CP-conserving 2HDM benchmarks

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The Higgs basis of the 2HDM

Start with the 2HDM scalar doublet, hypercharge-one fields, Φ_1 and Φ_2, in a generic basis, where $\langle\Phi_i\rangle = v_i$, and $v^2 \equiv |v_1|^2 + |v_2|^2 = (246\ \text{GeV})^2$. It is convenient to define new Higgs doublet fields:

$$H_1 = \begin{pmatrix} H_1^+ \\ H_1^0 \end{pmatrix} \equiv \frac{v_1^*\Phi_1 + v_2^*\Phi_2}{v}\,, \qquad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} \equiv \frac{-v_2\Phi_1 + v_1\Phi_2}{v}\,.$$

It follows that $\langle H_1^0\rangle = v/\sqrt{2}$ and $\langle H_2^0\rangle = 0$. This is the *Higgs basis*, which is uniquely defined up to $H_2 \to e^{i\chi}H_2$. The scalar potential is:

$$\begin{aligned}\mathcal{V} = {} & Y_1 H_1^\dagger H_1 + Y_2 H_2^\dagger H_2 + [Y_3 H_1^\dagger H_2 + \text{h.c.}] + \tfrac{1}{2}Z_1(H_1^\dagger H_1)^2 \\ & +\tfrac{1}{2}Z_2(H_2^\dagger H_2)^2 + Z_3(H_1^\dagger H_1)(H_2^\dagger H_2) + Z_4(H_1^\dagger H_2)(H_2^\dagger H_1) \\ & + \left\{\tfrac{1}{2}Z_5(H_1^\dagger H_2)^2 + \left[Z_6(H_1^\dagger H_1) + Z_7(H_2^\dagger H_2)\right]H_1^\dagger H_2 + \text{h.c.}\right\}\,,\end{aligned}$$

where Y_1, Y_2 and $Z_1, \ldots, Z_4$ are real and uniquely defined, whereas Y_3, Z_5, Z_6 and Z_7 are complex and transform under the rephasing of H_2,

$$[Y_3, Z_6, Z_7] \to e^{-i\chi}[Y_3, Z_6, Z_7] \quad \text{and} \quad Z_5 \to e^{-2i\chi}Z_5\,.$$

After minimizing the scalar potential, $Y_1 = -\frac{1}{2}Z_1 v^2$ and $Y_3 = -\frac{1}{2}Z_6 v^2$. This leaves 11 free parameters: 1 vev, 8 real parameters, Y_2, $Z_{1,2,3,4}$, $|Z_{5,6,7}|$, and two relative phases.

The charged Higgs boson is the charged component of the Higgs-basis doublet H_2, and its mass is given by $m^2_{H^\pm} = Y_2 + \frac{1}{2}Z_3 v^2$. The three physical neutral Higgs boson mass-eigenstates are determined by diagonalizing a 3×3 real symmetric squared-mass matrix that is defined in the Higgs basis

$$\mathcal{M}^2 = v^2 \begin{pmatrix} Z_1 & \mathrm{Re}(Z_6) & -\mathrm{Im}(Z_6) \\ \mathrm{Re}(Z_6) & \frac{1}{2}Z_{345} + Y_2/v^2 & -\frac{1}{2}\mathrm{Im}(Z_5) \\ -\mathrm{Im}(Z_6) & -\frac{1}{2}\mathrm{Im}(Z_5) & \frac{1}{2}Z_{345} - \mathrm{Re}(Z_5) + Y_2/v^2 \end{pmatrix},$$

where $Z_{345} \equiv Z_3 + Z_4 + \mathrm{Re}(Z_5)$.The diagonalizing matrix is a 3×3 real orthogonal matrix that depends on three angles: θ_{12}, θ_{13} and θ_{23}. The corresponding neutral Higgs masses will be denoted: m_1, m_2 and m_3. Under the rephasing $H_2 \to e^{i\chi} H_2$,

θ_{12} , θ_{13} are invariant, and $\quad \theta_{23} \to \theta_{23} - \chi$.

The CP-conserving 2HDM

Here, we will focus on the case of a CP-conserving scalar potential and vacuum. In this case, one can choose χ such that Y_3, Z_5, Z_6 and Z_7 are all real. The so-called *real Higgs basis* is not unique since we can still redefine $H_2 \to -H_2$. We shall use this freedom to fix $Z_6 > 0$, (the case of $Z_6 = 0$ must be treated separately) after which the real Higgs basis is unique. Then, we can identify

$$c_{12} = \sin(\beta - \alpha)\,,$$

$$s_{12} = -\cos(\beta - \alpha)\,,$$

$$\theta_{13} = \theta_{23} = 0\,,$$

where β and α refers to some generic basis which a priori has no special meaning, but $\beta - \alpha$ is an observable. Note that $m_2 > m_1$ implies that $\sin 2(\beta - \alpha) < 0$.

Notation: $c_{\beta-\alpha} \equiv \cos(\beta - \alpha)$ and $s_{\beta-\alpha} \equiv \sin(\beta - \alpha)$.

In addition, we shall consider particular models of the Higgs-fermion Yukawa couplings such that the neutral Higgs couplings to fermions are flavor-diagonal (eg. Type I or II). To implement this, we impose a $\mathbb{Z}_2$ symmetry on the dimension-four interactions of the Higgs Lagrangian in some basis $\{\Phi_1\,,\,\Phi_2\}$. With respect to this basis, we can define $\tan\beta = \langle\Phi_2^0\rangle/\langle\Phi_1^0\rangle$. The existence of this $\mathbb{Z}_2$ symmetry imposes the following constraint on the Higgs basis scalar potential parameters:

$$(Z_6+Z_7)(Z_2-Z_1)(Z_1+Z_2-2Z_{345})+(Z_6-Z_7)\left[(Z_2-Z_1)^2-4(Z_6+Z_7)^2\right]=0\,,$$

where $Z_{345}\equiv Z_3+Z_4+Z_5$. The parameter β is also determined (by convention, $0\leq\beta\leq\frac{1}{2}\pi$),

$$\tan 2\beta = \frac{2(Z_6+Z_7)}{Z_2-Z_1}$$

The case of $Z_1 = Z_2$ and $Z_6 = -Z_7$ must be treated separately. In this case, the existence of a $\mathbb{Z}_2$ symmetry is guaranteed, and the corresponding value of β is determined from the following quadratic equation,

$$(Z_1 - Z_{345})\tan 2\beta + 2Z_6(1 - \tan^2 2\beta) = 0\,.$$

This special case arises in the case of the MSSM Higgs sector, where

$$Z_1 = Z_2 = \tfrac{1}{4}(g^2 + g'^{\,2})\cos^2 2\beta\,, \qquad Z_3 = Z_5 + \tfrac{1}{4}(g^2 - g'^{\,2})\,, \qquad Z_4 = Z_5 - \tfrac{1}{2}g^2\,,$$
$$Z_5 = \tfrac{1}{4}(g^2 + g'^{\,2})\sin^2 2\beta\,, \qquad Z_7 = -Z_6 = \tfrac{1}{4}(g^2 + g'^{\,2})\sin 2\beta\cos 2\beta\,.$$

Ingredients for the CP-conserving 2HDM benchmarks

Case 1: Identify h with the observed Higgs boson, with $m_h \simeq 126$ GeV.

1. Choose $c_{\beta-\alpha} \ll 1$ to give SM-like hVV couplings.

2. Z_1 is determined by

$$Z_1 v^2 = m_h^2 - Z_6 v^2 \frac{c_{\beta-\alpha}}{s_{\beta-\alpha}}$$

3. Z_2 is determined in terms of β, Z_6 and Z_7

4. Imposing the $\mathbb{Z}_2$ symmetry, Z_{345} is determined in terms of Z_6 and Z_7 (once Z_1 and Z_2 are fixed).

5. Scan in the couplings Z_4, Z_5, Z_6 and Z_7 [where $Z_6 > 0$ and $s_{\beta-\alpha} c_{\beta-\alpha} < 0$ by convention].

The masses m_H, m_A and $m_{H^\pm}$ are determined by:

$$
\begin{aligned}
m_H^2 &= m_h^2 - \frac{Z_6 v^2}{s_{\beta-\alpha} c_{\beta-\alpha}}\,, \\
m_A^2 &= m_H^2 + \left[\frac{c_{\beta-\alpha}}{s_{\beta-\alpha}} Z_6 - Z_5\right] v^2\,, \\
m_{H^\pm}^2 &= m_A^2 - \tfrac{1}{2}(Z_4 - Z_5) v^2\,.
\end{aligned}
$$

In the case of $Z_6 = c_{\beta-\alpha} = 0$,

$$
\begin{aligned}
m_h^2 &= Z_1 v^2\,, \\
m_{H,A}^2 &= Y_2 + \tfrac{1}{2}(Z_3 + Z_4 \pm Z_5) v^2\,, \\
m_{H^\pm}^2 &= Y_2 + \tfrac{1}{2} Z_3 v^2\,.
\end{aligned}
$$

Remark:

Note that $c_{\beta-\alpha} \ll 1$ if either $m_A \gg m_h$ and/or $|Z_6| \ll 1$.

Case 2: Identify H with the observed Higgs boson, with $m_H \simeq 126$ GeV.

1. Choose $s_{\beta-\alpha} \ll 1$ to give SM-like HVV couplings.

2. Z_1 is determined by
$$Z_1 v^2 = m_H^2 + Z_6 v^2 \frac{s_{\beta-\alpha}}{c_{\beta-\alpha}}$$

3. Z_2 is determined in terms of β, Z_6 and Z_7

4. Imposing the $\mathbb{Z}_2$ symmetry, Z_{345} is determined in terms of Z_6 and Z_7 (once Z_1 and Z_2 are fixed).

5. Scan in the couplings Z_4, Z_5, Z_6 and Z_7 [where $Z_6 > 0$ and $s_{\beta-\alpha} c_{\beta-\alpha} < 0$ by convention]. Choose a value for $Z_6/s_{\beta-\alpha}$ such that $m_h^2 > 0$ consistent with the LEP Higgs bounds.

The masses m_H, m_A and $m_{H^\pm}$ are determined by:

$$
\begin{aligned}
m_h^2 &= m_H^2 + \frac{Z_6 v^2}{s_{\beta-\alpha} c_{\beta-\alpha}}\,,\\
m_A^2 &= m_H^2 + \left[\frac{c_{\beta-\alpha}}{s_{\beta-\alpha}} Z_6 - Z_5\right] v^2\,,\\
m_{H^\pm}^2 &= m_A^2 - \tfrac{1}{2}(Z_4 - Z_5) v^2\,.
\end{aligned}
$$

In the case of $Z_6 = s_{\beta-\alpha} = 0$,

$$
\begin{aligned}
m_H^2 &= Z_1 v^2\,,\\
m_{h,A}^2 &= Y_2 + \tfrac{1}{2}(Z_3 + Z_4 \pm Z_5) v^2\,,\\
m_{H^\pm}^2 &= Y_2 + \tfrac{1}{2} Z_3 v^2\,.
\end{aligned}
$$

Remark:

Note that $s_{\beta-\alpha} \ll 1$ if $|Z_6| \ll 1$ and $Z_1 v^2 > Y_2 + \frac{1}{2}\big(Z_3 + Z_4 + Z_5\big) v^2$.