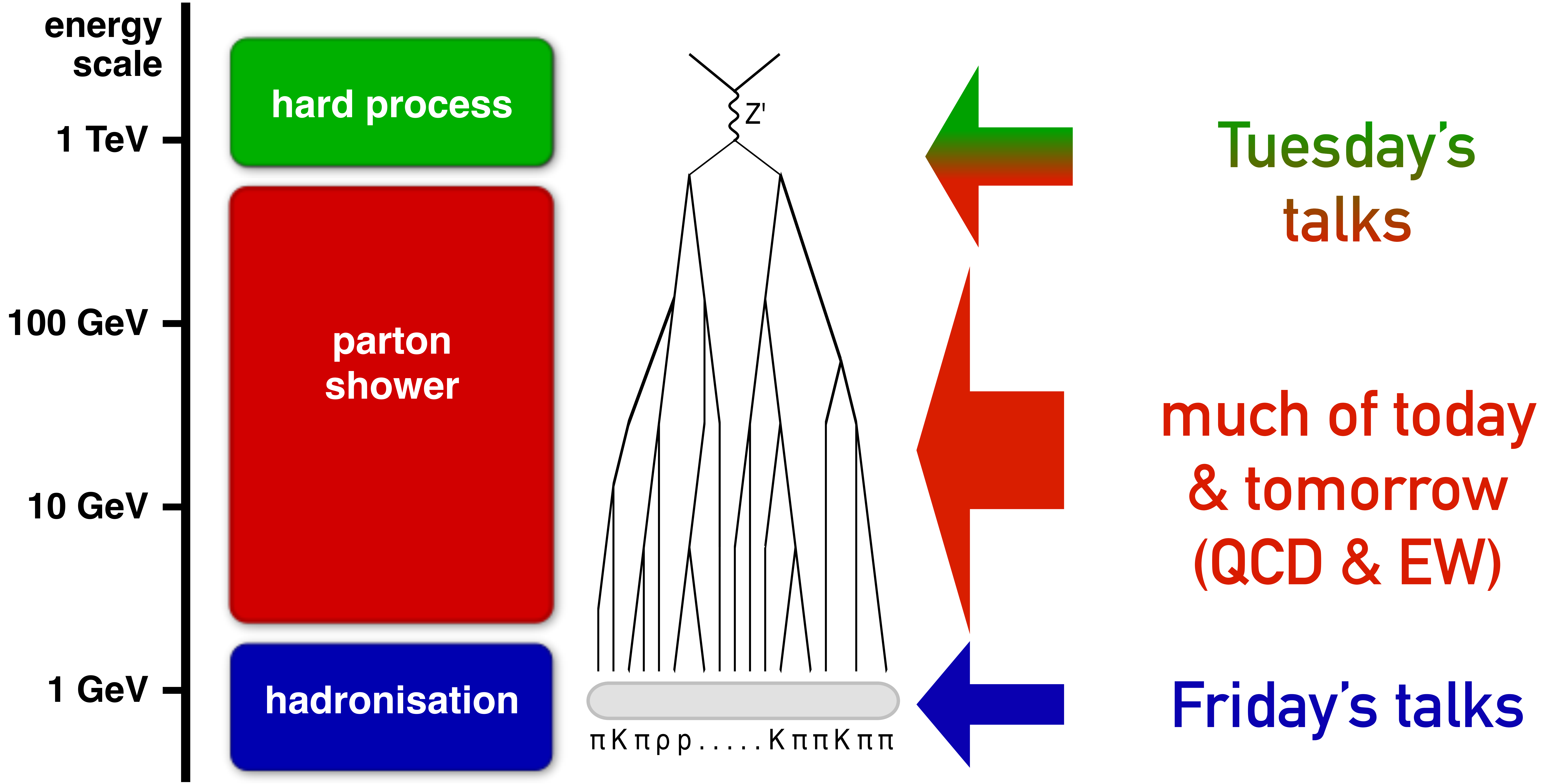
incoming beam particle

intermediate particle (quark or gluon)

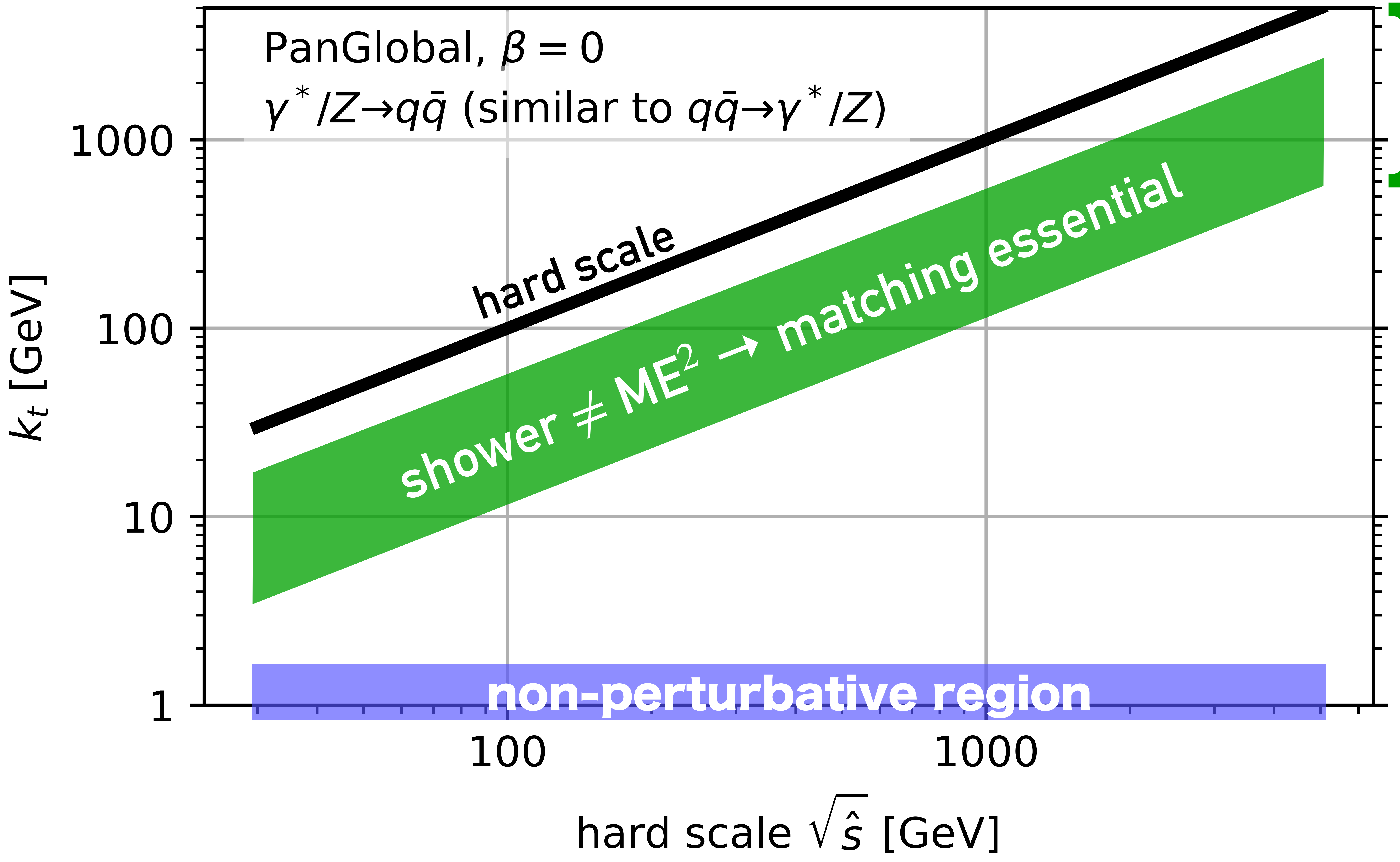
final particle (hadron)

Event evolution spans 7 orders of magnitude in space-time

[This is a Pythia8 event, reinterpreted as a time-sequence with gen-k_t (p=1/2) clustering]


energy scale
1 TeV
100 GeV
10 GeV
1 GeV
hard process
parton shower
hadronisation
Z'
π K π ρ p . . . . . K π π K π π
Tuesday's talks
much of today & tomorrow (QCD & EW)
Friday's talks

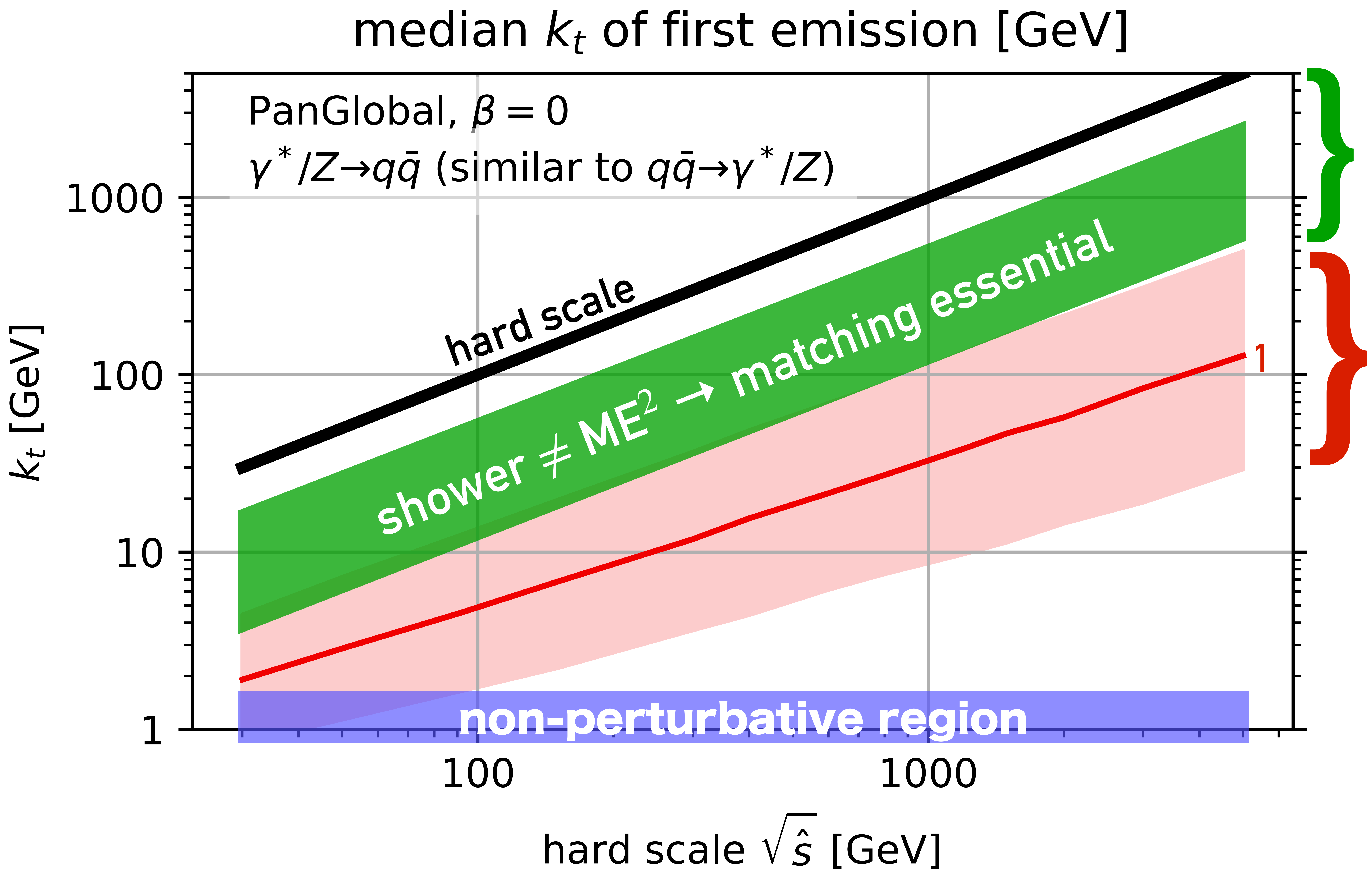
Where is shower accuracy useful / necessary?



Full matrix-element needed for $k_t \gtrsim 0.1\sqrt{\hat{s}}$?

It might be interesting to understand the scaling with k_t of (shower/ME^2 – 1).

Where is shower accuracy useful / necessary?

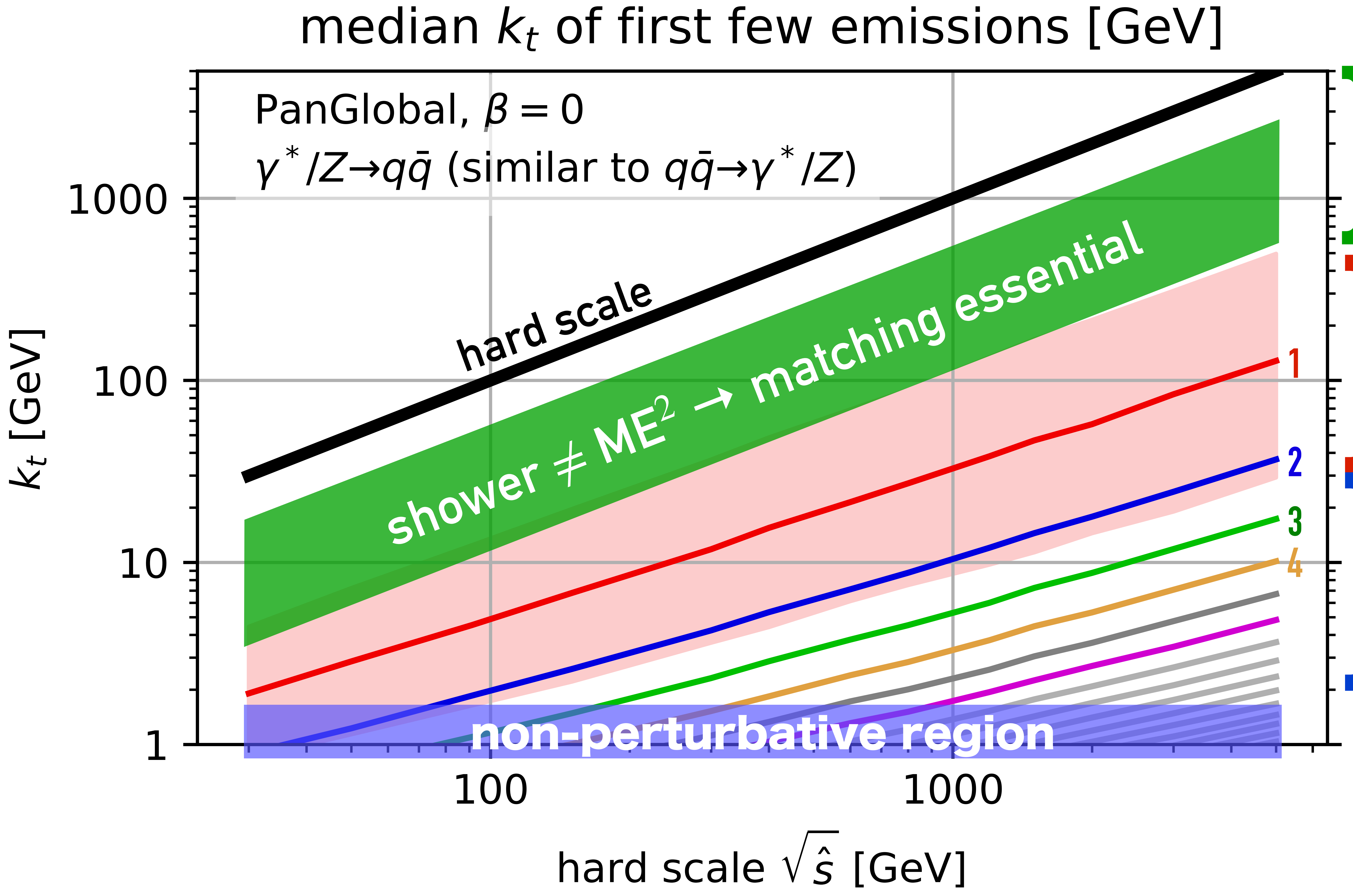


Full matrix-element needed for $k_t \gtrsim 0.1\sqrt{\hat{s}}$?

k_t of first emission (median and 68% interval)

That first emission often in a region where $k_t \ll \sqrt{\hat{s}}$ (i.e. a shower may be a good approx.)

Where is shower accuracy useful / necessary?



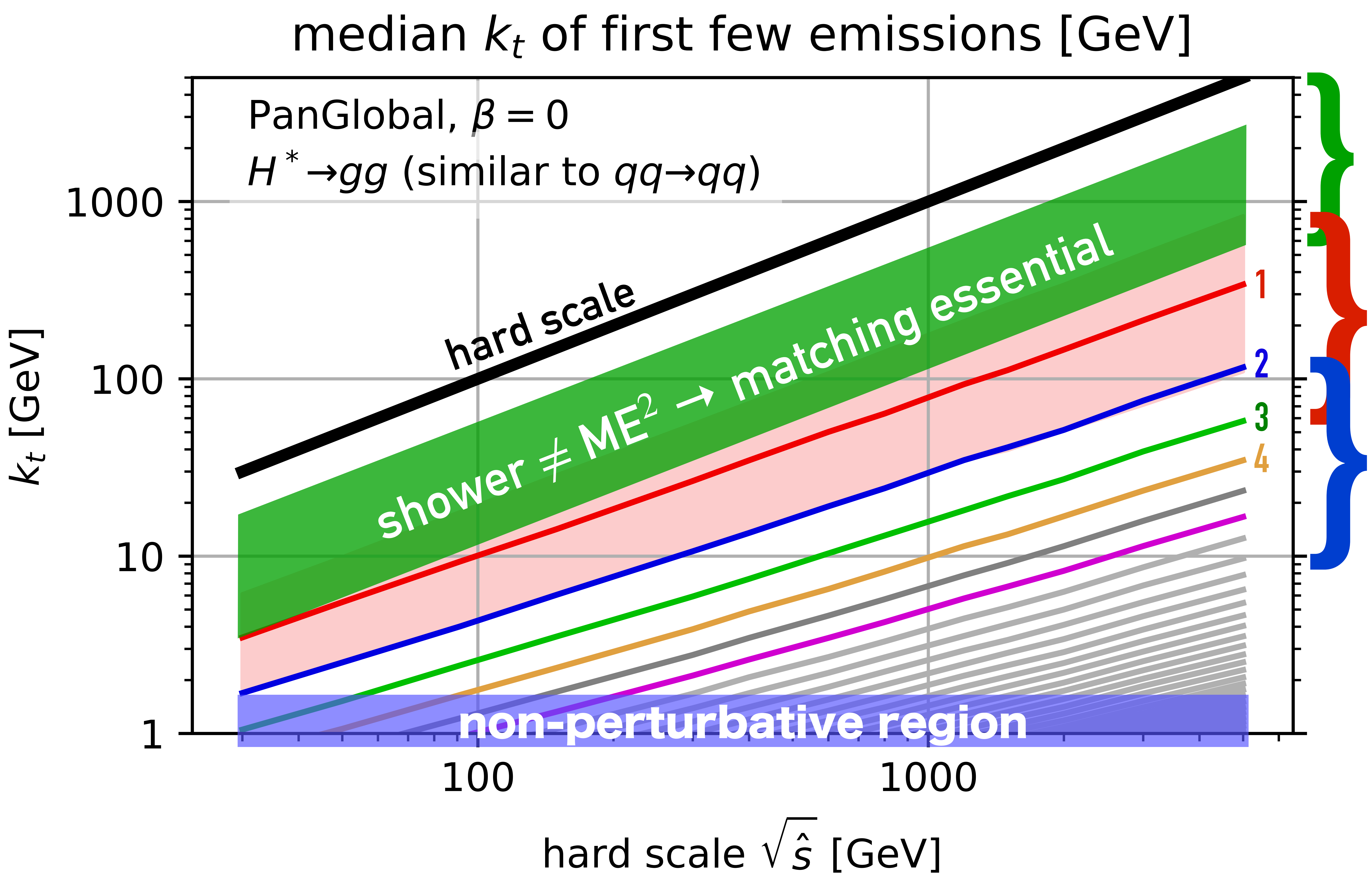
Full matrix-element needed for $k_t \gtrsim 0.1\sqrt{\hat{s}}$?

k_t of first emission (median and 68% interval)

median k_t of 2nd, 3rd, etc. emissions

the shower will be attempting to get all of these "right", together with the virtual corrections

Where is shower accuracy useful / necessary?



Full matrix-element needed for $k_t \gtrsim 0.1\sqrt{\hat{s}}$?

k_t of first emission (median and 68% interval)

median k_t of 2nd, 3rd, etc. emissions

the shower will be attempting to get all of these "right", together with the virtual corrections

what **should** a parton shower achieve?

not just a question of ingredients,
but also the final result of assembling them together

it's a complicated issue…

- For a total cross section, e.g. for Higgs production, it's easy to talk about systematic improvements (LO, NLO, NNLO, …). But they're restricted to that one observable
- With a parton shower (+hadronisation) you produce a "realistic" full set of particles. You can ask questions of arbitrary complexity:
  - **the multiplicity of particles**
  - **the total transverse momentum with respect to some axis (broadening)**
  - **the angle of 3rd most energetic particle relative to the most energetic one**

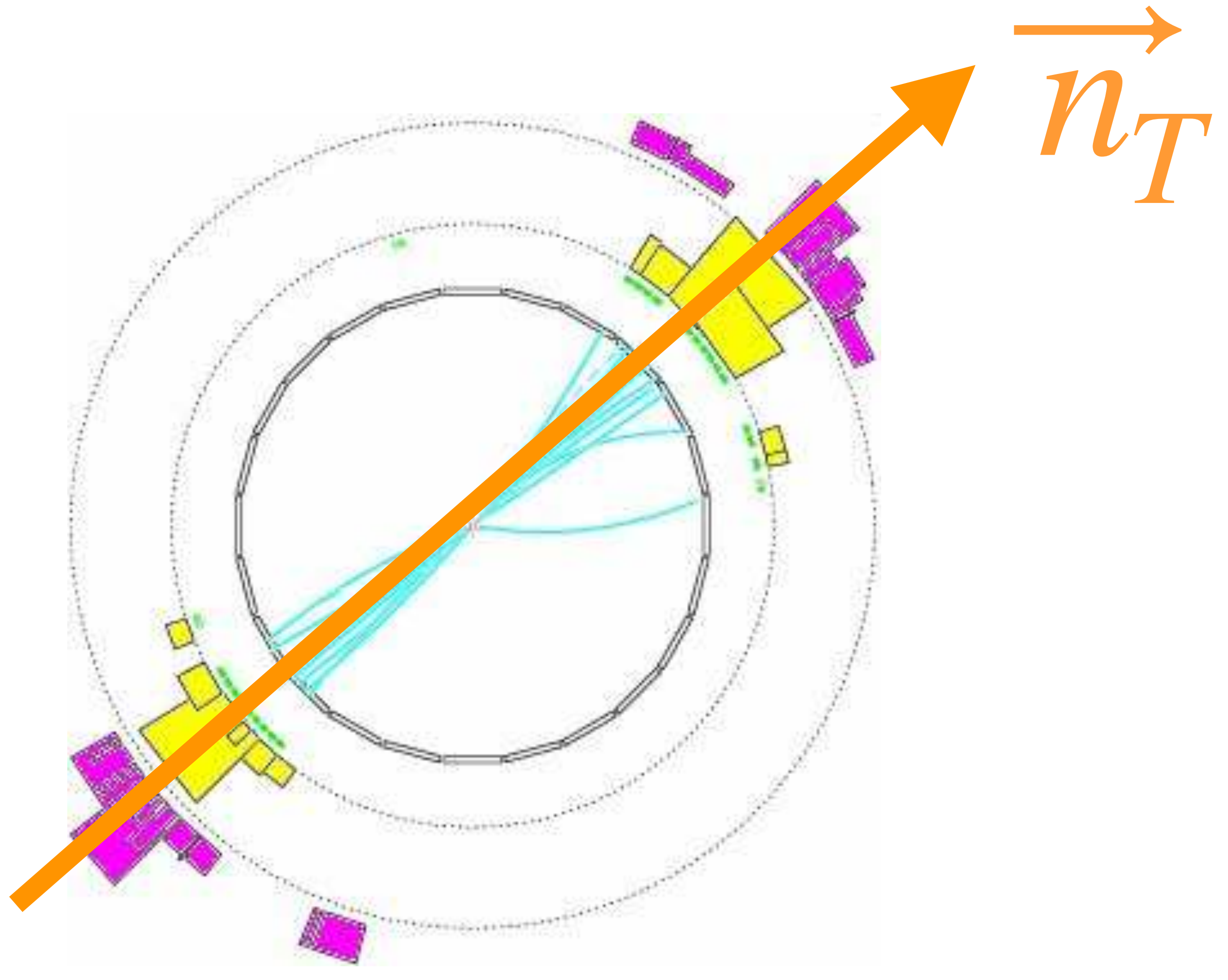
  [machine learning might "learn" many such features]

how can you prescribe correctness & accuracy of the answer, when the questions you ask can be arbitrary?

NLL means controlling O(1) terms

It's been common to hear that **showers are Leading Logarithmic (LL)** accurate.

That language, widespread for multiscale problems, comes from analytical resummations. E.g. transverse momentum broadening



$$B = \frac{\sum_i |\vec{p}_i \times \vec{n}_T|}{\sum_i |\vec{p}_i|}$$

You can resum cross section for B to be very small (as it is in most events)

$$\sigma(\ln B < -L) = \sigma_{tot} \exp\left[L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \alpha_s^2 g_4(\alpha_s L) + \cdots\right]$$

$[\alpha_s \ll 1, \alpha_s L \sim 1]$

LL ~ $O(\frac{1}{\alpha})$ NLL ~ $O(1)$ NNLL ~ $O(\alpha)$ N^3LL ~ $O(\alpha^2)$

Thrust: Catani, Trentadue, Turnock & Webber '93 → NLL

Thrust: Becher & Schwartz '08 → N^3LL

PanScales proposal for investigating shower accuracy

Resummation

Establish logarithmic accuracy for main classes of resummation:

- global event shapes (thrust, broadening, angularities, jet rates, energy-energy correlations, …)
- non-global observables (cf. Banfi, Corcella & Dasgupta, hep-ph/0612282)
- fragmentation / parton-distribution functions
- multiplicity, cf. original Herwig angular-ordered shower from 1980's

Matrix elements

Establish in what sense iteration of (e.g. 2→3) splitting kernel reproduces N-particle tree-level matrix elements *for any N*.
Because this kind of info is exploited by machine-learning algorithms.

I view this as a working proposal, rather than the ultimate classification

Baseline "NLL" requirements

Aim for NLL, control of $\alpha_s^n L^n$

Aim for NDL, i.e. $\alpha_s^n L^{2n-1}$

Aim for correctness when all particles well separated in Lund diagram

Some core principles for NLL showers

1. for a new emission k, when it is generated far in the Lund diagram from any other emission ($|d_{ki}^{Lund}| \gg 1$), **it should not modify the kinematics (Lund coordinates) of any preceding emission** by more than an amount $\exp(-p\,|d_{ki}^{Lund}|)$, where $p = \mathcal{O}(1)$

2. when k is distant from other emissions, generate it with matrix element and phasespace (and associated Sudakov)

$$\frac{d\Phi_k}{d\Phi_{k-1}} \frac{|M_{1\ldots k}|^2}{|M_{1\ldots(k-1)}|^2}$$

[simple forms known from factorisation properties of matrix-elements]

3. emission k **should not impact $d\Phi \times |M|^2$ ratio for subsequent distant emissions *unless***

   a. they are at commensurate angle (or on k's Lund "leaf"), or

   b. k was a hard collinear splitting, which can affect other hard collinear splittings (cross-talk on same leaf ≡ DGLAP, cross-talk on other leaves ≡ spin correlations)

candidate NLL final-state showers

PanScales, FHP & Deductor

all based on colour dipoles

all split the dipoles ~ in event centre-of-mass

[other dipole/antenna showers split in dipole centre-of-mass]

Deductor	FHP	PanLocal	PanGlobal
$k_t\theta$ ("Λ") ordered	k_t ordered	$k_t\sqrt{\theta}$ ordered	k_t or $k_t\sqrt{\theta}$ ordered
Recoil	**Recoil**	**Recoil**	**Recoil**
$\perp$: local	$\perp$: global	$\perp$: local	$\perp$: global
+: local	+: local	+: local	+: local
–: global	–: global	–: local	–: local
Tests	**Tests**	**Tests**	**Tests**
analytical / numerical for thrust	analytical for thrust & multiplicity	numerical for many observables	numerical for many observables

Nagy & Soper <u>*2011.04777*</u> *(+past decade)*

Forshaw, Holguin & Plätzer <u>*2003.06400*</u>

Dasgupta, Dreyer, Hamilton, Monni, GPS & Soyez <u>*2002.11114*</u>

Deductor: thrust checks (numerics at 2nd & 3rd order + all-order analytics)

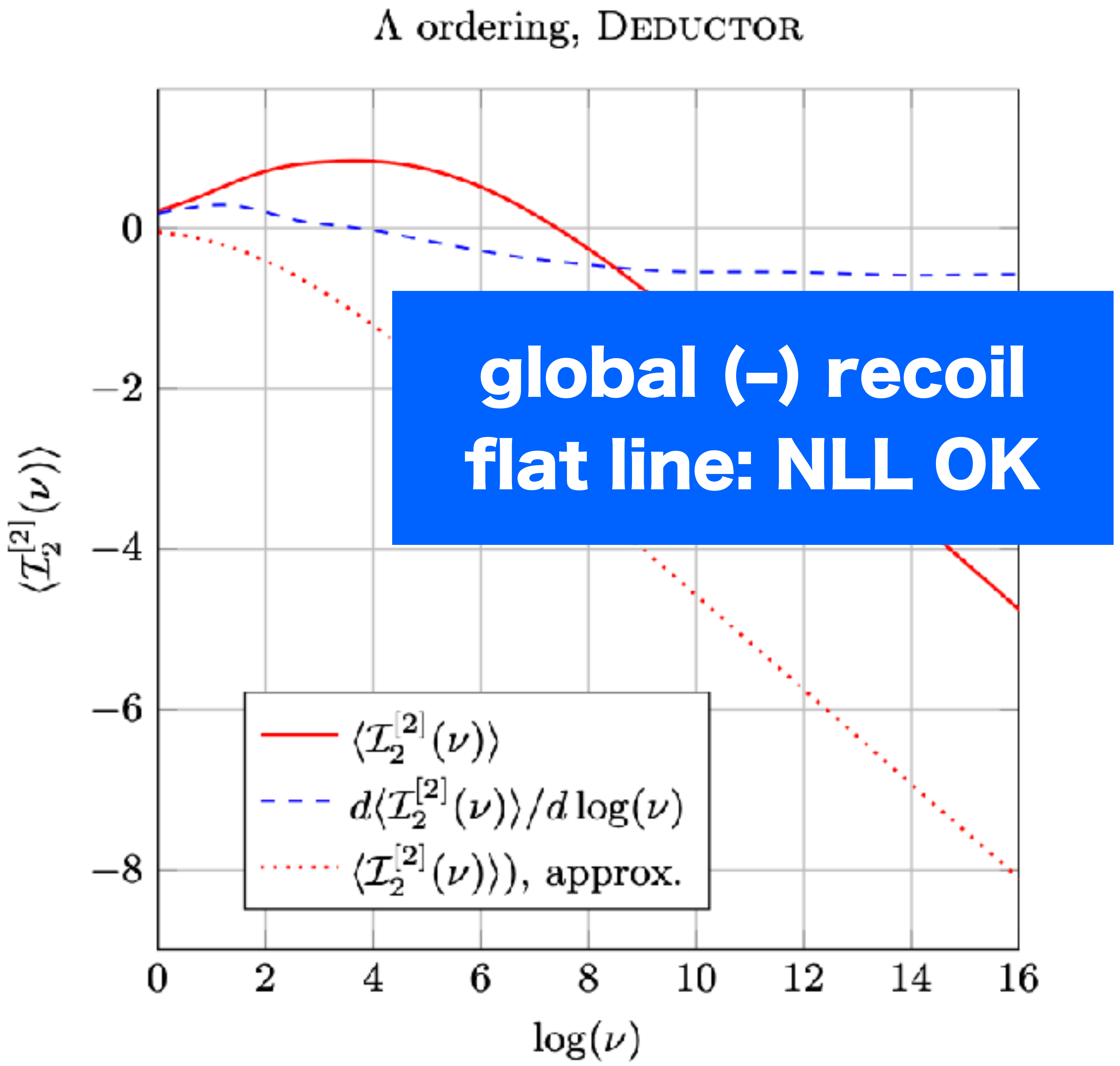


FIG. 1. Plot of $\langle \mathcal{I}_2^{[2]}(\nu)\rangle$, Eqs. (151) and (152), versus $\log(\nu)$ (solid red curve). For large $\log(\nu)$ the graph is approximately a straight line, corresponding to only one factor of $\log(\nu)$, indicating that the shower generates $\langle \mathcal{I}_2^{[2]}(\nu)\rangle$ at NLL accuracy. The dashed blue curve is $d\langle \mathcal{I}_2^{[2]}(\nu)\rangle/d\log(\nu)$. The dotted red curve shows an approximate version of $\langle \mathcal{I}_2^{[2]}(\nu)\rangle$ described in the text.

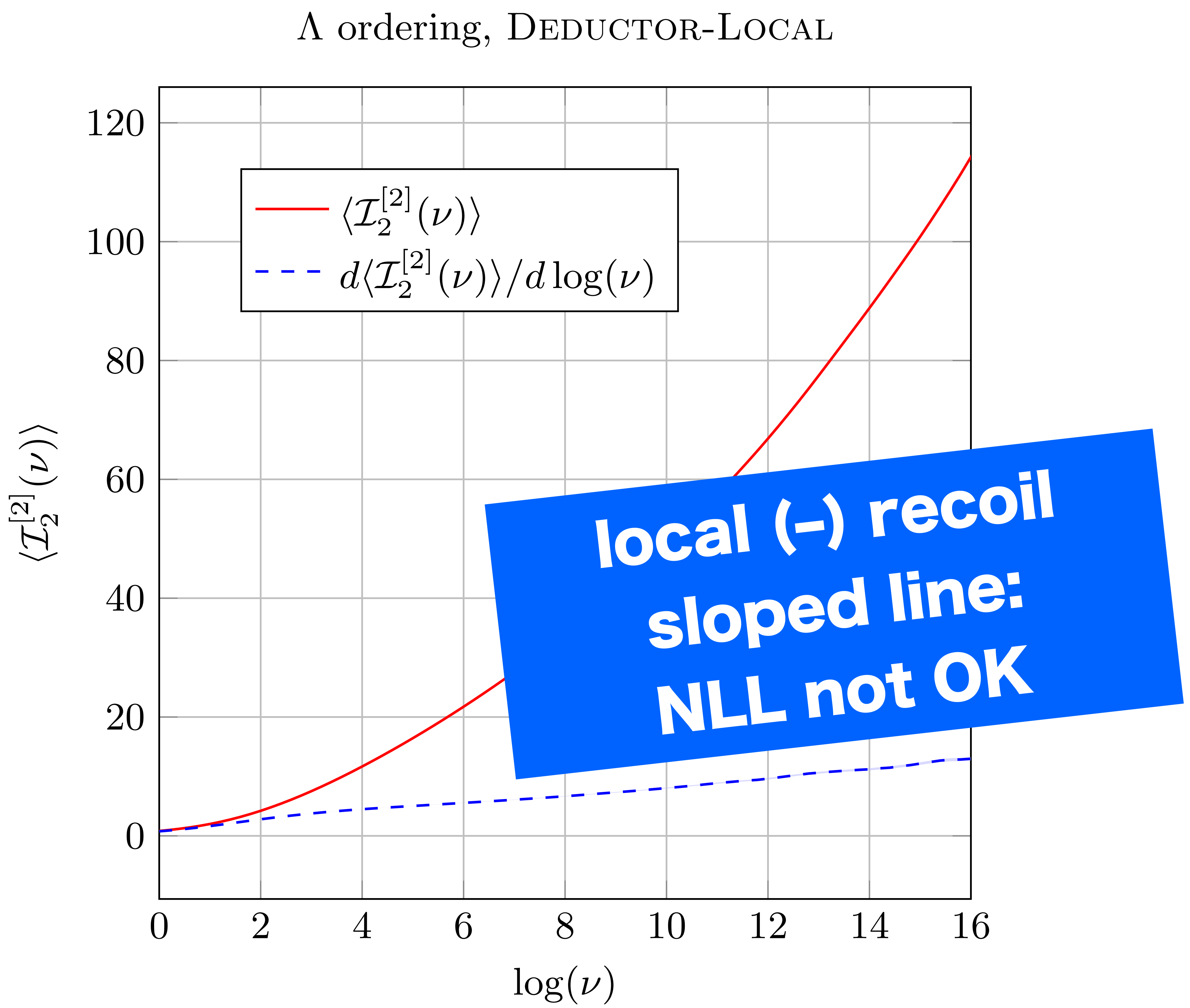


FIG. 9. Plot of $\langle \mathcal{I}_2^{[2]}(\nu)\rangle$, as in Fig. 1, for the DEDUCTOR splitting functions with the Catani-Seymour local momentum mapping [23]. $\langle \mathcal{I}_2^{[2]}(\nu)\rangle$ is approximately quadratic in $\log(\nu)$, indicating that $\mathcal{I}_2^{[2]}(\nu)$ that changes the NLL result.

Nagy & Soper, 2011.04777

FHP: analytic checks

Forshaw, Holguin & Plätzer 2003.06400

thrust

[NB: formulas here show NDL rather than NLL]
[multiplicity is only known to NDL]

$$\delta\Sigma(L) \lesssim \sum_{n=2}^{\infty} \frac{\alpha_s^n}{(2n-2)!} \left(\sum_{i=0}^{2n-2} \boxed{\tilde{A}_{i,n} \ln(1-T)^{2n-2-i} \mathrm{Li}_{2+i}\left(\frac{(1-T)\epsilon}{2}\right)} + \tilde{B}_n \mathrm{Li}_{2n}\left(\frac{\epsilon}{2}\right) \right), \quad \text{(D.8)}$$

subjet multiplicity

$$\begin{aligned}\phi_q(u,Q) =& \phi_q(u,q_{\perp 1})\Delta_q(q_{\perp 1},Q) \\ &+ \frac{\alpha_s}{2\pi}\int_{q_{\perp 1}}^{Q} \frac{dq_\perp}{q_\perp} \Delta_q(q_\perp,Q) \int_{\frac{q_\perp}{2Q}}^{1-\frac{q_\perp}{2Q}} dz\, \mathcal{P}_{qq}(z)\, \tilde{\phi}_q(u,q_\perp)\tilde{\phi}_g(u,q_\perp). \end{aligned} \quad \text{(D.17)}$$

This expression is correct at LL accuracy with complete colour and only requires the coupling to run as $\alpha_s(z(1-z)q_\perp)$ in order to capture the full NLL ($\alpha_s^n L^{2n-1}$) result. We

PanScales showers: all-order $\alpha_s \to 0$ limits

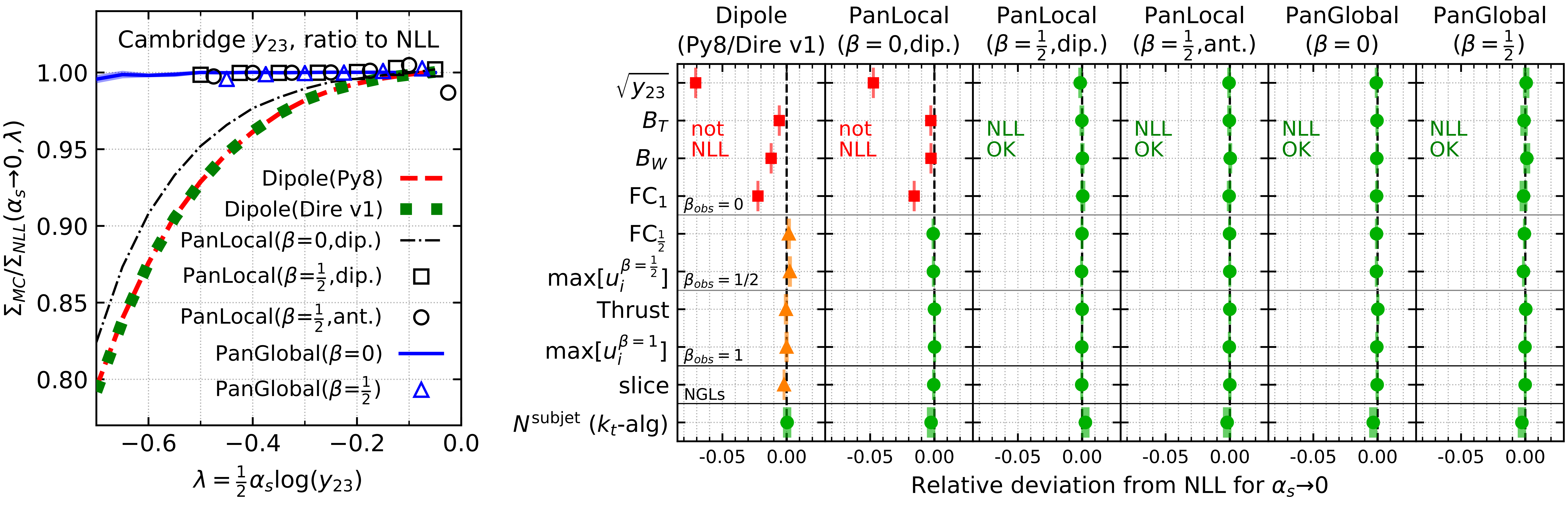


FIG. 2. Left: ratio of the cumulative y_{23} distribution from several showers divided by the NLL answer, as a function of $\alpha_s \ln y_{23}/2$, for $\alpha_s \to 0$. Right: summary of deviations from NLL for many shower/observable combinations (either $\Sigma_{\rm shower}(\alpha_s \to 0, \alpha_s L = -0.5)/\Sigma_{\rm NLL} - 1$ or $(N^{\rm subjet}_{\rm shower}(\alpha_s \to 0, \alpha_s L^2 = 5)/N^{\rm subjet}_{\rm NLL} - 1)/\sqrt{\alpha_s}$). Red squares indicate clear NLL failure; amber triangles indicate NLL fixed-order failure that is masked at all orders; green circles indicate that all NLL tests passed.

Dasgupta, Dreyer, Hamilton, Monni, GPS & Soyez 2002.11114

Herwig angular-ordered showers

Logarithmic Accuracy of Angular-Ordered Parton Showers 1904.11866

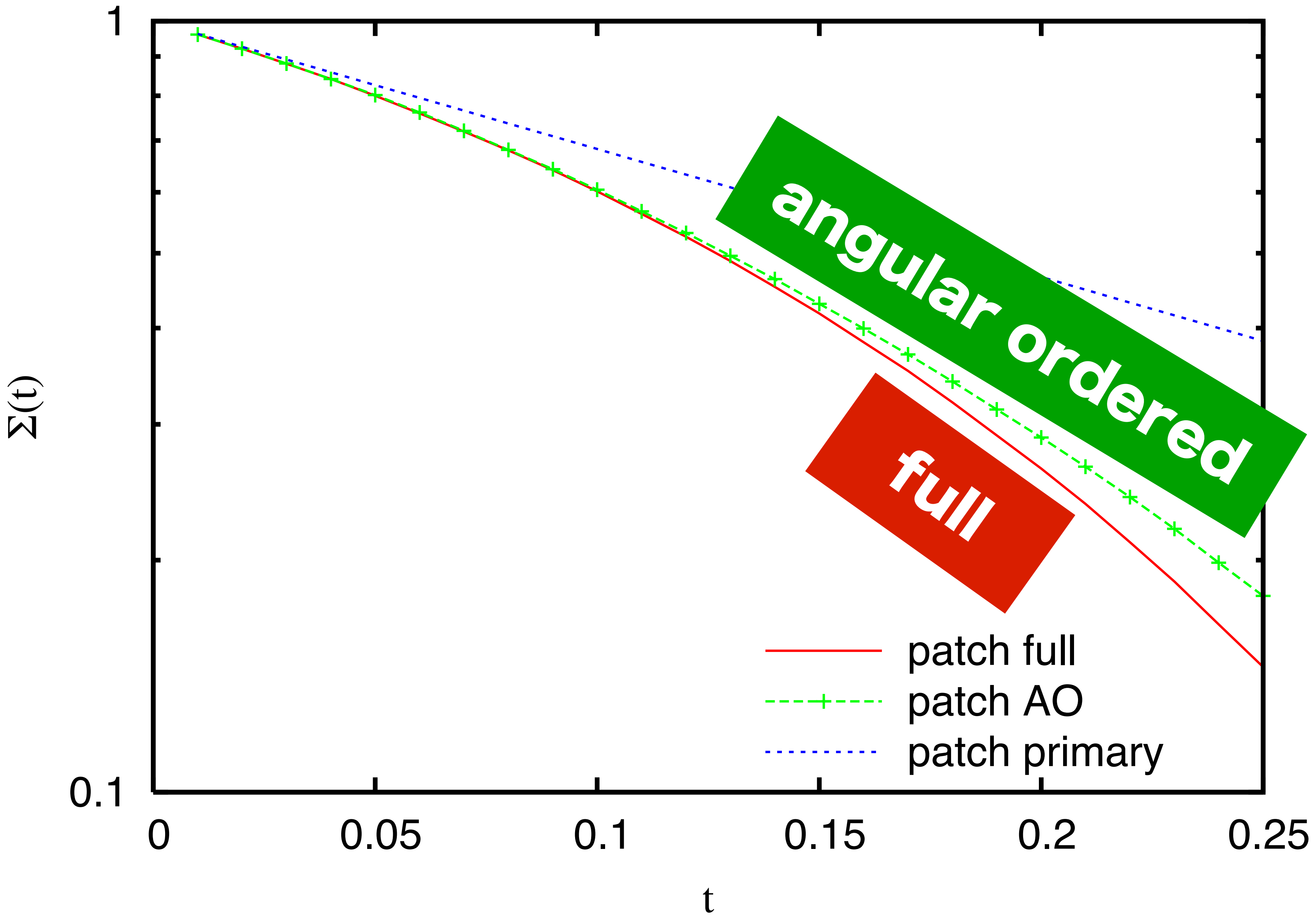
Initial State Radiation in the Herwig 7 Angular-Ordered Parton Shower 2107.04051

Gavin Bewick[a] **Silvia Ferrario Ravasio**[a,b] **Peter Richardson**[a,c] **Michael H. Seymour**[d]

Angular ordered showers can't get exact non-global logarithms (with ideas so far), but numerically not too bad an approximation; it seems conceivable they do get everything else right at NLL/NDL — and they have the advantage of being available in Herwig & tuned. Should they be the interim go-to "almost" NLL shower?

Banfi, Corcella, Dasgupta, hep-ph/0612282

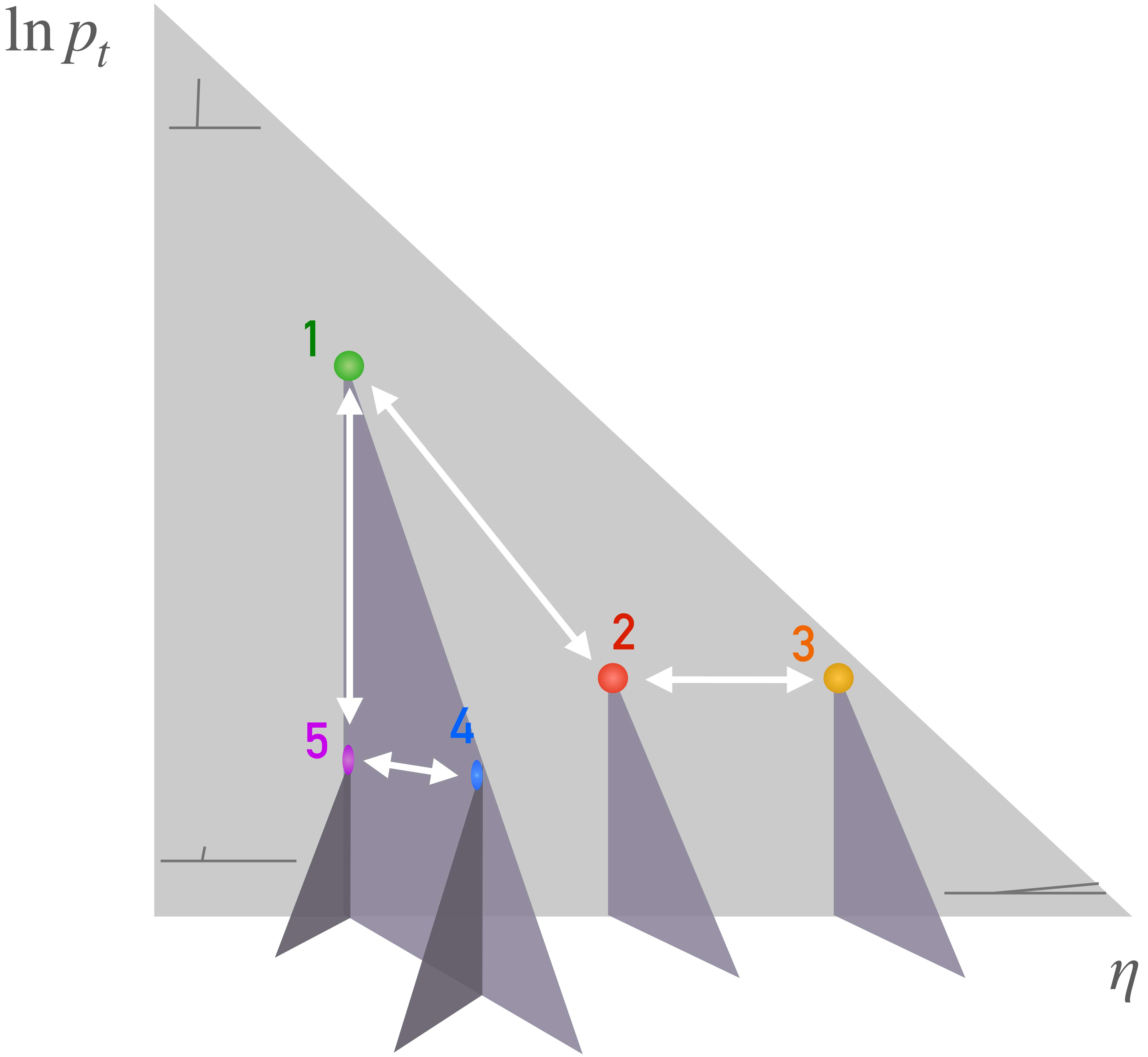
Energy flow into a patch



quantum v classical?

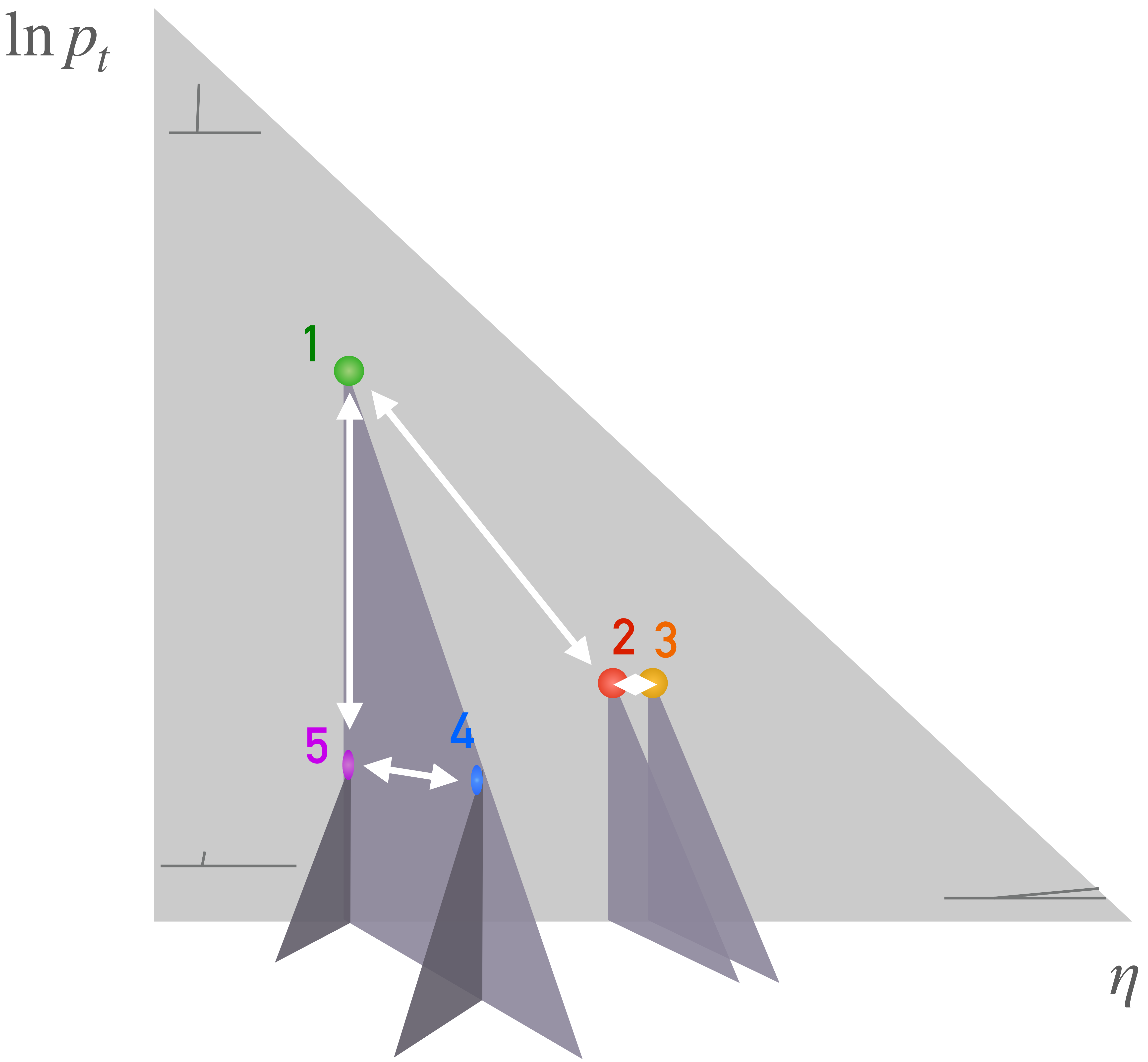
spin & colour

NLL: when should effective shower $|M^2|$ to be correct?



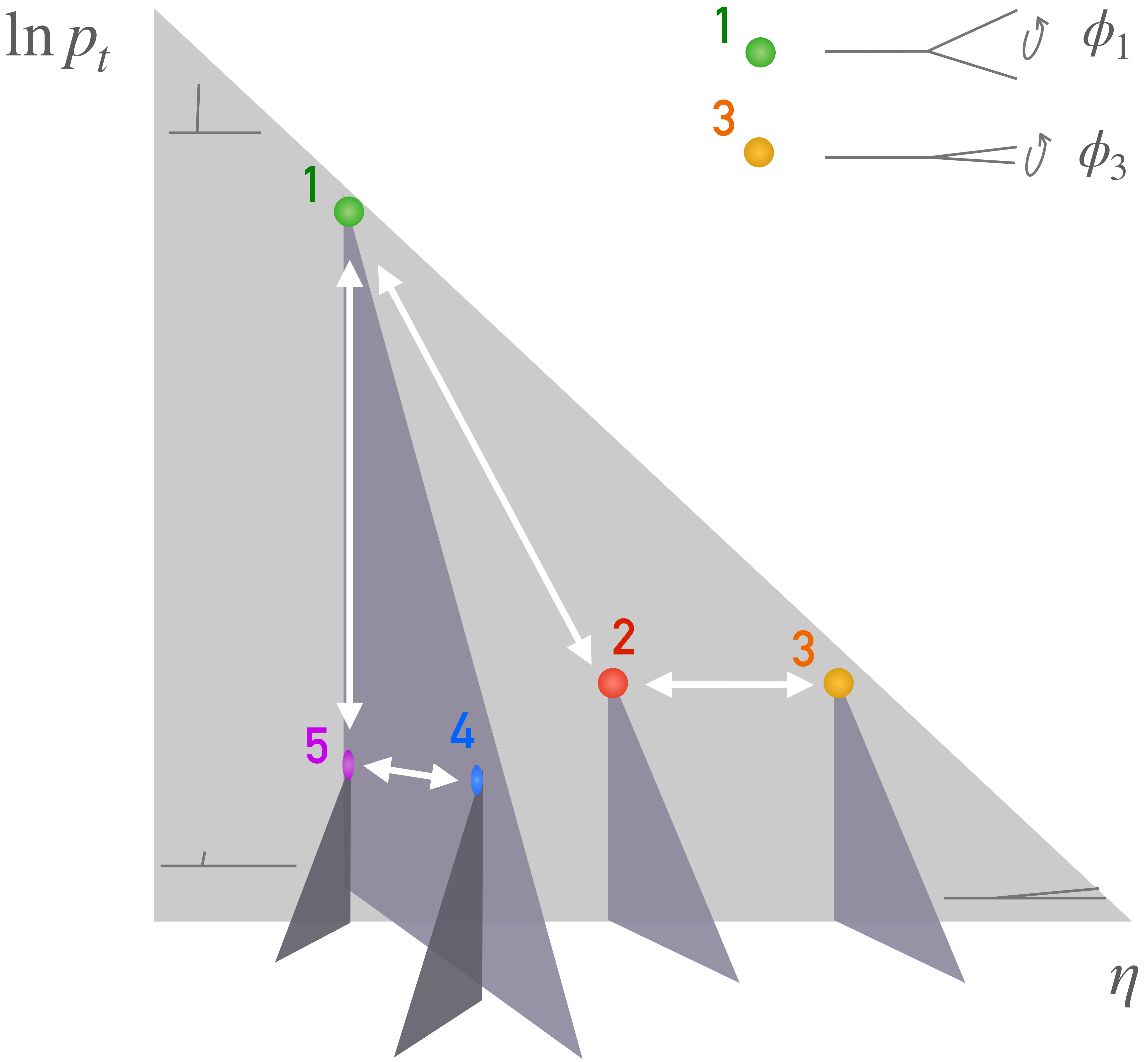
- a shower with simple 1→2 or 2→3 splittings can't reproduce full matrix element
- but QCD has amazing factorisation properties — simplifications in presence of energy or angular ordering
- **we should be able to reproduce $|M^2|$ when all emissions well separated in Lund diagram**
  $d_{12} \gg 1$, $d_{23} \gg 1$, $d_{15} \gg 1$, etc.

NLL: when should effective shower $|M^2|$ to be correct?



- a shower with simple 1→2 or 2→3 splittings can't reproduce full matrix element
- but QCD has amazing factorisation properties — simplifications in presence of energy or angular ordering
- **we allow ourselves to make a mistake (by $\mathcal{O}(1)$ factor) when a pair is close by,** e.g. $d_{23} \sim 1$

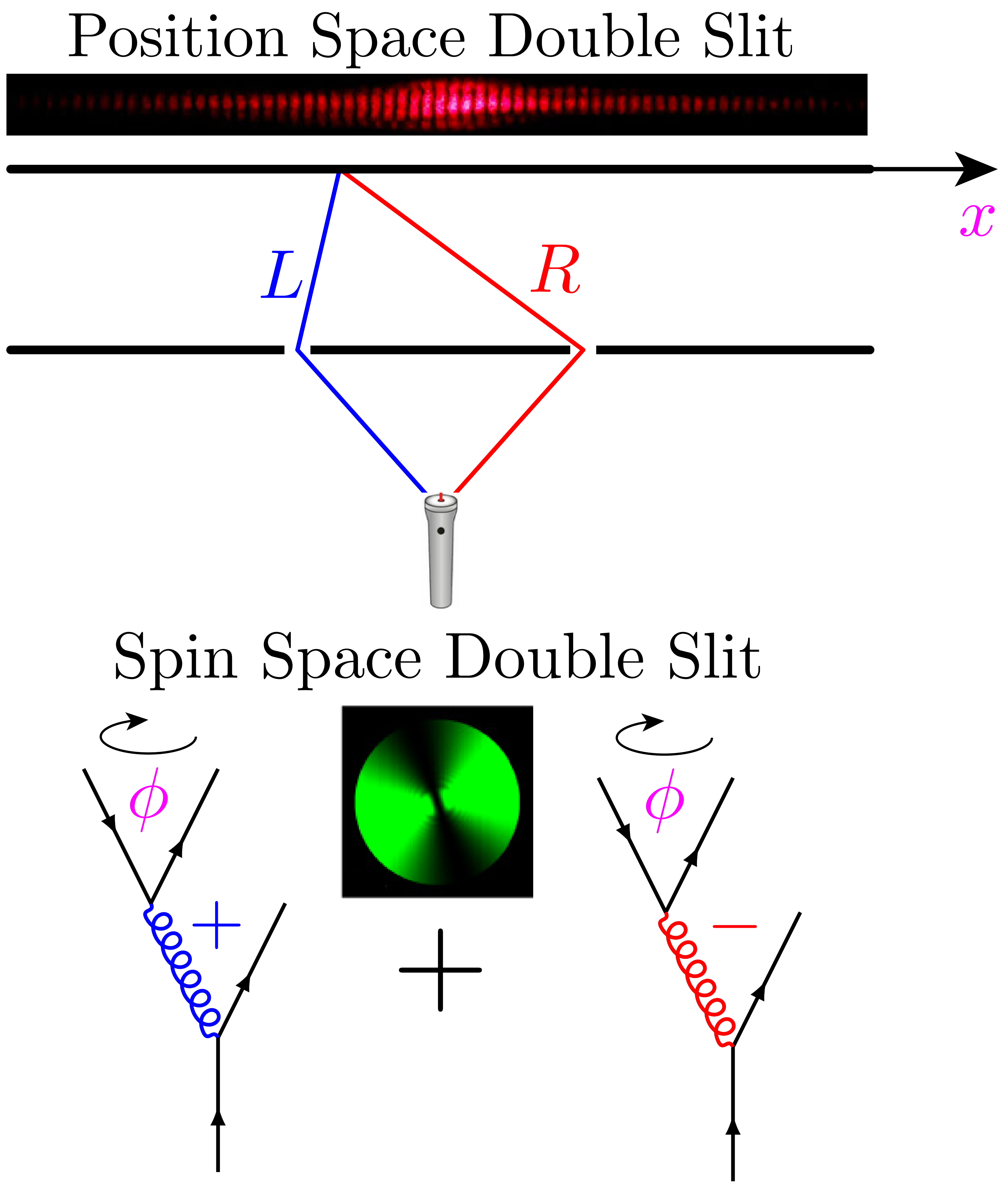
NLL: when should effective shower $|M^2|$ to be correct?



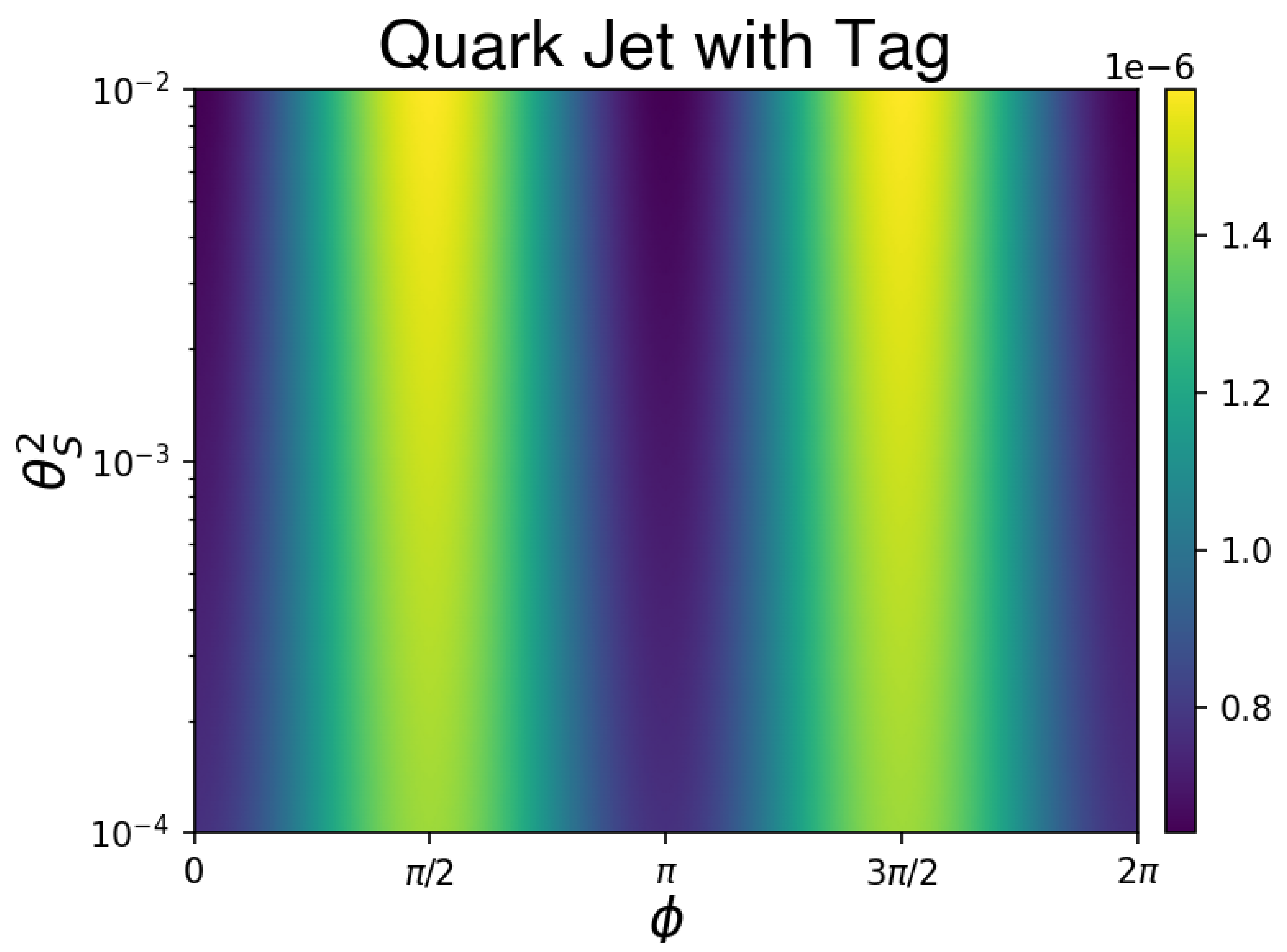
not just "classical" factorisation: 1 and 3 are far apart in the Lund plane but their azimuthal angles are strongly correlated by **quantum mechanical effects.**

Collins algorithm makes this straightforward
it's in Herwig & PanScales (and others?)
cf. Karlberg's talk tomorrow

Quantum spin correlations: first analytical resummations



Chen, Moult & Zhu, 2011.02492



$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2)\mathcal{E}(\hat{n}_3) = \frac{1}{(2\pi)^2}\frac{2}{\theta_S^2}\frac{2}{\theta_L^2}\vec{J}\left[\widehat{C}_{\phi_S}(2) - \widehat{C}_{\phi_S}(3)\right]\left[\frac{\alpha_s(\theta_L Q)}{\alpha_s(\theta_S Q)}\right]^{\frac{\widehat{\gamma}^{(0)}(3)}{\beta_0}}\left[\widehat{C}_{\phi_L}(3) - \widehat{C}_{\phi_L}(4)\right]\left[\frac{\alpha_s(Q)}{\alpha_s(\theta_L Q)}\right]^{\frac{\widehat{\gamma}^{(0)}(4)}{\beta_0}}\vec{\mathbb{O}}^{[4]}(\hat{n}_1) \cdot \quad (11)$$

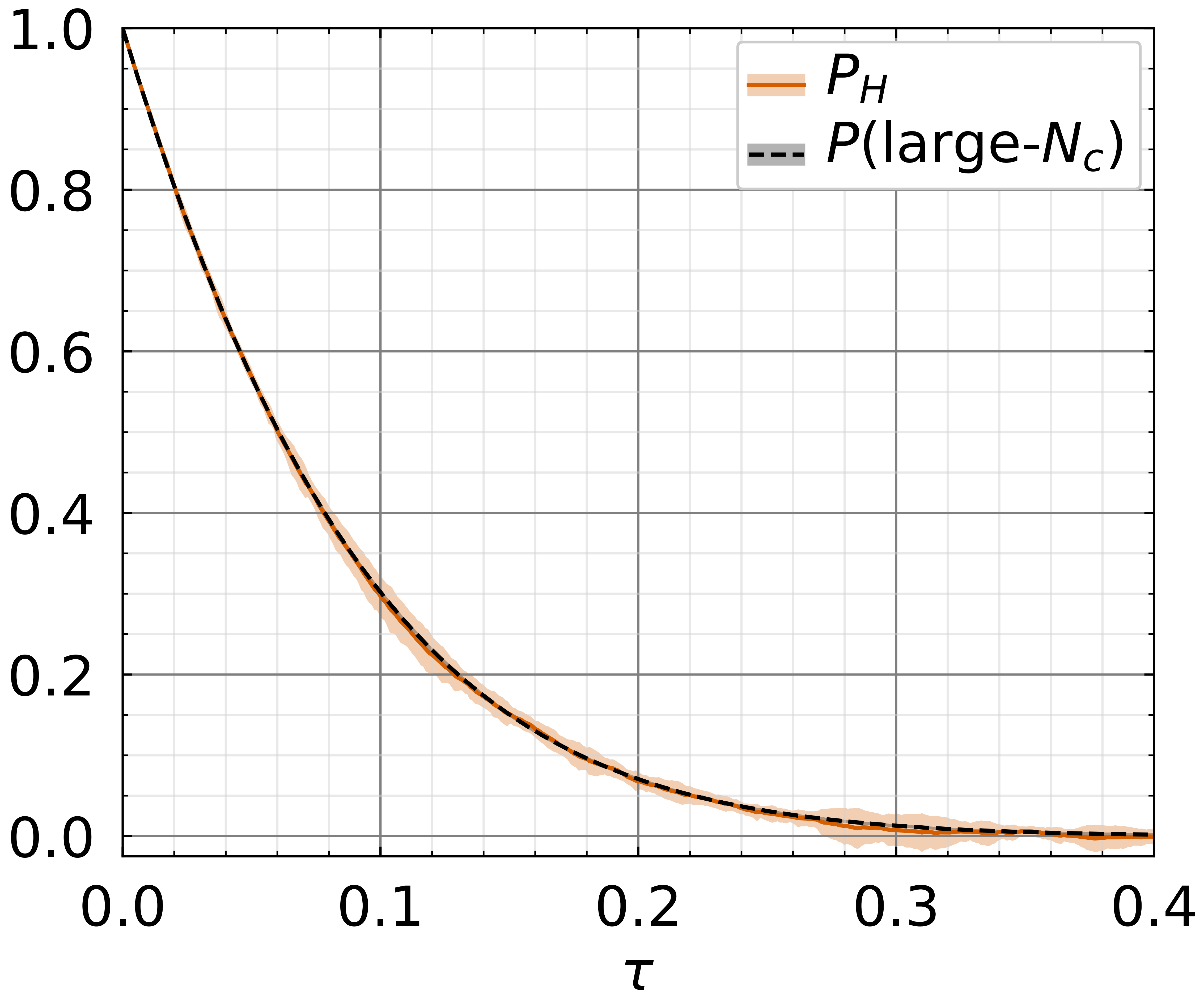
Colour: several talks this afternoon (Frixione, Plätzer, Scyboz)

- Leading colour is the default for dipole showers
- General full colour is intrinsically **quantum & expensive** [quantum spin is cheap]
  - But: full colour is easy for final-state emissions when they are all well-separated in angle (angular ordering tells you whether to use C_F or C_A)
  - It's harder when >2 emissions are at commensurate angles
  - It's harder when you mix collinear ISR and large-angle soft radiation (Forshaw, Kyrieleis & Seymour, hep-ph/0604094)
- **Can one limit the situations where one needs the full quantum input?**
- **Non-shower reference calculations provide important guidance / reference**

Crucial non-shower calculations in 2020/21

- E.g. using method of "random walk of Wilson lines"
- Intriguing results that large-N_c limits (or simple modifications of large-N_c) agree amazingly well with the full-colour calculations
- It would be interesting to understand why, because it might simply route to efficient systematic inclusion of higher order ($1/N_c^2$) effects

Gap survival prob. in H→ gg



Hatta & Ueda, 2011.04154

Crucial non-shower calculations in 2020/21

- super-leading logs are one place where simple modifications of leading-N_c results won't be enough
- because these terms have a log structure ($\alpha_s^n L^{2n-3}$) that is absent at leading-N_c (higher terms: $\alpha_s^n L^n$)
- Becher, Neubert & Shao have a (relatively) simple algorithm to generate these terms to all orders
- see also Nagy & Soper, 1908.11420

super-leading logs

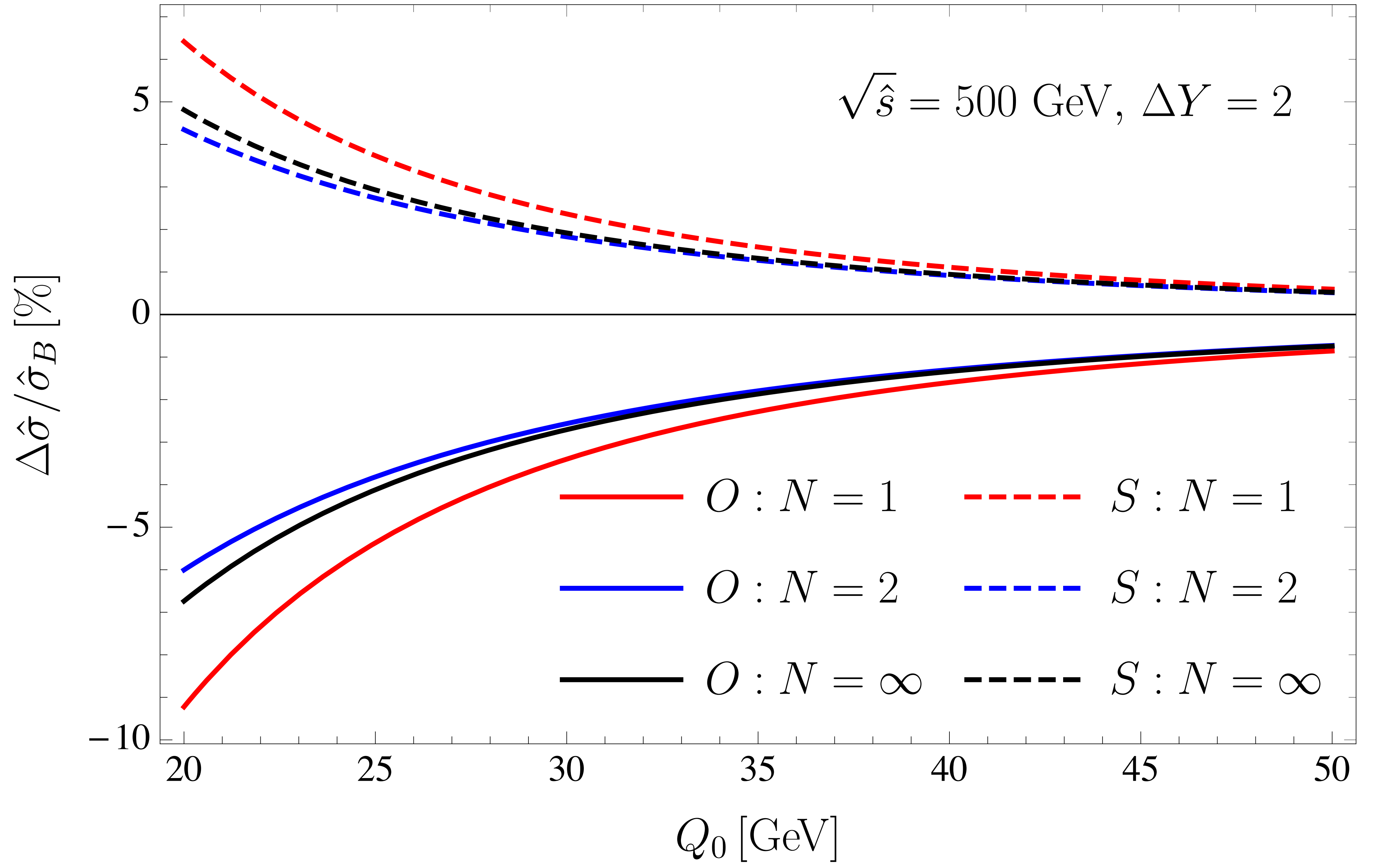


FIG. 2. Super-leading logarithms in quark-quark scattering summed up to four-loop (red), five-loop (blue) and infinite order (black). The solid and dashed lines refer to the color octet and singlet channel, respectively.

Becher, Neubert & Ding-Yu Shao, 2107.01212

Beyond NLL

Beyond NLL

- One question is how to build wisdom already obtained (e.g. Li & Skands, 1611.00013; Dulat, Höche & Prestel, 1805.03757 + earlier works) into showers that have NLL accuracy.
- NNLL will be considerably more complicated than NLL: calculations to test individual limits will be especially crucial, e.g.
  - soft-drop calculations (e.g. Frye et al, 1603.09338; Kardos et al, 2002.00942 ;Anderle et al, 2007.10355)
  - non-global calculations (Banfi, Dreyer & Monni, 2104.06416)
  - others that don't yet exist…
- Are there more fundamental constraints on recoil (e.g. Caola, Ferrario Ravasio, Limatola, Melnikov & Nason, 2108.08897; and many sub-Eikonal papers)

Mrinal Dasgupta
Manchester

Frédéric Dreyer
Oxford

Keith Hamilton
Univ. Coll. London

Pier Monni
CERN

GPS
Oxford

Grégory Soyez
IPhT, Saclay

since 2017

Emma Slade
Oxford (PhD) → GSK.ai

2018–20

Basem El-Menoufi
Manchester

Alexander Karlberg
Oxford

Rok Medves
Oxford (PhD)

Ludovic Scyboz
Oxford

Rob Verheyen
Univ. Coll. London

since 2019

The thoughts in this talk are thanks to many discussions with my PanScales collaborators over the past years (any errors are mine!)

Melissa van Beekveld
Oxford

Silvia Ferrario Ravasio
Oxford

Alba Soto Ontoso
IPhT, Saclay

since 2020

Conclusions

- Shower limitations appear to matter experimentally, and probably increasingly so as the LHC accumulates luminosity & exploits more event information, e.g. via machine learning
- Extent to which there is a parton shower depends on number/colour of hard legs and value of hard scale: 5 TeV $qq \to qq$ has lots of shower, on shell $Z \to q\bar{q}$ relatively little
- Classification of "NLL" shower accuracy a bit fuzzy, but the PanScales working definition places useful constraints — several showers are on the road to achieving / demonstrating NLL accuracy
- Many open directions: NLL-pp, efficient+accurate subleading colour, contributions beyond NNLL

backup

Q [GeV]	$\alpha_s(Q)$	$p_{t,\min}$ [GeV]	$\xi = \alpha_s L^2$	$\lambda = \alpha_s L$
91.2	0.1181	1.0	2.4	-0.53
91.2	0.1181	3.0	1.4	-0.40
91.2	0.1181	5.0	1.0	-0.34
1000	0.0886	1.0	4.2	-0.61
1000	0.0886	3.0	3.0	-0.51
1000	0.0886	5.0	2.5	-0.47
4000	0.0777	1.0	5.3	-0.64
4000	0.0777	3.0	4.0	-0.56
4000	0.0777	5.0	3.5	-0.52
20000	0.0680	1.0	6.7	-0.67
20000	0.0680	3.0	5.3	-0.60
20000	0.0680	5.0	4.7	-0.56