A "Periodic Table" of Jet Energy Flow Observables

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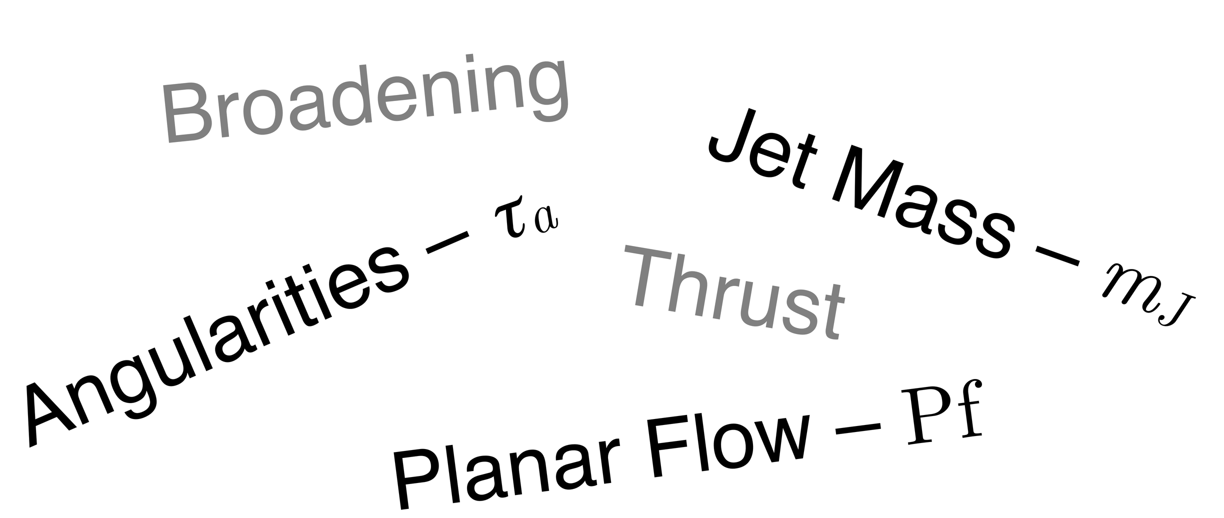
In a Nutshell

Jet substructure can help identify new physics at the LHC. Energy-flow observables depend on the jet energy distribution. Many observables have been defined, but they have not been organized:



We suggest a classification of these observables for narrow jets. Observables are IR-collinear safe and arranged in orders of detector resolution.

The first few observables are well known, followed by **new observables** that might be useful for new physics searches.

Constructing Observables

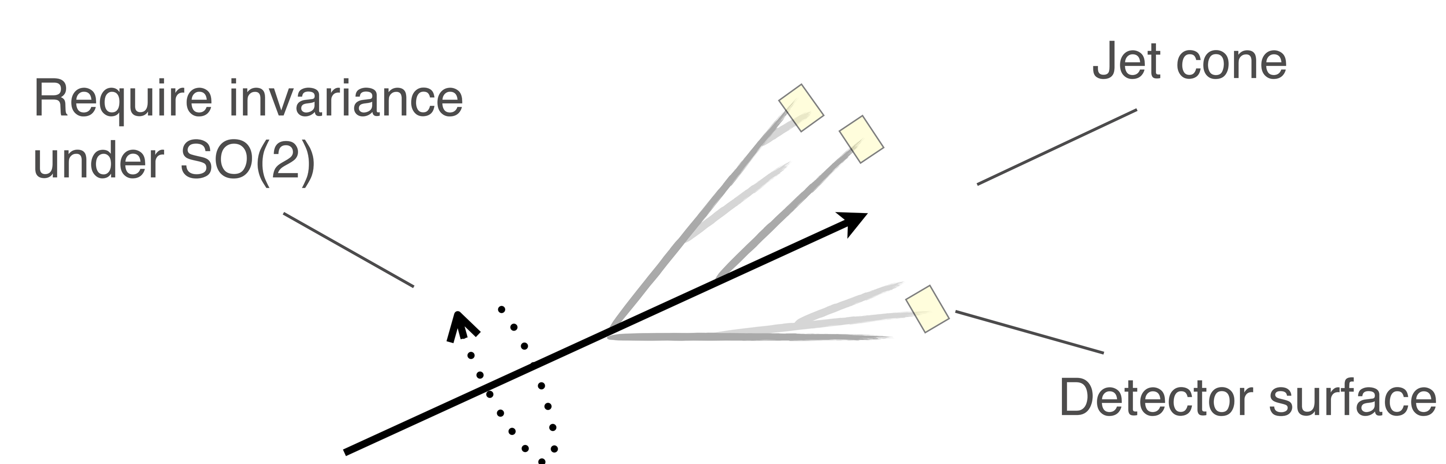
1. Calorimeter measures the **energy distribution** $E(x)$ on its surface. Describe the distribution by its **moments,**

$$I_{i_1 \dots i_n} = \int d^2x \, E(x) \, x_{i_1} \cdots x_{i_n} \, .$$

Direction, $i = 1, 2$

Coordinate on detector surface

2. Use moments to construct observables. Require invariance under **jet-axis rotations** — Lorentz transformations that keep jet's cone and momentum fixed.



3. Form **rotation-invariant observables** by multiplying moments and contracting indices:

$$I_{ii} \, , \; I_{iijj} \, , \; \epsilon_{ij}\epsilon_{kl} I_{ik} I_{jl} \, , \; I_{iijjkk} \, , \; \epsilon_{ij} I_{ik} I_{jkll} \, , \; \dots$$

4. The lowest orders recover well-known observables:

Mass: $I_{ii} = \int d^2x \, E(x) \, x^2 \sim m_J^2$

Angularity: $I_{iijj} = \int d^2x \, E(x) \, x^4 \sim \tau_{-2}$

Planar Flow: $\epsilon_{ij}\epsilon_{kl} I_{ik} I_{jl} \sim \det I_2 \sim m_J^2 \cdot \mathrm{Pf}$

5. Higher-order observables are sensitive to detector **energy and angular resolution.**

$$\epsilon_{ij} I_{ik} I_{jkll}$$

Energy resolution: $\mathcal{O} \sim I^2 \sim E^2 \;\Rightarrow\; \Delta\mathcal{O} \sim 2\Delta E$

Angular resolution: $\mathcal{O} \sim x^6 \;\Rightarrow\; \Delta\mathcal{O} \sim 6\Delta x$

Arrange observables by expansion orders, and read off the leading variables to describe N-body jets.

$\#\Delta x \backslash \#\Delta E$	1	2	3
2	I_{ii}	–	–
4	I_{iijj}	$\epsilon_{ij}\epsilon_{kl} I_{ik} I_{jl}$	–
6	I_{iijjkk}	$\epsilon_{ij} I_{ik} I_{jkll}, \dots$	$\dots$

Example: 2-body jets depend on two variables ⇒ best described with mass, angularity τ_{-2} .

The most intriguing new variable — a pseudoscalar, contributes at LO to 3-body jets:

$$\mathcal{O} = \epsilon_{ij} I_{ik} I_{jkll} \, .$$

Will be interesting to explore its usefulness for new physics searches.

Complementary Description — Zernike Polynomials

Expand the energy distribution in **orthogonal polynomials** $R_{m,n}$ on the detector surface,

$$E(x) = a_{0,0} + a_{2,0} R_{2,0}(x) + a_{2,2} R_{2,2}(x) \cos(2\phi) + a_{2,-2} R_{2,-2}(x) \sin(2\phi) + \cdots$$

The coefficients $a_{n,m}$ **are observables,** with simple relations to the moment expansion:

$$m_J^2 \sim a_{2,0} + 3a_{0,0} \, , \quad \tau_{-2} \sim a_{4,0} + 5a_{2,0} + 10a_{0,0} \, , \quad m_J^4(1 - \mathrm{Pf}) \sim a_{2,2}^2 + a_{2,-2}^2$$