„Multipole" exact solutions in relativistic hydrodynamics
(& higher order flow coefficients)

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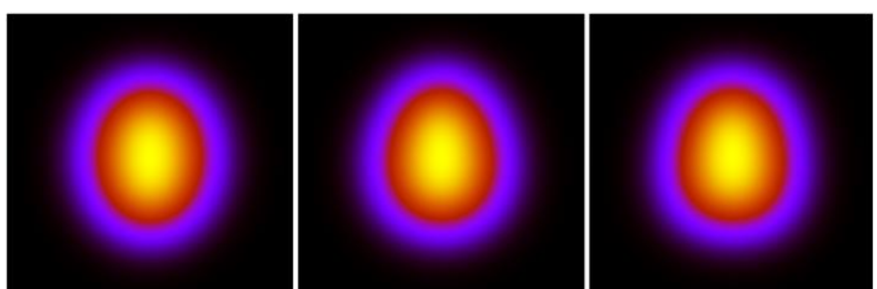
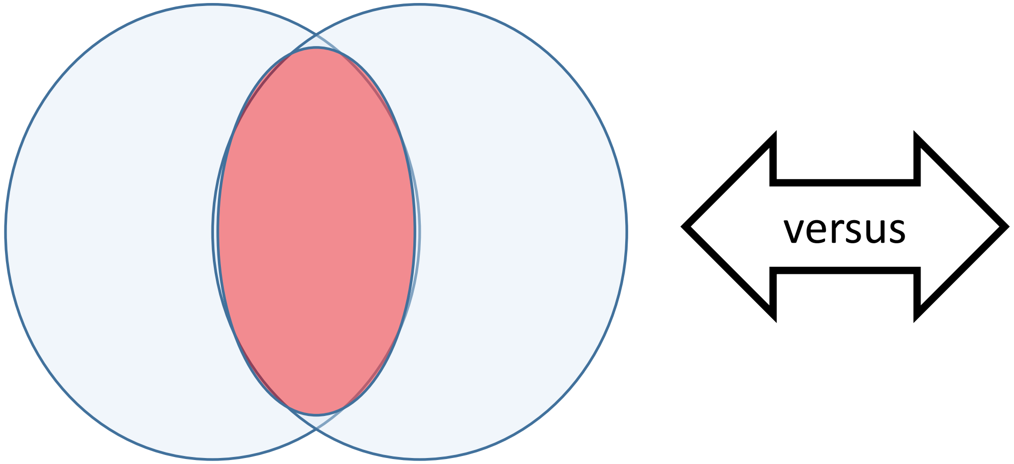
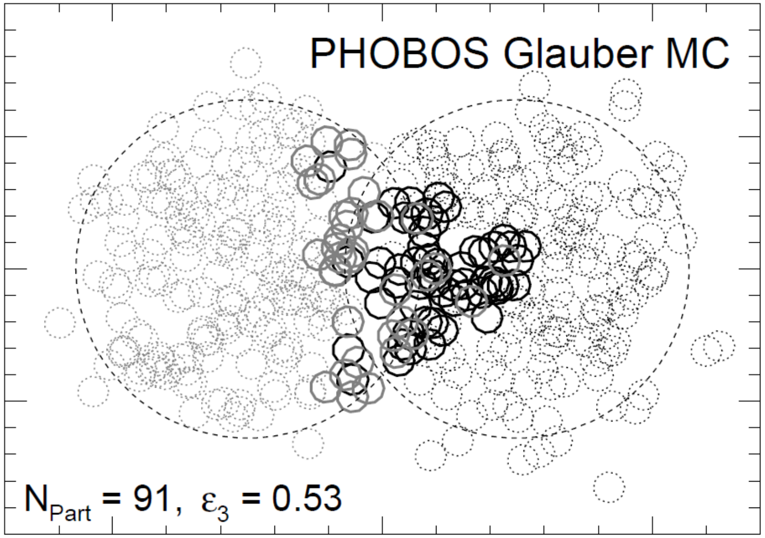


Higher order flow and event-by-event hydro

- Basic picture: elliptical region, v_2 most important
- Finite number of nucleons: event-by-event fluctuating initial condition

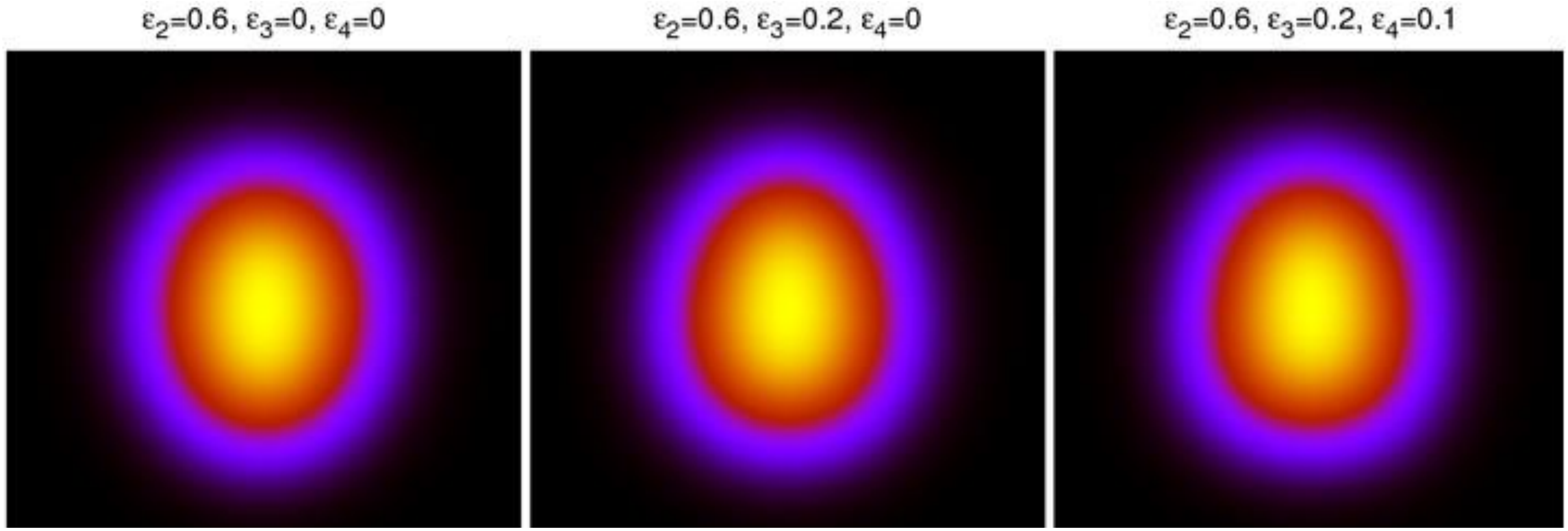




- Higher order asymmetry (flow) coefficients arise from this effect!
- How to handle this in hydrodynamics?
- Numerically (see e.g. Broniowski et al.): implement fluctuating initial cond.
- Analytic hydro?

New „multipole” solutions

- Based on Csörgő, Csernai, Hama, Kodama, Heavy Ion Phys. A21, 73 (2004)
- As $\dot{R} = \text{const.}$, Hubble-like velocity field: $u^\mu = \frac{x^\mu}{\tau}$, and $\frac{\tau_0}{\tau} = \frac{\gamma R_0}{R}$
- At given τ, temp. constant on scale variable defined surfaces, arbitrary profile
- Typical choice of temperature profile: $\exp[-bs]$, with $b = \left\langle \frac{\Delta T}{T} \right\rangle_r$
- Several asymmetries superimposed, with various ϵ_N coefficients
- Phase shifts define Nth order reaction plane, its angle is ψ_N



Multipole symmetries in 2D

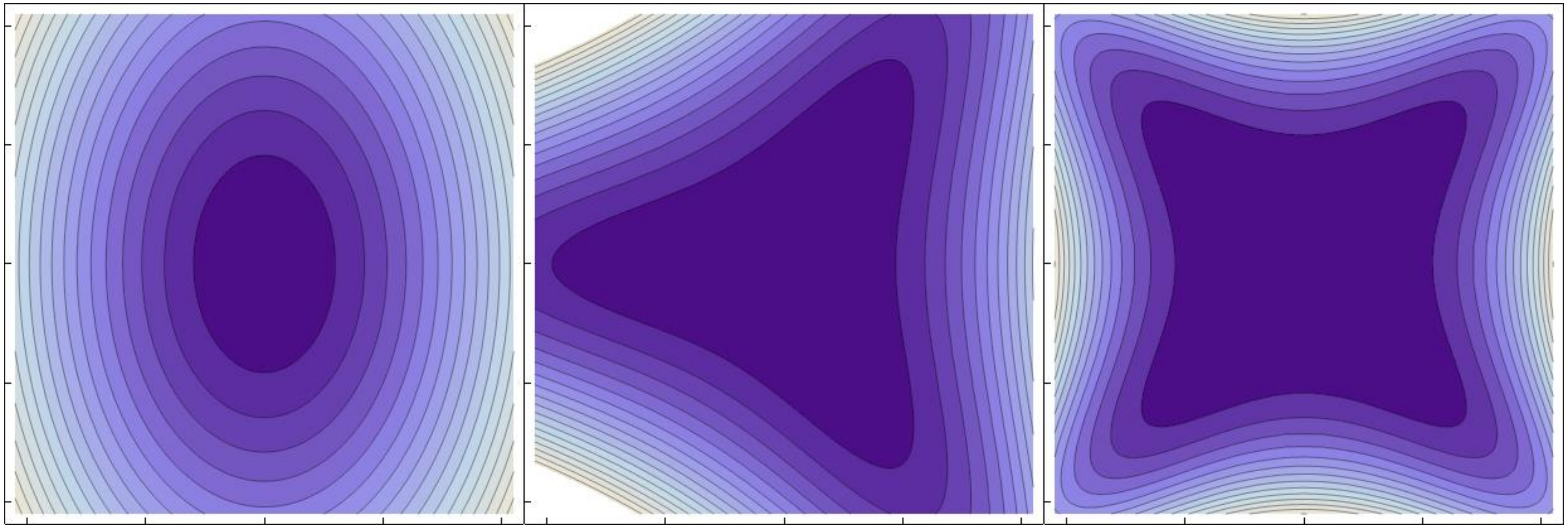
- Ellipsis in polar coordinates:

$$\frac{r_x^2}{X^2}+\frac{r_y^2}{Y^2}=\frac{r^2}{R^2}\times[1+\epsilon\cos(2\phi)], \text{ with } \frac{1}{X^2}+\frac{1}{Y^2}=\frac{2}{R^2} \text{ and } \epsilon=\frac{X^2-Y^2}{X^2+Y^2}$$

- Generalizing that to multipole symmetries:

$$\frac{r^N}{R^N}\times[1+\epsilon\cos(N\phi)]$$

$N=2$ $\quad$ $N=3$ $\quad$ $N=4$



- Are there hydro solutions that reflect these symmetries?

Non-ellipsoidal solutions of relativistic hydro?

- Quite few non-spherical relativistic solutions
- Even those assume elliptical (ellipsoidal) symmetry
- Basics: thermodynamical quantities constant on ellipsoidal surfaces
- Defined by scaling variable:

$$s = \frac{r_x^2}{X(t)^2} + \frac{r_y^2}{Y(t)^2} + \frac{r_z^2}{Z(t)^2}$$

- How to modify this in the transverse plane?
- Ellipsis in polar coordinates:

$$\frac{r_x^2}{X(t)^2} + \frac{r_y^2}{Y(t)^2} = \frac{r^2}{R(t)^2}\left[1 + \epsilon_2(t)\cos(2\phi)\right]$$

- Generalization to „multipole" symmetries, with $\Psi_N$reaction plane angles):

$$s = \frac{r^N}{R(t)^N}\left[1 + \epsilon_N(t)\cos(N(\phi - \Psi_N))\right]$$

- Works even superimposing many of these

$$s = \sum_N \frac{r^N}{R(t)^N}\left[1 + \epsilon_N(t)\cos(N(\phi - \Psi_N))\right]$$

Exact „multipole" hydro solutions

- A solution in axial coordinates:

$$s=\frac{r^N}{R^N}\left[1+\epsilon\cos\bigl(N(\phi-\psi)\bigr)\right]+\frac{z^N}{R^N}$$

$$u^\mu=\gamma\left(1,\frac{\dot{R}}{R}r\cos\phi\,,\frac{\dot{R}}{R}r\sin\phi\,,\frac{\dot{R}}{R}z\right)=\frac{x^\mu}{\tau}$$

$$T=T_0\left(\frac{\tau_0}{\tau}\right)^{\frac{3}{\kappa}}f(s)\text{ , with arbitrary } f \text{ function}$$

- Here $R=u_t t$, i.e. $\dot{R}=u_t$ = const., as well as ψ = const., ϵ = const.
- Constant EoS (but κ cancels from hadronic results)
- Works with other scaling variables as well, eg. (spherical coordinates):

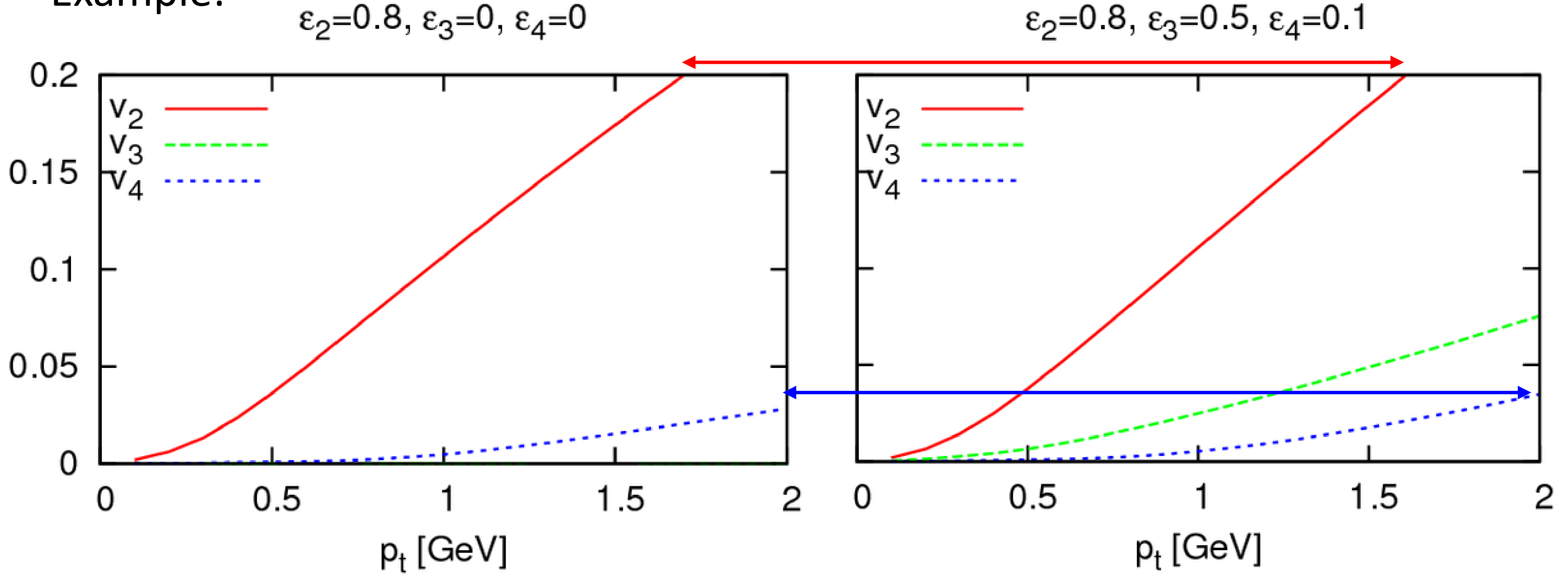
$$s=\frac{r^N}{R^N}\left[1+\epsilon\cos\bigl(N(\phi-\psi)\bigr)(1-\cos N\theta)+\chi\cos N\theta\right]$$

- Works combining many symmetries with different (constant) ϵ's and ψ's

$$s=\sum_{N=2}\left[\frac{r^N}{R(t)^N}\left(1+\epsilon_N\cos(N(\phi-\psi_N))\right)+\frac{z^N}{R(t)^N}\right]$$

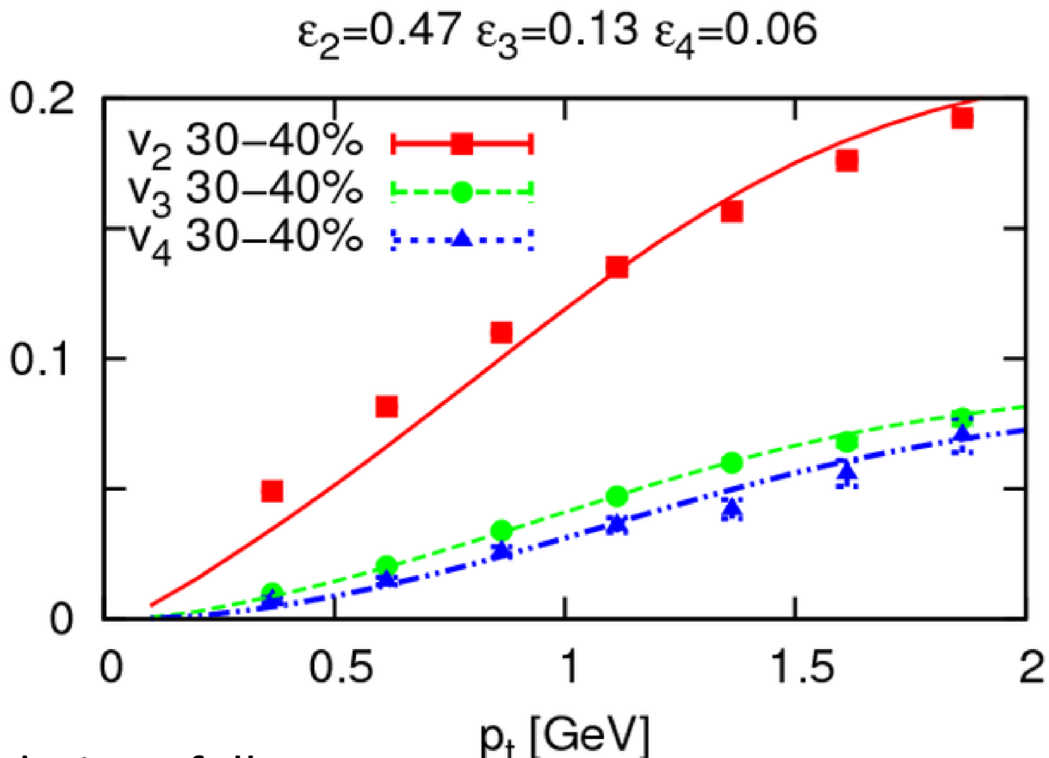
Observables from the new solutions

- Define hadronic freeze-out, results independent of EoS (κ)
- Azimuthally integrated observables (spectra) are the same!
- Earlier results applicable here
  - Spectra, HBT and flow fits shown in Eur.Phys.J. A44 (2010) 473
- Odd v_N coefficients arise naturally here, even coefficients change slightly
- Relative orientation of ψ_N planes plays no role in the results
- Example:



Comparison to data

- PHENIX data of higher order flow coefficients in several centrality bins:
  Phys. Rev. Lett. 107, 252301 (2011)
- Tune ϵ_N asymmetry parameters to data (others from previous hadron fits):



- Detailed analysis to follow

Summary

Higher order flow coefficients measure deviation from elliptical symmetry

Arise due to fluctuating initial conditions (c.f. wounded nucleon model)

Previously: predominantly in numerical hydro

This work: **exact analytic solutions for initial state fluctuations**

~ special case of T. Cs, Csernai, Hama, Kodama, Heavy Ion Phys. A21, 73 (2004)

New: Azimuthally integrated observables: nearly unchanged

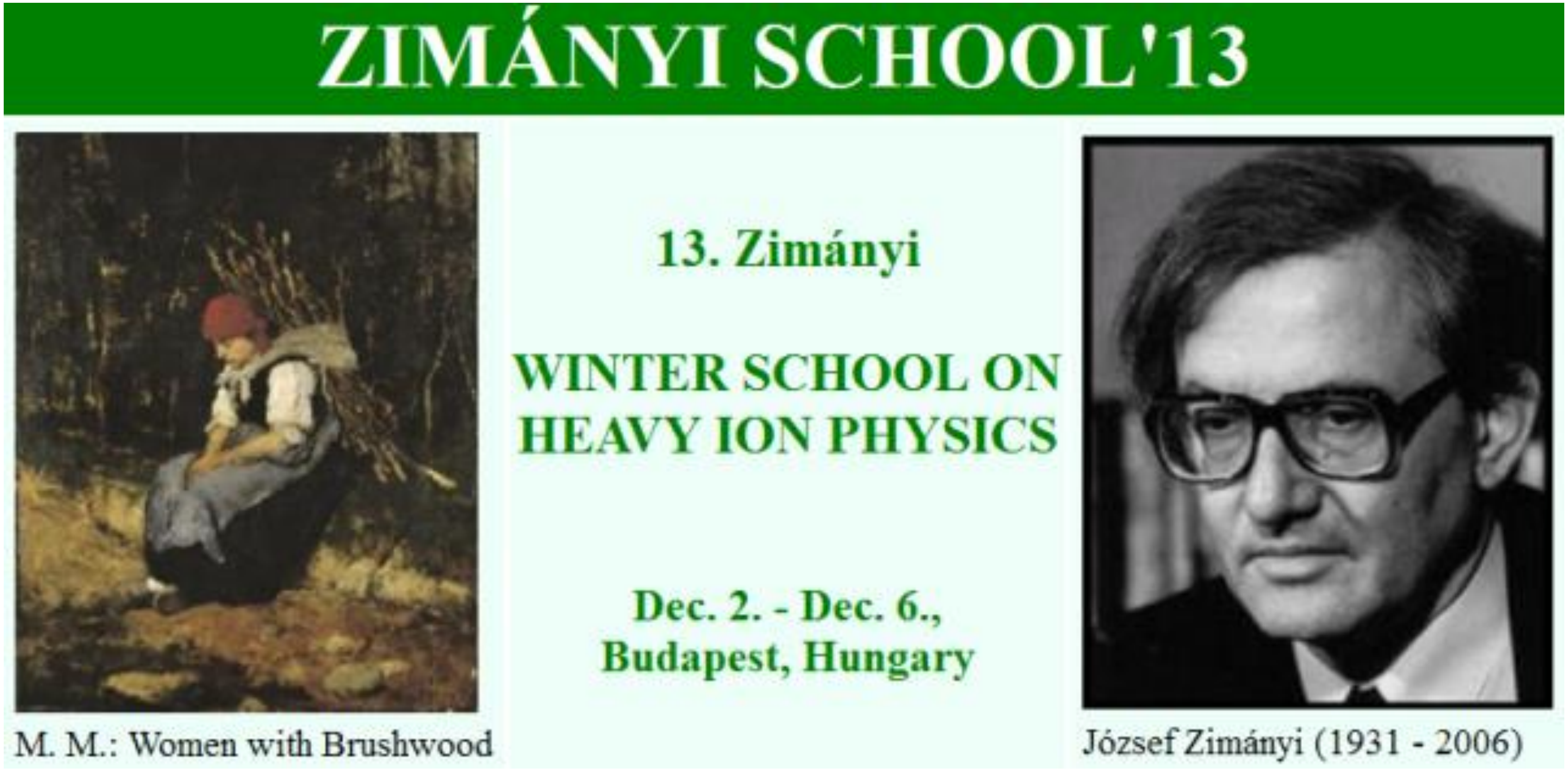
Realistic higher order (odd) flow coefficients calculable

Detailed investigations to follow:

connection to other solutions, asHBT, higher order HBT, lattice QCD EOS)

Thank you for your attention

And let me invite you to the 13th Zimanyi School



http://zimanyischool.kfki.hu/13

YOUR INVITATION TO WPCF 2014

KRF, Gyöngyös, foot of the Mátra hills, Hungary

Last week of August 2014

T. Csörgő & T. Novák (Wigner RCP – KRF)