

# Tsallis fitting of the CMS data

**Ferenc Siklér**

*Wigner RCP, Budapest*

for the CMS Collaboration



Heavy Ion Forum  
1 February 2013

# Introduction

---

- p-p analyses

- spectra of charged hadrons in p-p at  $\sqrt{s} = 0.9, 2.36, \text{ and } 7 \text{ TeV}$   
JHEP 02 (2010) 041 Phys Rev Lett 105 (2010) 022002
- spectra of charged hadrons in p-p up to high  $p_T$  at  $\sqrt{s} = 0.9 \text{ and } 7 \text{ TeV}$   
JHEP 08 (2011) 086
- spectra of identified neutral hadrons in p-p at  $\sqrt{s} = 0.9 \text{ and } 7 \text{ TeV}$   
JHEP 05 (2011) 064
- spectra of identified charged hadrons in p-p at  $\sqrt{s} = 0.9, 2.76, \text{ and } 7 \text{ TeV}$   
Eur Phys J C 72 (2012) 2164

- Why Tsallis?

- empirically describes both the low- $p_T$  exponential and the high- $p_T$  power-law behaviors
- flexible enough, still with only two parameters
- has a physics-motivated origin

All shown results are from **p-p collisions**

# Tsallis-Pareto-type distributions

---

By now a well known functional form, success at RHIC – LHC:

$$\frac{d^2 N}{dy dp_T} = \frac{dN}{dy} \cdot C \cdot p_T \left[ 1 + \frac{(m_T - m)}{nT} \right]^{-n}$$

where

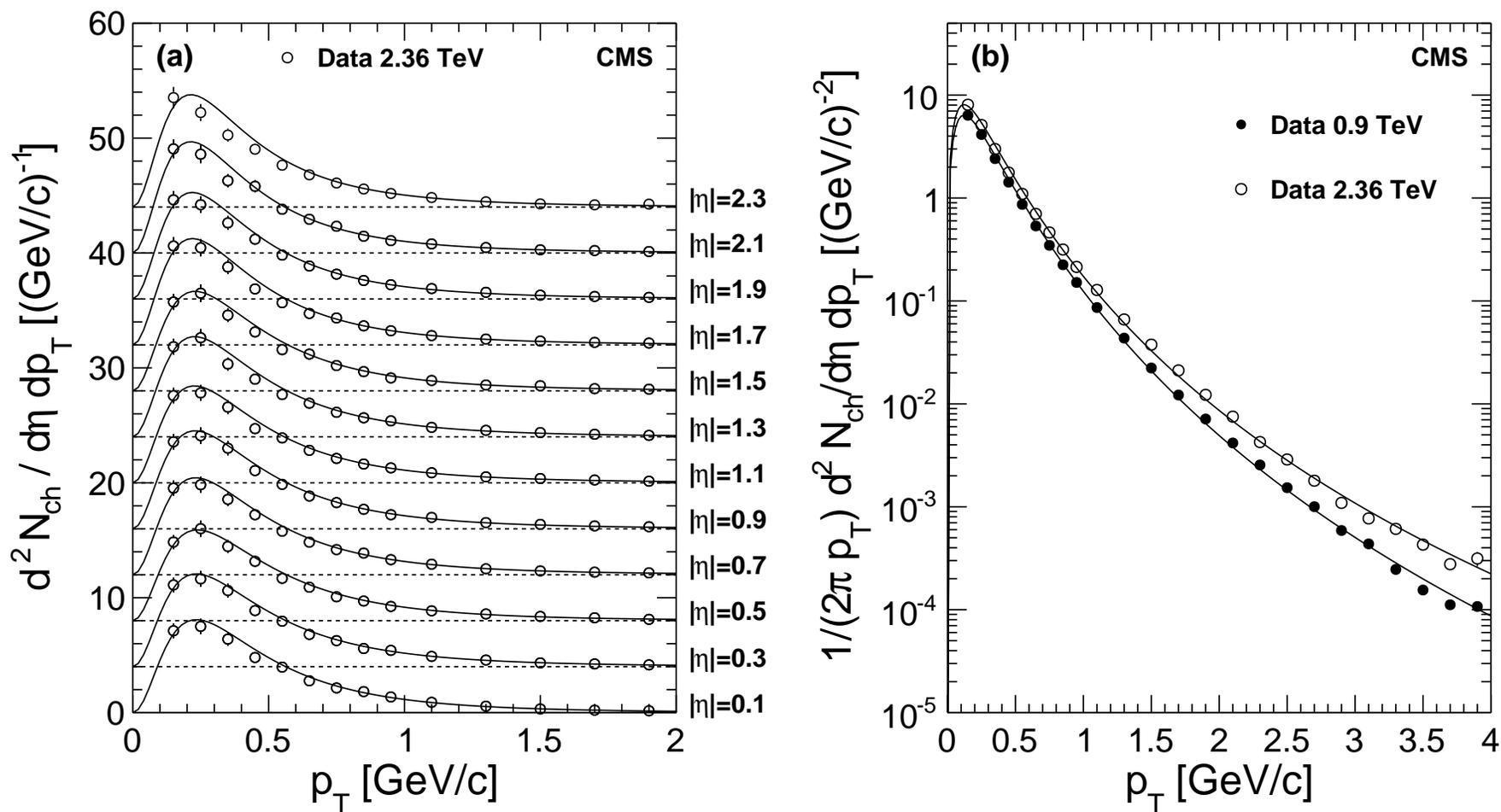
$$C = \frac{(n-1)(n-2)}{nT[nT + (n-2)m]}, \quad m_T = \sqrt{m^2 + p_T^2}.$$

With roots in non-extensive statistics; a Lévy function

$n$  – exponent,  $T$  – inverse slope parameter,  $m$  – particle mass  
 $\langle p_T \rangle$  is calculable with Monte Carlo integration

- $T$  is determined by the low  $p_T$  part, while  $n$  is by the tail
- $n$  and  $T$  can be highly correlated, especially if we don't have enough  $p_T$  range

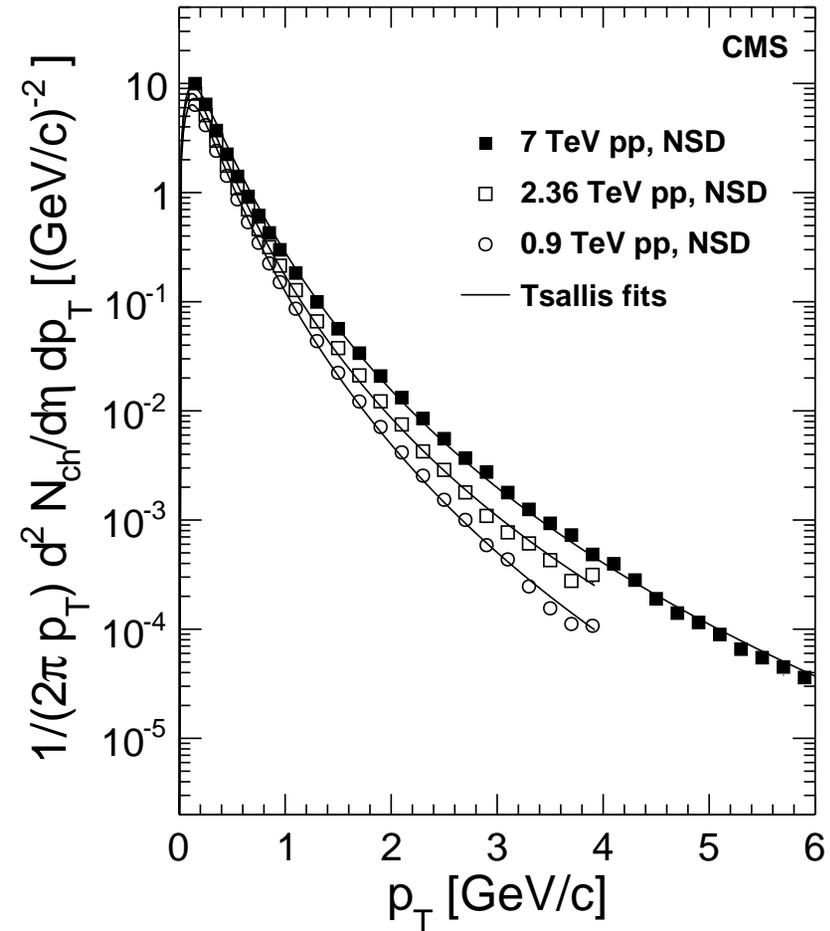
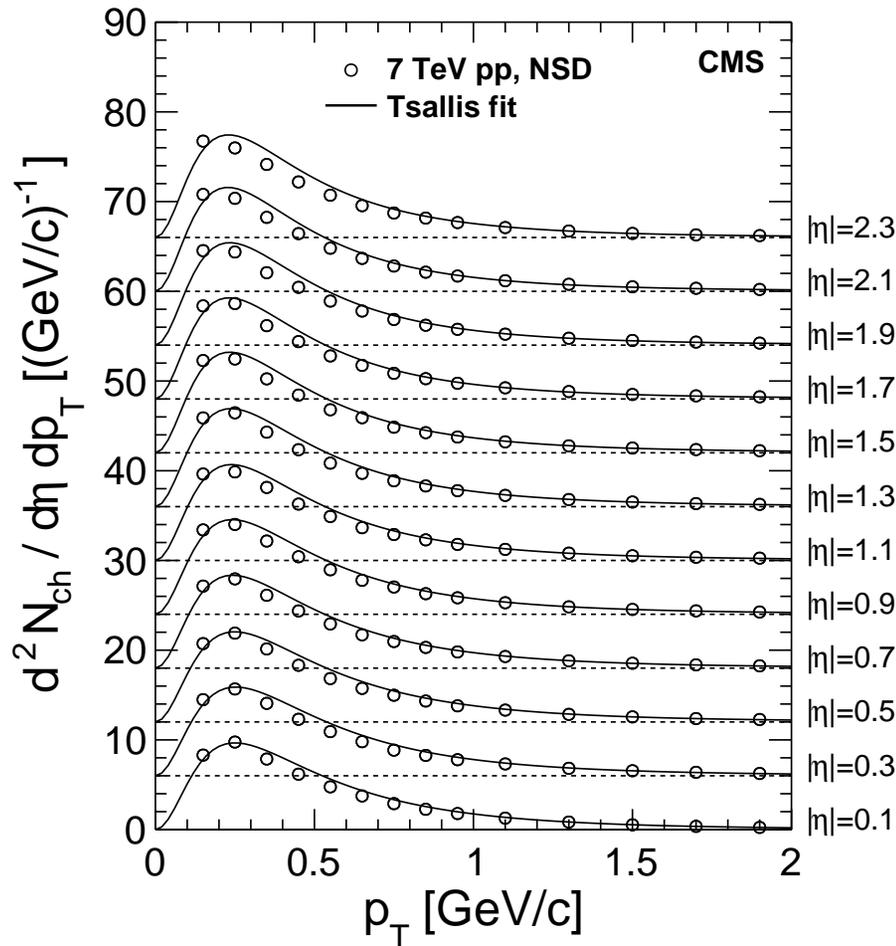
# Results – charged hadrons



Unidentified charged hadrons, both charges  
Fits in narrow  $|\eta|$  bins at 0.9 and 2.36 TeV

Good fits

# Results – charged hadrons



Unidentified charged hadrons, both charges  
Right plot, p-p at 0.9, 2.36, and 7 TeV

# Results – charged hadrons

---

$$E \frac{d^3 N_{\text{ch}}}{dp^3} = \frac{1}{2\pi p_T} \frac{E}{p} \frac{d^2 N_{\text{ch}}}{d\eta dp_T} = C(n, T, m) \frac{dN_{\text{ch}}}{dy} \left(1 + \frac{E_T}{nT}\right)^{-n}$$

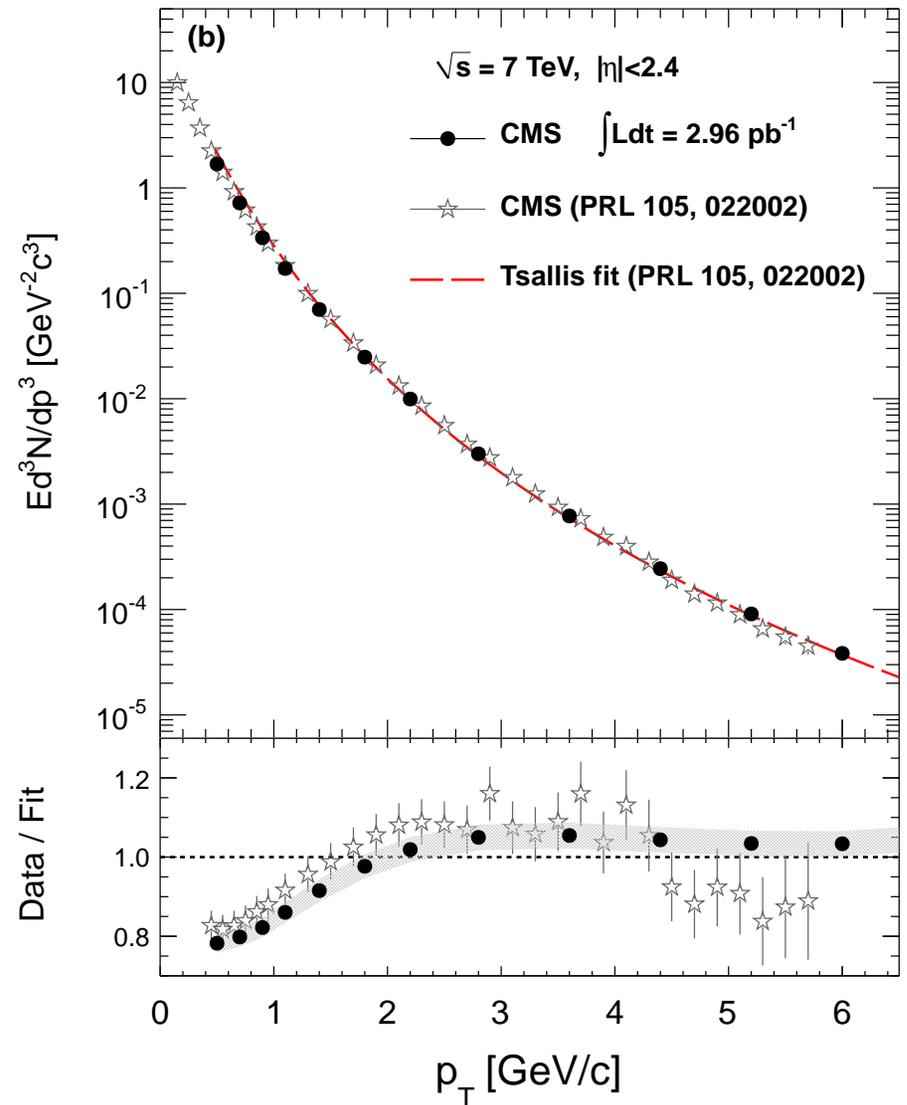
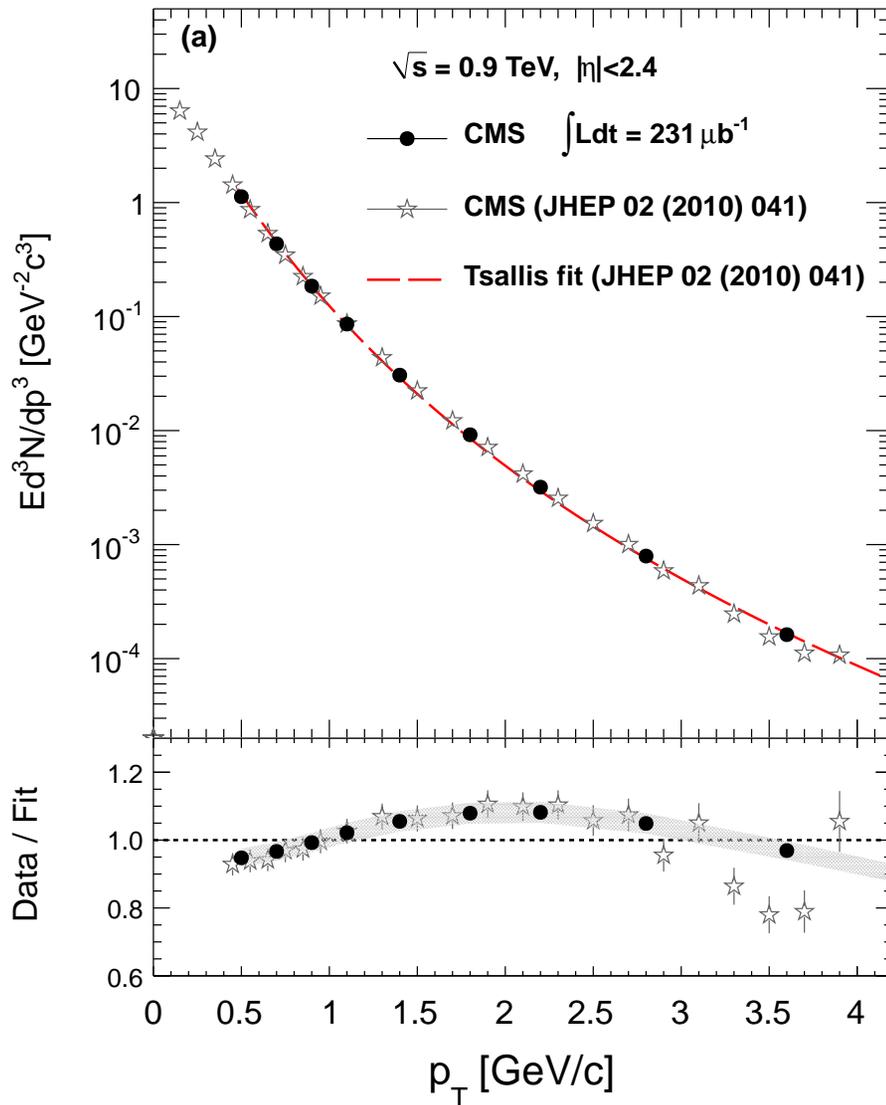
$$E_T = \sqrt{m^2 + p_T^2} - m$$

The  $(y, p_T)$  plan was transformed to  $(\eta, p_T)$ , taking  $m$  the charged pion mass

0.9 TeV	$T = 0.13 \pm 0.01$	GeV	$n = 7.7 \pm 0.2$
2.36 TeV	$T = 0.14 \pm 0.01$	GeV	$n = 6.7 \pm 0.2$
7 TeV	$T = 0.145 \pm 0.005$	GeV	$n = 6.6 \pm 0.2$

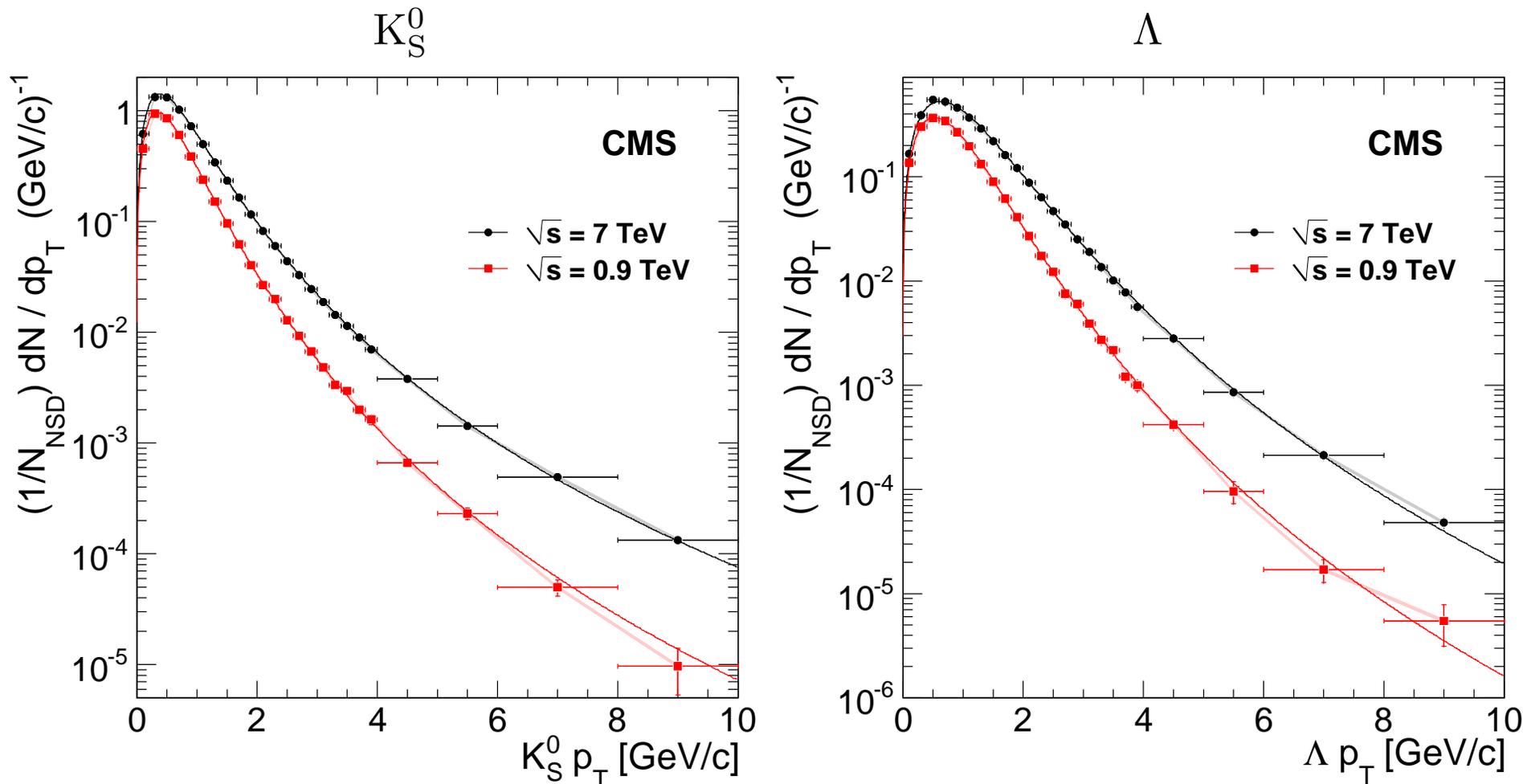
Slow evolution with  $\sqrt{s}$

# Results – high- $p_T$ charged hadrons



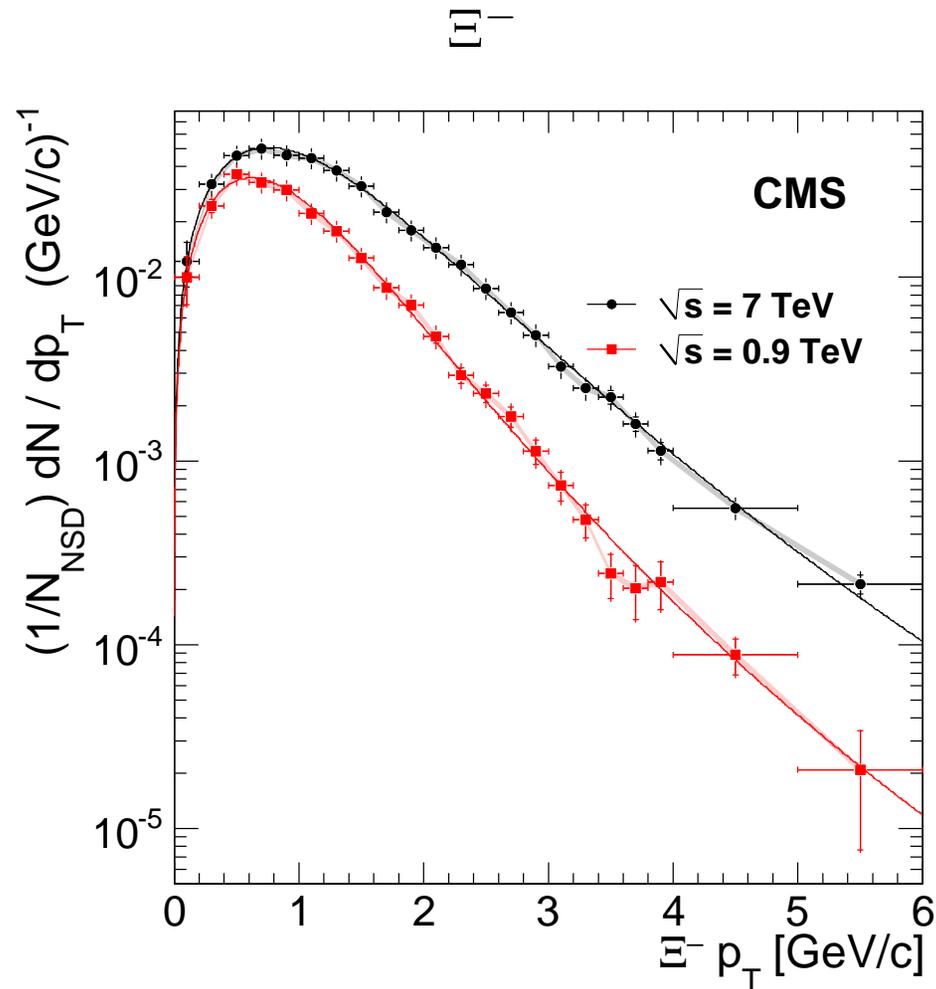
Comparison; good match with the Tsallis fit obtained for  $p_T \approx 0 - 4 \text{ GeV}/c$

# Results – strange hadrons



Particle production per NSD event  
The (correlated) normalization uncertainty is shown as a band  
Good fits to most of the samples

# Results – strange hadrons



Particle production per NSD event  
The (correlated) normalization uncertainty is shown as a band

# Results – strange hadrons

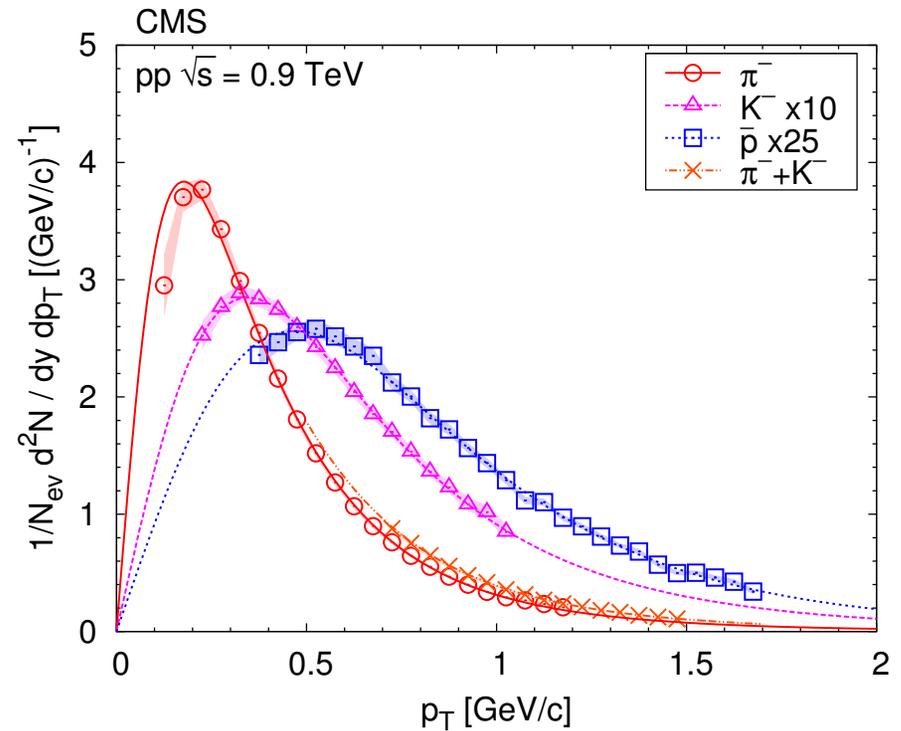
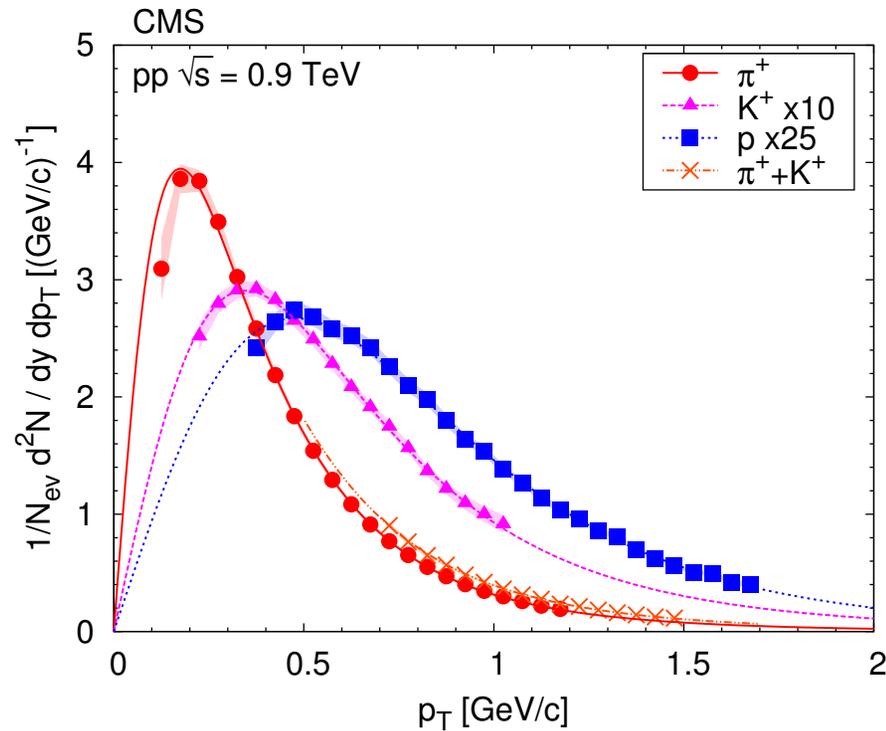
$$\frac{1}{N_{\text{NSD}}} \frac{dN}{dp_{\text{T}}} = C \frac{(n-1)(n-2)}{nT[nT + m(n-2)]} p_{\text{T}} \left[ 1 + \frac{\sqrt{p_{\text{T}}^2 + m^2} - m}{nT} \right]^{-n}$$

Particle	$\sqrt{s}$ (TeV)	$C$	$T$ (MeV)	$n$	$\chi^2/\text{NDF}$
$K_{\text{S}}^0$	0.9	$0.776 \pm 0.002 \pm 0.042$	$187 \pm 1 \pm 4$	$7.79 \pm 0.07 \pm 0.26$	19/21
$\Lambda$	0.9	$0.395 \pm 0.002 \pm 0.041$	$216 \pm 2 \pm 11$	$9.3 \pm 0.2 \pm 1.1$	32/21
$\Xi^-$	0.9	$0.043 \pm 0.001 \pm 0.006$	$250 \pm 8 \pm 48$	$10.1 \pm 0.9 \pm 4.7$	19/19
$K_{\text{S}}^0$	7	$1.329 \pm 0.001 \pm 0.062$	$220 \pm 1 \pm 3$	$6.87 \pm 0.02 \pm 0.09$	50/21
$\Lambda$	7	$0.696 \pm 0.002 \pm 0.058$	$292 \pm 1 \pm 10$	$9.3 \pm 0.1 \pm 0.5$	128/21
$\Xi^-$	7	$0.080 \pm 0.001 \pm 0.012$	$361 \pm 7 \pm 72$	$11.2 \pm 0.7 \pm 4.9$	21/19

Table 2. Results of fitting the Tsallis function to the data. In the  $C$ ,  $T$ , and  $n$  columns, the first uncertainty is statistical and the second is systematic. The parameter values and  $\chi^2/\text{NDF}$  are obtained from fits to the data with only the statistical uncertainty included.

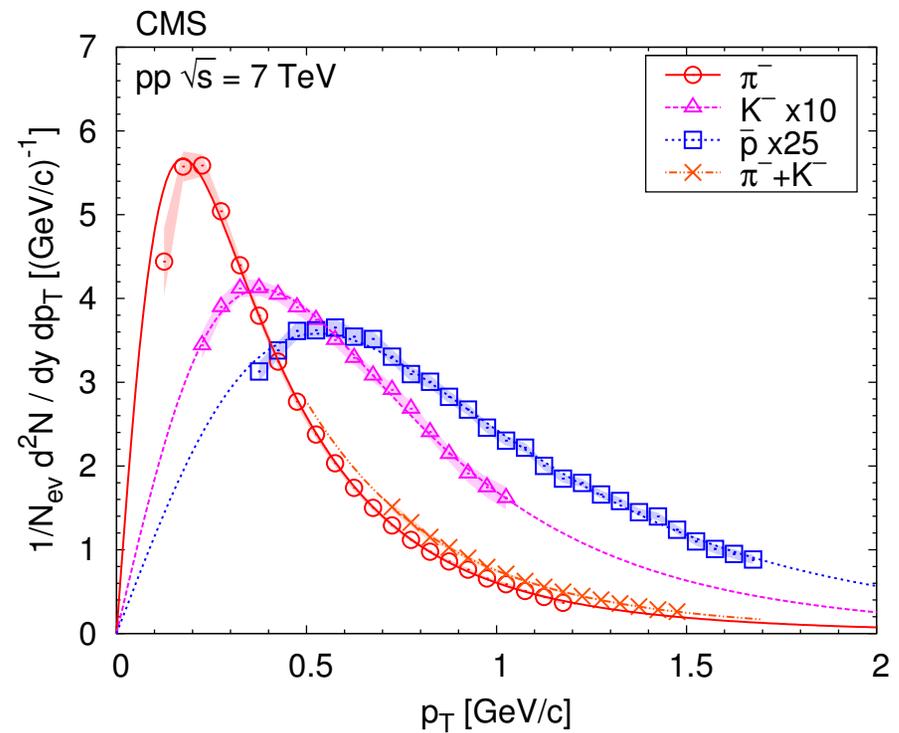
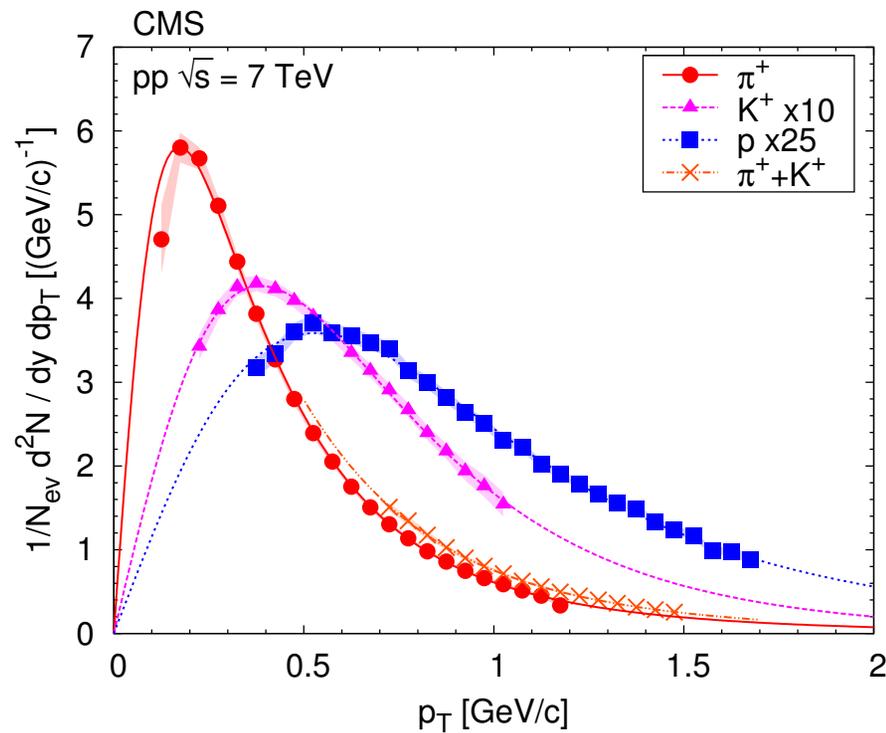
$T$  increases with particle mass and centre-of-mass energy  
 Much steeper fall off ( $n$ ) for the two baryons than for the  $K_{\text{S}}^0$

# Results – $\pi/K/p$ spectra



Results are corrected to a double-sided selection (DS):  
at least one particle with  $E > 3$  GeV on both sides  
( $-5 < \eta < -3$  and  $3 < \eta < 5$ )

# Results – $\pi/K/p$ spectra

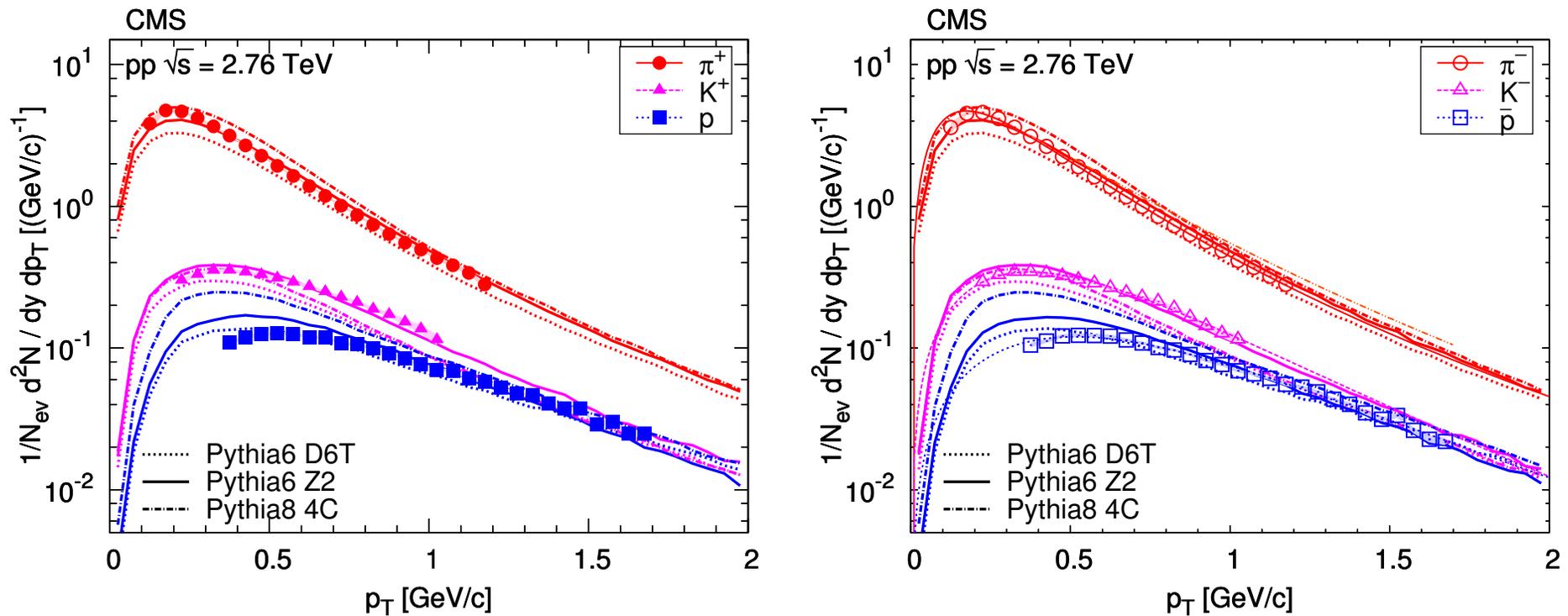


The fits are usually of good quality

With  $\chi^2/\text{ndf}$  values in the range 0.6-1.5 for pions,  
0.6-2.1 for kaons, and 0.4-1.1 for protons

Some deviations for the lowest- $p_T$  pions

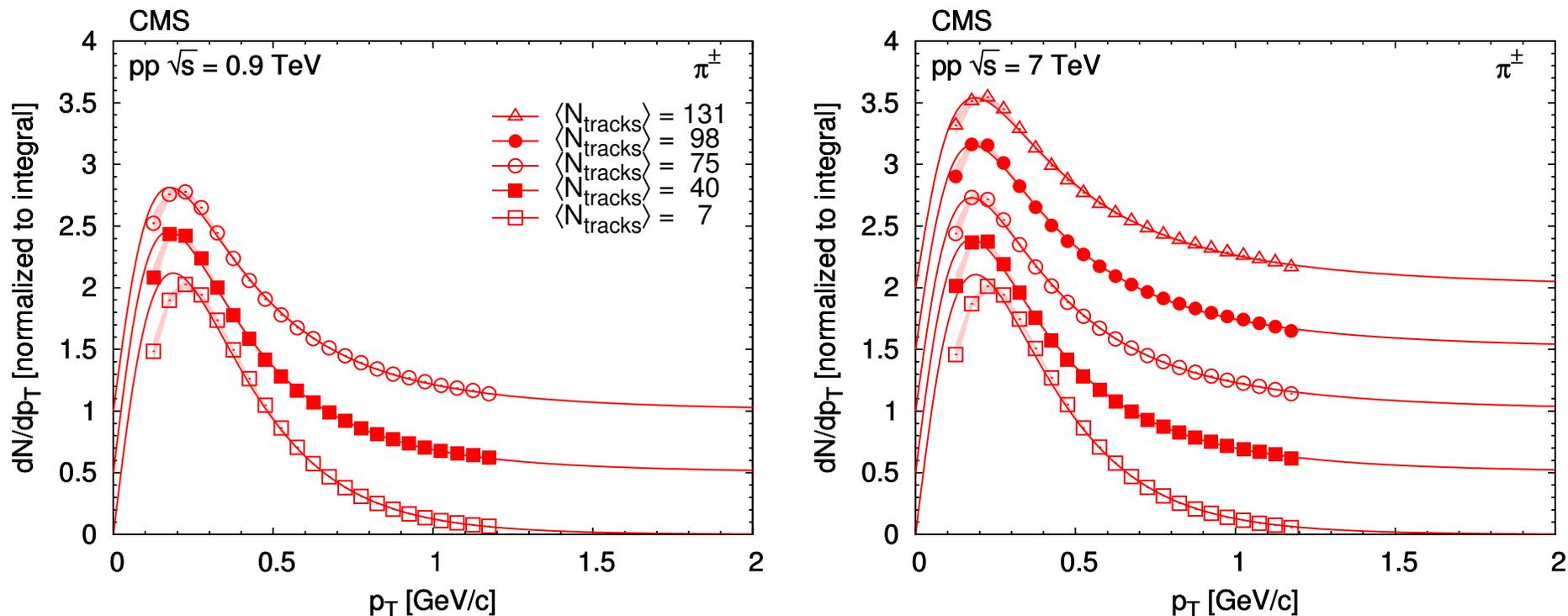
# Results – $\pi/K/p$ spectra



Logarithmic scale, comparison to models

Pythia6 D6T and Pythia8 4C tend to systematically under/overshoot the spectra  
Pythia6 Z2 is generally closer to the measurements (except for low- $p_T$  protons)

# Results – multiplicity dependence – pions

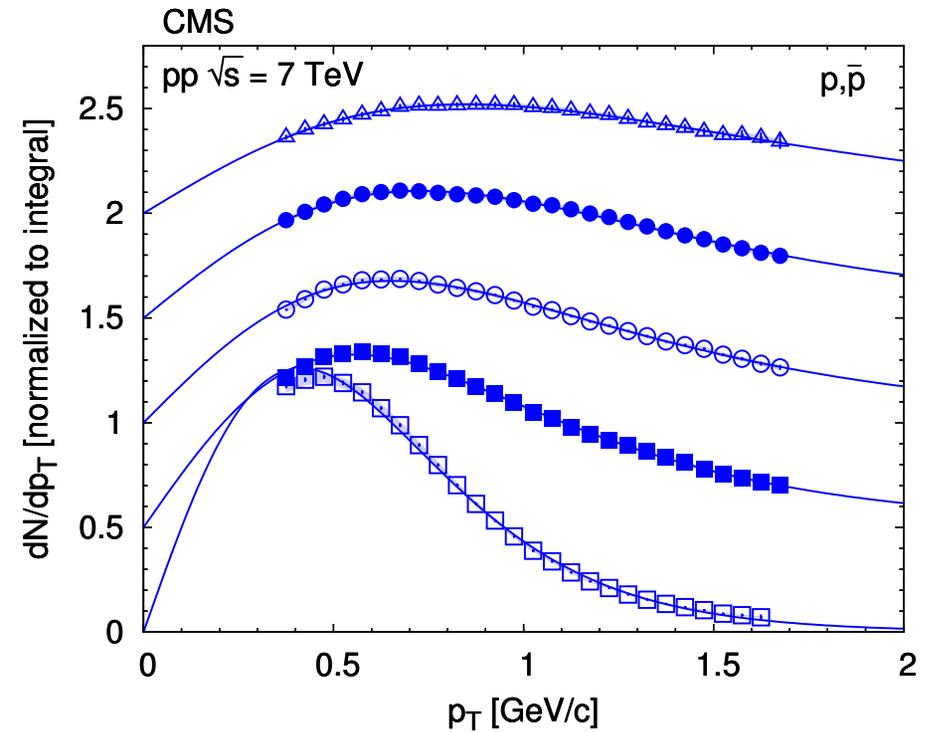
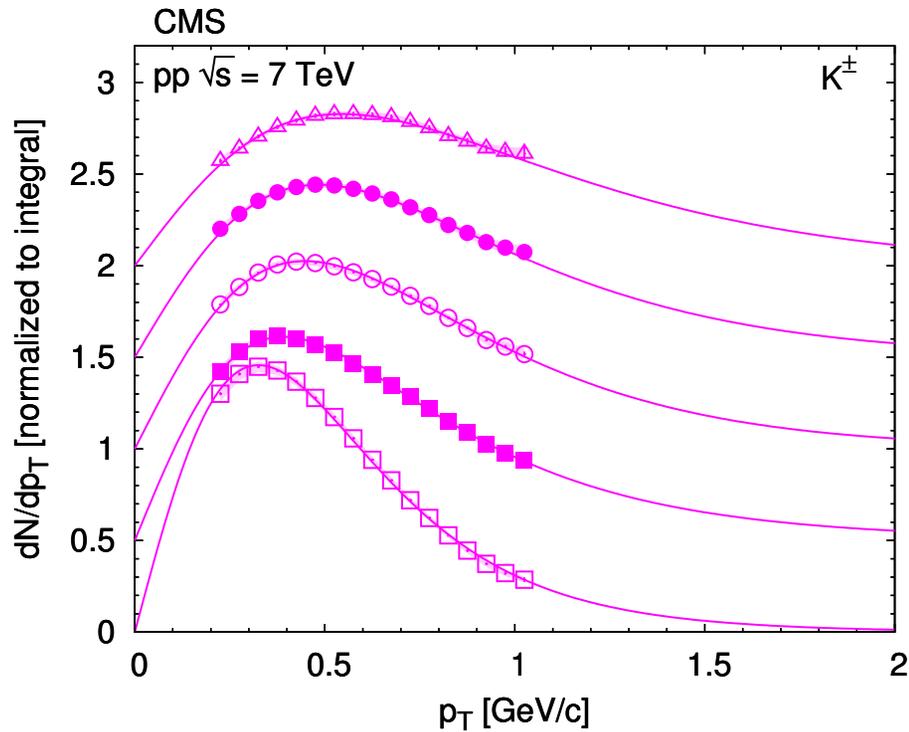


Multiplicity classes: relate measured  $N_{\text{rec}}$  to true  $\langle N_{\text{tracks}} \rangle$  in  $|\eta| < 2.4$

$N_{\text{rec}}$	0-9	10-19	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99	100-109	110-119
$\langle N_{\text{tracks}} \rangle$	7	16	28	40	52	63	75	86	98	109	120	131

Shapes?

# Results – multiplicity dependence – kaons and protons



Harder spectral shape with increasing multiplicity

Increasing  $\langle p_T \rangle$  with increasing multiplicity

# Summary

---

- CMS and Tsallis
  - Fits from very low to high  $p_T$  with only two parameters ( $T$  and  $n$ )
  - Used in all p-p spectra analyses
- Why so successful?
  - many applications in the natural and social sciences
  - physics origins? non-extensivity?
  - maybe the fractal structure and dynamic nature of particle production?
  - in p-p collisions the phase-space has to be filled with particles in a very short time