

TMD evolution and parton showers

27.08.2021

A. Bermúdez Martínez on behalf of the TMD PB collaboration



Why TMDs?

R. A. Martinez et al. [[APP B46 \(2015\) 12, 2501–2534](#)]

- Small transverse momentum phenomena
- Small-x phenomena
- DY, and semi-inclusive DIS
- Transverse momentum effects from intrinsic k_t and evolution

Parton Branching (PB) method

- Evolution of TMDs (and collinear PDFs)
- Resummation of soft gluons at LL and NLL
- Solution valid at LO, NLO and NNLO
- Determination of TMDs from the fully exclusive solution
- **Backward evolution fully determines the TMD shower**

FH et al. [[PLB 772 \(2017\) 446–451](#)]

FH et al. [[JHEP 2018, 70 \(2018\)](#)]

ABM et al. [[PRD 99, 074008 \(2019\)](#)]

 consistently treats perturbative and non-perturbative transverse momentum effects

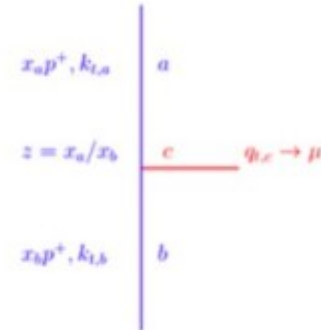
PB formulation of TMD evolution

[slide by M. van Kampen]

JHEP 01 (2018) 070 [arXiv:1708.03279]

PB evolution equation for TMDs $\tilde{\mathcal{A}}_a(x, k_t^2, \mu^2)$ can be solved iteratively with the Monte Carlo method:

$$\begin{aligned} \tilde{\mathcal{A}}_a(x, k_t^2, \mu^2) = & \Delta_a(\mu^2, \mu_0^2) \tilde{\mathcal{A}}_a(x, k_{t,0}^2, \mu_0^2) + \\ & + \sum_b \left[\int \frac{d^2 \mu'}{\pi \mu'^2} \int_x^{z_M(\mu')} dz \Theta(\mu^2 - \mu'^2) \Theta(\mu'^2 - \mu_0^2) \right. \\ & \times \left. \frac{\Delta_a(\mu^2, \mu_0^2)}{\Delta_a(\mu'^2, \mu_0^2)} P_{ab}^{(R)}(\alpha_s(q_t), z) \tilde{\mathcal{A}}_b\left(\frac{x}{z}, \underbrace{k_{t,b} - q_{t,c}}_{k_{t,a}}, \mu'^2\right) \right] \end{aligned}$$



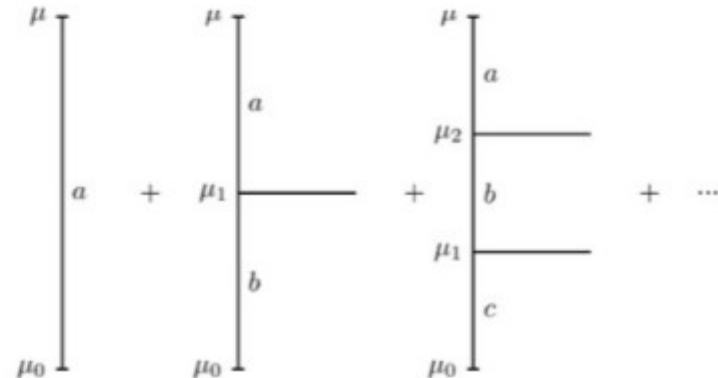
Kinematics in each branching governed by momentum conservation: $k_{t,b} = k_{t,a} + q_{t,c}$

$P_{ab}^{(R)}(\alpha_s, z)$ real splitting function (resolvable branching probability),
 $\Delta_a(\mu^2, \mu_0^2)$ Sudakov (no branching probability)

Angular ordering condition: $q_t^2 = (1-z)^2 \mu'^2$

$$P_{ab}^{(R)}(\alpha_s, z) = \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{2\pi} \right)^n P_{ab}^{(R)n-1}(z)$$

$$\Delta_a(\mu^2, \mu_0^2) = \exp \left(- \sum_b \int \frac{d\mu^2}{\mu^2} \int_0^{z_M} dz z P_{ab}^{(R)}(z, \alpha_s) \right)$$



PB formulation of TMD evolution

[slide by M. van Kampen]

Backward evolution with PB method

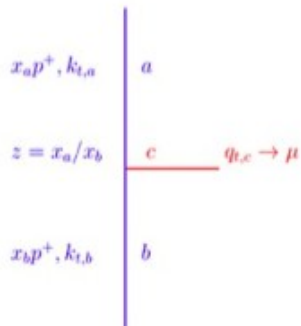
The TMD evolution equation can be used to do a backward evolution:

$$\frac{\partial}{\partial \ln \mu^2} \left(\frac{\tilde{A}_a(x, k_t, \mu)}{\Delta_a(\mu)} \right) = \sum_b \int_x^{z_M} dz P_{ab}^{(R)} \frac{\tilde{A}_b(x/z, k'_t, \mu)}{\Delta_a(\mu)},$$

normalize to $\frac{\tilde{A}_a(x, k_t, \mu)}{\Delta_a(\mu)}$ and integrate over μ' from μ_i down to μ_{i-1}

$$\Delta_{bw}(x, k_t, \mu_i, \mu_{i-1}) = \exp \left\{ - \sum_b \int_{\mu_{i-1}^2}^{\mu_i^2} \frac{d\mu'^2}{\mu'^2} \int_x^{z_M} dz P_{ab}^{(R)} \frac{\tilde{A}_b(x/z, k'_t, \mu')}{\tilde{A}_a(x, k_t, \mu')} \right\}.$$

This Sudakov is used as the no-branching probability in the TMD parton shower.

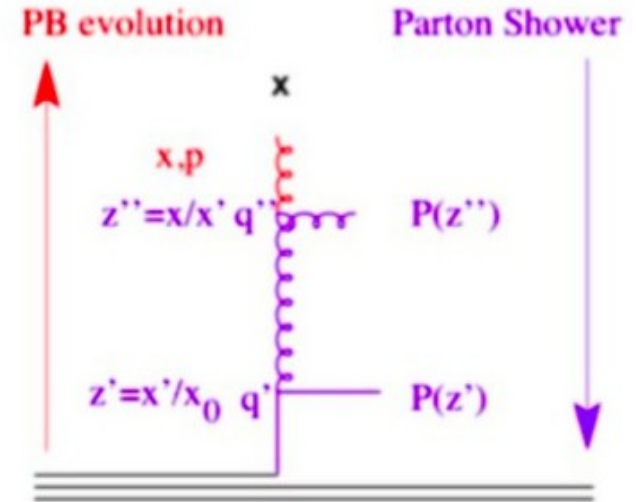


- In each splitting

$$\begin{aligned} \mathbf{k}_{t,b} &= \mathbf{k}_{t,a} + \mathbf{q}_{t,c} \\ &= \mathbf{k}_{t,a} + (1-z)\boldsymbol{\mu} \end{aligned}$$

- Total transverse momentum:

$$\mathbf{k}_t = \mathbf{k}_{t,0} + \sum_c \mathbf{q}_{t,c}$$

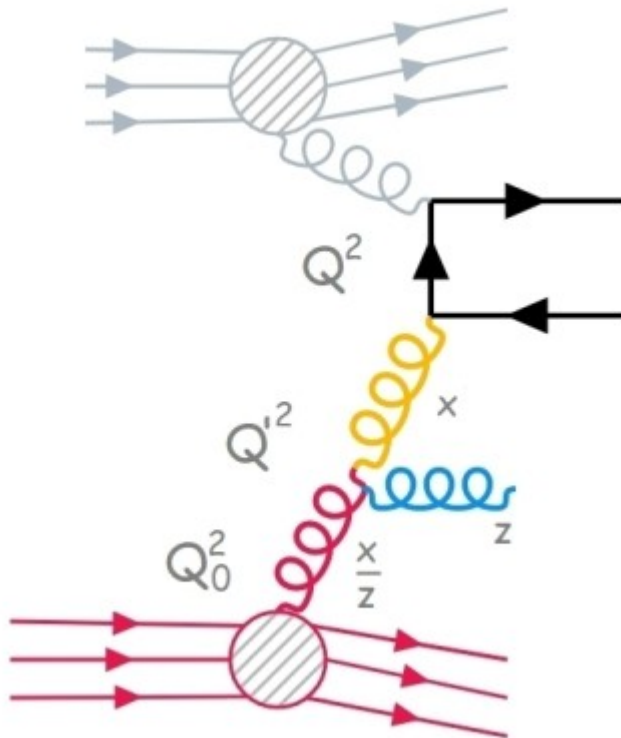


Implemented in the **CASCADE** event generator

S. Baranov et al. [Eur. Phys. J. C 81 (2021) 425]

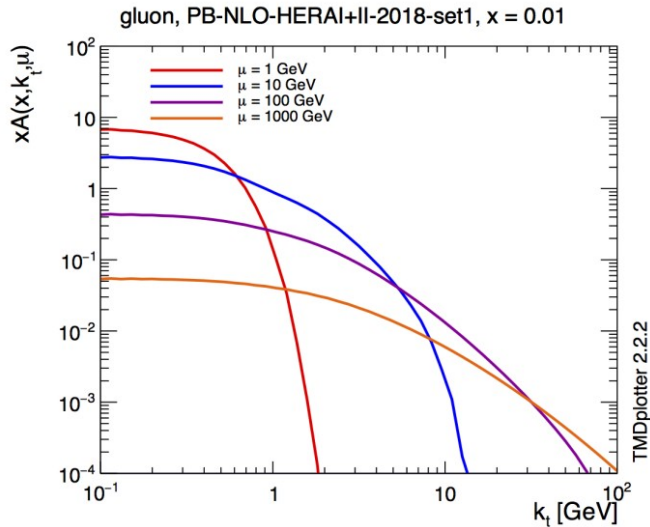
PB method recap

$$A_a^{(1)}(x, \mathbf{k}_t; Q^2) = \Delta_a(Q^2, Q_0^2) A_a(x, \mathbf{k}_t; Q_0^2) + \sum_b \int_{Q_0^2}^{Q^2} \frac{d^2 Q'}{\pi Q'^2} \frac{\Delta_a(Q^2, Q_0^2)}{\Delta_a(Q'^2, Q_0^2)} \int_x^{z_M} dz P_{ab}^{(R)}(z, \alpha_s(Q'^2)) \Delta_b(Q'^2, Q_0^2) A_b\left(\frac{x}{z}, \mathbf{k}_t + (1-z)\mathbf{Q}'; Q_0^2\right)$$



- kinematics of the splittings is known
- physics \rightarrow mapping of evolution variables to splitting kinematics
- TMD from cumulative k_t of the branchings in forward PB evolution
- Automatically includes resummation at NLL
- **Initial-state shower fully determined by TMD and its backward PB evolution**
- **Parton shower exactly matches the evolution of the TMD**

Pert. and non-pert. PB TMD contributions



- ISR broadens initial distribution

ABM et al. [PRD 99, 074008 (2019)]

- DIS measurements from HERA I+II
- fitting procedure in a nutshell:
 - parametrize the integrated distribution at Q_0
 - with the PB method evolve the TMD to $Q > Q_0$ (implemented in xFitter)
 - fit the measurements and extract the initial parametrization
 - store the TMD in a grid for later use (TMDlib, complementary slides)

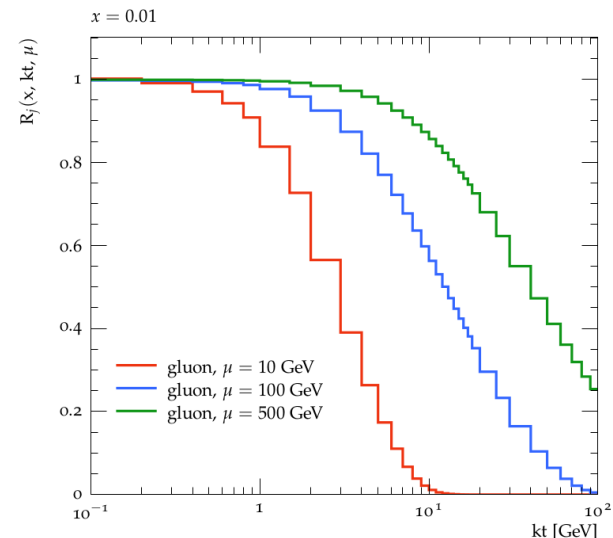
N. A. Abdulov et al. [Eur. Phys. J. C 81 (2021) 752]

Consider the integrated distribution above the jet pT scale:

$$a_j(x, \mathbf{k}, \mu^2) = \int \frac{d^2 \mathbf{k}'}{\pi} A_j(x, \mathbf{k}', \mu^2) \Theta(\mathbf{k}'^2 - \mathbf{k}^2)$$

- e.g. probability of 0.3 that the gluon develops a kt larger than 20 GeV, for $\mu = 100$ GeV

- TMD evolution effects crucial at describing jet production



ABM et al. [arXiv:2107.01224]

Z pT in a wide range of DY mass

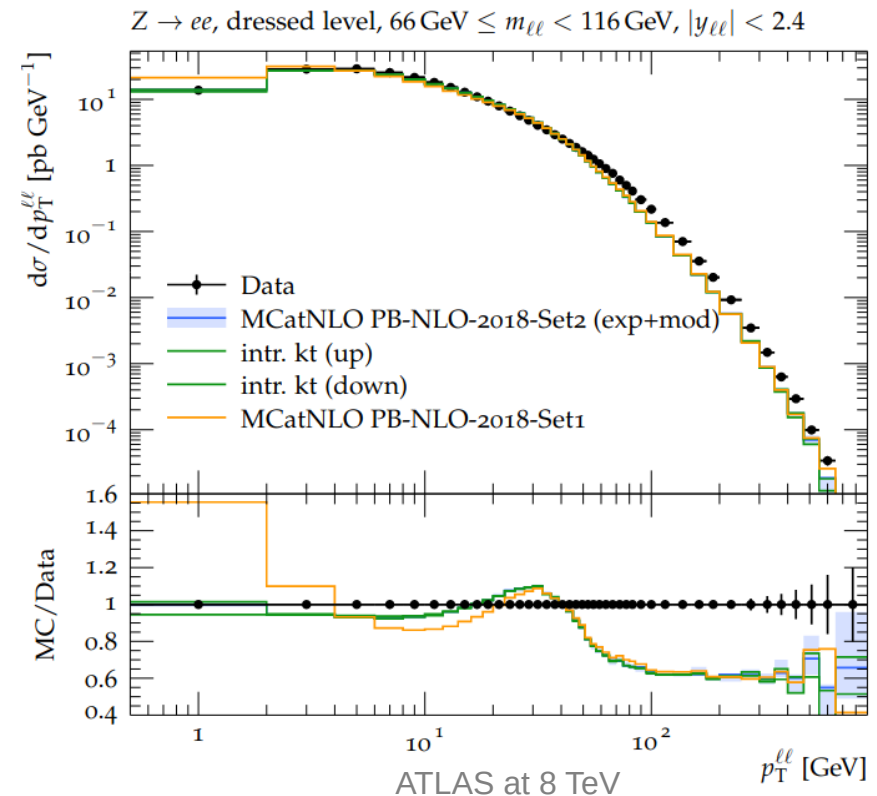
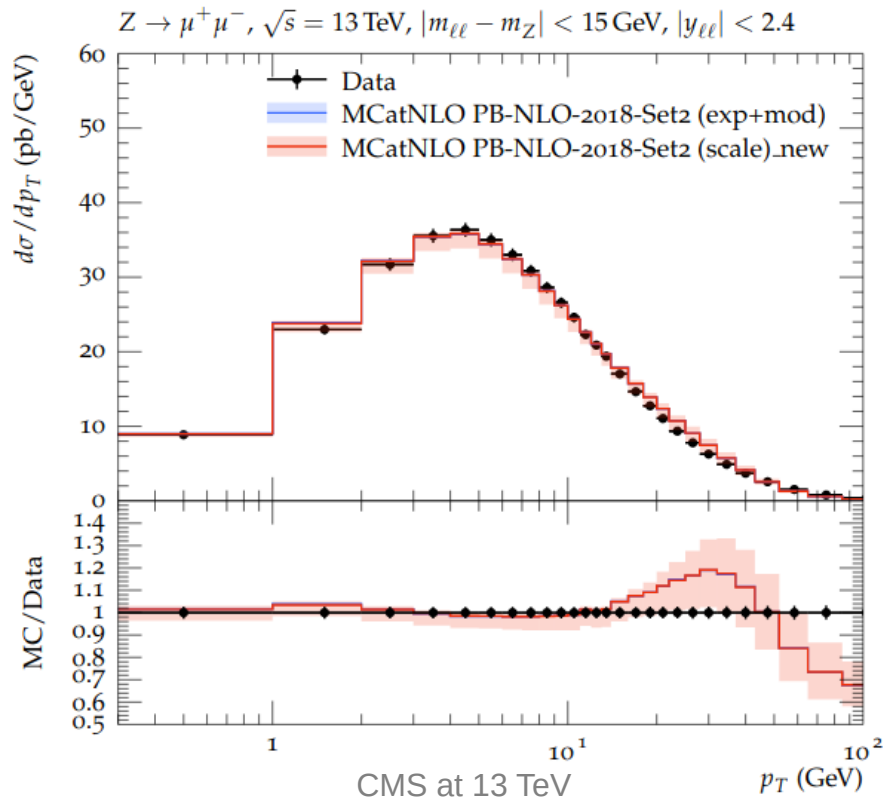
Application to high mass DY production

DY p_T spectrum

- Combined with MC@NLO
- Excellent description of DY p_T spectrum
- Non-perturbative TMD effects not significant at high p_T
- Missing contributions at high p_T

ABM et al. [PRD 100, 074027 (2019)]

ABM et al. [EPJC 80, 598 (2020)]



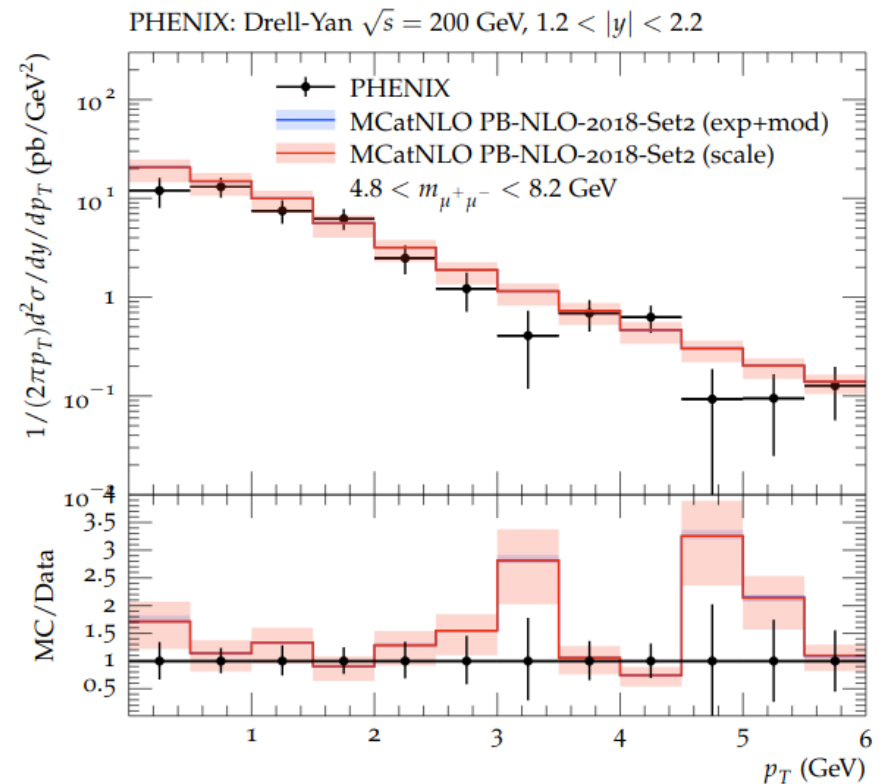
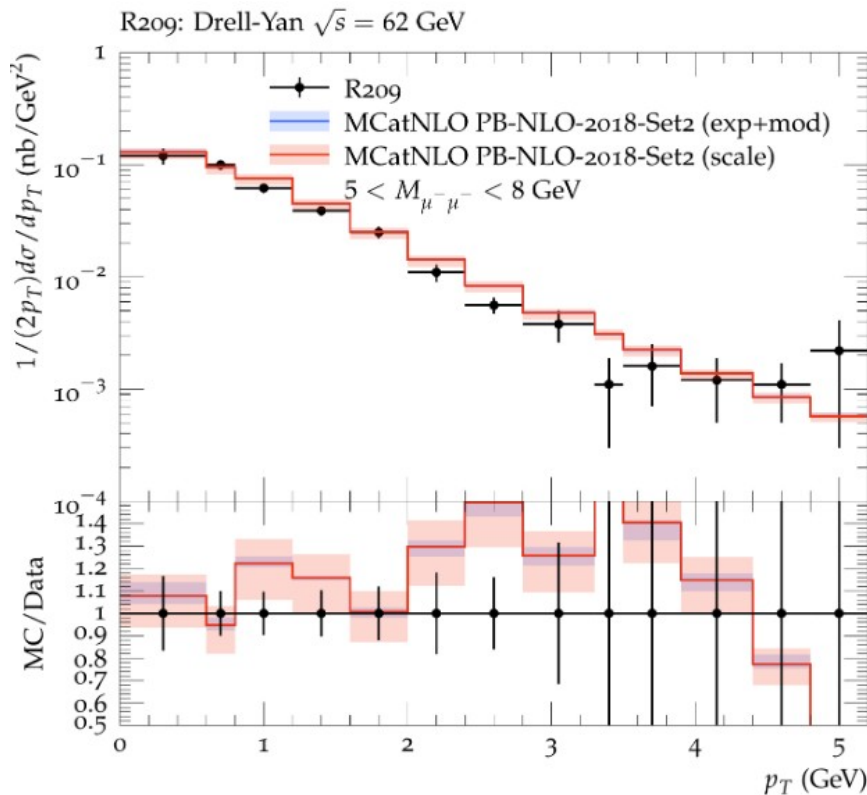
Application to low mass DY production

DY p_T spectrum

- Combined with MC@NLO
- Excellent description of DY p_T spectrum
- **First simultaneous description of both low and high-mass DY p_T spectrum**
- **No more low p_T crisis** Bacchetta et al. [PRD 100 (2019) 014018]; ABM et al. [EPJC 80, 598 (2020)]

ABM et al. [PRD 100, 074027 (2019)]

ABM et al. [EPJC 80, 598 (2020)]

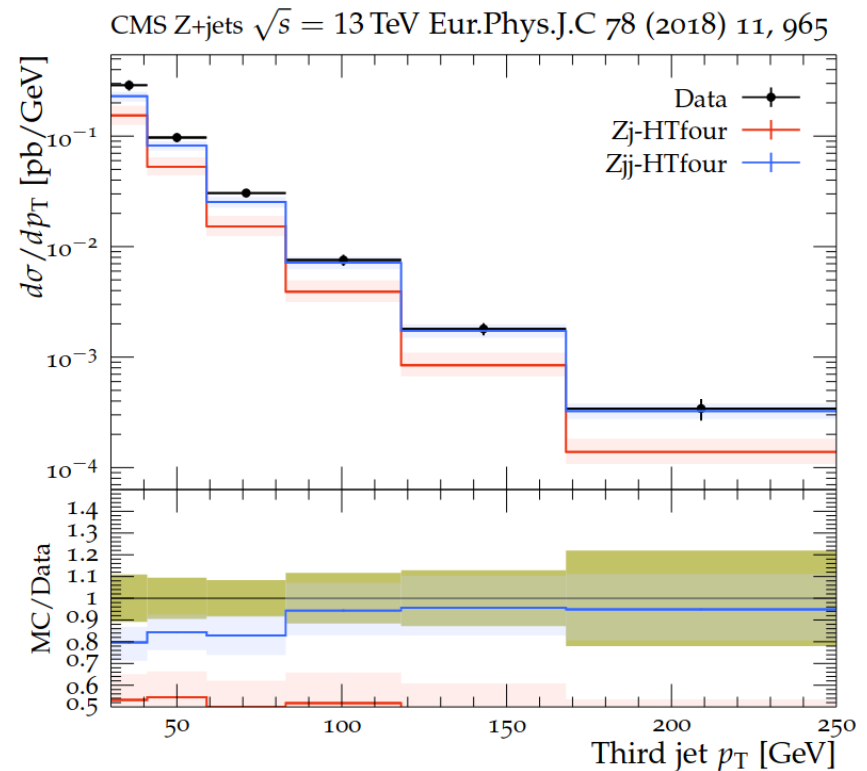
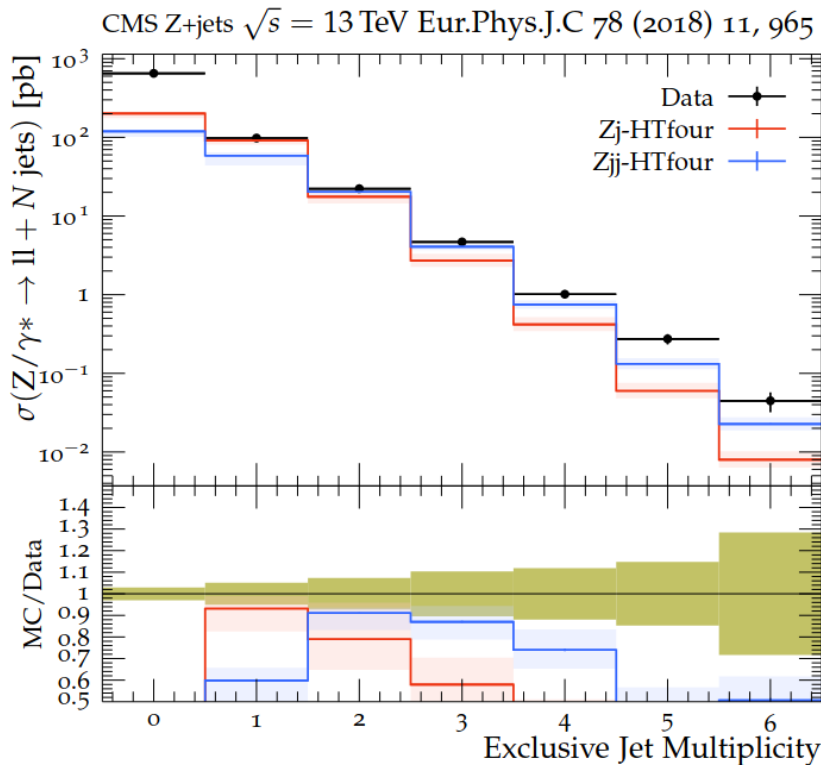


Jet production

Z + jets production

Jet multiplicity and third jet p_T spectrum

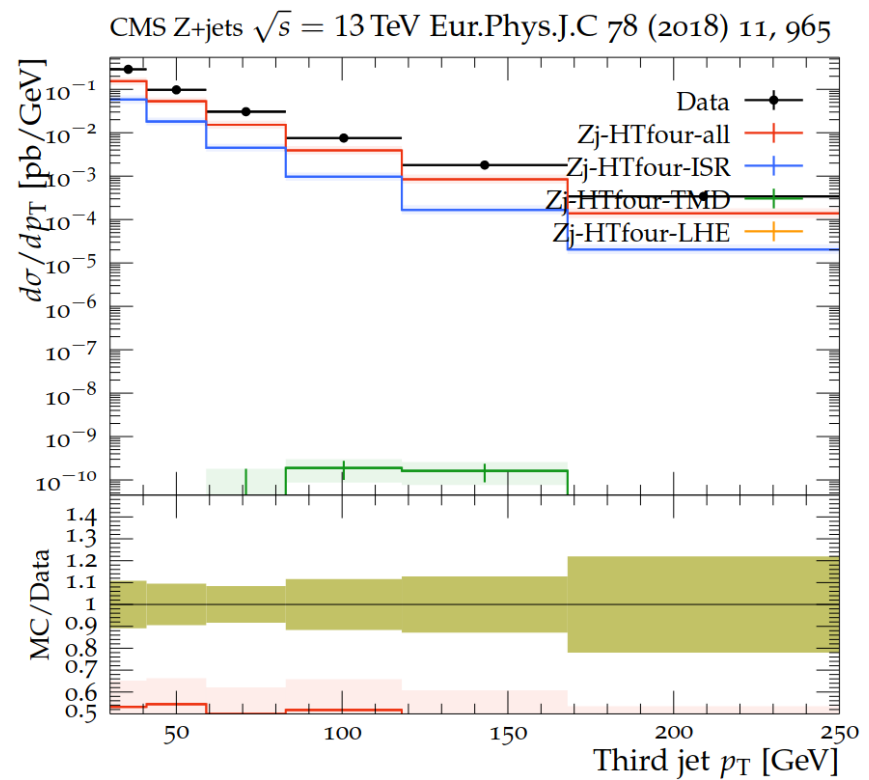
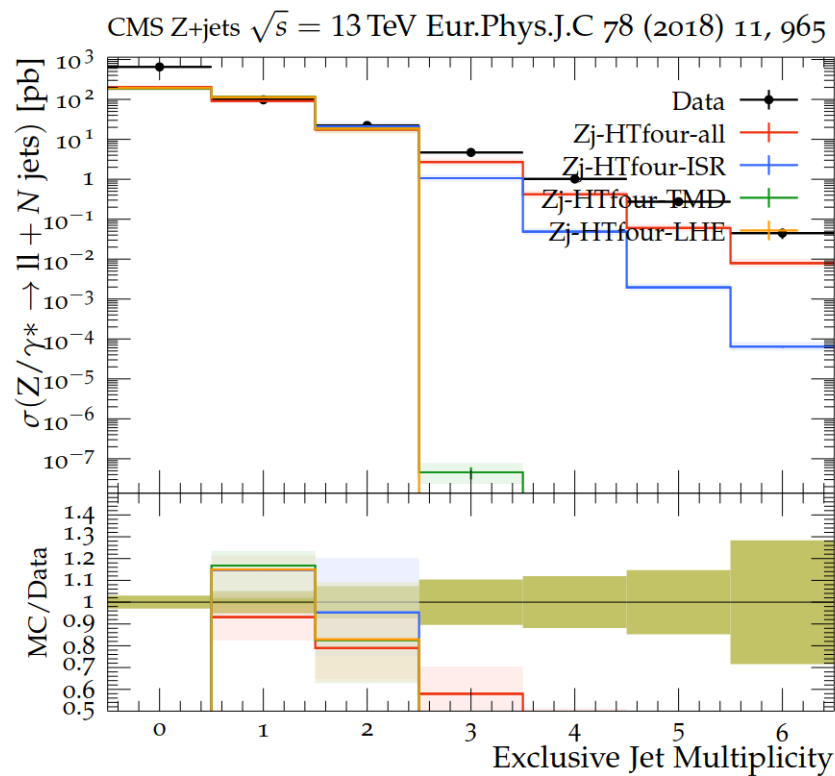
- PB shower combined with Z + j and Z + jj MC@NLO
- Suffers at higher multiplicities
- **Good description of third jet p_T spectrum given by higher orders**



Z + jets production

Jet multiplicity and third jet p_T spectrum

- Significant contribution from perturbative part of TMD
- Final state emissions important at higher multiplicities

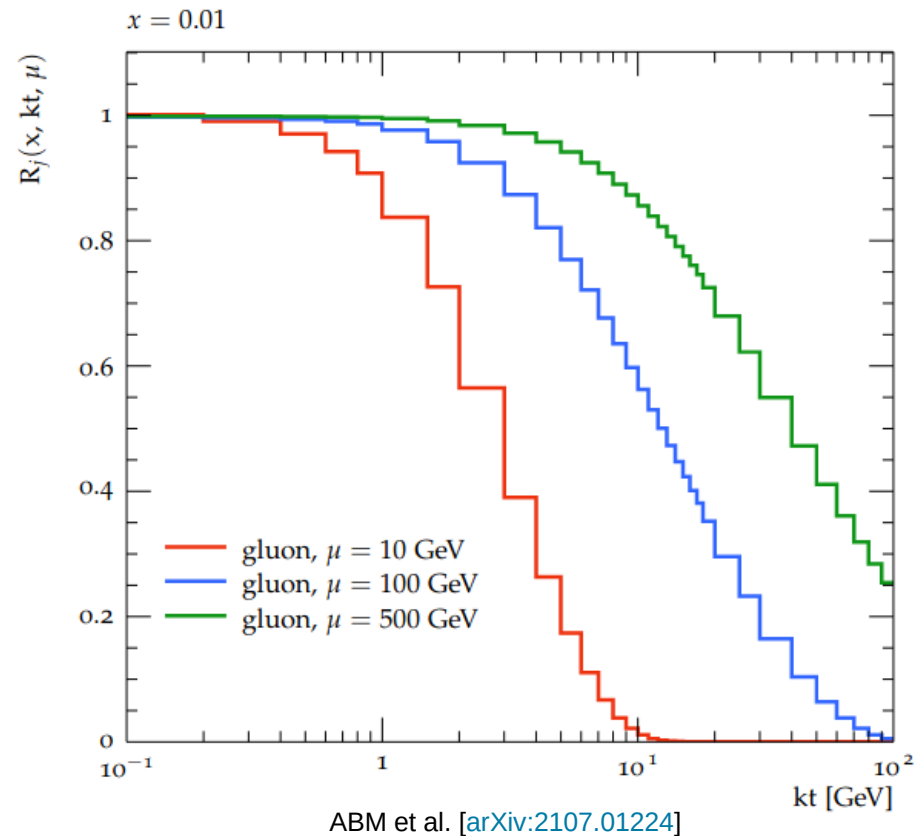
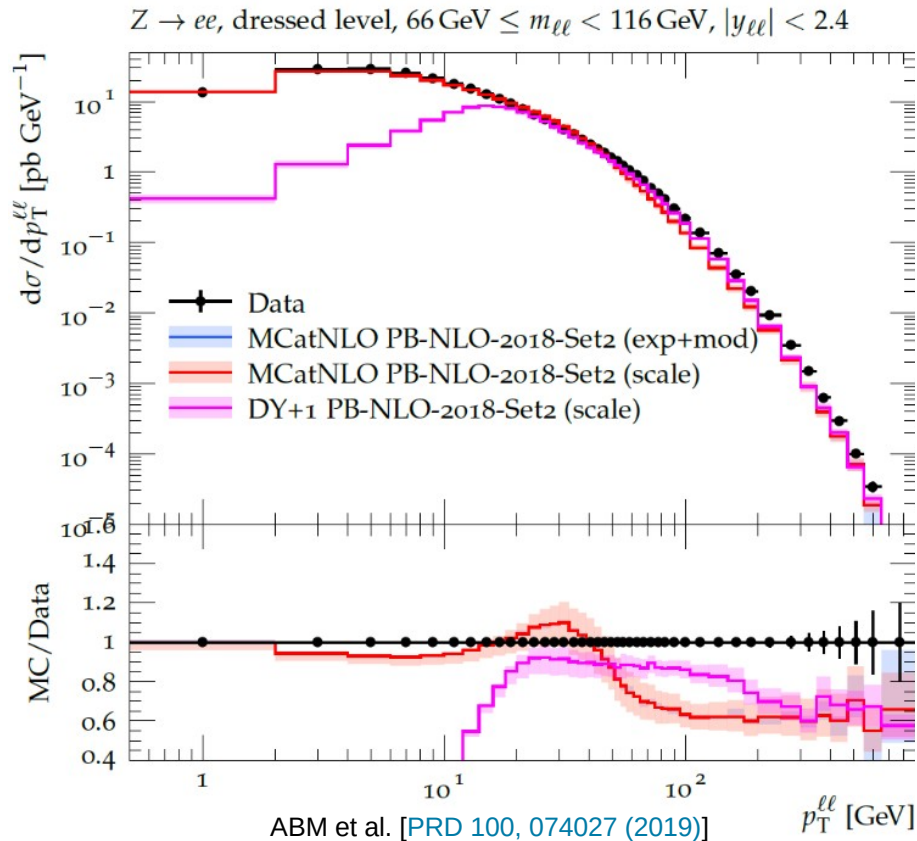


Combining TMD shower with higher orders

Trying TMD shower with higher orders

DY pt spectrum

- Important deficit at high p_T with Z at NLO
- Potentially large corrections by higher orders
- **Try combining high p_T TMD effects with multiple higher orders**



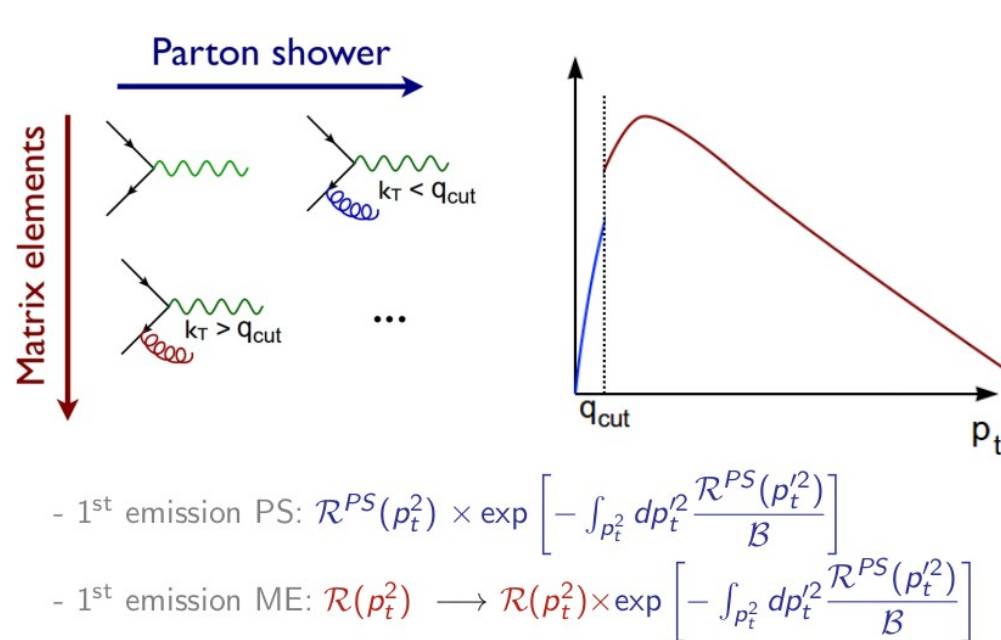
What we want:

- Treat perturbative and non-perturbative TMD effects
- Include soft gluon resummation
- Include corrections from higher-order fixed-order calculations

➔ **Develop a method to combine PB-TMDs with multi-jet calculations**

Multi-jet merging

- Make higher-order ME exclusive by Sudakov suppression
 - Avoid double counting between PS and ME
- ↓
- Improvement of hard, wide-angle emissions
 - Description of high- p_T phenomena

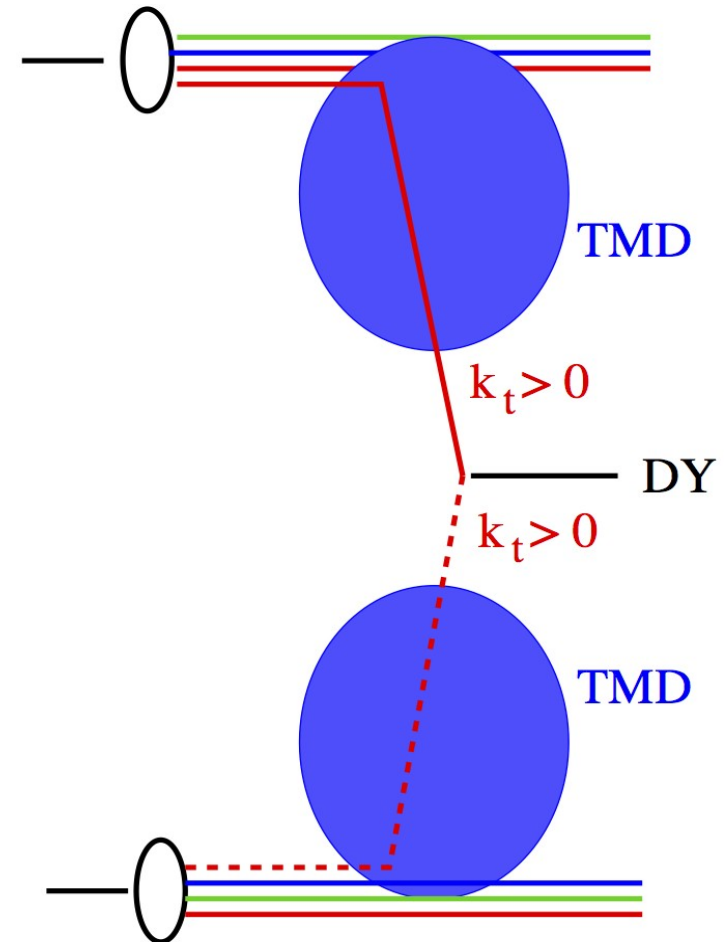


TMD merging method

ABM et al. [arXiv:2107.01224]

- Evaluate the ME for n-jet cross sections
- Reweight the strong coupling according to shower history
- **Evolve the ME using the TMD PB evolution**
- **Shower the events using the backward PB evolution for ISR**
- **Apply the MLM^[1] prescription between the PB-evolved ME and the showered events**

NB: The method could also be applied to merging criteria other than MLM



New merging procedure applicable to TMDs!

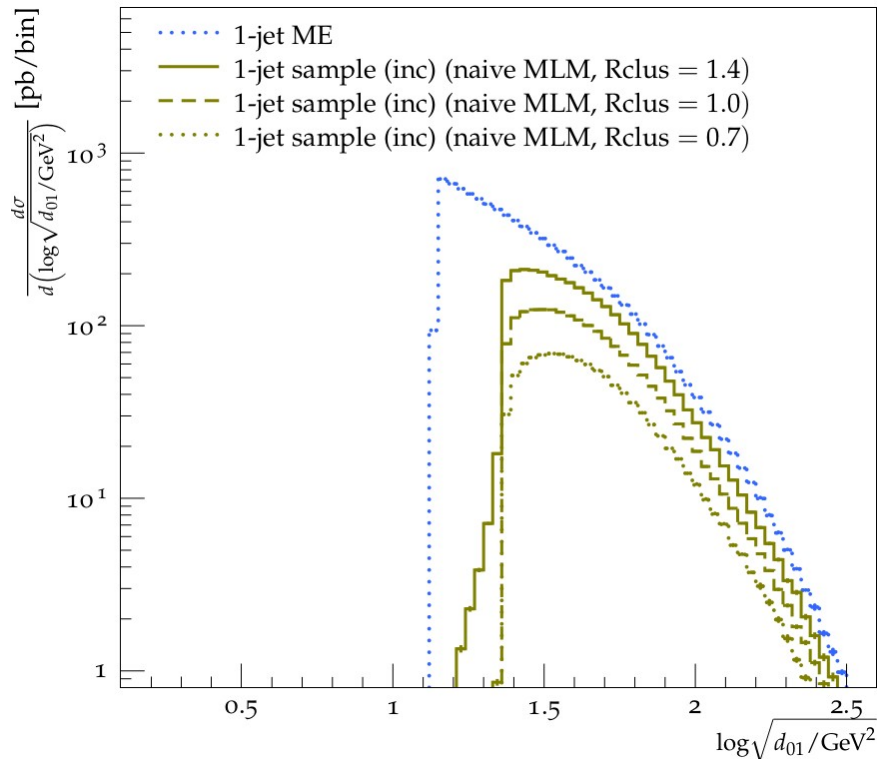
[1] M. L. Mangano [NPB 632 (2002) 343–362]

From MLM to TMD merg.

ABM et al. [paper in preparation]

What about the original MLM applied to TMD events?

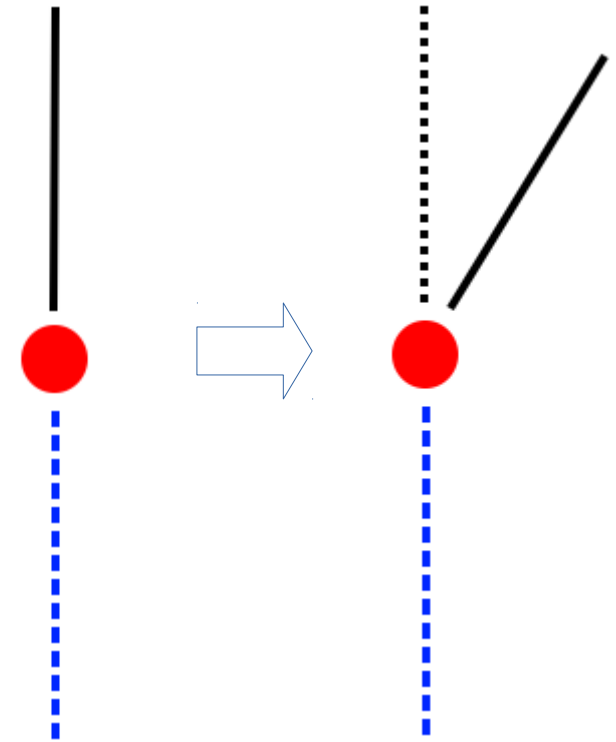
- very strong dependence on R_{clus}
- **at large scales ME accuracy lost!**



$d(n,n+1)$: square of scale at which $(n+1)$ -jet configuration becomes n -jet

at ME level

after shower

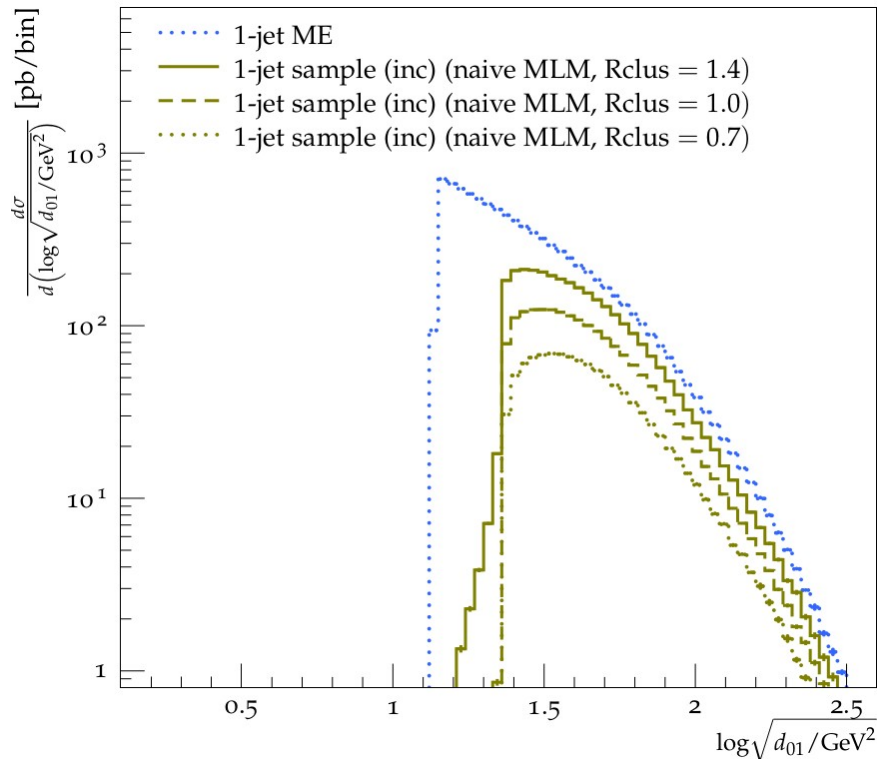


From MLM to TMD merg.

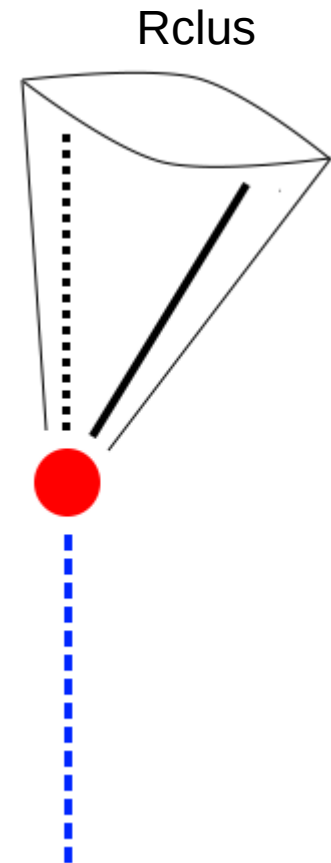
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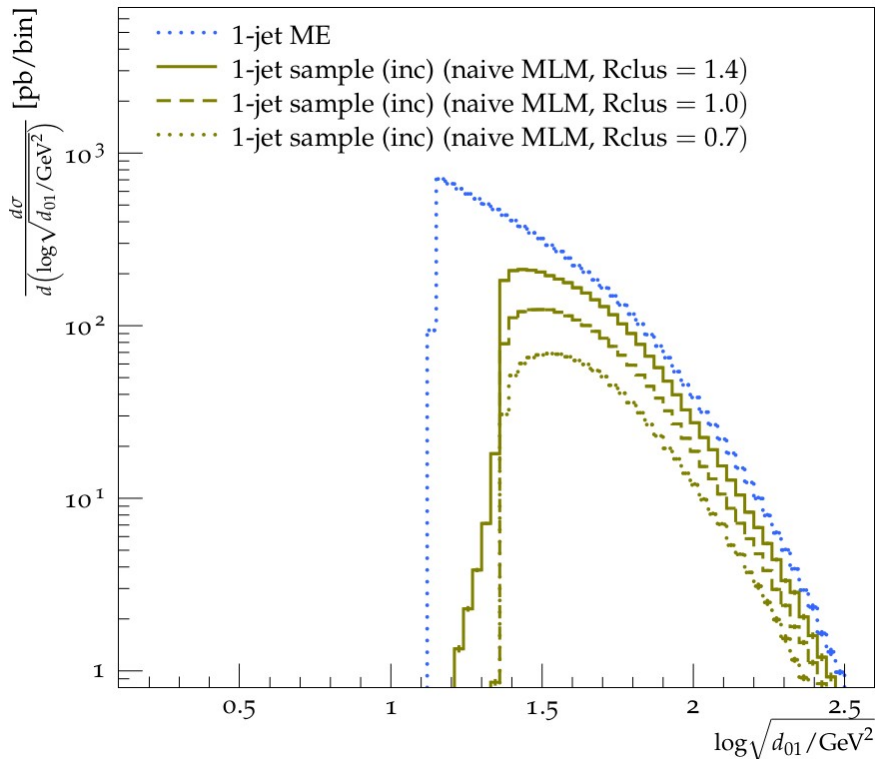
reject non-matched events

From MLM to TMD merg.

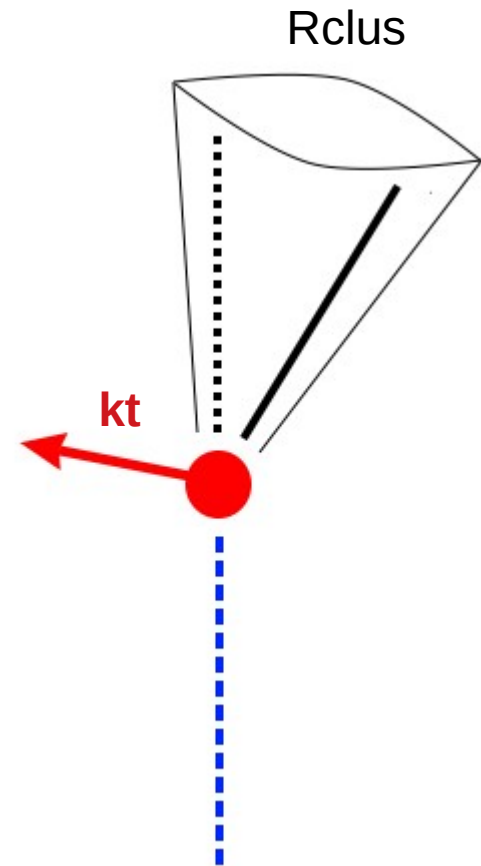
ABM et al. [paper in preparation]

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$d(n,n+1)$: square of scale at which $(n+1)$ -jet configuration becomes n -jet



$$kt_{\text{max}} = kt_{\text{max}}(R_{\text{clus}}) !$$

R_{clus} translates into a maximum TMD evolution scale

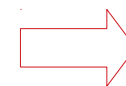
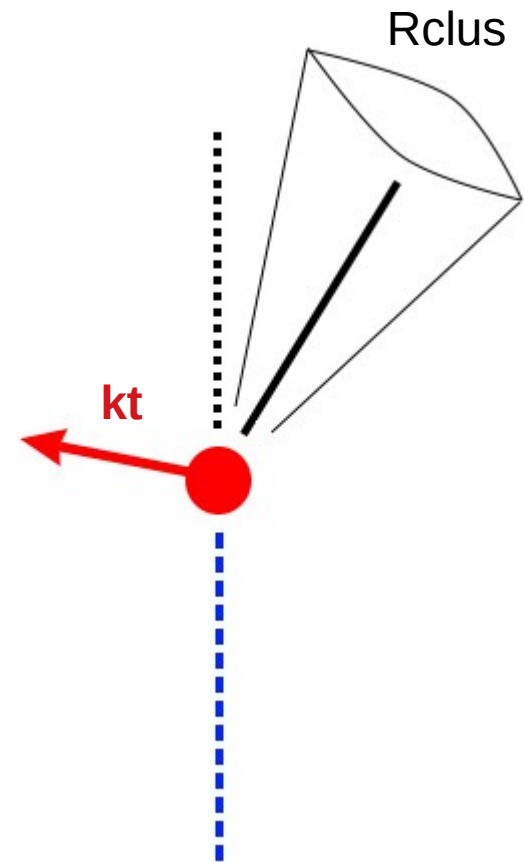
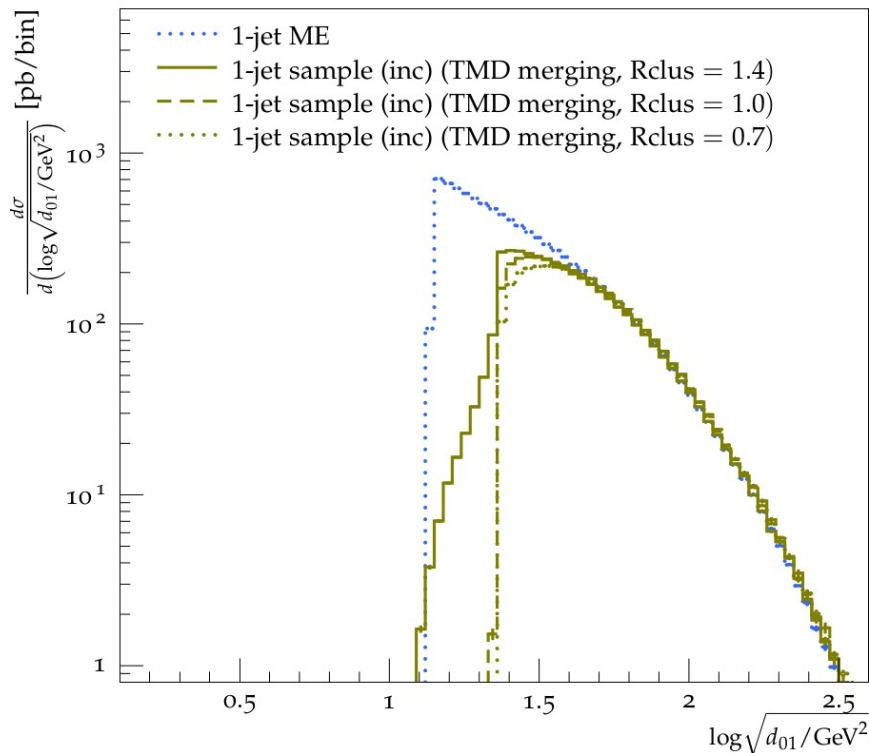
➡ naive MLM incompatible with PB TMD evolution

From MLM to TMD merg.

ABM et al. [paper in preparation]

TMD merging

- little dependence on R_{clus}
- at large scales ME accuracy recovered!

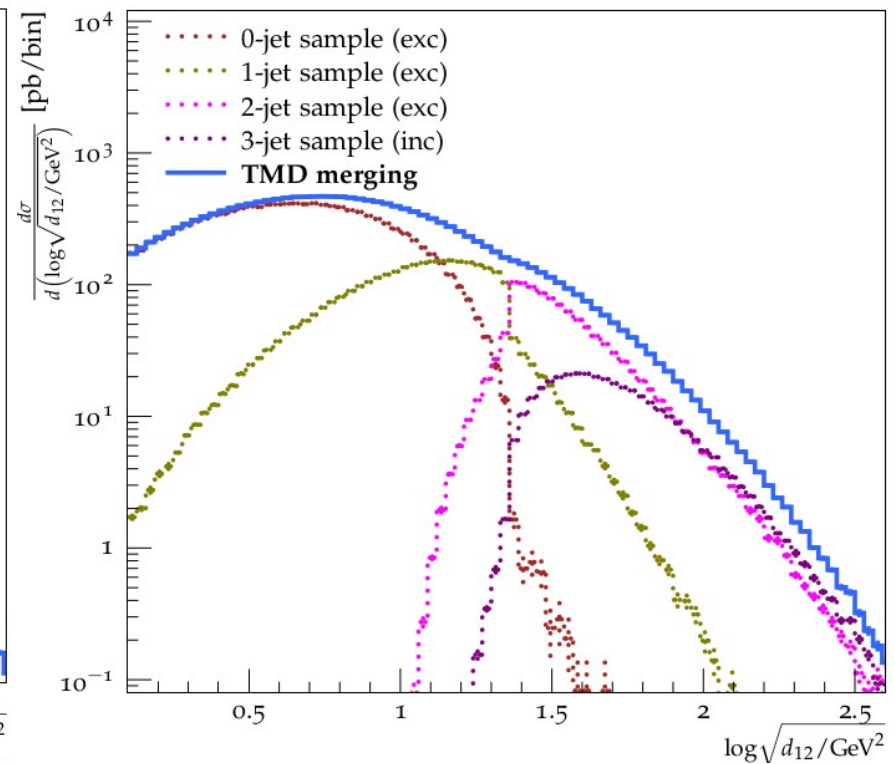
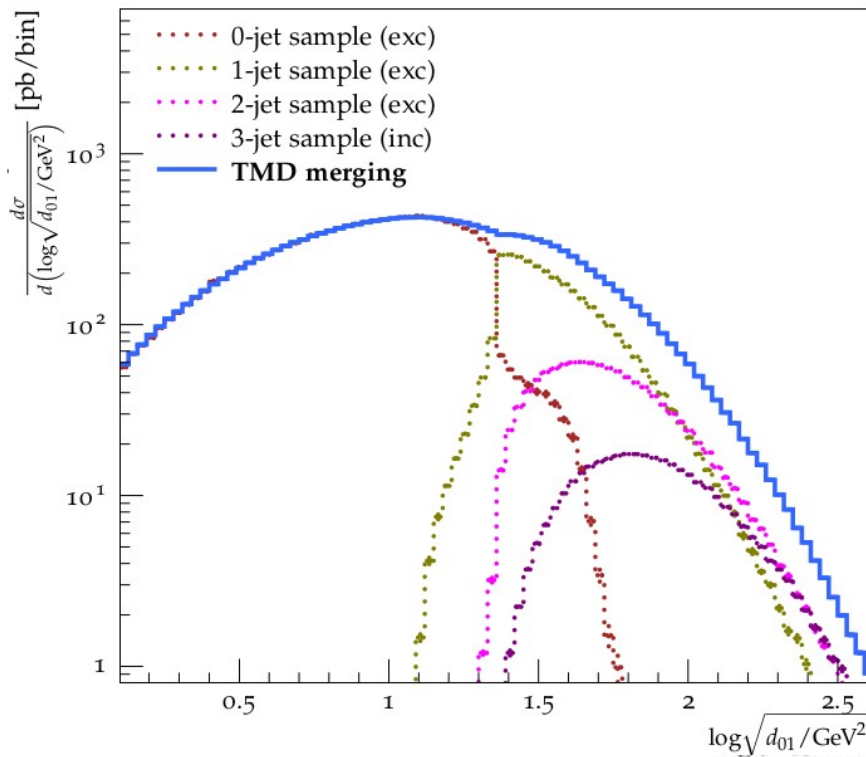


TMD PB evolution decouples from MLM matching

Combining TMD shower with higher orders

ABM et al. [paper in preparation]

$d(n,n+1)$: scale at which $(n+1)$ -jet configuration becomes n -jet



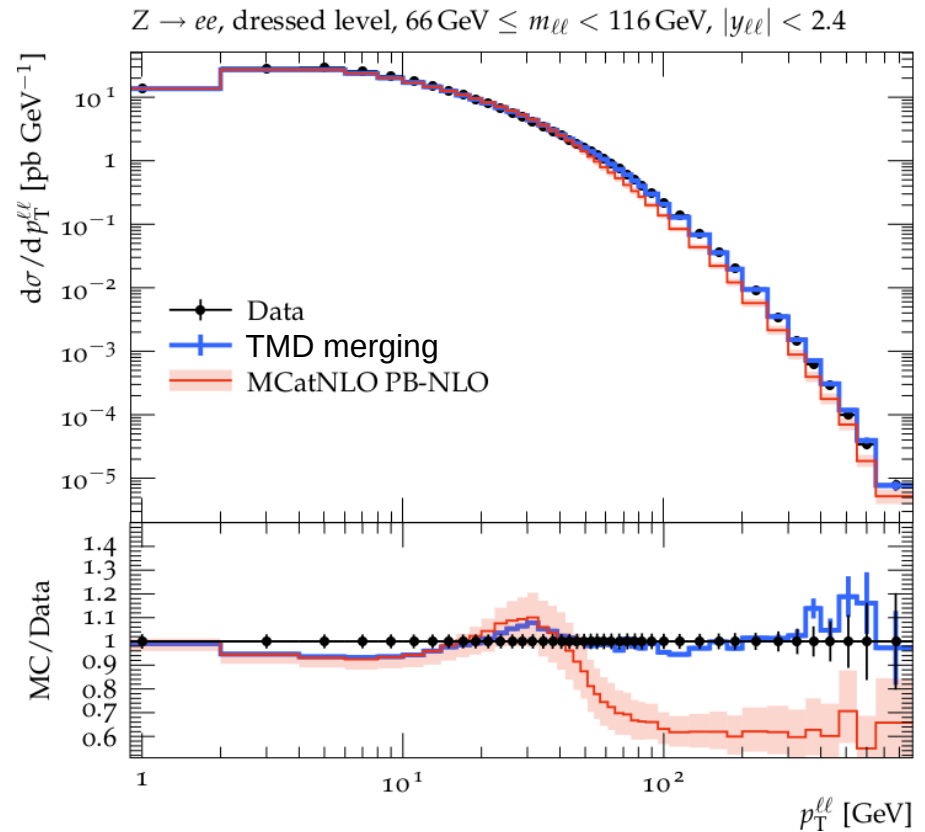
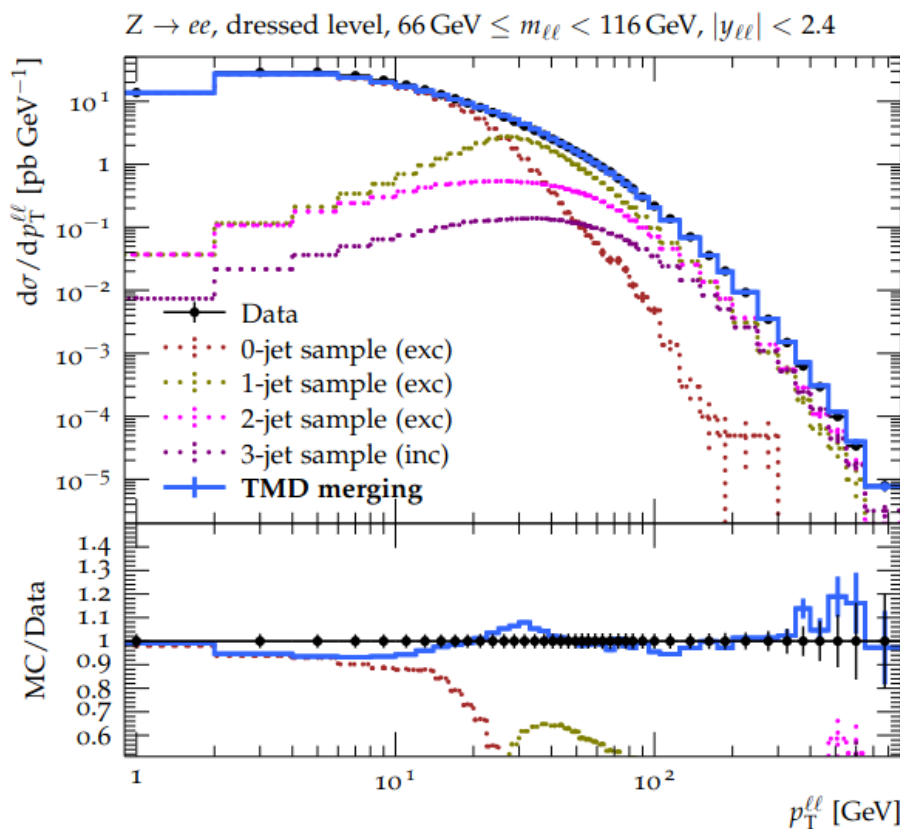
- Smoothness \longrightarrow merging follows shower Sudakov suppression
- Merging scale divides phase space for different jet multiplicities avoiding double counting

Combining TMD shower with higher orders

DY pt spectrum

- TMD evolution with multi-jet merging achieved at LO
- Low as well as high-pt now nicely described
- Consistent with MCatNLO PB-NLO at low pT

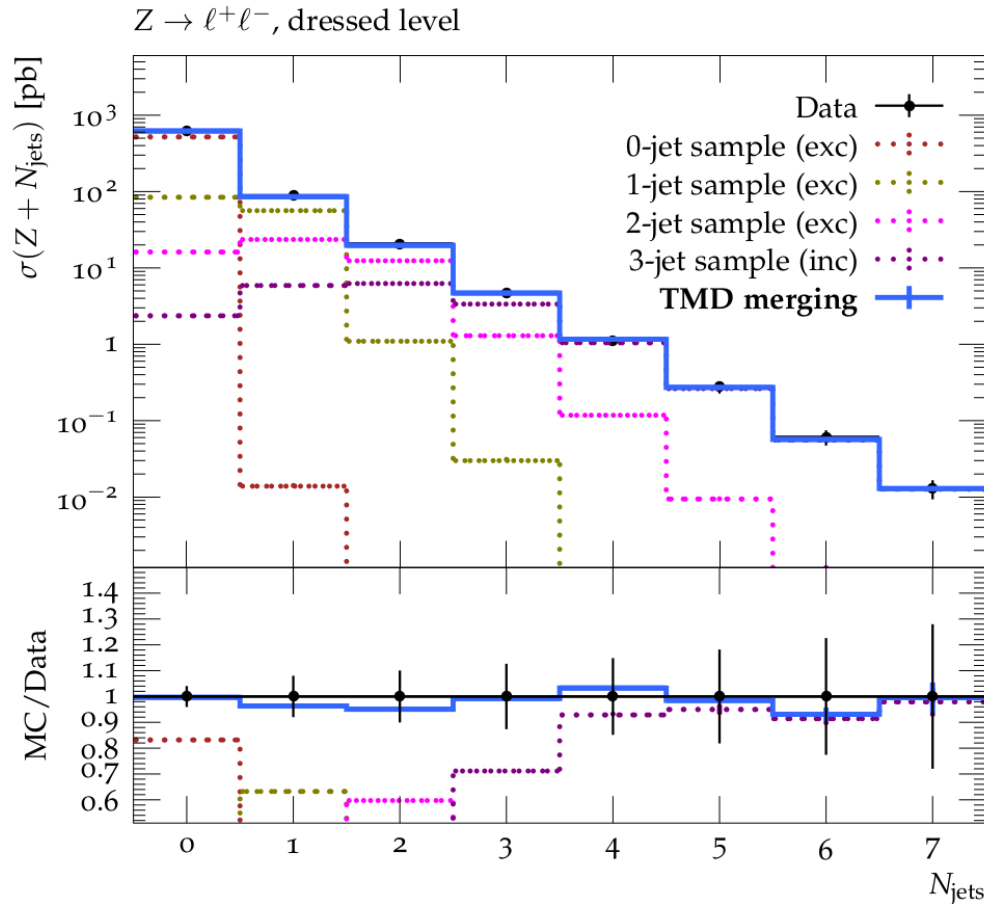
New! ABM et al. [arXiv:2107.01224]



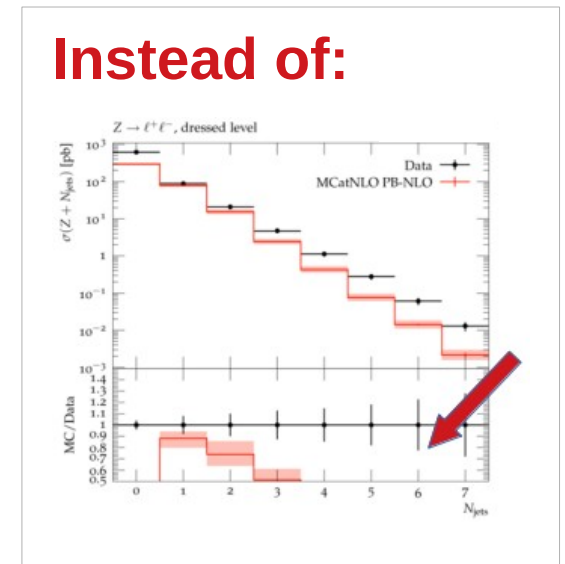
Combining TMD shower with higher orders

Exclusive jet multiplicity in Z events

ATLAS data: [Eur. Phys. J. C77 (2017) 361]



Instead of:



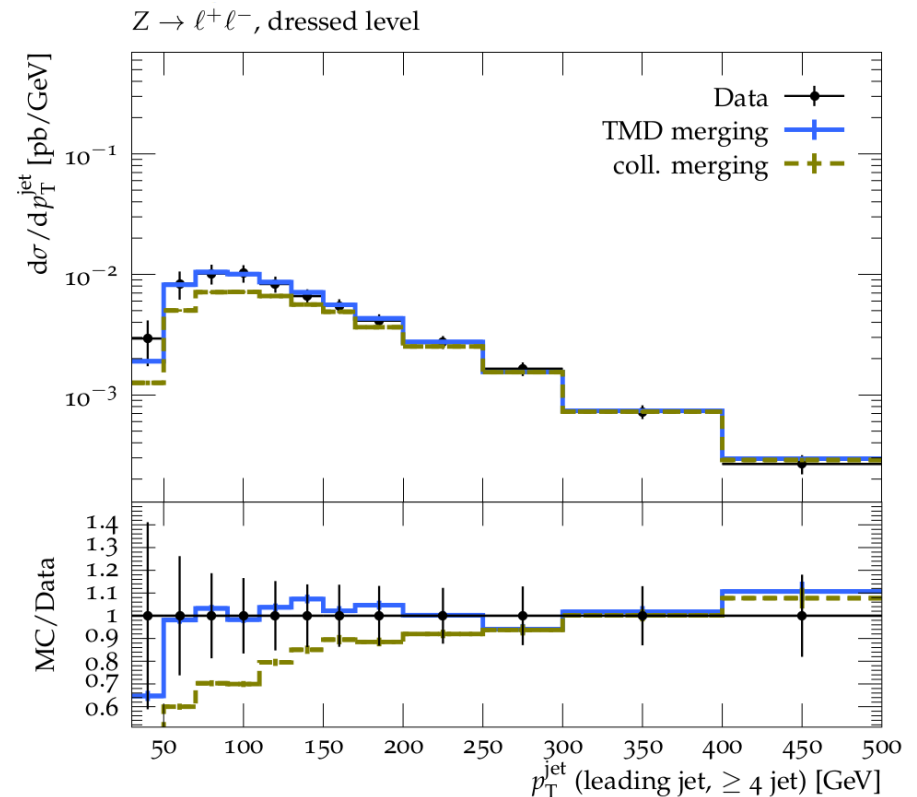
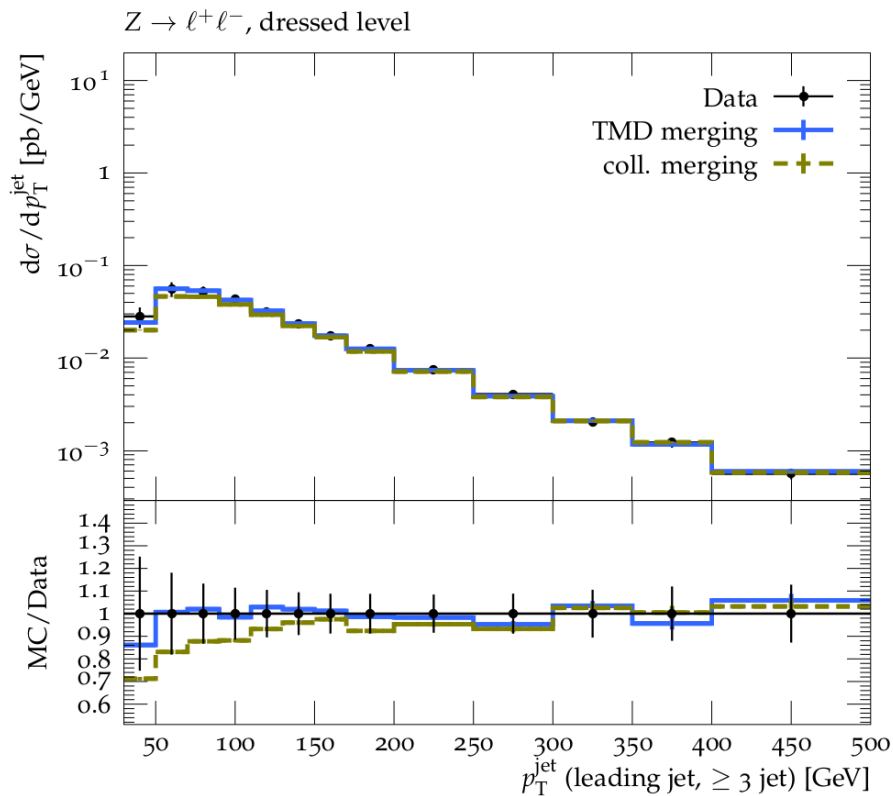
- Not only the overall recoil but also the number of jets are described

Combining TMD shower with higher orders

Jets pt spectrum

New! ABM et al. [[arXiv:2107.01224](https://arxiv.org/abs/2107.01224)]

- Not only overall recoil but also jet pT
- The description of jet pT improves at high multiplicities



Systematics

Multi-jet cross section in Z production

Merging scale [GeV]	$\sigma[\text{tot}]$ [pb]	$\sigma[\geq 1 \text{ jet}]$ [pb]	$\sigma[\geq 2 \text{ jet}]$ [pb]	$\sigma[\geq 3 \text{ jet}]$ [pb]	$\sigma[\geq 4 \text{ jet}]$ [pb]
23	572.98	87.26	20.27	4.84	1.18
33	563.04	86.15	20.48	4.86	1.19

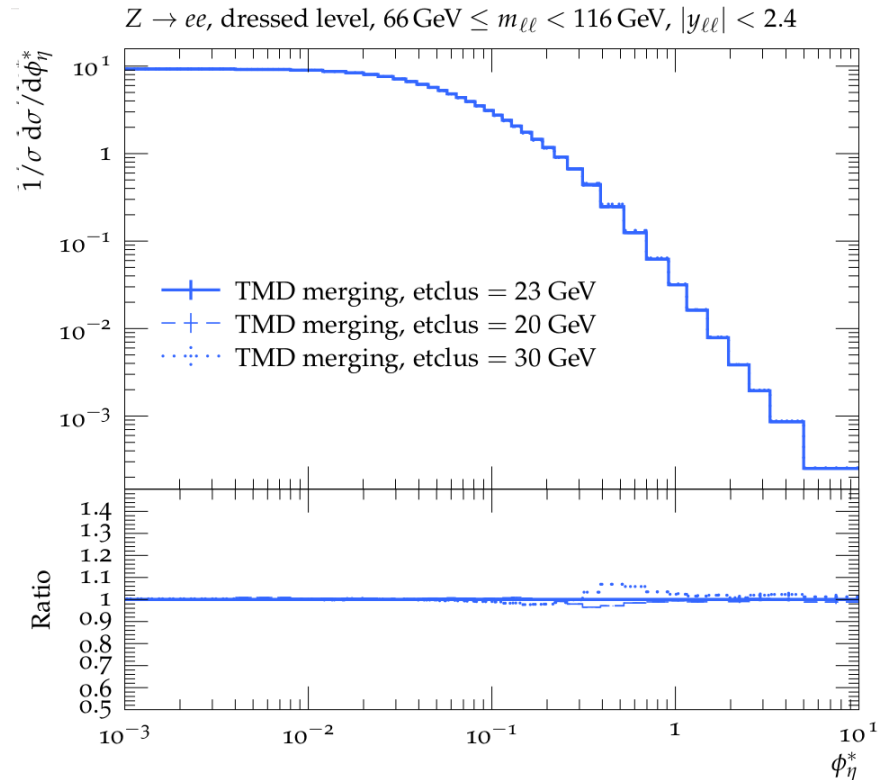
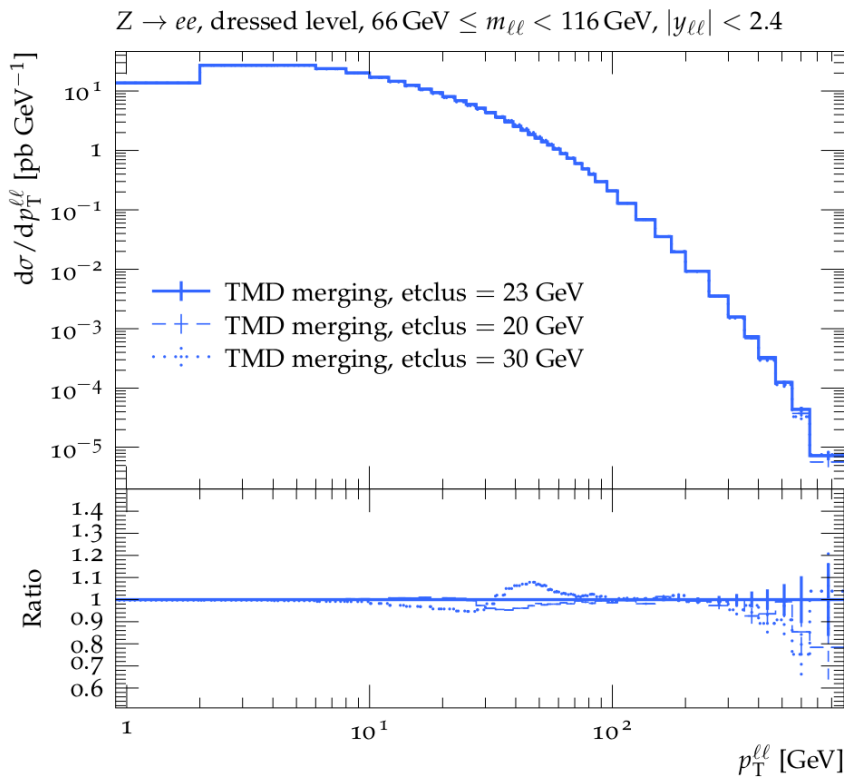
- 10 GeV variation gives < **2% change** in jets cross sections
- Standard merging algorithms can give over 10 % change for the same variation of the merging scale CF: J. Alwall et al. [EPJC 53, 473–500 (2008)]



Dependence on merging scale reduced by treating transverse momentum in the initial state

Systematics

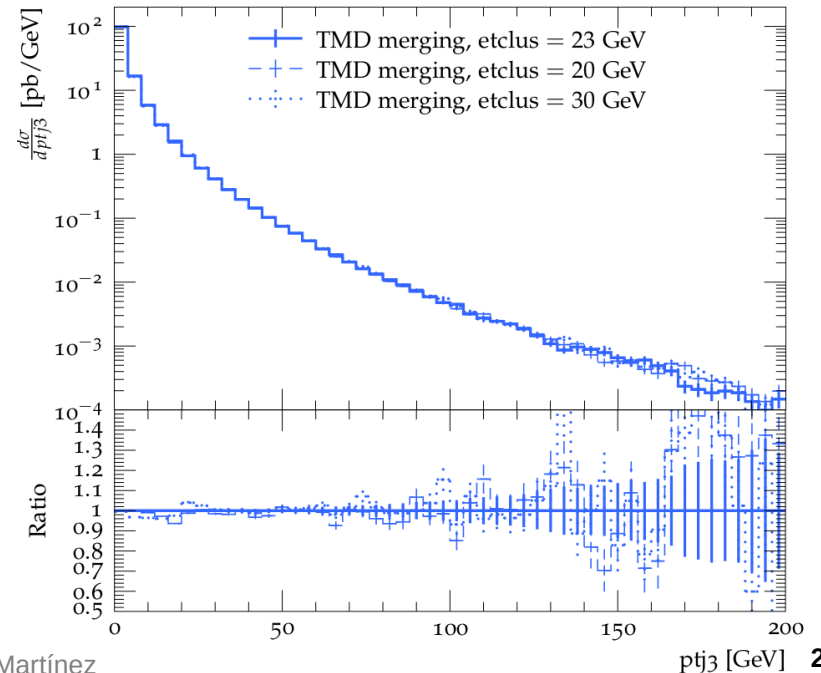
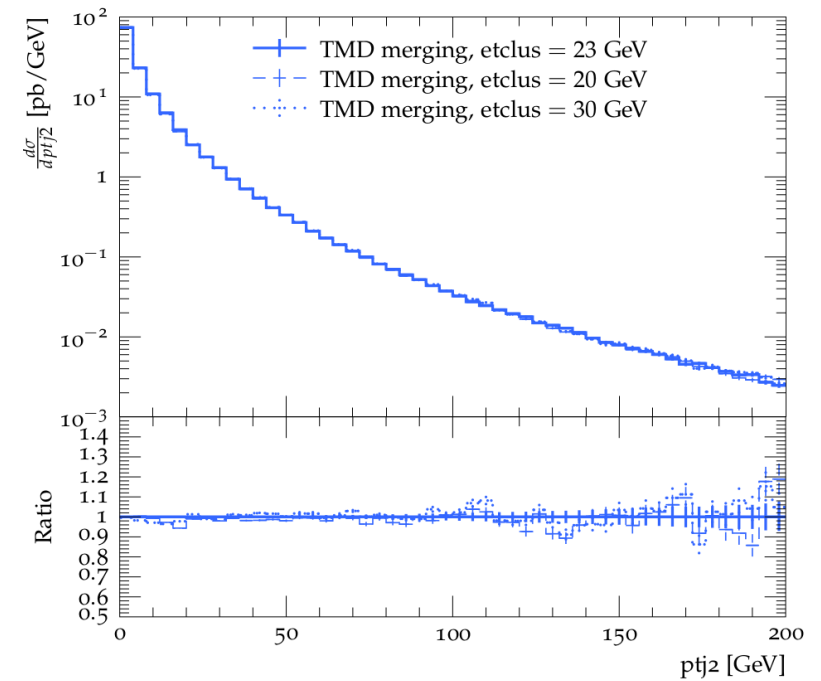
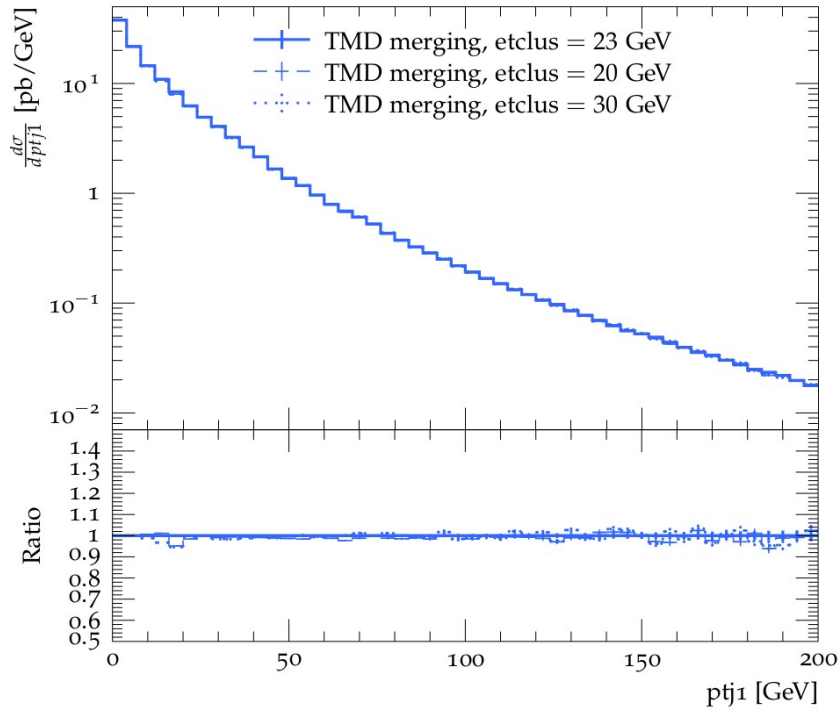
Z pt and phi*



Less than 10% effect localized around the merging scale

Systematics

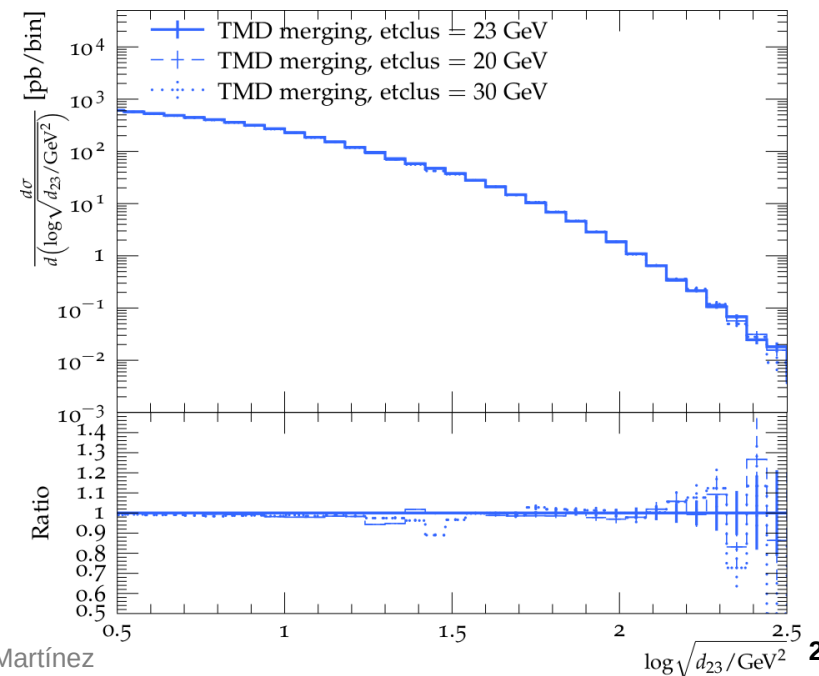
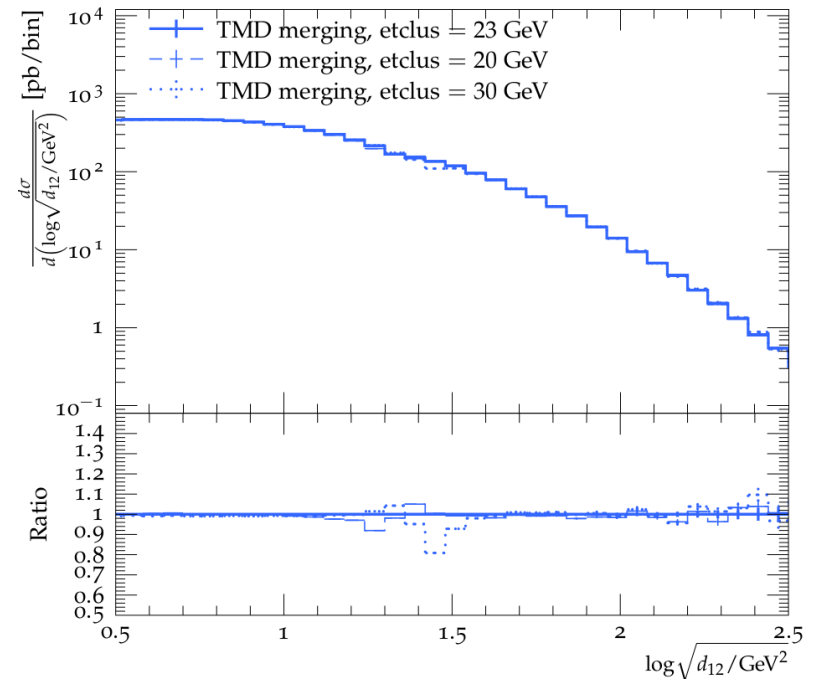
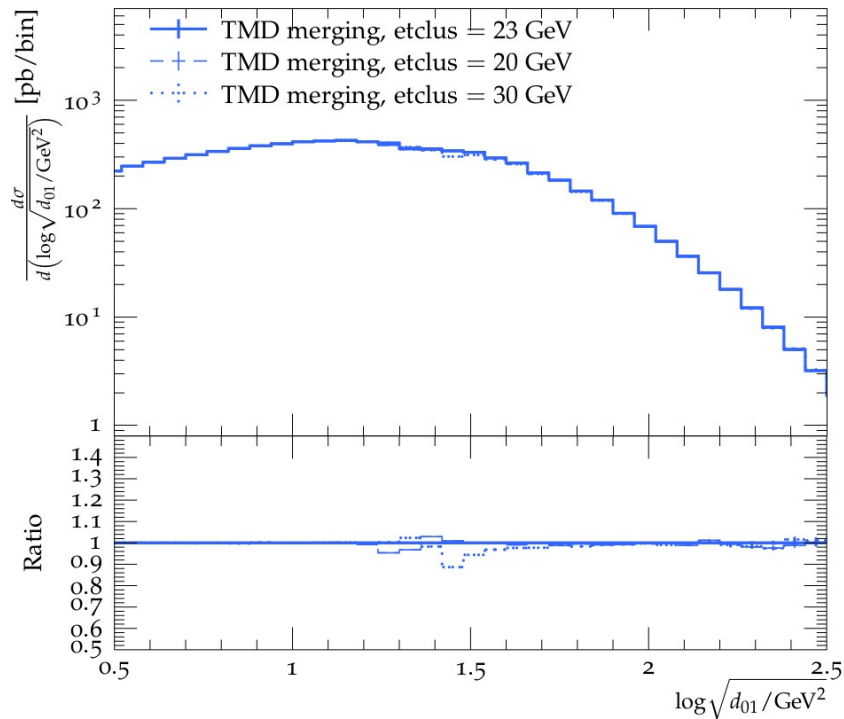
Pt of the jets



Less than 10% effect localized around the merging scale

Systematics

Differential jet rates



Up to 20% effect localized around the merging scale

Conclusions

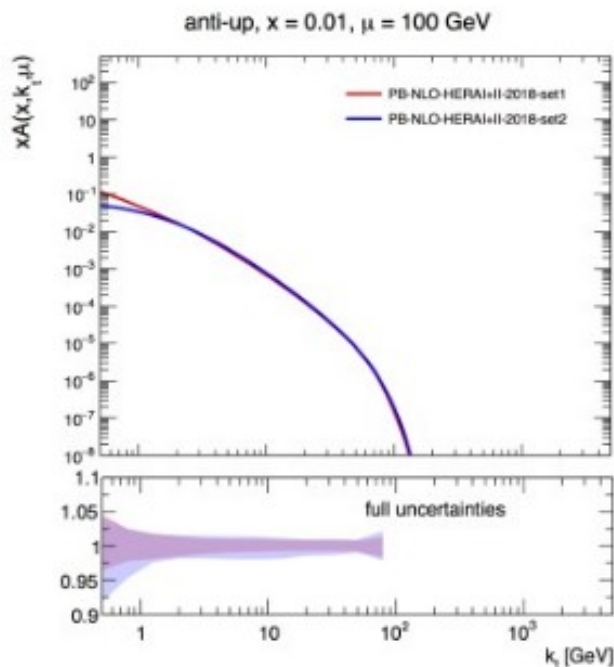
- PB TMD evolution provides excellent description of DY pt spectrum in a wide range of DY mass
- Parton shower from PB TMD evolution have significant contribution to jet multiplicity and jet pt spectra
- Higher fix-order contributions to PB TMD evolution are significant
- First combination of TMD evolution effects with multi-jet merging for Z pt and jet spectra
- Dependence of the results on the merging scale are smaller than that of standard algorithms

Thank you

PB framework

Phys. Lett. B 772:446451 (2017)
JHEP 01:070 (2018)
Phys. Rev. D 99, 074008 (2019)

- TMD determined, no extra parameters
- Full access to splitting kinematics
- TMD evolution implemented in xFitter



Where to find them:

arXiv:2103.09741 (accepted for publication in EPJC)

- TMDlib: library of parametrization of TMDs and uPDFs
- TMDplotter: TMD plotting tool

The screenshot shows the 'Integrated PDF plotter' web interface. It features a navigation bar with 'Home', 'TMD Plotter', 'Publications', and 'HEP Links'. The main content area is divided into several sections: 'Parameters' with input fields for $q^2 = 25$ GeV², $Y_{min} = 1.0E-5$, $Y_{max} = 100$, $X_{min} = 1.0E-5$, and $X_{max} = 1$; 'PDFs' with a list of four entries (gluon, gluon, photon, gluon) and their corresponding PDF sets and weights; and 'Output' with a 'Format' dropdown set to 'ps' and checkboxes for 'display ratio' and 'display command line'. A 'Plot' button is visible. On the right, a plot shows the distribution for $q^2 = 25$ GeV² on a log-log scale. The footer contains the copyright information '© 2012-2016 Deutsches Elektronen-Synchrotron (DESY)', the version 'UNPDF 0.1.4 and TMDlib 1.0.0', and logos for DESY and the Helmholtz Association.