Neutrino Theory

Boris Kayser
FPCP
May 25, 2011

NASA Hubble Photo

Points of Contact

Quark flavor physics and neutrino physics both —

- Feature mixing phenomena
- Want to find the source of CP violation behind the baryon-antibaryon asymmetry of the universe

We will focus on the first of these points of contact.

A Common Treatment of *Neutrino Oscillation* and *B-Factory* Experiments

With quantum entanglement.

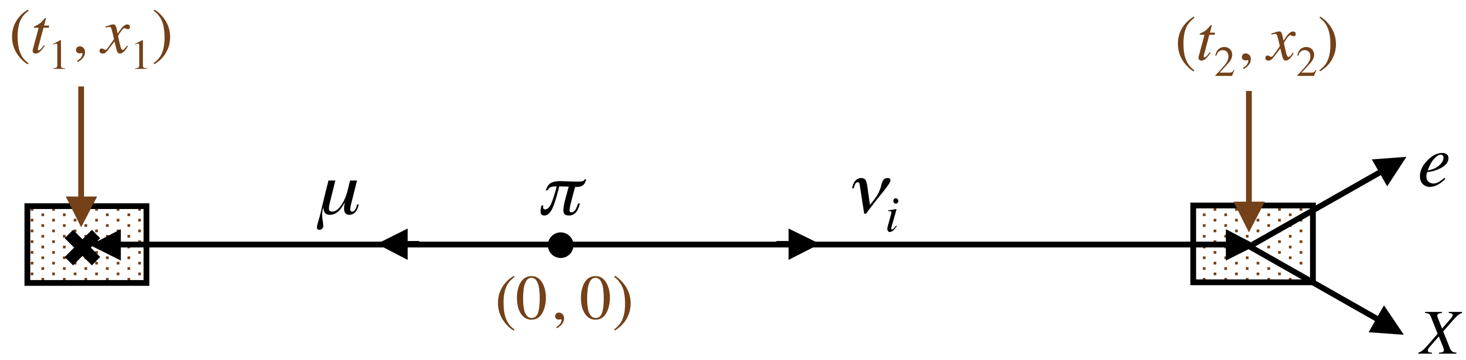
But avoiding Einstein-Podolsky-Rosen puzzles,

and non-intuitive kinematical assumptions.

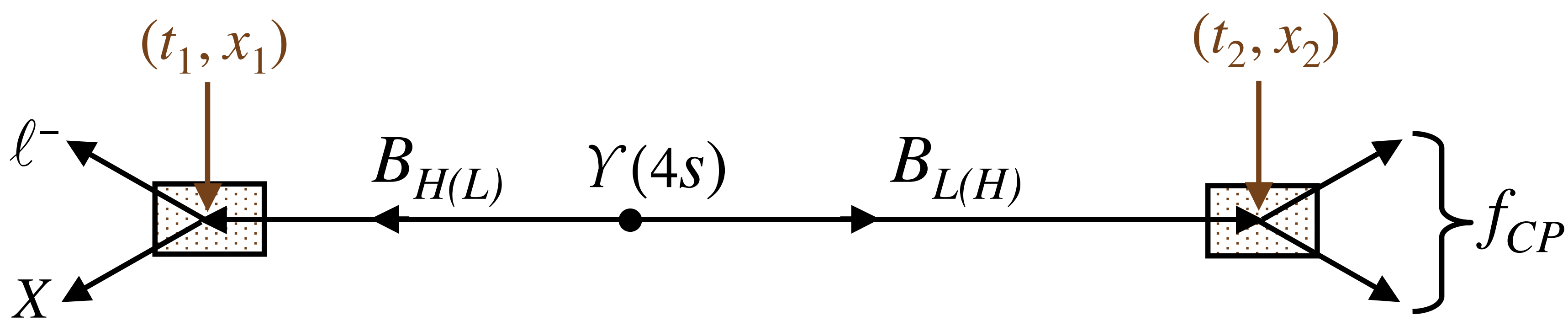
We assume there are just 3 neutrino mass eigenstates ν_i.

There are 2 neutral (non-strange) B mass eigenstates, $B_{H(eavy)}$ and $B_{L(ight)}$.

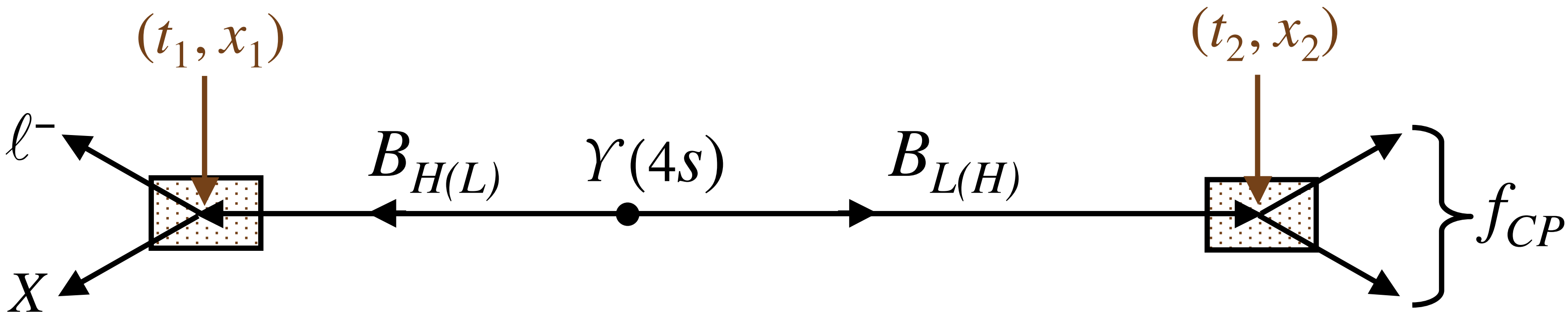
An accelerator neutrino oscillation experiment



A B-factory experiment



A B-factory experiment



We calculate the *amplitude* for this whole process.

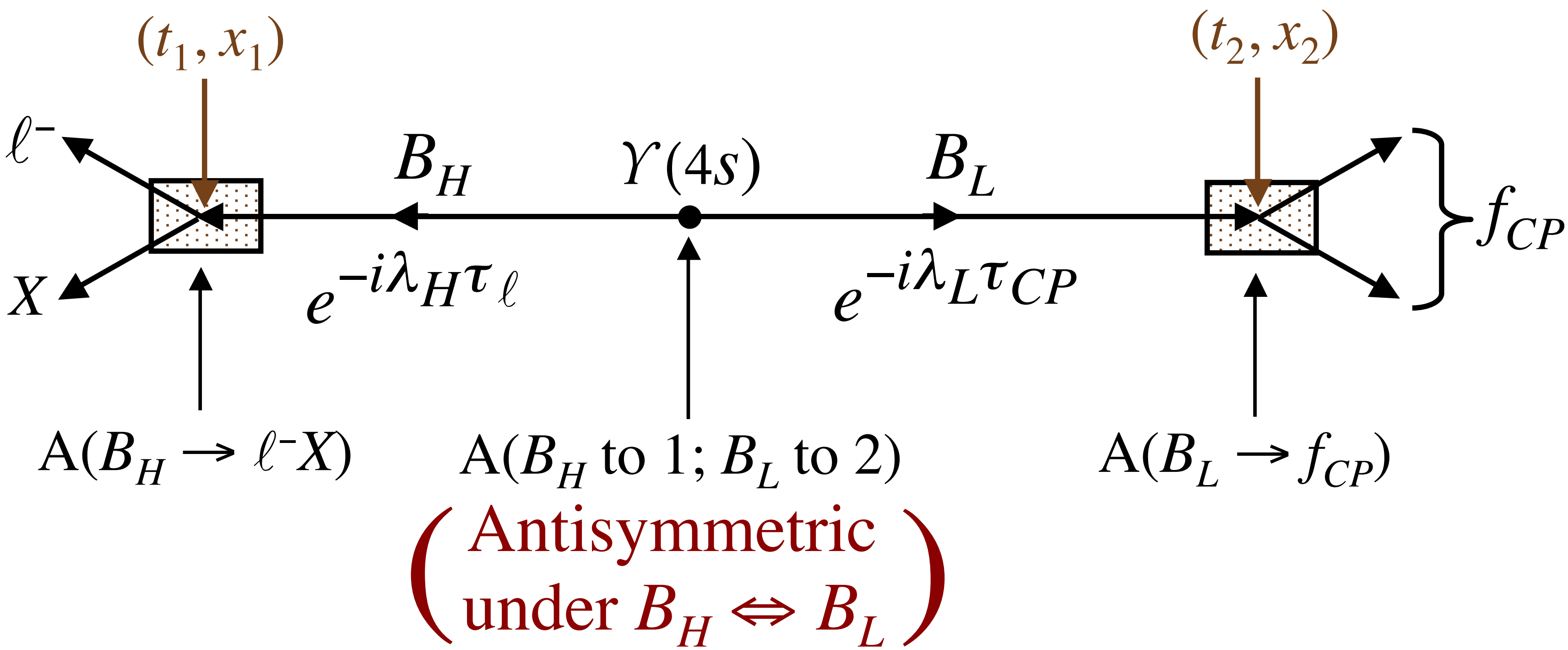
(B. K., Stodolsky)

We use —

Amplitude (Particle of mass $\lambda = m - i\frac{\Gamma}{2}$

propagates for a proper time τ) $= \exp(-\,i\lambda\tau)$

$$\{ \ \exp\left[i\left(px - Et\right)\right] = \exp\left(-im\tau\right) \ \}$$



$$\mathrm{Amp} = e^{-i\lambda_H \tau_\ell}\, e^{-i\lambda_L \tau_{CP}}\, \mathrm{A}(B_H \to \ell^- X)\, \mathrm{A}(B_L \to f_{CP})$$

$$- B_H \Leftrightarrow B_L$$

Using —

B_H and B_L have ~ the same width

$$\lambda_{H,L} = m \pm \frac{\Delta m}{2} - i\frac{\Gamma}{2}$$

$$B_{H,L} = \frac{1}{\sqrt{2}}\left[B_d \pm e^{-2i\delta_{CKM}^{mix}}\,\overline{B}_d\right]$$

Strong phase

$$A\left(B_d \to f_{CP}\right) = Me^{i\delta_{CKM}^{f}}\,e^{i\alpha_{ST}}$$

$$A\left(\overline{B}_d \to f_{CP}\right) = \eta_f\,Me^{-i\delta_{CKM}^{f}}\,e^{i\alpha_{ST}}$$

CP parity of f_{CP}

And defining —

$$2\left(\delta_{CKM}^{mix} + \delta_{CKM}^{f}\right) \equiv \phi \,,$$

one finds that —

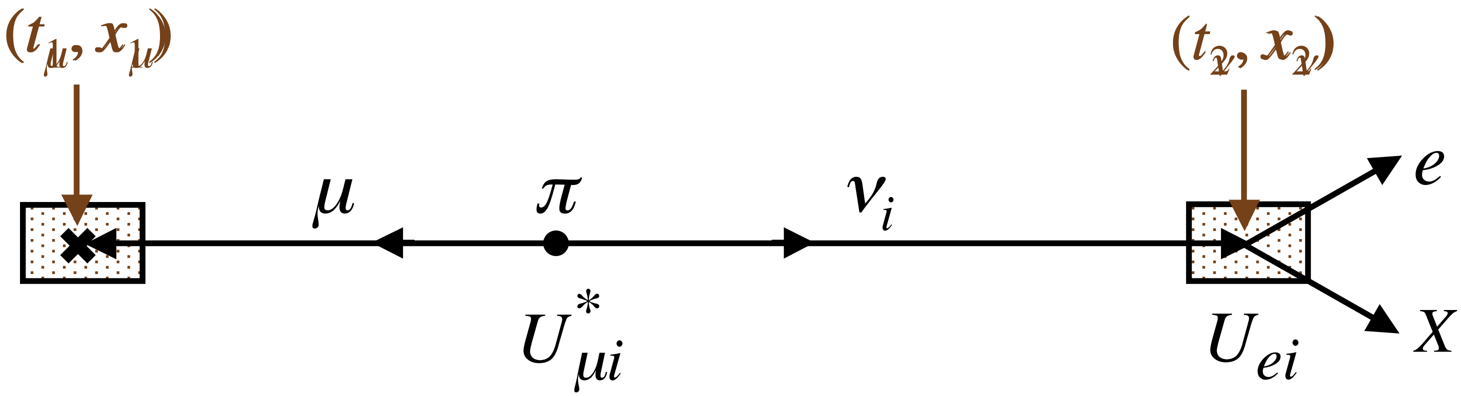
$$\Gamma\left(\text{One } B \to \ell^- X \text{ after } \tau_\ell;\ \text{Other } B \to f_{CP} \text{ after } \tau_{CP}\right) = |\text{Amp}|^2$$

$$\propto e^{-\Gamma\left(\tau_{CP}+\tau_\ell\right)}\left\{1 - \eta_f \sin\phi \sin\left[\Delta m\left(\tau_{CP} - \tau_\ell\right)\right]\right\}$$

This is the usual result, except that times in the $\Upsilon(4s)$ rest frame are replaced by proper times in the *B* rest frames.

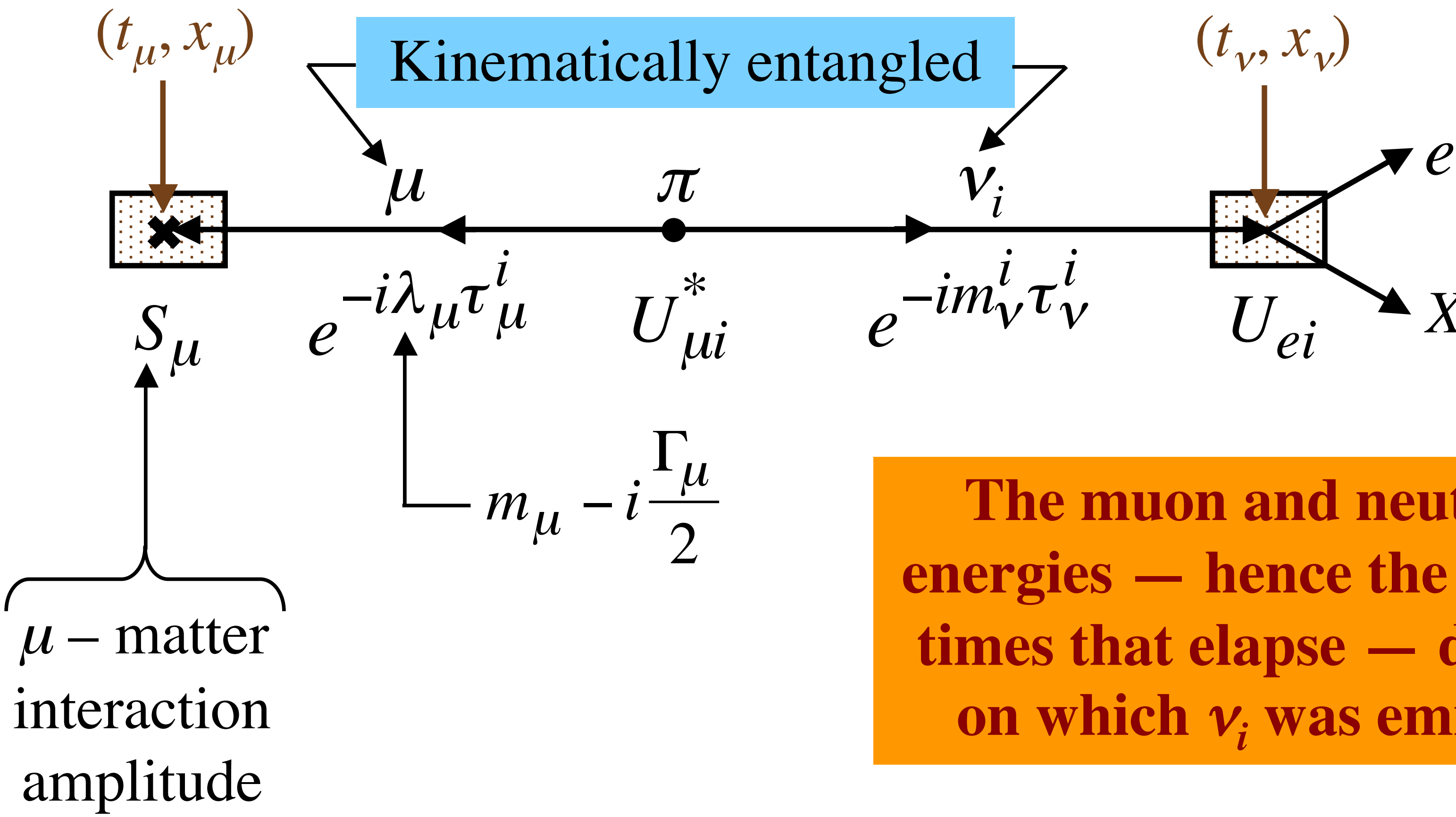
No need to think in terms of a collapsing wave function.

An accelerator neutrino oscillation experiment



We shall make use of the leptonic analogue of the CKM quark mixing matrix —

$$U_{PMNS} = \begin{array}{cc} & \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\ \begin{array}{c} e \\ \mu \\ \tau \end{array} & \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \end{array} \quad .$$



The muon and neutrino energies — hence the proper times that elapse — depend on which ν_i was emitted.

$$\text{Amp} = \sum_{i=1,2,3} S_\mu \, e^{-i\left(m_\mu - i\frac{\Gamma_\mu}{2}\right)\tau_\mu^i} U_{\mu i}^* \, e^{-i m_\nu^i \tau_\nu^i} U_{ei}$$

$$\text{Amp} = \sum_{i=1,2,3} S_\mu \, e^{-i\left(m_\mu - i\frac{\Gamma_\mu}{2}\right)\tau_\mu^i} U_{\mu i}^* \, e^{-im_\nu^i \tau_\nu^i} U_{ei}$$

How do the kinematical phase factors depend on *i*?

In the phase factor for the recoiling *muon*,

$$\tau_\mu^i = \frac{1}{m_\mu}\left(E_\mu^i t_\mu - p_\mu^i x_\mu\right)$$

Energy and momentum of muon in π rest frame

Choosing $x_\mu = v_\mu t_\mu = \dfrac{p_\mu}{E_\mu}$ to avoid (Event rate) = 0,

Velocity, momentum, and energy of muon in π rest frame for massless neutrinos

And using —

$$E_\mu^i = \frac{m_\pi^2 + m_\mu^2 - \left(m_\nu^i\right)^2}{2m_\pi} \quad \text{and} \quad \left(p_\mu^i\right)^2 = \left(E_\mu^i\right)^2 - m_\mu^2 \,,$$

we find that to lowest (first) order in $\Delta m_{ij}^2 \equiv \left(m_\nu^i\right)^2 - \left(m_\nu^j\right)^2$,

$$\tau_\mu^i - \tau_\mu^j = 0 \,.$$

That is, to lowest order, the muon phase factor

$$\boxed{e^{-i\left(m_\mu - i\frac{\Gamma_\mu}{2}\right)\tau_\mu^i}}$$

actually does not depend on i, so it will not influence the $|\mathrm{Ampl}|^2$, and can be dropped.

(First noticed neglecting muon decay by Akhmedov and Smirnov)

In the phase factor for the *neutrino*, $e^{-im_\nu^i \tau_\nu^i}$,

$$m_\nu^i \tau_\nu^i = E_\nu^i t_\nu - p_\nu^i x_\nu .$$

Energy and momentum of neutrino in π-rest-frame

Since in practice neutrinos are ultra relativistic,
we choose $t_\nu = x_\nu \equiv L$ to avoid (Event rate) = 0.

Using —

$$E_\nu^i = \frac{m_\pi^2 + \left(m_\nu^i\right)^2 - m_\mu^2}{2m_\pi} \quad \text{and} \quad \left(p_\nu^i\right)^2 = \left(E_\nu^i\right)^2 - \left(m_\nu^i\right)^2,$$

we find that to lowest (first) order in $\Delta m_{ij}^2 \equiv \left(m_\nu^i\right)^2 - \left(m_\nu^j\right)^2$,

Distance ν travels in the π rest frame

$$m_\nu^i \tau_\nu^i - m_\nu^j \tau_\nu^j = \Delta m_{ij}^2 \frac{L}{2E}$$

Energy ν would have in the π rest frame if it were massless

Thus, we may take the neutrino phase factor, $e^{-i m_\nu^i \tau_\nu^i}$, to be —

$$\boxed{e^{-i\left(m_\nu^i\right)^2 \frac{L}{2E}}} \;.$$

We had —

$$\text{Amp} = \sum_{i=1,2,3} \cancel{S_\mu}\, e^{-i\cancel{\left(m_\mu - \frac{i\Gamma_\mu}{2}\right)}\tau_\mu^i} U_{\mu i}^* \, e^{-im_\nu^i \tau_\nu^i} U_{ei} \;\; ,$$

— a Lorentz-invariant amplitude.

In terms of the π-rest-frame variables L and E,

$$\text{Amp} = \sum_{i=1,2,3} U_{\mu i}^* \, e^{-i\left(m_\nu^i\right)^2 \frac{L}{2E}} U_{ei}$$

From $\Delta p \Delta x \geq \hbar$, we cannot observe ν oscillation vs. travel distance in the lab unless there is a spread in lab-frame π momenta, so that the π is somewhat localized.

Because neutrinos are ultra-relativistic, when the parent π is moving in the lab, the ν travel distance and energy in the lab frame, L' and E', are related to their π-rest-frame counterparts, L and E, by —

$$\frac{L'}{E'} = \frac{L}{E}$$

Thus, in terms of lab-frame variables,

$$\text{Amp} = \sum_{i=1,2,3} U_{\mu i}^{*} \, e^{-i\left(m_{\nu}^{i}\right)^{2} \frac{L'}{2E'}} \, U_{ei}$$

This leads to —

$$P\left(\nu_\mu \to \nu_e\right) = \left|\mathrm{Amp}\right|^2 = -4\sum_{i>j} \mathrm{Re}\left(U^*_{\mu i} U_{ei} U_{\mu j} U^*_{ej}\right) \sin^2\left(\Delta m^2_{ij} \frac{L'}{4E'}\right)$$

$$+ 2\sum_{i>j} \mathrm{Im}\left(U^*_{\mu i} U_{ei} U_{\mu j} U^*_{ej}\right) \sin\left(\Delta m^2_{ij} \frac{L'}{2E'}\right)$$

This is the usual result.

We derived it now in the same way as we treat B-factory experiments.

We allowed for the $\nu - \mu$ kinematical entanglement, which proved to be irrelevant.

We didn't need to make any assumption about how the energies of the different neutrino mass eigenstates are related.

Previous consideration of entanglement in processes with mixing

B. K., Stodolsky
Goldman
Nauenberg
Dolgov, Morozov, Okun, Schepkin
Burkhardt, Lowe, Stephenson, Goldman
Lowe, Bassalleck, Burkhardt, Rusek, Stephenson
Cohen, Glashow, Ligeti
B. K., Kopp, Robertson, Vogel
Akhmedov, Smirnov

Conclusion

B **– factory and neutrino oscillation experiments can be treated in the same way.**

While $B - \bar{B}$ **EPR correlations are really present, one need not think in terms of them.**

The treatment of neutrino oscillation that allows for entanglement yields the standard result.