RADIATION OF A CHARGE FLYING FROM VACUUM INTO ANISOTROPIC DISPERSIVE MEDIUM

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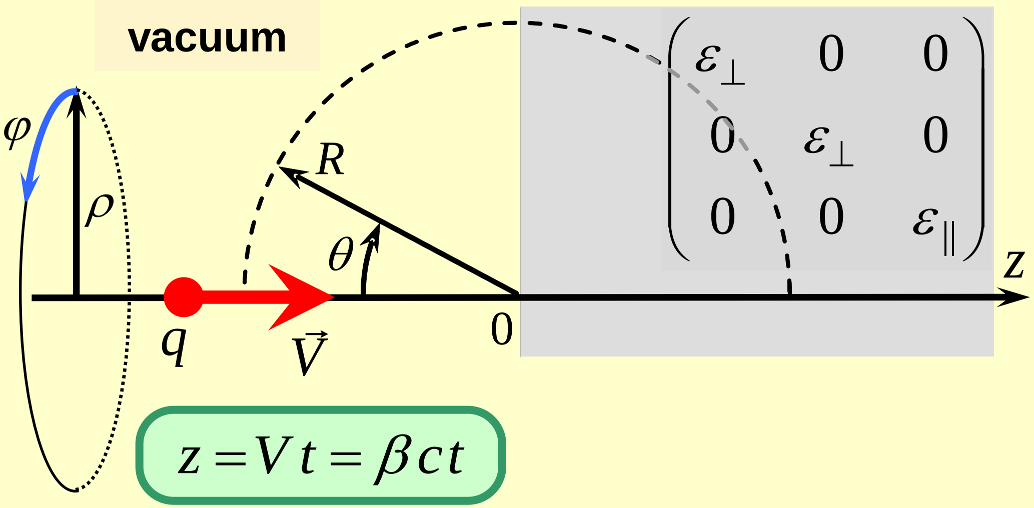
Formulation of the problem

Vacuum – anisotropic plasma-like medium interface



$$\begin{pmatrix} \varepsilon_\perp & 0 & 0 \\ 0 & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_\parallel \end{pmatrix}$$

$$z = Vt = \beta c t$$

$$\mu = 1 \qquad \varepsilon_\perp = 1 - \frac{\omega_{p\perp}^2}{\omega^2 + 2i\omega\omega_{d\perp}},$$

$$\varepsilon_\parallel = 1 - \frac{\omega_{p\parallel}^2}{\omega^2 + 2i\omega\omega_{d\parallel}}$$

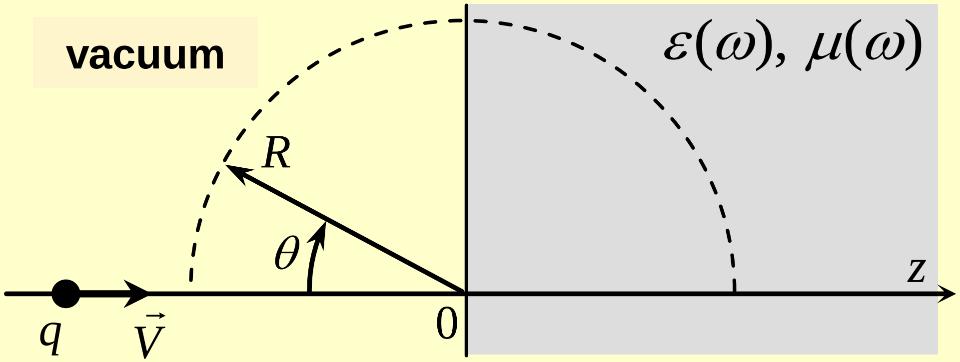
Goals:

Detailed investigation of the structure of the electromagnetic field

Taking into account the specific frequency dispersion and losses

Motivation

Vacuum – "left-handed medium" (LHM) interface



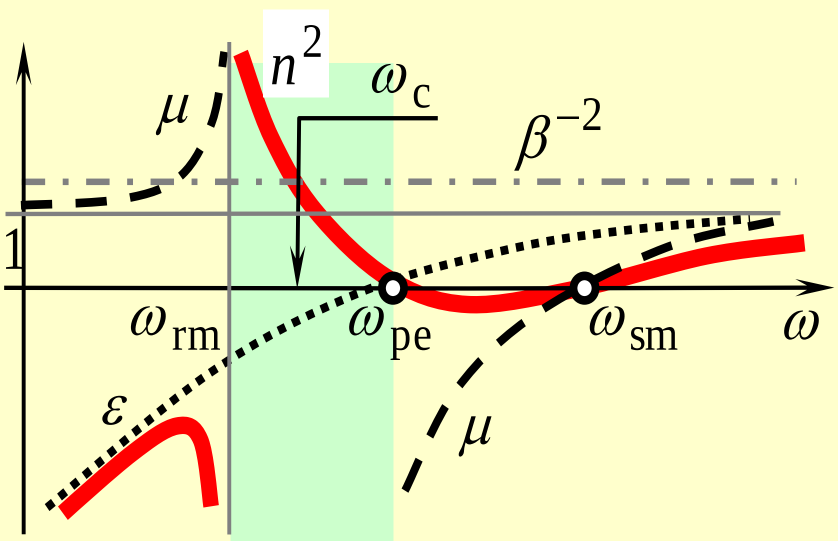
Reversed Cherenkov-transition radiation (RCTR) occurs in both vacuum and medium

Isotropic LHM:

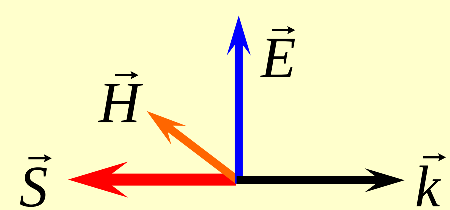
$$\varepsilon(\omega) = 1 - \frac{\omega_{\mathrm{pe}}^2}{\omega^2 + 2\,\mathrm{i}\,\omega_{\mathrm{de}}\omega}, \quad \mu(\omega) = 1 + \frac{\omega_{\mathrm{pm}}^2}{\omega_{\mathrm{rm}}^2 - 2\mathrm{i}\omega_{\mathrm{dm}}\omega - \omega^2}$$

$\omega_{\mathrm{pe}} > \omega_{\mathrm{rm}}$

Left-handed frequency band



reversed VCR:

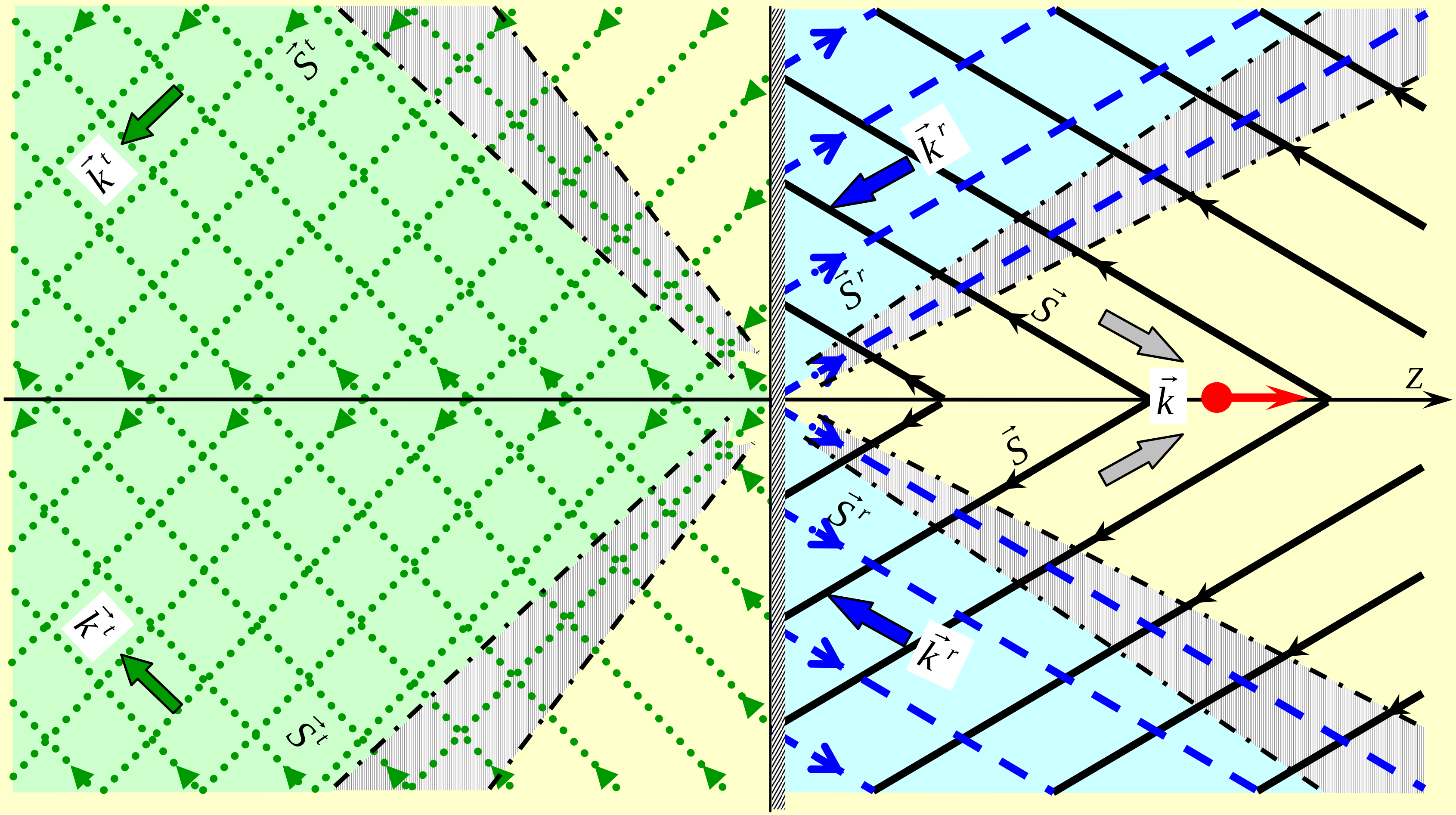


Motivation

Vacuum – “left-handed medium” (LHM) interface

Spatial distribution of the Fourier harmonic

$\beta_{CR} < \beta < \beta_{TIR}$



– lines parallel to the Poyting vector of VCR

– lines parallel to the Poyting vector of RCTR in medium

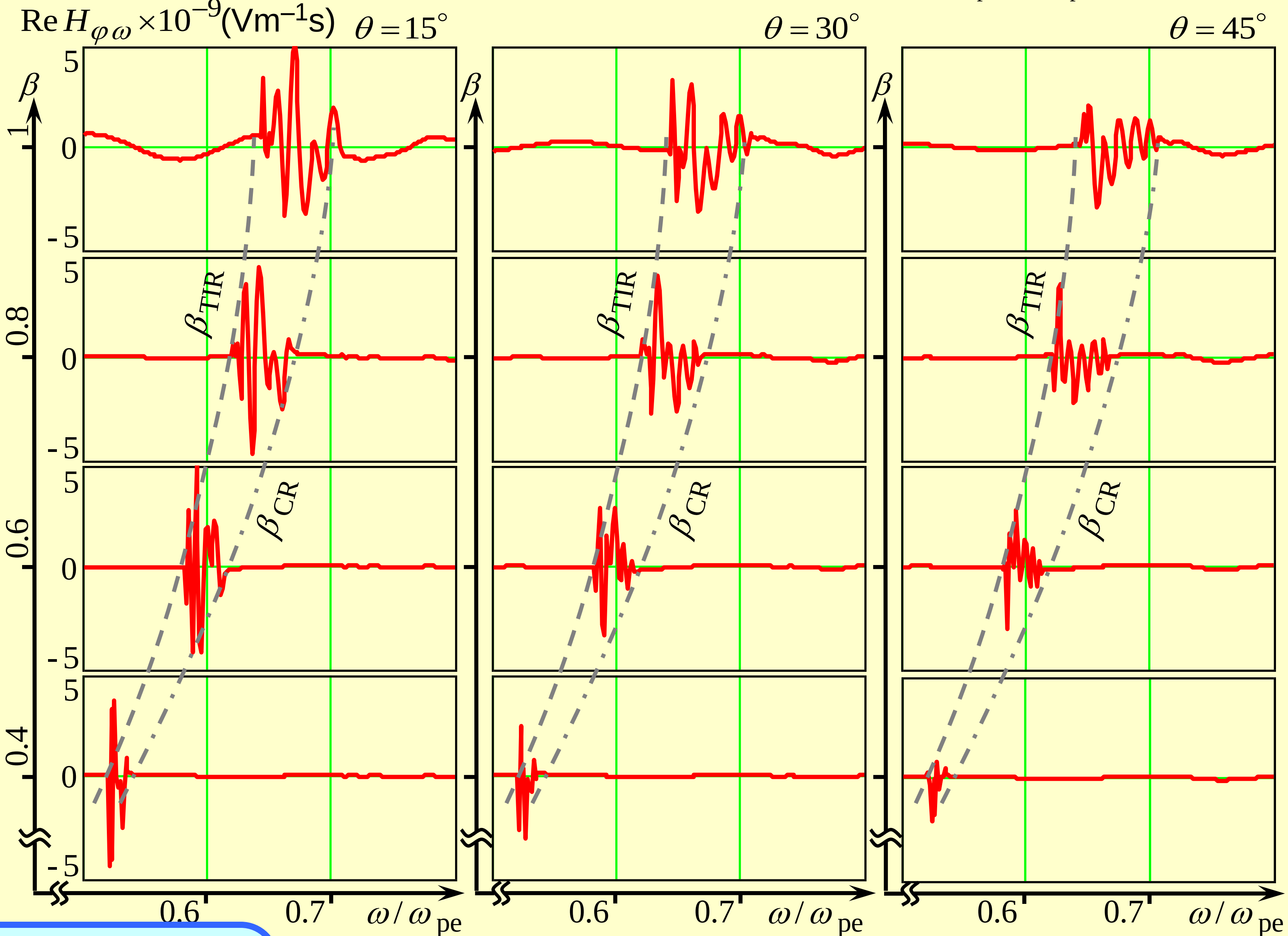
– lines parallel toi Poyting vector of RCTR in vacuum

Motivation

Field spectrum in vacuum

$q=-1$ nC $\quad R=14$ cm

$\omega_{pm2}=\omega_{pe2}=2\pi\cdot 10\cdot 10^9\,\mathrm{c}^{-1}$



$\beta_{CR}<\beta<\beta_{TIR}$

$\omega_{rm2}=0,\quad \omega_{de2}=\omega_{dm2}=10^{-3}\omega_{pe2}\,,\ \omega_{pe1}=10^{-2}\omega_{pe2}\,,\ \omega_{de1}=10^{-6}\omega_{pe2}$

Solution of the problem

Vacuum – anisotropic plasma-like medium interface

V.L. Ginzburg, V.N. Tsytovich. “Transition radiation and transition scattering»

$$E_\rho, E_z, H_\varphi \neq 0 \qquad H_\varphi^{(1,2)} = H_\varphi^{q(1,2)} + H_\varphi^{b(1,2)}$$

Self-field of the charge

$$H_\varphi^{q(1,2)} = \frac{q}{2\pi\beta c}\int_{-\infty}^{+\infty} H_{\varphi\omega}^{q(1,2)} \exp(i\omega\zeta V^{-1})\,d\omega, \qquad \zeta = z - Vt$$

$$H_{\varphi\omega}^{q(1,2)} = i\pi\beta s_{1,2} H_1^{(1)}\left(s_{1,2}\rho\right) \qquad s_1 = i\sqrt{\omega^2 V^{-2}\left(1-\beta^2\right)} \qquad s_2 = \sqrt{\omega^2 V^{-2}\varepsilon_\parallel \varepsilon_\perp^{-1}\left(\varepsilon_\perp \beta^2 - 1\right)}$$

$$\operatorname{Im} s_{1,2} > 0$$

Vavilov-Cherenkov radiation (VCR): $s - \mathbf{Real}$

$$\min(\omega_{p\parallel}, \omega_{p\perp}) < \omega < \max(\omega_{p\parallel}, \omega_{p\perp})$$

Scattered field

$$H_\varphi^{b(1,2)} = \frac{q}{2\pi\beta c}\int_{-\infty}^{+\infty} H_{\varphi\omega}^{b(1,2)} \exp(-i\omega t)\,d\omega$$

$$H_{\varphi\omega}^{b(1,2)} = \mp\int_{-\infty}^{+\infty} dk_\rho\, B^{(1,2)} k_\rho^2 [k_z^{(1,2)}]^{-1} H_1^{(1)}(\rho k_\rho)\exp(i k_z^{(1,2)}|z|)$$

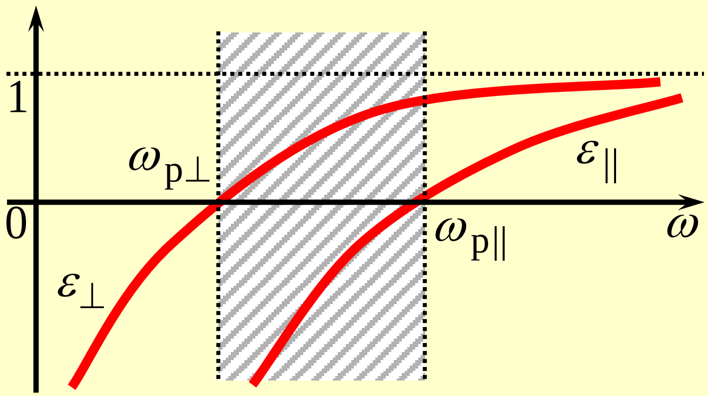
$$k_z^{(1)} = \sqrt{\omega^2 c^{-2} - k_\rho^2} \qquad k_z^{(2)} = \sqrt{\varepsilon_\perp \varepsilon_\parallel^{-1}(\omega^2 c^{-2}\varepsilon_\parallel - k_\rho^2)} \qquad \operatorname{Im} k_z^{(1,2)} \geq 0$$

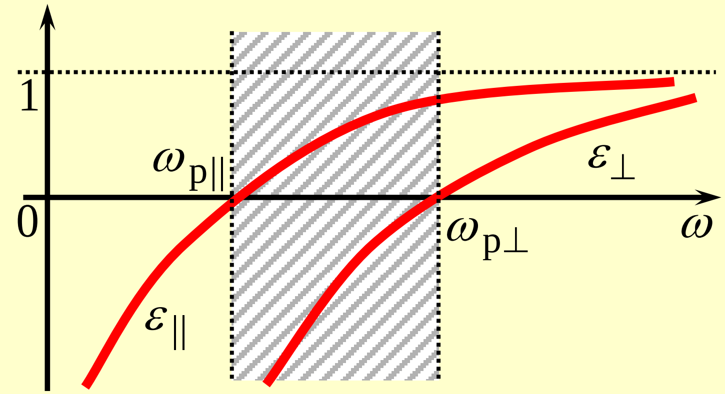
$$B^{(1)} = \frac{k_z^{(1)}}{k_z^{(2)} + \varepsilon_\perp k_z^{(1)}}\left(\frac{\beta k_z^{(2)} - \omega c^{-1}\varepsilon_\perp}{k_\rho^2 - s_1^2} + \frac{c\,\varepsilon_\perp \varepsilon_\parallel^{-1}\beta^2}{\omega\left(1+\sqrt{1 - c^2\beta^2\omega^{-2}\varepsilon_\perp\varepsilon_\parallel^{-1}(k_\rho^2 - s_2^2)}\right)}\right)$$

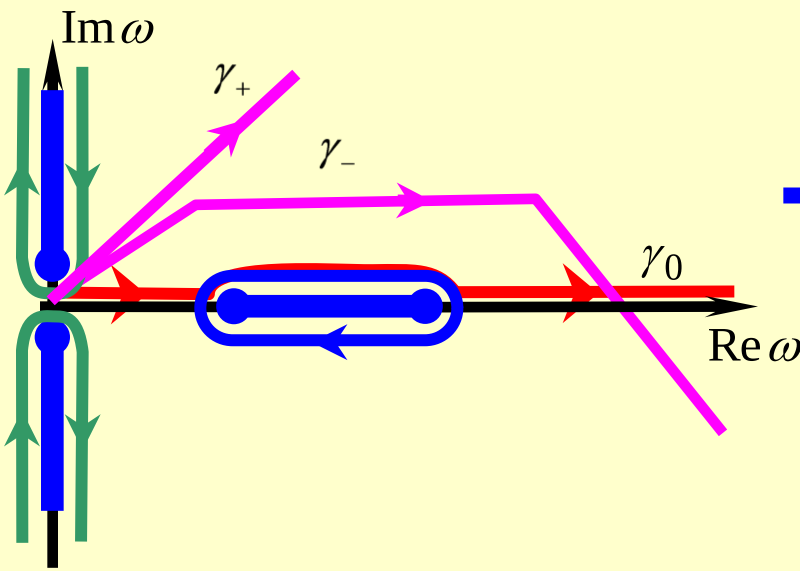
Transition radiation (TR) | **Reversed Cherenkov – transition radiation (RCTR)**

Self-field of the charge

Anisotropic plasma-like medium







– **branch points**

– **cuts** $\mathrm{Im}\, s_2 = 0$

$$H_{\varphi}^{\mathrm{q}(2)} = \boxed{H_{\varphi \mathrm{C}}^{\mathrm{q}(2)}} + \boxed{H_{\varphi \mathrm{W}}^{\mathrm{q}(2)}}$$

wave field (VCR)

$$H_{\varphi \mathrm{W}}^{\mathrm{q}(2)} = \frac{2q}{c} \int_{\omega_{\mathrm{p}||}}^{\omega_{\mathrm{p}\perp}} \left| s_2(\omega) \right| J_1\left(\rho \left| s_2(\omega) \right| \right) \sin\left(\omega \frac{|\zeta|}{V} \right) \mathrm{d}\omega \, \Theta(-\zeta)$$

guasistatic („quasiqulomb") field

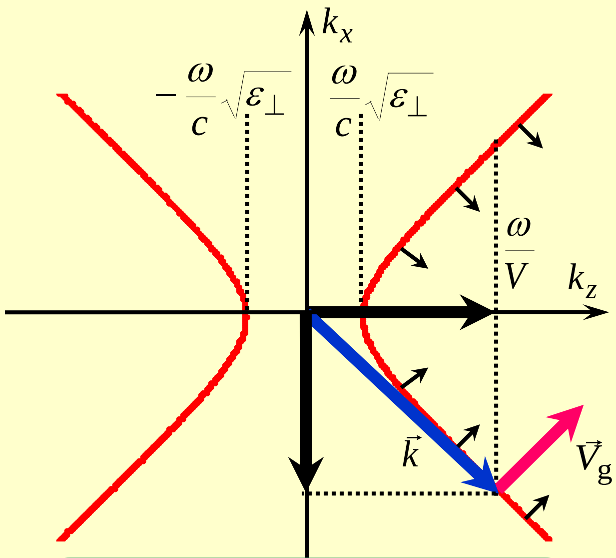
$$H_{\varphi \mathrm{C}}^{\mathrm{q}(2)} = \frac{q}{c} \int_{|\omega_{\mathrm{c}}|}^{\infty} \left| s_2(\mathrm{i}\tilde{\omega}) \right| J_1\left(\rho \left| s_2(\mathrm{i}\tilde{\omega}) \right| \right) \exp\left(-\tilde{\omega} \frac{|\zeta|}{V} \right) \mathrm{d}\tilde{\omega}$$

Self-field of the charge

Anisotropic plasma-like medium

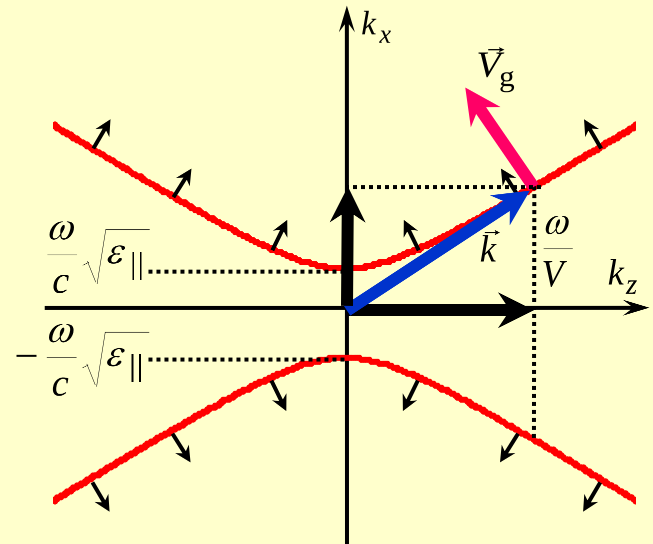
$$\Delta = k_x^2 / \varepsilon_{\parallel} + k_z^2 / \varepsilon_{\perp} - \omega^2 / c^2 = 0$$

$\omega_{p\parallel} > \omega_{p\perp}$



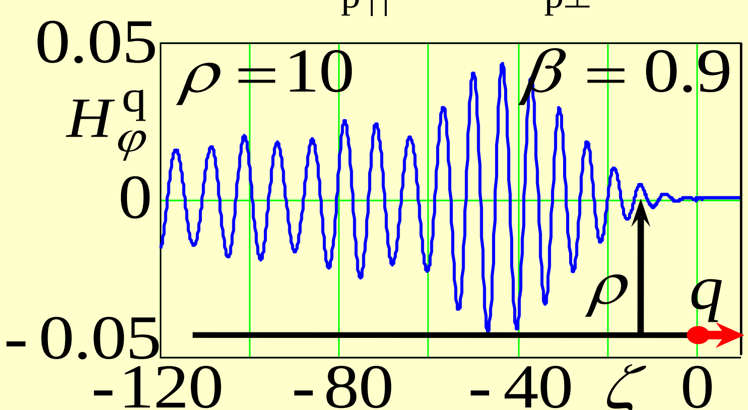
forward radiation

$\omega_{p\parallel} < \omega_{p\perp}$



backward radiation

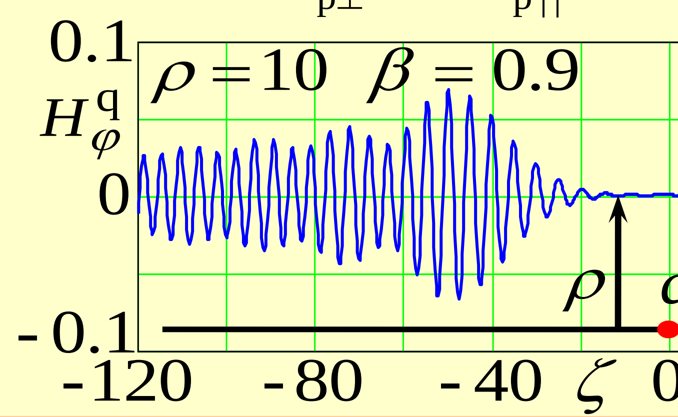
$\omega_{p||} = 1.5\omega_{p\perp}$



$\omega_{d||} = \omega_{d\perp} = 10^{-3}\omega_{p||}$

ζ, ρ in units of $c\omega_{p||}^{-1}$

$\omega_{p\perp} = 1.5\omega_{p||}$



Scattered field

Vacuum – anisotropic plasma-like medium interface

Analytical approach

Fourier harmonics of the scattered field

$$H_{\varphi\omega}^{\mathrm{b}(1,2)} = \mp \int_{-\infty}^{+\infty} \mathrm{d}k_\rho \, B^{(1,2)} \frac{k_\rho^2}{k_z^{(1,2)}} H_1^{(1)}(\rho k_\rho) \exp(\mathrm{i} k_z^{(1,2)} |z|)$$

Asymptotic representation in the far field zone (with respect to the incident point)

$$\sim R^{-1}, \ \sim \rho^{-1/2}$$

"Half-shadow" regions estimation

Impact of losses in medium on the radiation field

Numerical approach

Fourier harmonics of the scattered field

Analytical investigation of the integrands behavior, choosing the appropriate integration step and integration interval

Total scattered field

$$H_{\varphi}^{\mathrm{b}(1,2)} = \frac{q}{2\pi\beta c} \int_{-\infty}^{+\infty} \mathrm{d}\omega \, H_{\varphi\omega}^{\mathrm{b}(1,2)} \exp(-\mathrm{i}\omega t)$$

Numerical investigation of the integrands behavior, choosing the appropriate integration step and interval

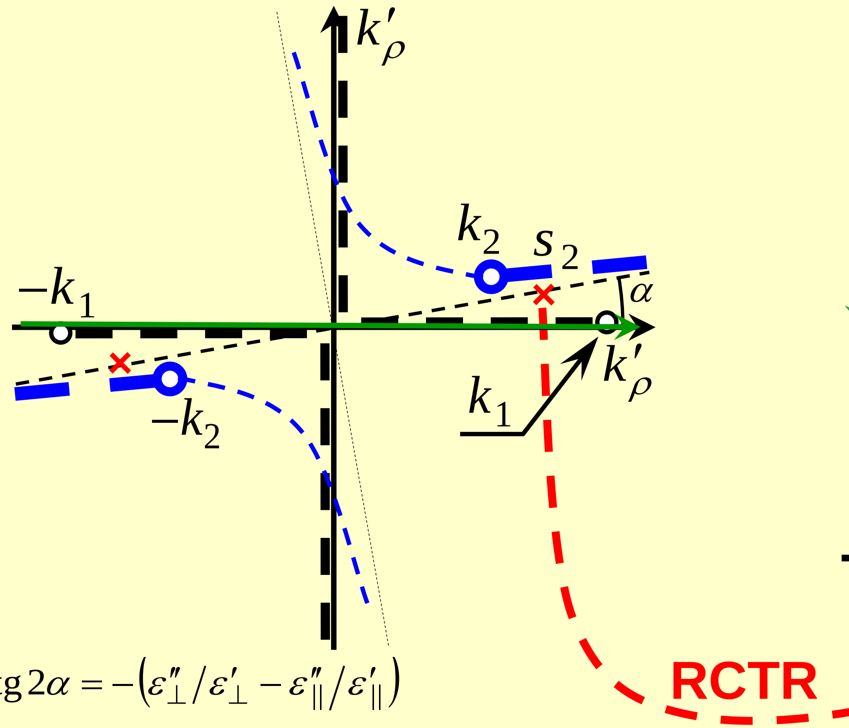
Scattered field

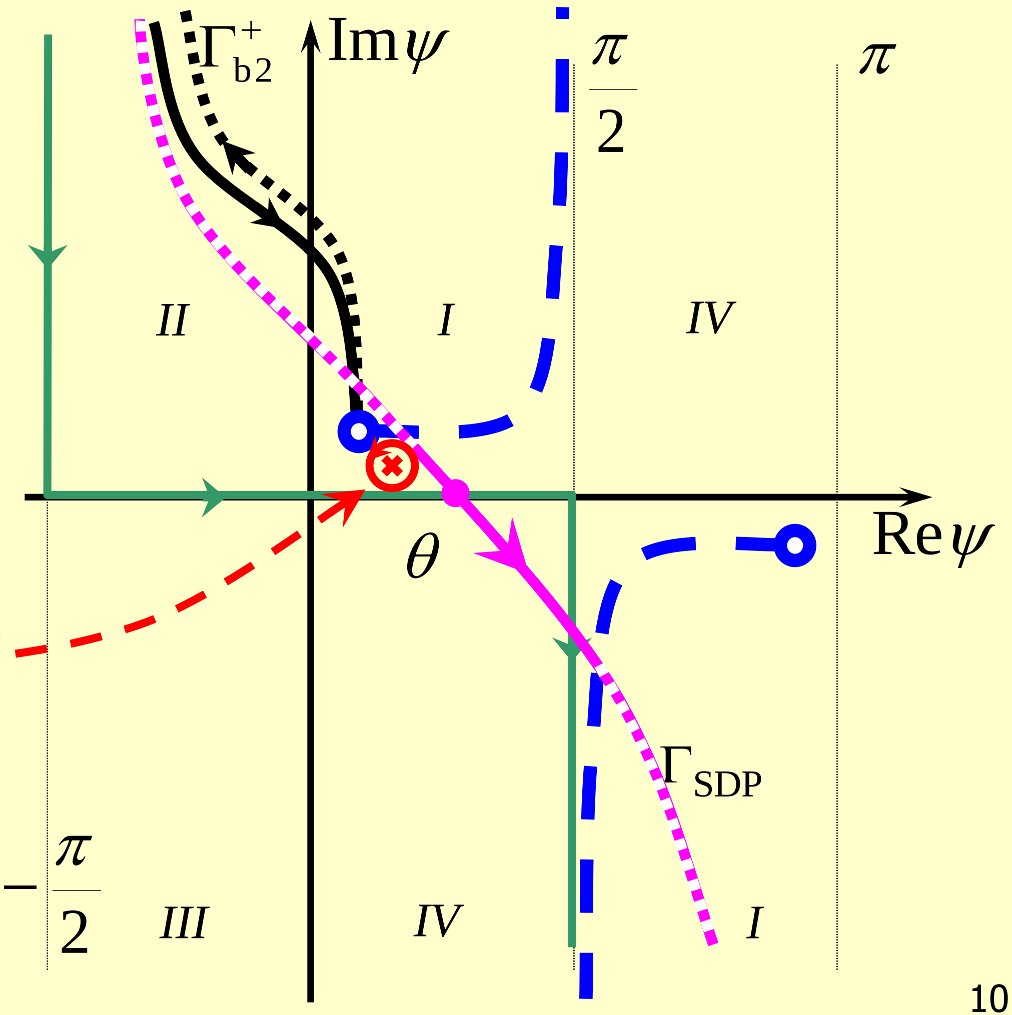
Vacuum – anisotropic plasma-like medium interface

Analytical approach

Field in vacuum, the case of backward VCR:

$\omega_{p\parallel} < \omega < \omega_{p\perp}$





$\mathrm{tg}\,2\alpha = -\left(\varepsilon''_\perp/\varepsilon'_\perp - \varepsilon''_\parallel/\varepsilon'_\parallel\right)$

$k_\rho = k_1 \sin\psi$

$k_1 = \sqrt{\omega^2 c^{-2} \varepsilon_1} \qquad k_2 = \sqrt{\omega^2 c^{-2} \varepsilon_\parallel}$

$\mathrm{Im}\, k_{1,2} > 0$

Results

Vacuum – anisotropic plasma-like medium interface

Conditions of the RCTR existence in vacuum

$$\omega_{\mathrm{p}\|} < \omega < \Omega(\beta) \qquad \beta > \beta_{\mathrm{RCTR}}(\omega)$$

$$\Omega^2 = \frac{\omega_{\mathrm{p}\|}^2(1-\beta^2)}{2} + \sqrt{\frac{\omega_{\mathrm{p}\|}^4(1-\beta^2)^2}{4} + \omega_{\mathrm{p}\|}^2\omega_{\mathrm{p}\perp}^2\,\beta^2}$$

$$\beta_{\mathrm{RCTR}}(\omega) = \sqrt{\omega^2(\omega^2-\omega_{\mathrm{p}\,\|}^2)\,\omega_{\mathrm{p}\|}^{-2}(\omega_{\mathrm{p}\perp}^2-\omega^2)^{-1}}$$

Field asymptotic in vacuum, $\sim R^{-1}$ $\sim \rho^{-1/2}$

$$H_{\varphi\omega}^{\mathrm{b}(1)} \approx H_{\varphi\omega}^{\mathrm{b}(1)\mathrm{S}} + H_{\varphi\omega}^{\mathrm{b}(1)\mathrm{P}}$$

Spherical wave of TR: $H_{\varphi\omega}^{\mathrm{b}(1)\mathrm{S}} \sim \dfrac{\exp(\mathrm{i}\,k_1 R)}{R}$ $\qquad k_1 R >> 1$

Cylindrical wave of RCTR:

$$H_{\varphi\omega}^{\mathrm{b}(1)\mathrm{P}} = \frac{\mathrm{i}\,q}{\beta c^2}\,\frac{-s_2\,\omega}{g_3^*(\omega)}\,H_1^{(1)}(\rho s_2)\,\exp(\mathrm{i}\,k_z^{(1)}(s_2)\,|\,z\,|)\;\Theta(\theta-\theta_1),$$

$$g_3^*(\omega) = -\omega/c\beta + \varepsilon_\perp k_z^{(1)}(s_2) \qquad \sin\theta_{10} = \sqrt{\varepsilon'_\|}\sqrt{1+\frac{1}{|\varepsilon'_\perp|\beta^2}}$$

$$\theta > \theta_1 = \theta_{10} + \delta\theta_1$$

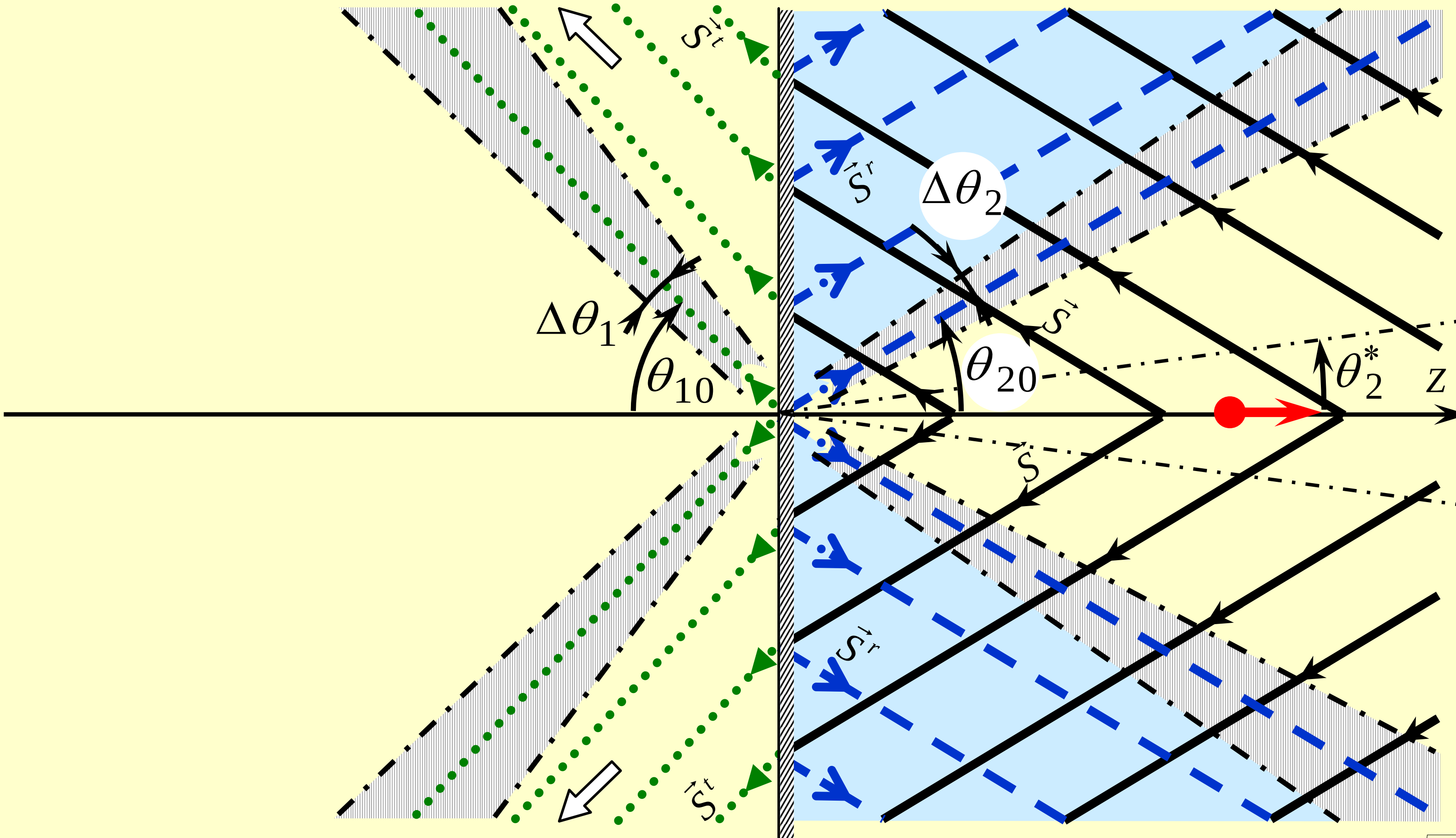
$$\delta\theta_1 = \sqrt{\frac{2c}{\omega R_1}} \qquad R_1 = 8\frac{c}{\omega}\left[\mathrm{tg}\,\theta_{10}\left(\frac{\varepsilon''_\|}{\varepsilon'_\|} + \frac{\varepsilon''_\perp}{|\varepsilon'_\perp|}\,\frac{1}{1+|\varepsilon'_\perp|\beta^2}\right)\right]^{-2}$$

“half-shadow” regions $\left|\theta-\theta_{10}\right| \le \Delta\theta_1$ $\qquad R \le R_1$ $\qquad \Delta\theta_1 = \sqrt{2c\omega^{-1}R^{-1}(1-R/R_1)}$

Results

Spatial distribution of the Fourier harmonic

$$\omega_{p\|} < \omega < \Omega(\beta)$$



$$\mathrm{tg}\,\theta_2^* = \sqrt{\left|\varepsilon'_\perp\right| / \varepsilon'_\|}$$

– lines parallel to the Poyting vector of VCR

– lines parallel to the Poyting vector of RCTR in medium

– lines parallel toi Poyting vector of RCTR in vacuum

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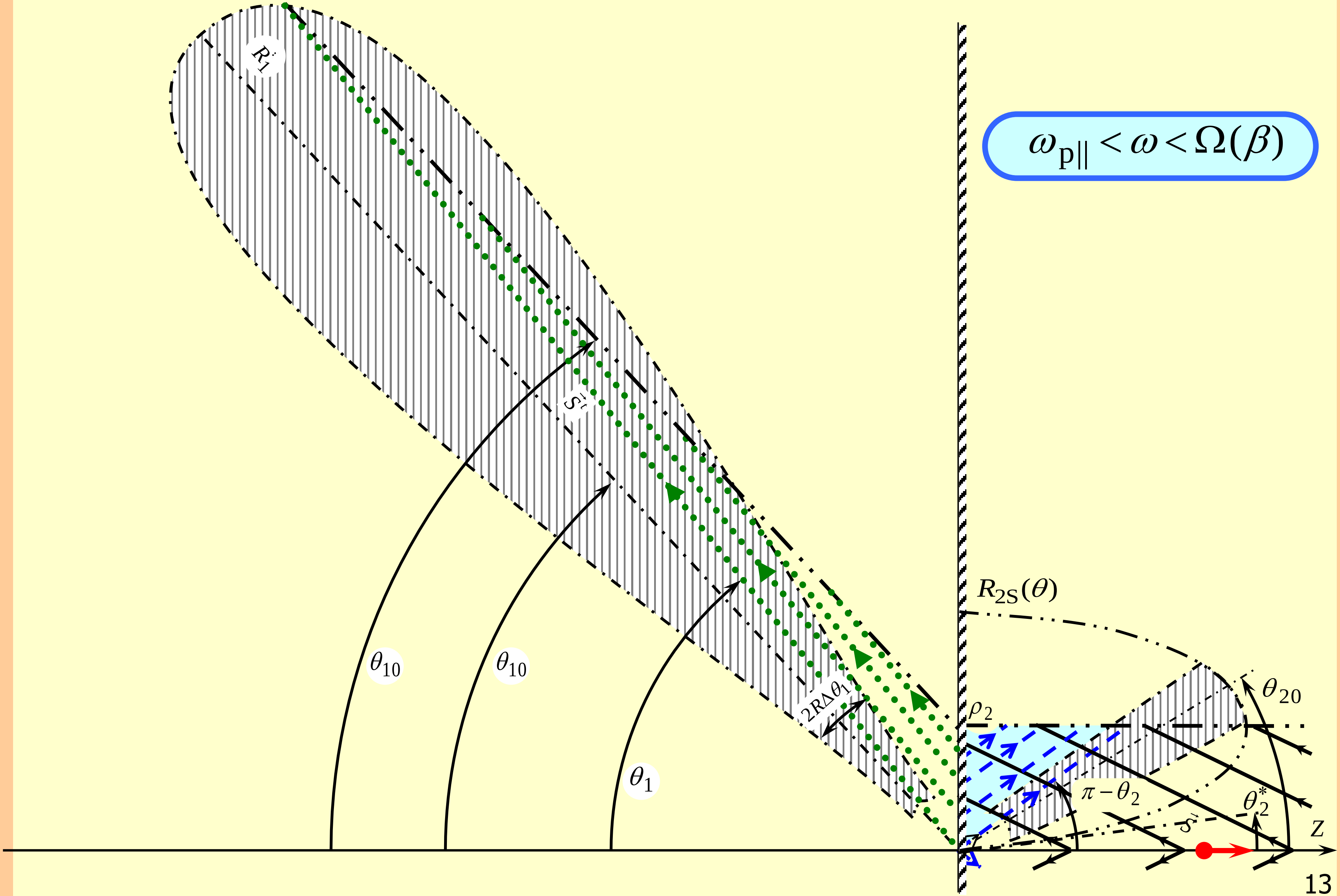
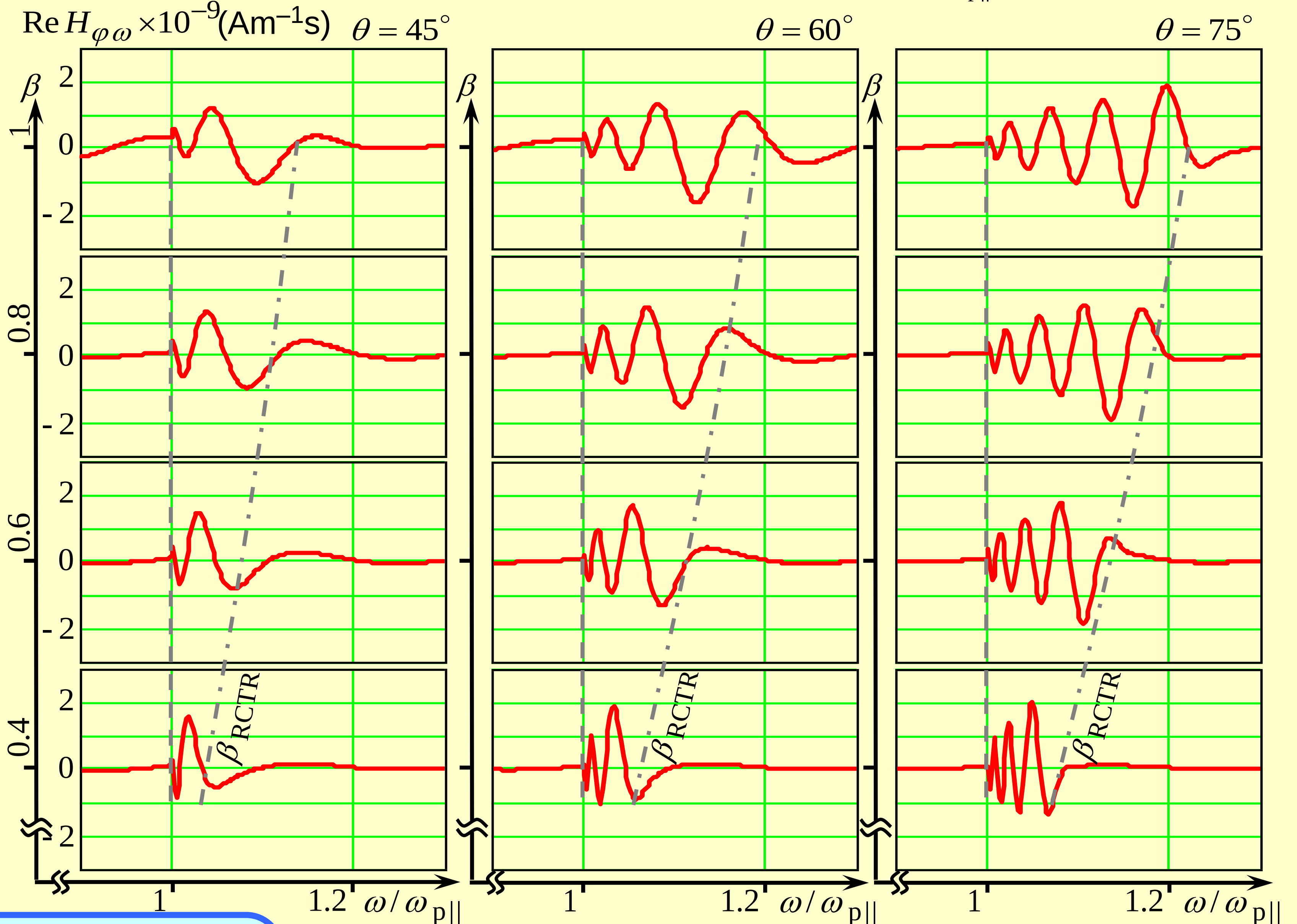


Results

Field spectrum in vacuum

$R = 14$ cm $\quad q = -1$ nC

$\omega_{p\|} = 2\pi \cdot 10^{10} c^{-1}, \quad \omega_{p\perp} = 1.5\omega_{p\|}$



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$\omega_{d\|} = \omega_{d\perp} = 10^{-3}\omega_{p\|}, \quad \omega_{pe1} = 10^{-2}\omega_{p\|}, \quad \omega_{de1} = 10^{-6}\omega_{p\|}$

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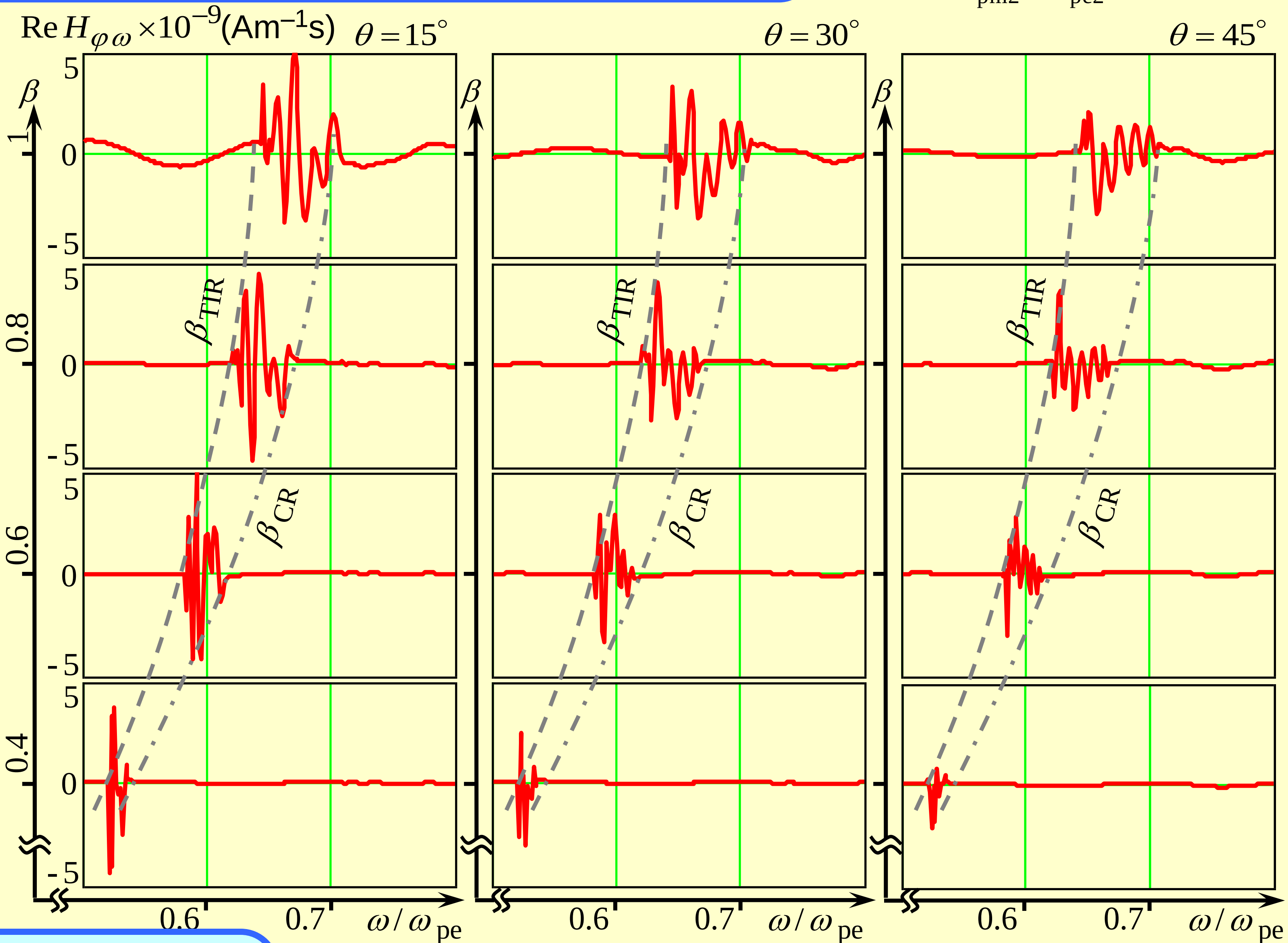
$\omega_{pm2}=\omega_{pe2}=2\pi\cdot 10\cdot 10^9\,c^{-1}$



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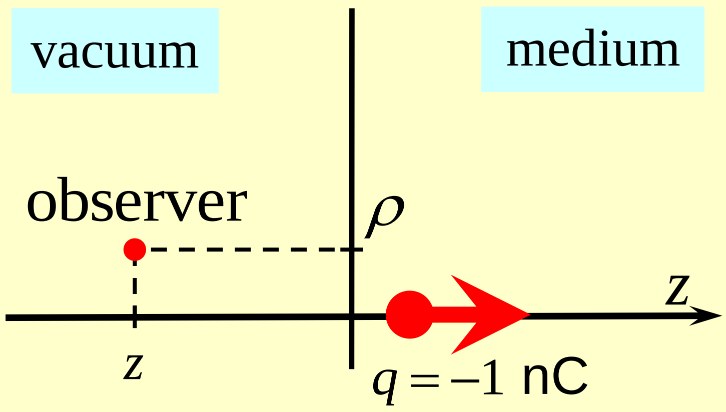
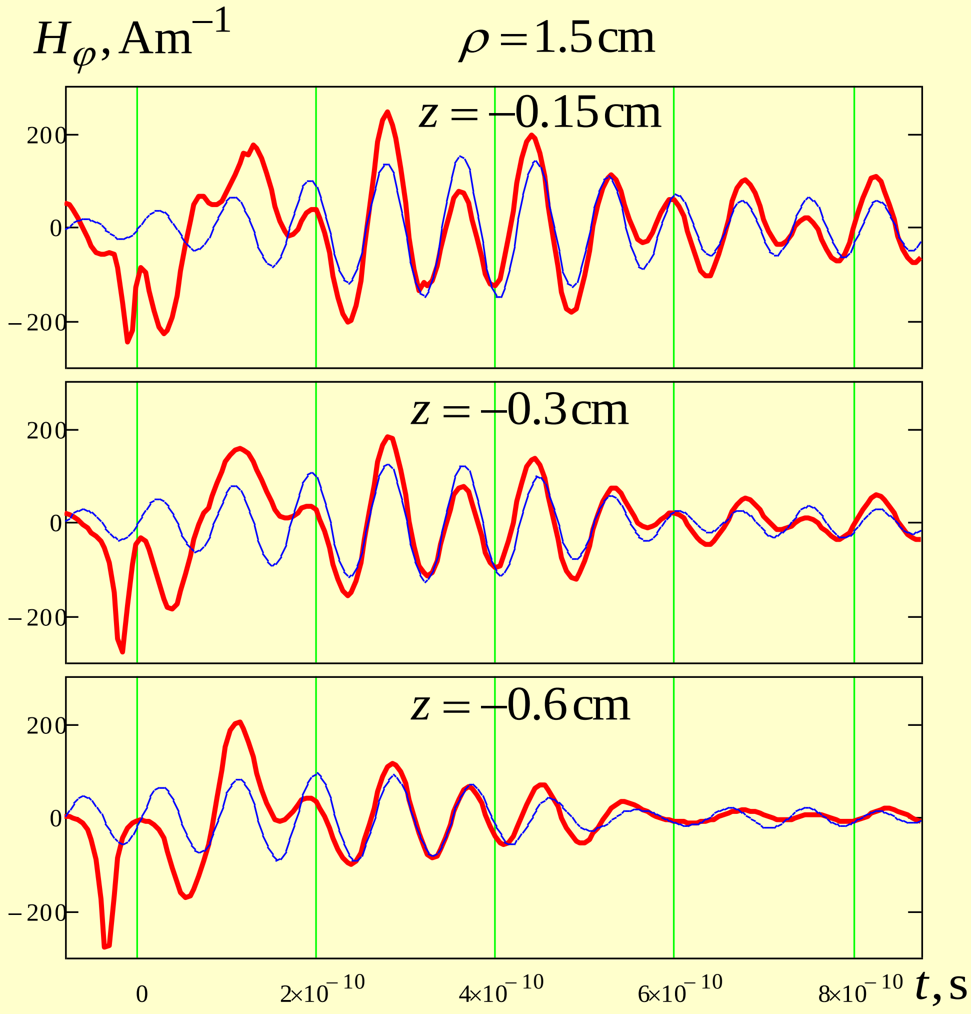
Results

Time evolution of the total field





Conclusion

Investigation of the electromagnetic field generated at a charge flight from vacuum into anisotropic plasma-like medium

Reversed Cherenkov-Transition Radiation (RCTR)

Analytical approach:

- **Rigorous condition of the RCTR presence**
- **Spatial structure of the far-field Fourier harmonics**
- **“Half-shadow” areas**
- **Impact of losses**

Numerical approach:

- **Field spectrum results**
- **Time evolution of the total field**
- **Possibility of the RCTR dominance in the total field**

Thank you for your attention!