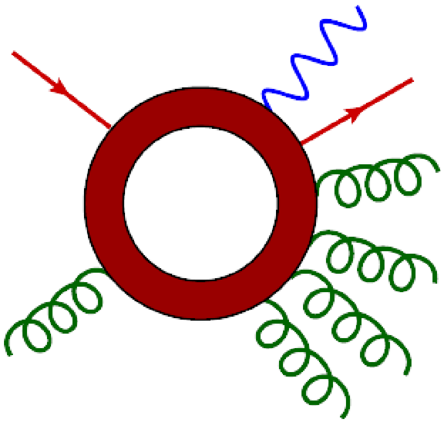
QCD at Colliders

Lecture 4
Modern QCD amplitude computation



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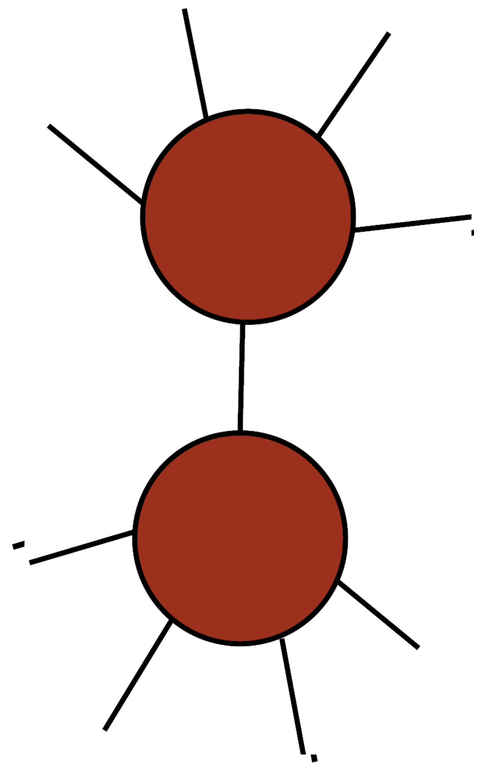
2012 European School
of High Energy Physics

Recycling "Plastic" Amplitudes

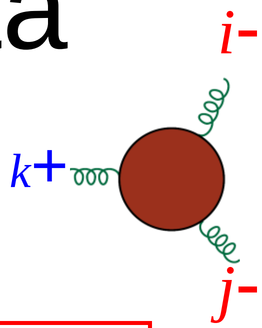
Amplitudes fall apart into simpler ones in special limits – pole information



Trees recycled into trees

BCFW recursion relations

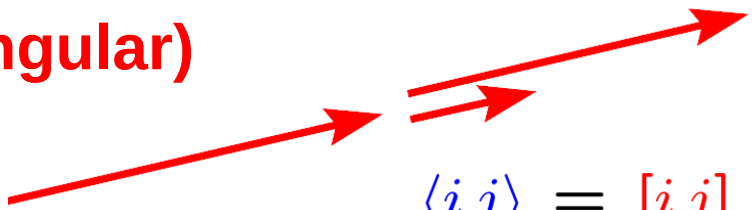
Utility of Complex Momenta



- Makes sense of most basic process: all 3 particles massless

$$s_{ij} = 2k_i \cdot k_j = (k_i + k_j)^2 = 0 \quad \forall\ i,j \qquad \langle i\,j\rangle [j\,i] = s_{ij}$$

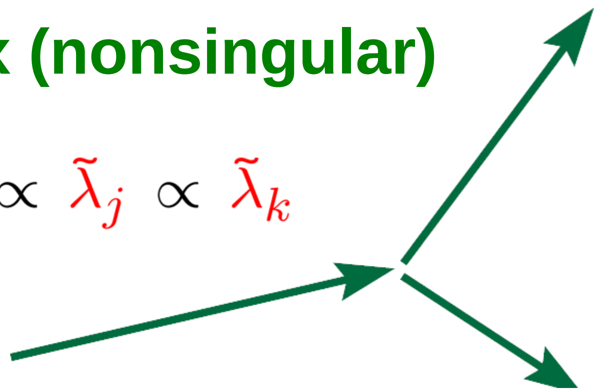
real (singular)



$$\langle i\,j\rangle = [i\,j] = s_{ij} = 0 \qquad \forall\ i,j$$

complex (nonsingular)

$$\tilde\lambda_i \propto \tilde\lambda_j \propto \tilde\lambda_k$$



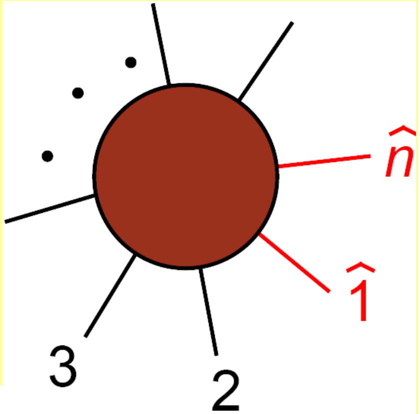
$$[i\,j] = 0 \ \text{ but } \ \langle i\,j\rangle \neq 0$$

$$\boxed{\frac{\langle i\,j\rangle^4}{\langle 1\,2\rangle\langle 2\,3\rangle\langle 3\,1\rangle} \text{ makes sense}}$$

use conjugate kinematics for ($\lambda_i \propto \lambda_j \propto \lambda_k \quad \langle i\,j\rangle = 0\,,\ [i\,j] \neq 0$

Tree-level plasticity
→ BCFW recursion relations

- BCFW consider a family of on-shell amplitudes $A_n(z)$ depending on a complex parameter z which shifts the momenta to complex values



- For example, the $[n,1\rangle$ shift:

$$\lambda_1 \to \hat{\lambda}_1 = \lambda_1 + z\lambda_n \qquad \tilde{\lambda}_1 \to \tilde{\lambda}_1$$

$$\lambda_n \to \lambda_n \qquad \tilde{\lambda}_n \to \hat{\tilde{\lambda}}_n = \tilde{\lambda}_n - z\tilde{\lambda}_1$$

- On-shell condition:

$$\begin{aligned}(\hat{k}_1)^\mu(\hat{k}_1)_\mu &= (\hat{k}_1)^{\alpha\dot{\alpha}}(\hat{k}_1)_{\dot{\alpha}\alpha} \\ &= \langle(\lambda_1 + z\lambda_n)(\lambda_1 + z\lambda_n)\rangle[1\,1] = 0\end{aligned}$$

similarly, $\hat{k}_n^2 = 0$

- Momentum conservation:

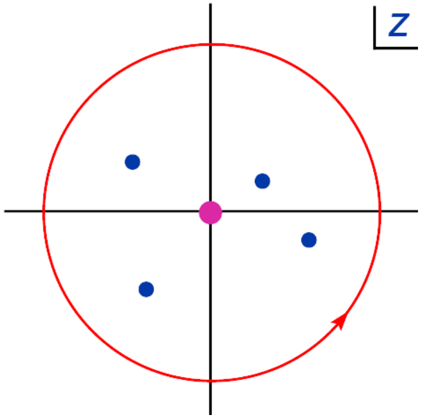
$$\hat{k}_1 + \hat{k}_n = (\lambda_1 + z\lambda_n)\tilde{\lambda}_1 + \lambda_n(\tilde{\lambda}_n - z\tilde{\lambda}_1) = k_1 + k_n$$

Analyticity → recursion relations

$$\hat{\lambda}_1 = \lambda_1 + z\lambda_n \qquad \hat{\tilde{\lambda}}_1 = \tilde{\lambda}_1$$
$$\hat{\lambda}_n = \lambda_n \qquad \hat{\tilde{\lambda}}_n = \tilde{\lambda}_n - z\tilde{\lambda}_1$$

$$\Rightarrow \quad A(0) \;\to\; A(z)$$

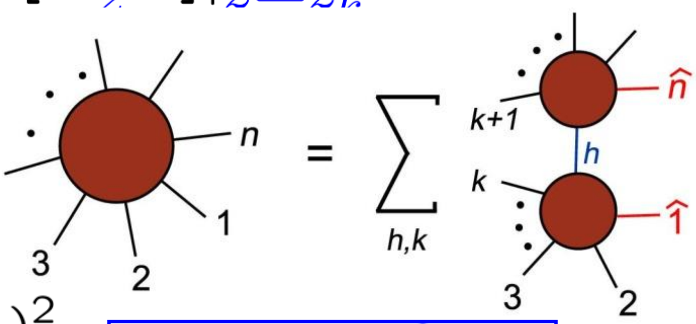
meromorphic function, each pole corresponds to one factorization



Cauchy: If $A(\infty) = 0$ then

$$0 = \frac{1}{2\pi i}\oint dz\, \frac{A(z)}{z} = A(0) + \sum_k \mathrm{Res}\left[\frac{A(z)}{z}\right]\Big|_{z=z_k}$$

Where are the poles? Require on-shell intermediate state,



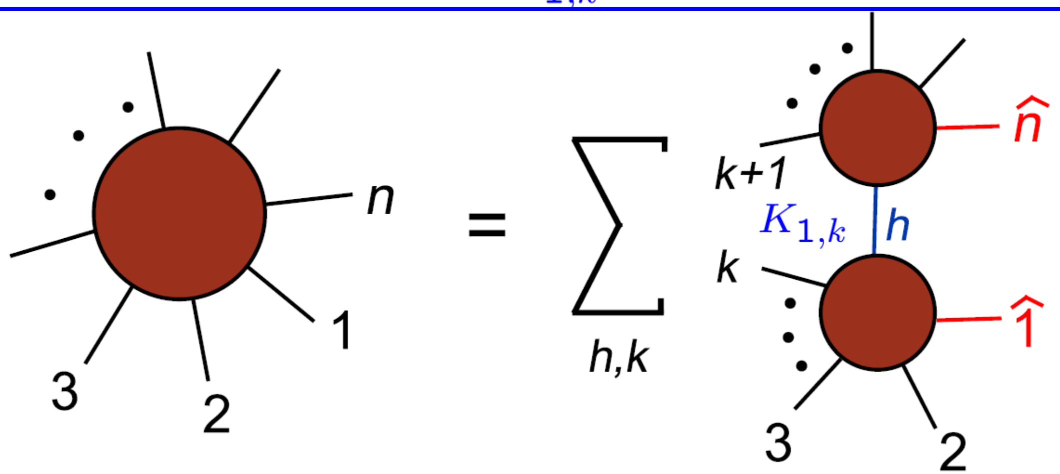
$$\begin{aligned} 0 &= (\hat{k}_1(z) + k_2 + \cdots + k_k)^2 = (z\lambda_n\tilde{\lambda}_1 + K_{1,k})^2 \\ &= z\langle n^-|\, \not{K}_{1,k}|1^-\rangle + K_{1,k}^2 \end{aligned}$$

$$\boxed{z_k = -\frac{K_{1,k}^2}{\langle n^-|\not{K}_{1,k}|1^-\rangle}}$$

Final formula

Britto, Cachazo, Feng, hep-th/0412308

$$A_n(1,2,\ldots,n) \;=\; \sum_{h=\pm}\sum_{k=2}^{n-2} A_{k+1}(\hat{1},2,\ldots,k,-\hat{K}_{1,k}^{-h}) \times \frac{i}{K_{1,k}^2} A_{n-k+1}(\hat{K}_{1,k}^{h},k+1,\ldots,n-1,\hat{n})$$



A_{k+1} and $A_{n\text{-}k+1}$ are on-shell **color-ordered** tree amplitudes with fewer legs, evaluated with 2 momenta shifted by a **complex** amount

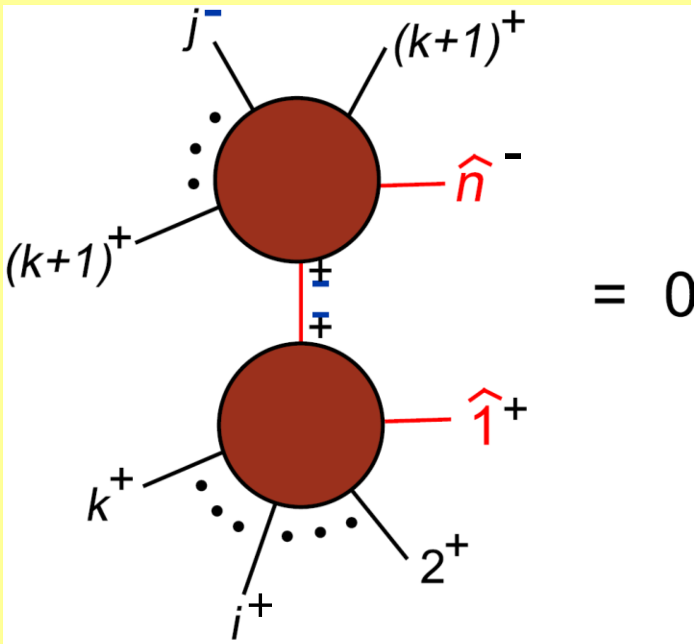
MHV example

- Apply the $[n,1\rangle$ BCFW formula to the MHV amplitude

$$A_n^{jn,\,\mathrm{MHV}} = A_n(1^+,2^+,\ldots,j^-,\ldots,n^-) = \frac{\langle j\, n\rangle^4}{\langle 1\,2\rangle\langle 2\,3\rangle\cdots\langle n\,1\rangle}$$

- The generic diagram vanishes because $2 + 2 = 4 > 3$
- So one of the two tree amplitudes is always zero
- The one exception is $k = 2$, which is different because

$$A_3(1^+,2^+,3^-) \neq 0$$



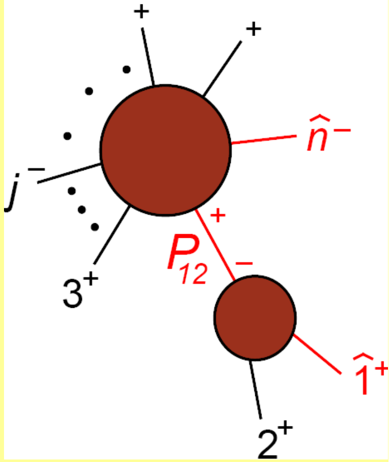
MHV example (cont.)

- For $k = 2$, we compute the value of z:

$$z_2 = -\frac{s_{12}}{\langle n^-|(1+2)|1^-\rangle} = -\frac{\langle 1\,2\rangle[2\,1]}{\langle n\,2\rangle[2\,1]} = -\frac{\langle 1\,2\rangle}{\langle n\,2\rangle}$$

- Kinematics are complex collinear

$$\langle \hat{1}\,2\rangle = \langle 1\,2\rangle + z_2\langle n\,2\rangle = 0 \qquad [\hat{1}\,2] = [1\,2] \neq 0$$
$$s_{\hat{1}2} = \langle \hat{1}\,2\rangle[2\,\hat{1}] = 0$$



- The only term in the BCFW formula is:

$$A_{n-1}(\hat{P}_{12}^+, 3^+, \ldots, j^-, \ldots, n^-)\frac{1}{s_{12}}A_3(\hat{1}^+, 2^+, -\hat{P}_{12}^-)$$
$$= \frac{\langle j\,\hat{n}\rangle^4}{\langle \hat{P}\,3\rangle\langle 3\,4\rangle\cdots\langle n-1,\hat{n}\rangle\langle \hat{n}\,\hat{P}\rangle}\frac{1}{s_{12}}\frac{[\hat{1}\,2]^3}{[2\,\hat{P}][\hat{P}\,\hat{1}]}$$
$$= \frac{\langle j\,n\rangle^4}{\langle \hat{P}\,3\rangle\langle 3\,4\rangle\cdots\langle n-1,n\rangle\langle n\,\hat{P}\rangle}\frac{1}{s_{12}}\frac{[1\,2]^3}{[2\,\hat{P}][\hat{P}\,1]}$$

note
$A_3(+,+,+) = 0$

MHV example (cont.)

- Using
$$\langle n\, \widehat{P}\rangle[\widehat{P}\, 2] \;=\; \langle n^-|(1+2)|2^-\rangle + z\langle n\, n\rangle[1\,2] = \langle n\, 1\rangle[1\,2]$$
$$\langle 3\, \widehat{P}\rangle[\widehat{P}\, 1] \;=\; \langle 3^-|(1+2)|1^-\rangle + z\langle 3\, n\rangle[1\,1] = \langle 3\, 2\rangle[2\,1]$$

one confirms

$$\begin{aligned}
& \frac{\langle j\, n\rangle^4}{\langle \widehat{P}\, 3\rangle\langle 3\, 4\rangle\cdots\langle n-1,n\rangle\langle n\, \widehat{P}\rangle}\frac{1}{s_{12}}\frac{[1\,2]^3}{[2\, \widehat{P}][\widehat{P}\, 1]} \\
= \; & \frac{\langle j\, n\rangle^4[1\,2]^3}{(\langle 1\,2\rangle[2\,1])([1\,2]\langle 2\,3\rangle)(\langle n\,1\rangle[1\,2])\langle 3\,4\rangle\cdots\langle n-1,n\rangle} \\
= \; & \frac{\langle j\, n\rangle^4}{\langle 1\,2\rangle\langle 2\,3\rangle\cdots\langle n-1,n\rangle\langle n\,1\rangle} \\
= \; & A_n^{jn,\,\mathrm{MHV}}
\end{aligned}$$

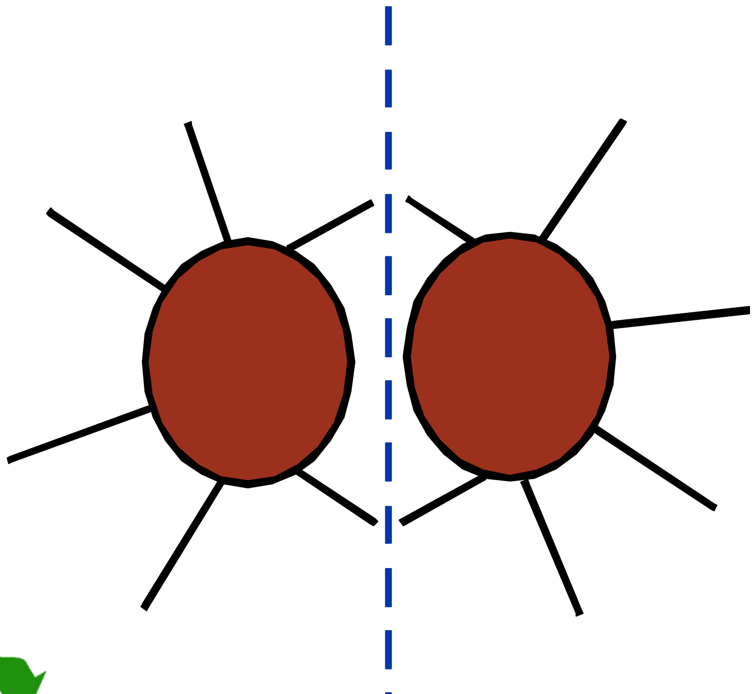
- This proves the Parke-Taylor formula by induction on n.

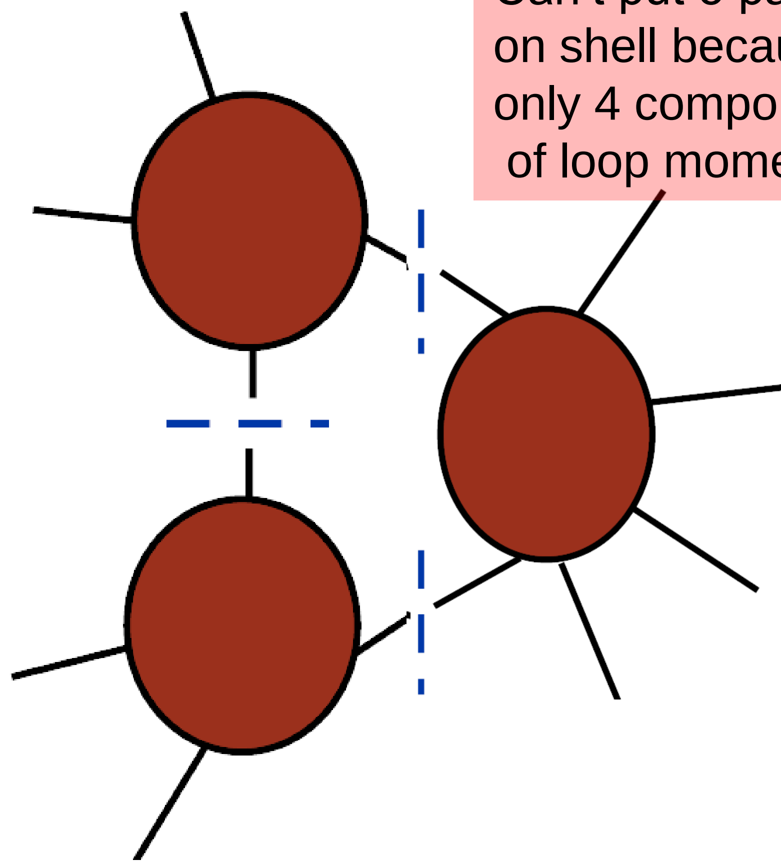
Branch cut information → Generalized Unitarity (One-loop fluidity)

Ordinary unitarity:
put 2 particles on shell

Generalized unitarity:
put 3 or 4 particles on shell

Can't put 5 particles on shell because only 4 components of loop momentum





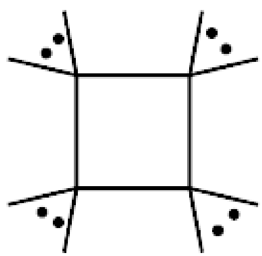
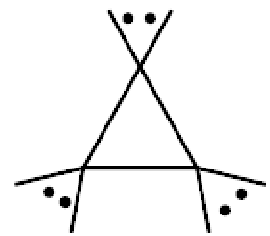
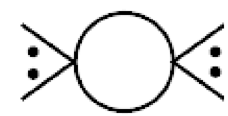
Trees recycled into loops!

One-loop amplitudes reduced to trees

When all external momenta are in $D = 4$, loop momenta in $D = 4\text{-}2\varepsilon$ (dimensional regularization), one can write:

Bern, LD, Dunbar, Kosower (1994)

coefficients are all rational functions – determine algebraically from products of trees using (generalized) unitarity

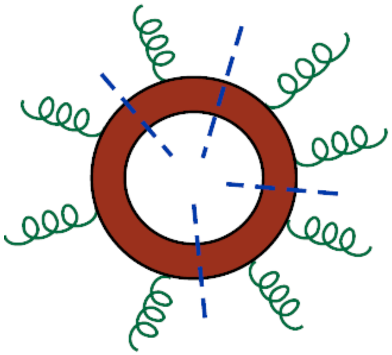
$$A^{1\text{-loop}} = \sum_i d_i$$

$$+ \sum_i c_i$$

$$+ \sum_i b_i$$

$$+ R + \mathcal{O}(\epsilon)$$

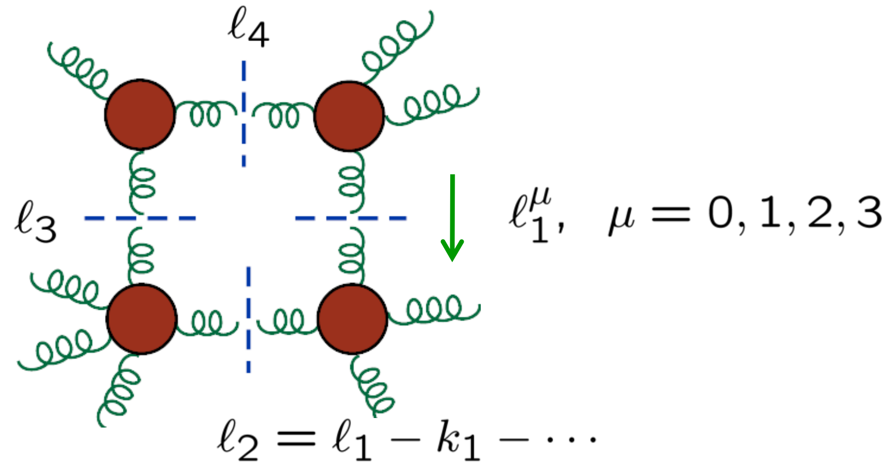
rational part

known **scalar** one-loop integrals, same for all amplitudes

Generalized Unitarity for Box Coefficients d_i

Britto, Cachazo, Feng, hep-th/0412308





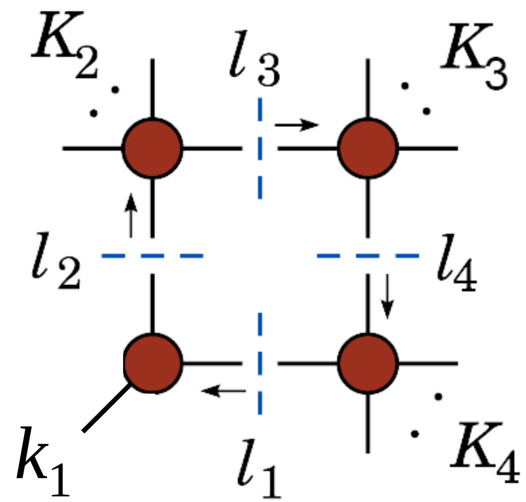
$$\boxed{\begin{aligned} & \int d^4\ell \; \delta(\ell_1^2 - m_1^2)\, \delta(\ell_2^2 - m_2^2) \\ & \qquad \times \delta(\ell_3^2 - m_3^2)\, \delta(\ell_4^2 - m_4^2) \times A^{1-\mathrm{loop}}(\ell_i) \\ = & \sum_{\pm} A_1^{\mathrm{tree}}(\ell_0^\pm) A_2^{\mathrm{tree}}(\ell_0^\pm) A_3^{\mathrm{tree}}(\ell_0^\pm) A_4^{\mathrm{tree}}(\ell_0^\pm) \\ = & \; d_i^+ + d_i^- \end{aligned}}$$

No. of dimensions = 4 = no. of constraints → discrete solutions

Easy to code, numerically very stable

Box coefficients d_i (cont.)

- General solution involves a quadratic formula
- Solutions simplify (and are more stable numerically) when all internal lines are massless, and at least one external line (k_1) is massless:



$$(l_1^{(\pm)})^\mu = \frac{\langle 1^\mp | \not{K}_2 \not{K}_3 \not{K}_4 \gamma^\mu | 1^\pm \rangle}{2 \langle 1^\mp | \not{K}_2 \not{K}_4 | 1^\pm \rangle}, \qquad (l_2^{(\pm)})^\mu = -\frac{\langle 1^\mp | \gamma^\mu \not{K}_2 \not{K}_3 \not{K}_4 | 1^\pm \rangle}{2 \langle 1^\mp | \not{K}_2 \not{K}_4 | 1^\pm \rangle},$$

$$(l_3^{(\pm)})^\mu = \frac{\langle 1^\mp | \not{K}_2 \gamma^\mu \not{K}_3 \not{K}_4 | 1^\pm \rangle}{2 \langle 1^\mp | \not{K}_2 \not{K}_4 | 1^\pm \rangle}, \qquad (l_4^{(\pm)})^\mu = -\frac{\langle 1^\mp | \not{K}_2 \not{K}_3 \gamma^\mu \not{K}_4 | 1^\pm \rangle}{2 \langle 1^\mp | \not{K}_2 \not{K}_4 | 1^\pm \rangle}.$$

Exercise: Show

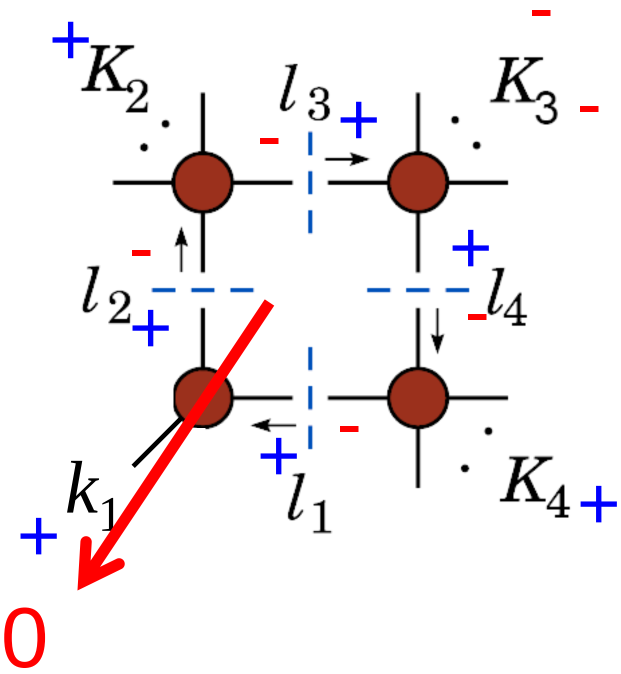
$l_2 - l_3 = K_2$, $l_3 - l_4 = K_3$, $l_4 - l_1 = K_4$

BH, 0803.4180;
Risager 0804.3310

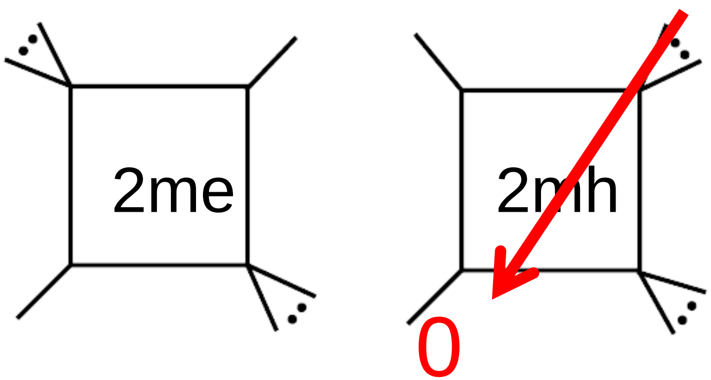
Example of MHV amplitude

All 3-mass boxes (and 4-mass boxes) vanish trivially – not enough (-) helicities

Have 2 + 4 = 6 (-) helicities, but need 2 + 2 + 2 + 1 = 7

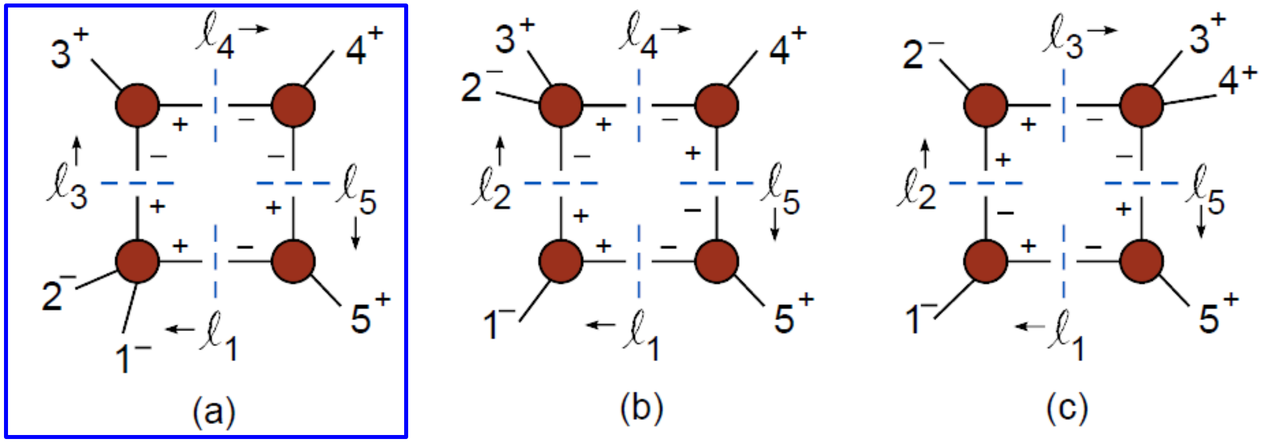


2-mass boxes come in two types:



5-point MHV Box example

For (--+++), 3 inequivalent boxes to consider

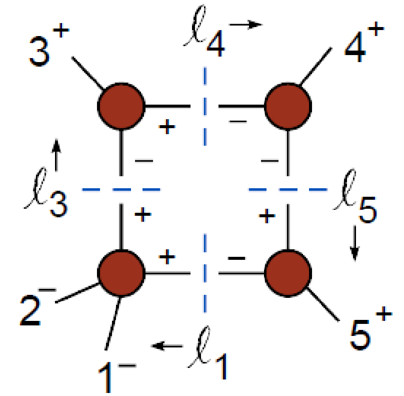


Look at this one. Corresponding integral in dim. reg.:

$$\mathcal{I}(K_{12}) = \mu^{2\epsilon} \int \frac{d^{4-2\epsilon}\ell}{(2\pi)^{4-2\epsilon}} \frac{1}{\ell^2(\ell-K_{12})^2(\ell-K_{123})^2(\ell+k_5)^2}$$

$$= \frac{-2i\, c_\Gamma}{s_{34}s_{45}} \Bigg\{ -\frac{1}{\epsilon^2}\Bigg[\left(\frac{\mu^2}{-s_{34}}\right)^{\epsilon} + \left(\frac{\mu^2}{-s_{45}}\right)^{\epsilon} - \left(\frac{\mu^2}{-s_{12}}\right)^{\epsilon}\Bigg] + \mathrm{Li}_2\left(1-\frac{s_{12}}{s_{34}}\right) + \mathrm{Li}_2\left(1-\frac{s_{12}}{s_{45}}\right) + \frac{1}{2}\ln^2\left(\frac{-s_{34}}{-s_{45}}\right) + \frac{\pi^2}{6}\Bigg\} + \mathcal{O}(\epsilon)\,,$$

5-point MHV Box example



$$\ell_4^\mu = \tfrac{1}{2}\xi_4 \left\langle 3^- \middle| \gamma^\mu \middle| 4^- \right\rangle .$$

The constant ξ_4 is fixed by the last of the four on-shell equations,

$$\ell_1^2 = (\ell_4 - K_{45})^2 = -\xi_4 \left\langle 3^- \middle| 5 \middle| 4^- \right\rangle + s_{45} = 0 \,,$$

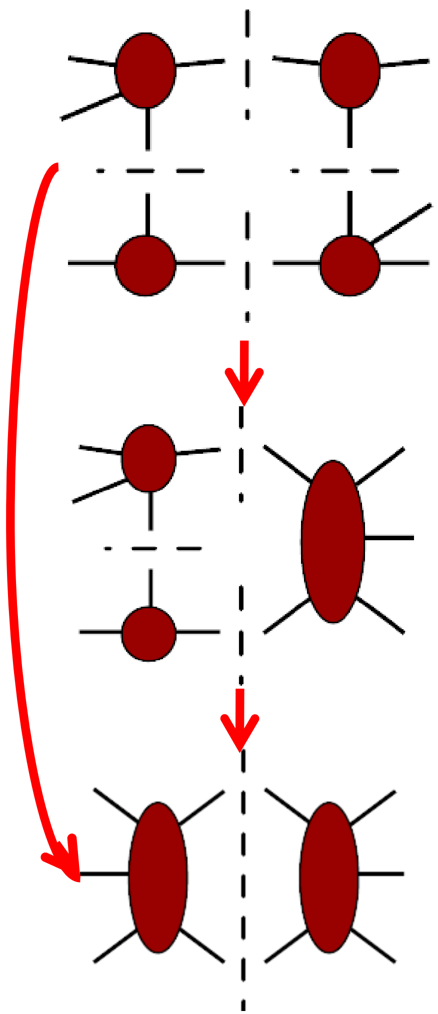
to have the value $\xi_4 = \langle 4\,5\rangle \,/\, \langle 3\,5\rangle$.

$$\begin{aligned} c_{12} &= \frac{1}{2} A_4^{\rm tree}(-\ell_1^+, 1^-, 2^-, \ell_3^+) A_3^{\rm tree}(-\ell_3^-, 3^+, \ell_4^+) A_3^{\rm tree}(-\ell_4^-, 4^+, \ell_5^-) A_3^{\rm tree}(-\ell_5^+, 5^+, \ell_1^-) \\ &= \frac{1}{2} \frac{\langle 1\,2\rangle^3}{\langle 2\,\ell_3\rangle \langle \ell_3\,(-\ell_1)\rangle \langle (-\ell_1)\,1\rangle} \frac{[3\,\ell_4]^3}{[\ell_4\,(-\ell_3)]\,[(-\ell_3)\,3]} \frac{\langle \ell_5\,(-\ell_4)\rangle^3}{\langle 4\,\ell_5\rangle \langle (-\ell_4)\,4\rangle} \frac{[(-\ell_5)\,5]^3}{[5\,\ell_1]\,[\ell_1\,(-\ell_5)]} \\ &= -\frac{1}{2} \frac{\langle 1\,2\rangle^3 \left\langle 3^+ \middle| \ell_4\ell_5 \middle| 5^- \right\rangle^3}{\left\langle 2^- \middle| \ell_3 \middle| 3^- \right\rangle \left\langle 4^- \middle| \ell_4\ell_3\ell_1 \middle| 5^- \right\rangle \left\langle 1^- \middle| \ell_1\ell_5 \middle| 4^+ \right\rangle} . \end{aligned}$$

$$\begin{aligned} c_{12} &= \frac{1}{2} \frac{\langle 1\,2\rangle^3 \left\langle 4^- \middle| \ell_4 \middle| 3^- \right\rangle^2 [4\,5]^3}{\left\langle 2^- \middle| \ell_4 \middle| 3^- \right\rangle \langle 3\,4\rangle\,[4\,5]\,\langle 1\,5\rangle \left\langle 4^- \middle| \ell_4 \middle| 5^- \right\rangle} \\ &= -\frac{1}{2} \frac{\langle 1\,2\rangle^3\, s_{34} s_{45}}{\langle 2\,3\rangle \langle 3\,4\rangle \langle 4\,5\rangle \langle 5\,1\rangle} \\ &= \frac{i}{2} s_{34} s_{45}\, A_5^{\rm tree}(1^-, 2^-, 3^+, 4^+, 5^+) . \end{aligned}$$

In fact, all 2me boxes in the 1-loop MHV amplitude in N=4 SYM (and many in QCD) are also proportional to $A_n^{\rm tree}$
BDDK (1994)

Full amplitude determined hierarchically



Each box coefficient comes uniquely from 1 "quadruple cut"

Britto, Cachazo, Feng, hep-th/0412103

Ossola, Papadopolous, Pittau, hep-ph/0609007;
Mastrolia, hep-th/0611091; Forde, 0704.1835;
Ellis, Giele, Kunszt, 0708.2398; Berger et al., 0803.4180;…

Each triangle coefficient from 1 triple cut, but "contaminated" by boxes

Each bubble coefficient from 1 double cut, removing contamination by boxes and triangles

Rational part depends on all of above

More complicated examples

$ggggg$

$$A(+++++) = \frac{i}{96\pi^2}\frac{s_{12}s_{23}+s_{23}s_{34}+s_{34}s_{45}+s_{45}s_{51}+s_{51}s_{12}+\varepsilon(1,2,3,4)}{\langle 1\,2\rangle\langle 2\,3\rangle\langle 3\,4\rangle\langle 4\,5\rangle\langle 5\,1\rangle},$$

$$A(-++++) = \frac{i}{48\pi^2}\frac{1}{[1\,2]\langle 2\,3\rangle\langle 3\,4\rangle\langle 4\,5\rangle[5\,1]}\Big[(s_{23}+s_{34}+s_{45})[2\,5]^2-[2\,4]\langle 4\,3\rangle[3\,5][2\,5]$$

$$-\frac{[1\,2][1\,5]}{\langle 1\,2\rangle\langle 1\,5\rangle}\Big(\langle 1\,2\rangle^2\langle 1\,3\rangle^2\frac{[2\,3]}{\langle 2\,3\rangle}+\langle 1\,3\rangle^2\langle 1\,4\rangle^2\frac{[3\,4]}{\langle 3\,4\rangle}+ \ldots$$

More legs, or massive legs, rapidly increases complexity!

$$V^g = -\frac{1}{\epsilon^2}\sum_{j=1}^{5}\left(\frac{\mu^2}{-s_{j,j+1}}\right)^{\epsilon}+\sum_{j=1}^{5}\ln\left(\frac{-s_{j,j+1}}{-s_{j+1,j+2}}\right)\ln\left(\frac{-s_{j+2,j-2}}{-s_{j-2,j-1}}\right)+\frac{5}{6}\pi^2-\frac{\delta_R}{3}$$

the following functions for the $(1^-,2^-,3^+,4^+,5^+)$ helicity configuration,

~ 1 page

$--+++$

$$V^f = -\frac{5}{2\epsilon}-\frac{1}{2}\left[\ln\left(\frac{\mu^2}{-s_{23}}\right)+\ln\left(\frac{\mu^2}{-s_{51}}\right)\right]-2, \qquad V^s = -\frac{1}{3}V^f+\frac{2}{9}$$

$$F^f = -\frac{1}{2}\frac{\langle 1\,2\rangle^2(\langle 2\,3\rangle[3\,4]\langle 4\,1\rangle+\langle 2\,4\rangle[4\,5]\langle 5\,1\rangle)}{\langle 2\,3\rangle\langle 3\,4\rangle\langle 4\,5\rangle\langle 5\,1\rangle}\frac{\mathrm{L}_0\left(\frac{-s_{23}}{-s_{51}}\right)}{s_{51}}$$

$$F^s = -\frac{1}{3}\frac{[3\,4]\langle 4\,1\rangle\langle 2\,4\rangle[4\,5](\langle 2\,3\rangle[3\,4]\langle 4\,1\rangle+\langle 2\,4\rangle[4\,5]\langle 5\,1\rangle)}{\langle 3\,4\rangle\langle 4\,5\rangle}\frac{\mathrm{L}_2\left(\frac{-s_{23}}{-s_{51}}\right)}{s_{51}^3}-\frac{1}{3}F^f$$
$$-\frac{1}{3}\frac{\langle 3\,5\rangle[3\,5]^3}{[1\,2][2\,3]\langle 3\,4\rangle\langle 4\,5\rangle[5\,1]}+\frac{1}{3}\frac{\langle 1\,2\rangle[3\,5]^2}{[2\,3]\langle 3\,4\rangle\langle 4\,5\rangle[5\,1]}+\frac{1}{6}\frac{\langle 1\,2\rangle[3\,4]\langle 4\,1\rangle\langle 2\,4\rangle[4\,5]}{s_{23}\langle 3\,4\rangle\langle 4\,5\rangle s_{51}}$$

Some helicity config's more complex than others

and the corresponding ones for the $(1^-,2^+,3^-,4^+,5^+)$ helicity configuration,

$-+-++$

$$V^f = -\frac{5}{2\epsilon}-\frac{1}{2}\left[\ln\left(\frac{\mu^2}{-s_{34}}\right)+\ln\left(\frac{\mu^2}{-s_{51}}\right)\right]-2, \qquad V^s = -\frac{1}{3}V^f+\frac{2}{9}$$

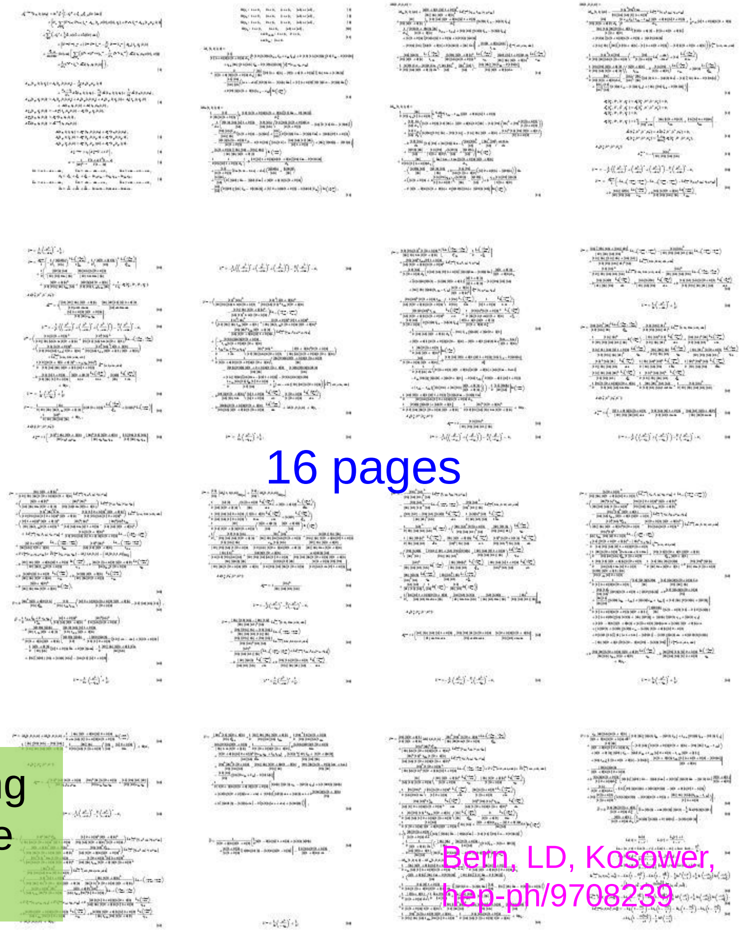
$$F^f = -\frac{\langle 1\,3\rangle^2\langle 4\,1\rangle[2\,4]^2}{\langle 4\,5\rangle\langle 5\,1\rangle}\frac{\mathrm{Ls}_1\left(\frac{-s_{23}}{-s_{51}},\frac{-s_{34}}{-s_{51}}\right)}{s_{51}^2}+\frac{\langle 1\,3\rangle^2\langle 5\,3\rangle[2\,5]^2}{\langle 3\,4\rangle\langle 4\,5\rangle}\frac{\mathrm{Ls}_1\left(\frac{-s_{12}}{-s_{34}},\frac{-s_{51}}{-s_{34}}\right)}{s_{34}^2}$$
$$-\frac{1}{2}\frac{\langle 1\,3\rangle^3(\langle 1\,5\rangle[5\,2]\langle 2\,3\rangle-\langle 3\,4\rangle[4\,2]\langle 2\,1\rangle)}{\langle 1\,2\rangle\langle 2\,3\rangle\langle 3\,4\rangle\langle 4\,5\rangle\langle 5\,1\rangle}\frac{\mathrm{L}_0\left(\frac{-s_{34}}{-s_{51}}\right)}{s_{51}}$$

$$F^s = -\frac{\langle 1\,2\rangle\langle 2\,3\rangle\langle 3\,4\rangle\langle 4\,1\rangle^2[2\,4]^2}{\langle 4\,5\rangle\langle 5\,1\rangle\langle 2\,4\rangle^2}\frac{2\,\mathrm{Ls}_1\left(\frac{-s_{23}}{-s_{51}},\frac{-s_{34}}{-s_{51}}\right)+\mathrm{L}_1\left(\frac{-s_{23}}{-s_{51}}\right)+\mathrm{L}_1\left(\frac{-s_{34}}{-s_{51}}\right)}{s_{51}^2}$$
$$+\frac{\langle 3\,2\rangle\langle 2\,1\rangle\langle 1\,5\rangle\langle 5\,3\rangle^2[2\,5]^2}{\langle 5\,4\rangle\langle 4\,3\rangle\langle 2\,5\rangle^2}\frac{2\,\mathrm{Ls}_1\left(\frac{-s_{12}}{-s_{34}},\frac{-s_{51}}{-s_{34}}\right)+\mathrm{L}_1\left(\frac{-s_{12}}{-s_{34}}\right)+\mathrm{L}_1\left(\frac{-s_{51}}{-s_{34}}\right)}{s_{34}^2}$$
$$+\frac{2}{3}\frac{\langle 2\,3\rangle^2\langle 4\,1\rangle^3[2\,4]^3}{\langle 4\,5\rangle\langle 5\,1\rangle\langle 2\,4\rangle}\frac{\mathrm{L}_2\left(\frac{-s_{23}}{-s_{51}}\right)}{s_{51}^3}-\frac{2}{3}\frac{\langle 2\,1\rangle^2\langle 5\,3\rangle^3[2\,5]^3}{\langle 5\,4\rangle\langle 4\,3\rangle\langle 2\,5\rangle}\frac{\mathrm{L}_2\left(\frac{-s_{12}}{-s_{34}}\right)}{s_{34}^3}$$
$$+\frac{\mathrm{L}_2\left(\frac{-s_{34}}{-s_{51}}\right)}{s_{51}^3}\left(\frac{1}{3}\frac{\langle 1\,3\rangle[2\,4][2\,5](\langle 1\,5\rangle[5\,2]\langle 2\,3\rangle-\langle 3\,4\rangle[4\,2]\langle 2\,1\rangle)}{\langle 4\,5\rangle}\right.$$
$$\left.+\frac{2}{3}\frac{\langle 1\,2\rangle^2\langle 3\,4\rangle^2\langle 4\,1\rangle[2\,4]^3}{\langle 4\,5\rangle\langle 5\,1\rangle\langle 2\,4\rangle}-\frac{2}{3}\frac{\langle 3\,2\rangle^2\langle 1\,5\rangle^2\langle 5\,3\rangle[2\,5]^3}{\langle 5\,4\rangle\langle 4\,3\rangle\langle 2\,5\rangle}\right)$$
$$+\frac{1}{6}\frac{\langle 1\,3\rangle^3(\langle 1\,5\rangle[5\,2]\langle 2\,3\rangle-\langle 3\,4\rangle[4\,2]\langle 2\,1\rangle)}{\langle 1\,2\rangle\langle 2\,3\rangle\langle 3\,4\rangle\langle 4\,5\rangle\langle 5\,1\rangle}\frac{\mathrm{L}_0\left(\frac{-s_{34}}{-s_{51}}\right)}{s_{51}}+\frac{1}{3}\frac{[2\,4]^2[2\,5]^2}{[1\,2][2\,3][3\,4]\langle 4\,5\rangle[5\,1]}$$
$$-\frac{1}{3}\frac{\langle 1\,2\rangle\langle 4\,1\rangle^2[2\,4]^3}{\langle 4\,5\rangle\langle 5\,1\rangle\langle 2\,4\rangle[2\,3][3\,4]s_{51}}+\frac{1}{3}\frac{\langle 3\,2\rangle\langle 5\,3\rangle^2[2\,5]^3}{\langle 5\,4\rangle\langle 4\,3\rangle\langle 2\,5\rangle[2\,1][1\,5]s_{34}}+\frac{1}{6}\frac{\langle 1\,3\rangle^2[2\,4][2\,5]}{s_{34}\langle 4\,5\rangle s_{51}}.$$

Bern, LD, Kosower, hep-ph/9302280

Luckily everything can now be done numerically!

$Vq\bar{q}gg$ $\quad V = W, Z, \gamma^*$



16 pages

Bern, LD, Kosower, hep-ph/9708239

Some Automated On-Shell One Loop Programs

Blackhat: Berger, Bern, LD, Diana, Febres Cordero, Forde, Gleisberg, Höche, Ita, Kosower, Maître, Ozeren, 0803.4180, 0808.0941, 0907.1984, 1004.1659, 1009.2338…
+ **Sherpa** → NLO *W,Z* + 3,4,5 jets pure QCD 4 jets

CutTools: Ossola, Papadopolous, Pittau, 0711.3596
NLO *WWW, WWZ*, ... Binoth+OPP, 0804.0350
NLO $t\bar{t}b\bar{b}$, $t\bar{t}$ + 2 jets,…
Bevilacqua, Czakon, Papadopoulos, Pittau, Worek, 0907.4723; 1002.4009

MadLoop: Hirschi, Frederix, Frixione, Garzelli, Maltoni, Pittau 1103.0621

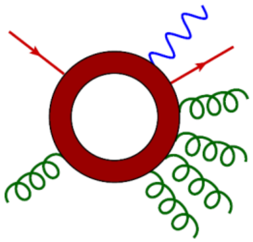
HELAC-NLO: Bevilacqua et al, 1110.1499

Rocket: Giele, Zanderighi, 0805.2152
Ellis, Giele, Kunszt, Melnikov, Zanderighi, 0810.2762
NLO *W* + 3 jets Ellis, Melnikov, Zanderighi, 0901.4101, 0906.1445
$W^+W^\pm$ + 2 jets Melia, Melnikov, Rontsch, Zanderighi, 1007.5313, 1104.2327

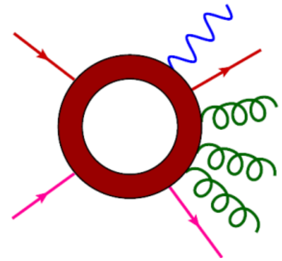
SAMURAI: Mastrolia, Ossola, Reiter, Tramontano, 1006.0710

NGluon: Badger, Biedermann, Uwer, 1011.2900

NLO $pp \to W + 5$ jets also feasible

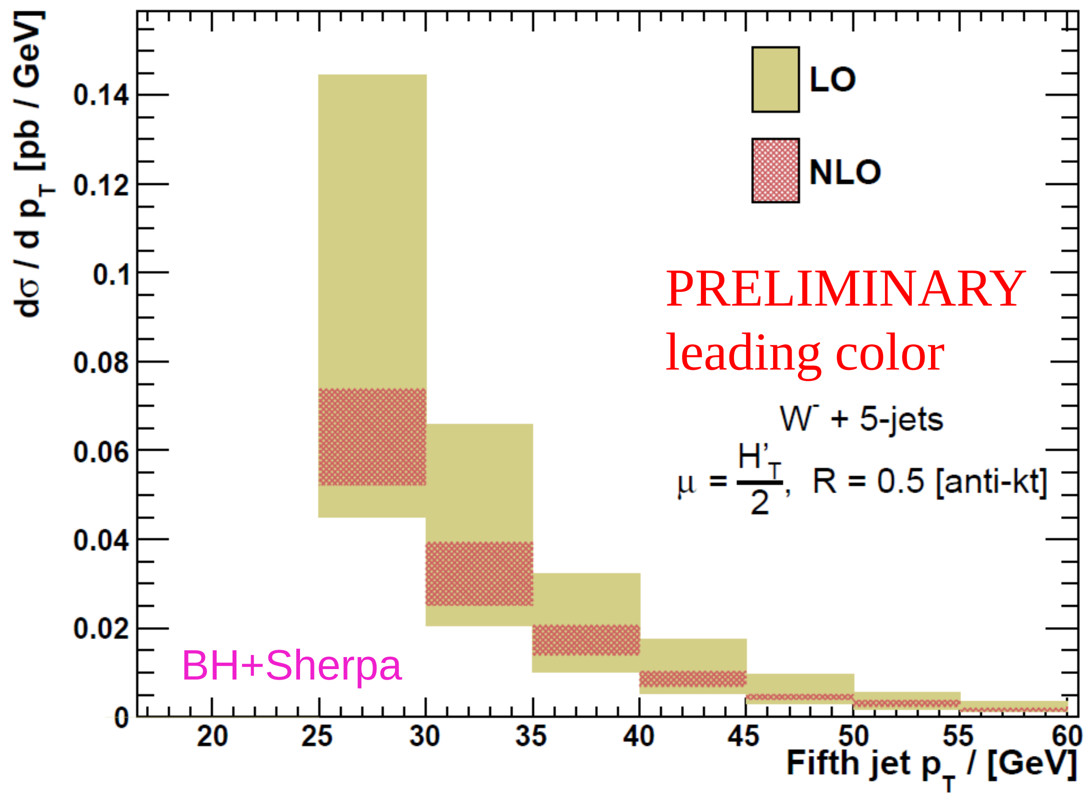


256,265 +



49,614 + …

Brand New!



Fixed order vs. MC

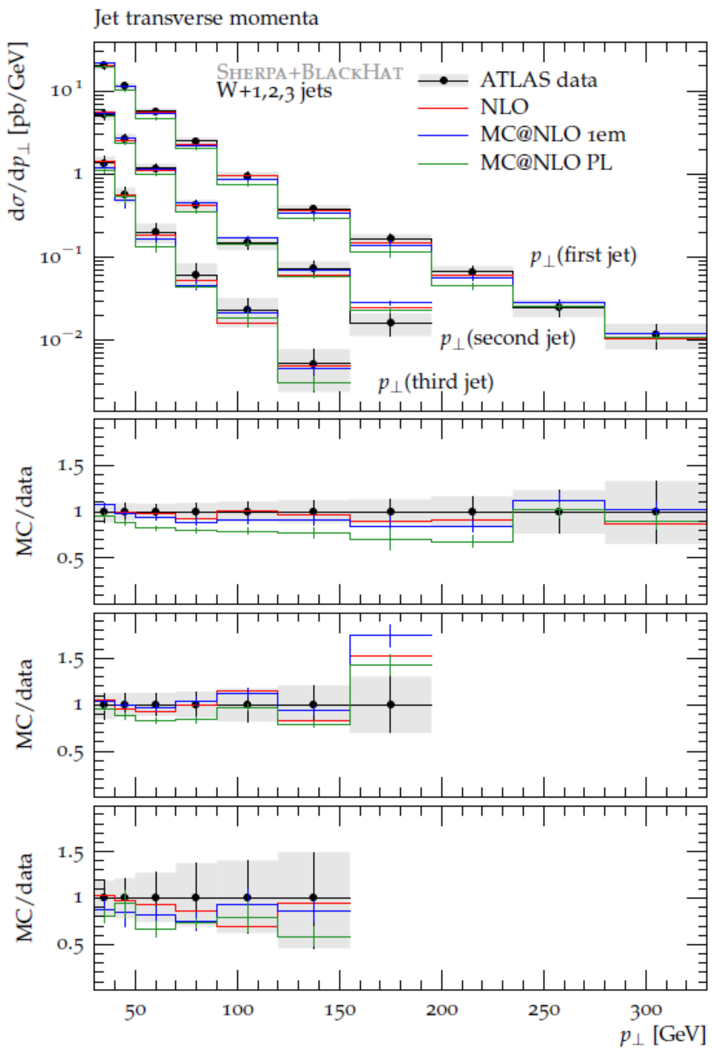
- State-of-art NLO calculations are almost always done first as fixed-order, parton level:

  no parton shower, no hadronization, no underlying event (except as estimated as corrections).
- Methods available for matching NLO parton-level results to parton showers, with NLO accuracy:
  - **MC@NLO** Frixione, Webber (2002) + SHERPA implementation
  - **POWHEG** Nason (2004); Frixione, Nason, Oleari (2007)
  - **GenEvA** Bauer, Tackmann, Thaler (2008)
- Recently implemented for increasingly complex final states!

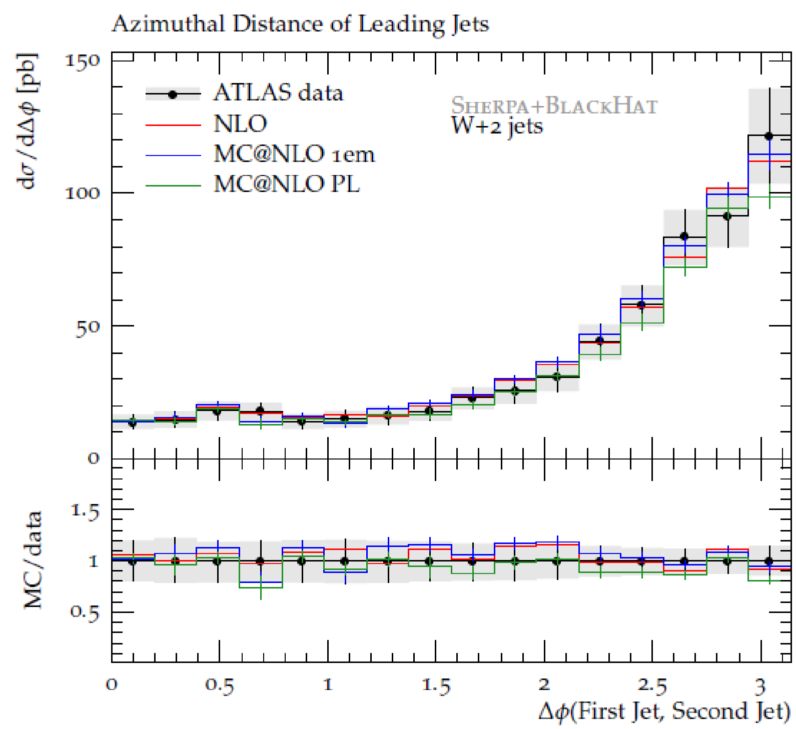
Remarkable NLO+MC progress

- Some recent NLO+shower processes:
  - 2 jets — Alioli, Hamilton Nason, Oleari, Re, 1012.3380 [POWHEG]
  - Z + 1 jet — Alioli, Nason, Oleari, Re, 1009.5594 [POWHEG]
  - $W + b\bar{b}$ — Oleari, Reina, 1105.4488 [POWHEG]; Frederix et al., 1106.6019 [aMC@NLO]
  - W^+W^+ + 2 jets — Jäger, Zanderighi 1108.0864 [POWHEG]
  - W + 2 jets — Frederix et al., 1110.5502 [aMC@NLO]
  - $t\bar{t}$ + 1 jet — Alioli, Moch, Uwer, 1110.5251 [POWHEG]
  - $t\bar{t}Z$ — Garzelli, Kardos, Papadopolous, Trócsányi et al., 1111.1444
  - W + 3 jets — Höche, Krauss, Schönherr, Siegert, 1201.5882 [SHERPA]

NLO MC for *W* + 1,2,3 jets vs. ATLAS data



Höche et al., 1201.5882



Topics glossed over

- Had no time to discuss:
- Details of real-emission contributions: subtraction methods
- NNLO results for *W*, *Z*, Higgs, $\gamma\gamma$
- Various types of soft-gluon resummations: threshold, p_T, other kinematic boundaries

Conclusions

- Understanding QCD at hadron colliders is important, not just in its own right, but as a tool for controlling important Standard Model backgrounds
- QCD dynamics: multi-scale, fractal behavior modified by slow breaking of scale invariance due to the running coupling.
- Need to have infrared-safe observables to compute reliably in perturbation theory → IR safe jet algorithms
- New and efficient ways to compute QCD amplitudes for LHC processes with complex final states
  - exploit **analyticity/unitarity**: build loop amplitudes out of trees
  - implemented **numerically** in several programs:
  - long and growing list of complex processes computed at NLO
- Incorporation into NLO Monte Carlos
- Good agreement with LHC measurements ☹ (so far ☺)