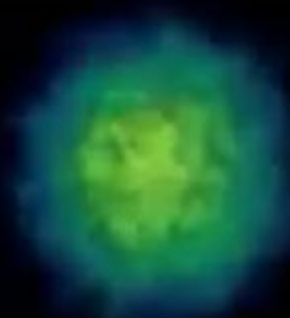


Imperial College  
London

# Magnetic Monopoles in the Cosmos and at the LHC



Arttu Rajantie  
MoEDAL Physics Workshop  
CERN, 20 June 2012

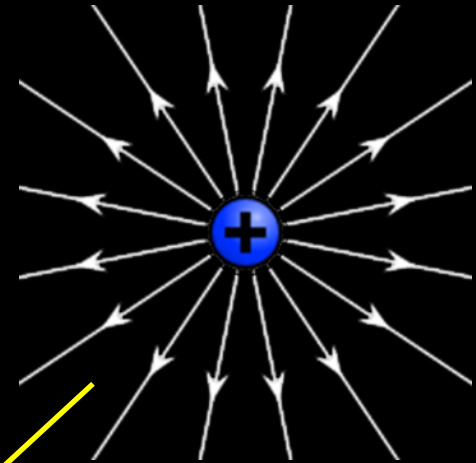
# Maxwell Equations

$$\vec{\nabla} \cdot \vec{E} = \rho_E$$

$$\vec{\nabla} \cdot \vec{B} = \rho_M$$

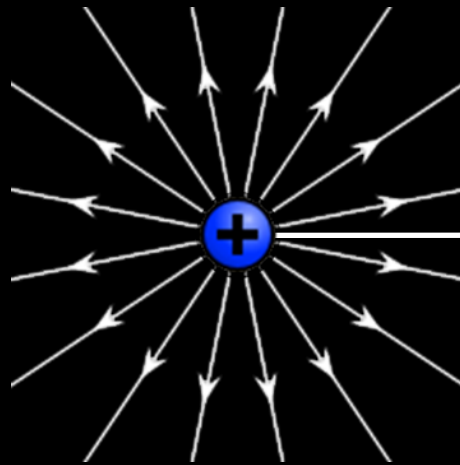
$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} - \vec{j}_M$$

$$\vec{\nabla} \times \vec{B} = \frac{\partial \vec{E}}{\partial t} + \vec{j}_E$$



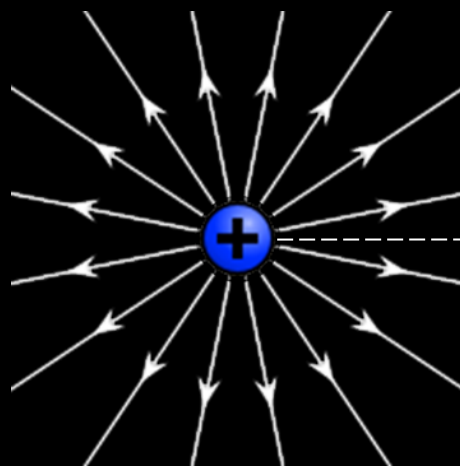
Duality  $\vec{E} \leftrightarrow \vec{B}$

# Dirac Monopole (1931)



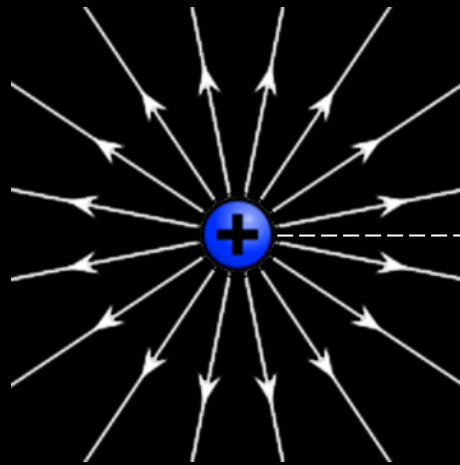
- ▶ Vector potential  $\vec{A}(\vec{r}) = \frac{g}{4\pi|\vec{r}|} \frac{\vec{r} \times \vec{n}}{|\vec{r}| - \vec{r} \cdot \vec{n}}$
- ▶ Dirac string: Singularity along  $\vec{n}$
- ▶ QM: Unobservable if  $g = 2\pi/e$

# Dirac Monopole (1931)



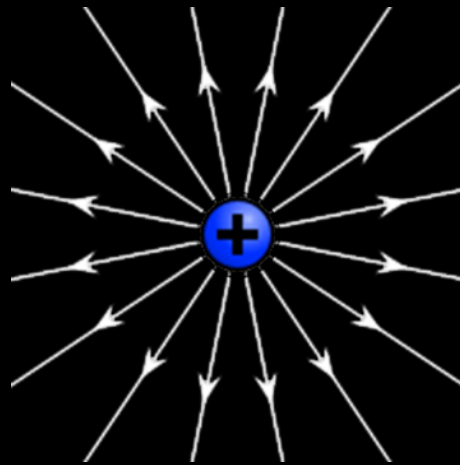
- ▶ Dirac quantisation condition:  
All electric and magnetic charges must satisfy  $\frac{eg}{2\pi} \in \mathbb{Z}$
- ▶ Existence of monopoles would explain observed quantisation of electric charge
- ▶ “...one would be surprised if Nature had made no use of it”

# Dirac Monopole (1931)



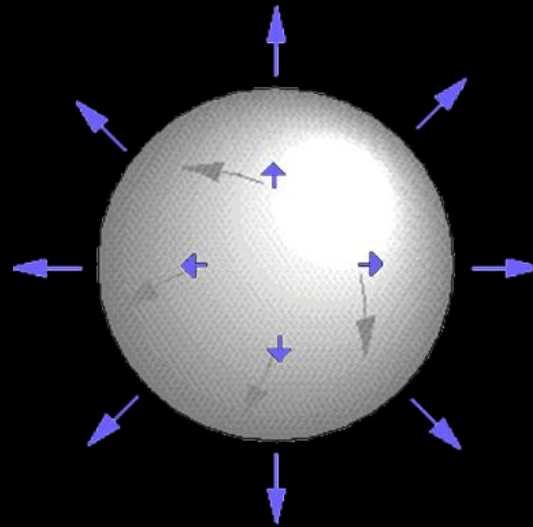
- ▶ Magnetic Coulomb field:  $\vec{B}(\vec{r}) = \frac{g}{4\pi} \frac{\vec{r}}{|\vec{r}|^3}$
- ▶ Magnetic charge localised at a point
- ▶ Divergent energy:  $E = \int d^3x \frac{\vec{B}^2}{2} \sim g^2 \Lambda \sim \frac{\Lambda}{e^2}$

# QFT of Monopoles



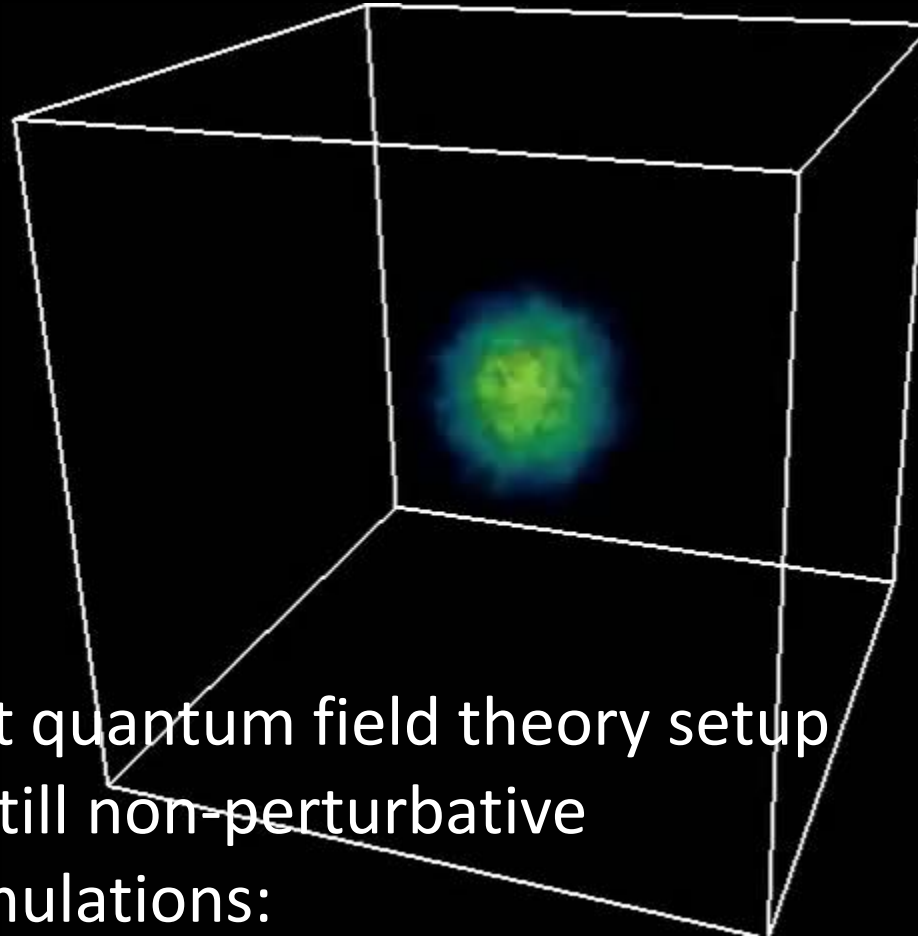
- ▶ Full quantum calculation: Monopole loops
- ▶ Difficult to formulate:  
Two vector potentials (Schwinger 1975)
- ▶ Strong coupling  $g = \frac{2\pi}{e} \gg 1$

# 't Hooft-Polyakov Monopole (1974)



- ▶ Smooth “hedgehog” solution in SU(2)+adjoint Higgs
- ▶ Magnetic charge  $g = 4\pi/e$
- ▶ Finite mass  $M \approx \frac{4\pi v}{e} \sim \frac{m}{e^2}$

# 't Hooft-Polyakov Monopole (1974)

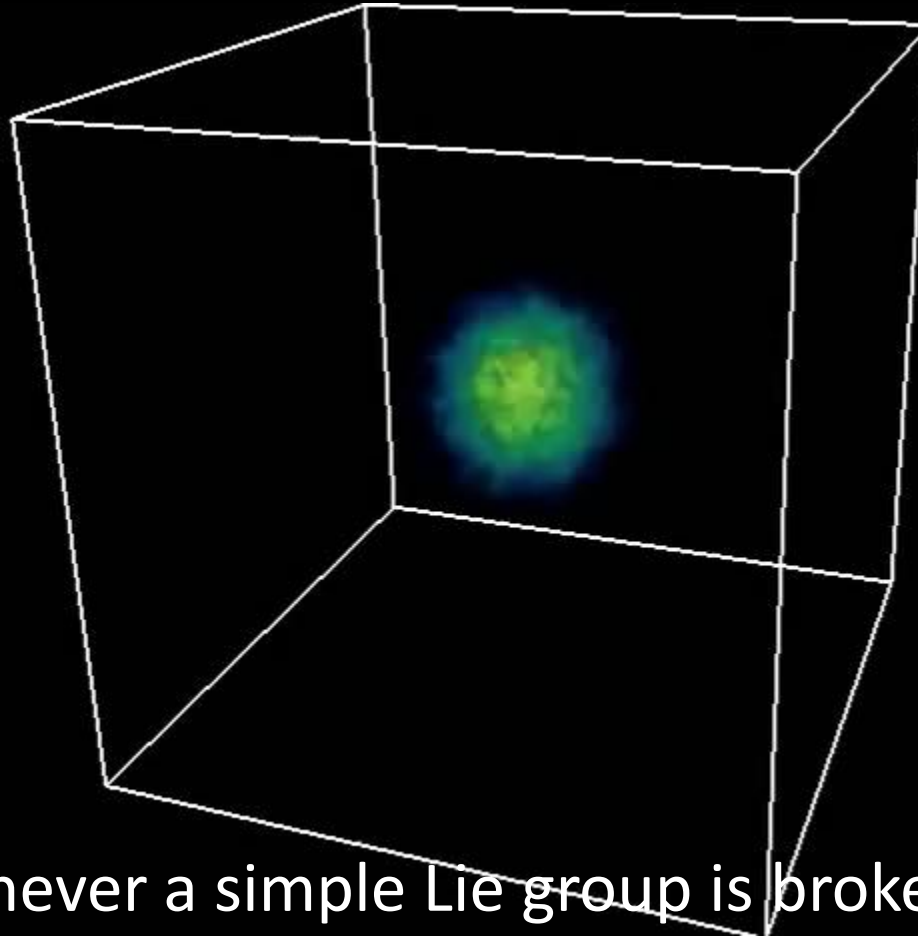


Visualisation: [www.vapor.ucar.edu](http://www.vapor.ucar.edu)

- ▶ Consistent quantum field theory setup
- ▶ Large  $g$ : Still non-perturbative
- ▶ Lattice simulations:  
Mass, scattering (AR&Weir 2011)



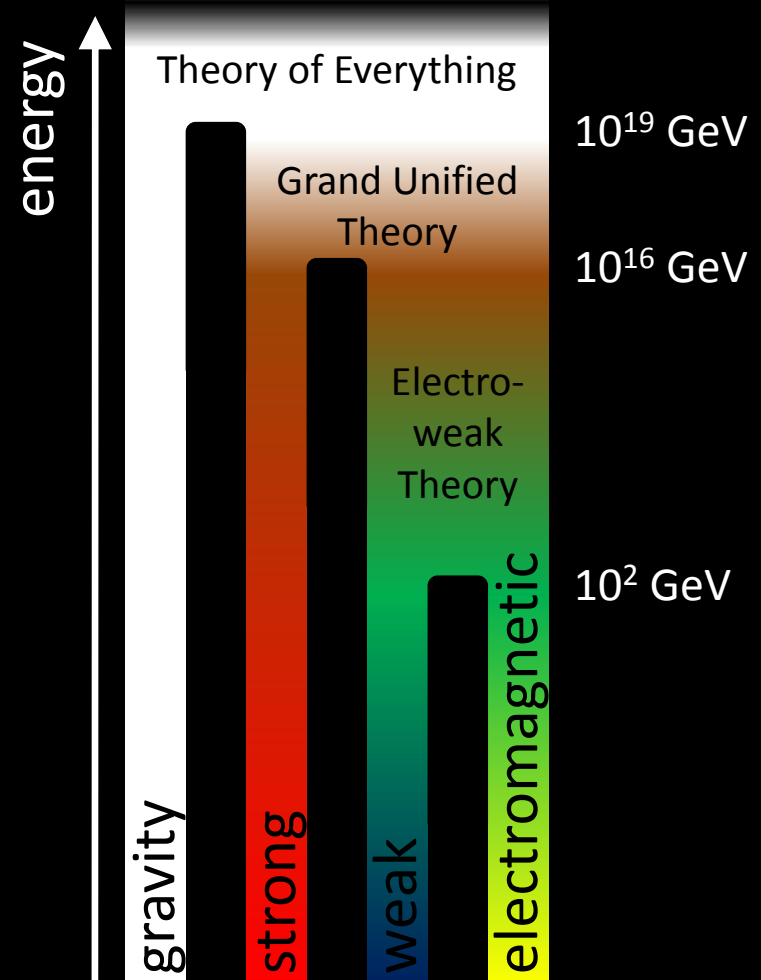
# 't Hooft-Polyakov Monopole (1974)



- ▶ Exist whenever a simple Lie group is broken to something with a  $U(1)$  factor: Grand Unification

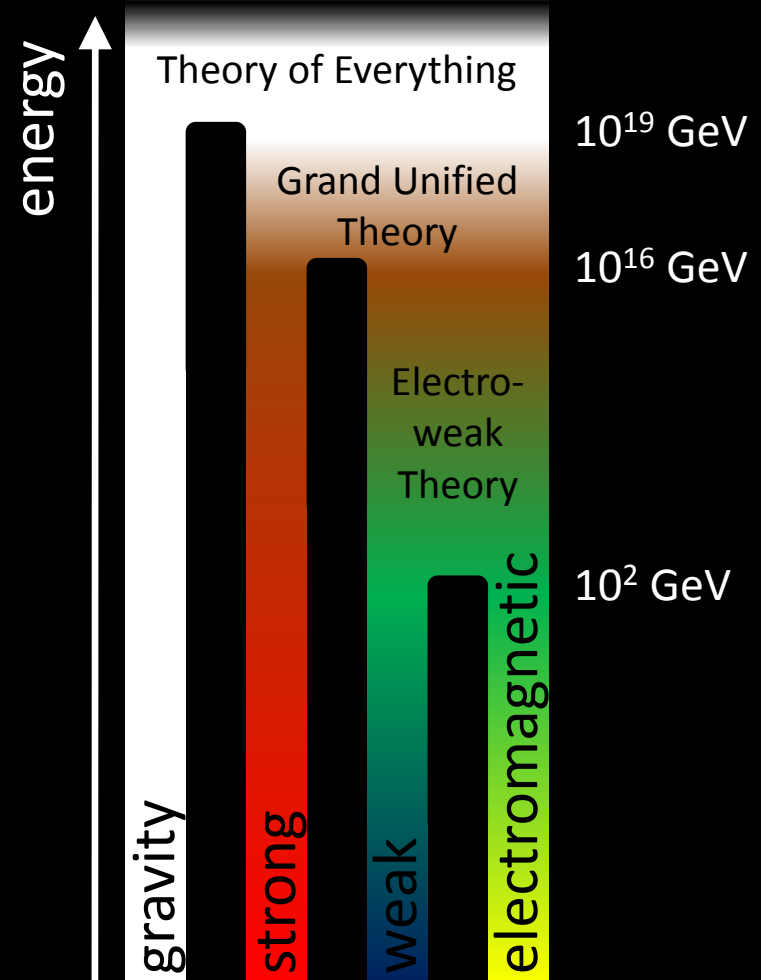
# Grand Unification

- ▶ Standard Model:  
EM & weak forces unified  
above 100 GeV
- ▶ Grand Unified Theory (GUT):  
Electroweak & strong forces  
unified above  $10^{16}$  GeV
  - e.g.  
 $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$



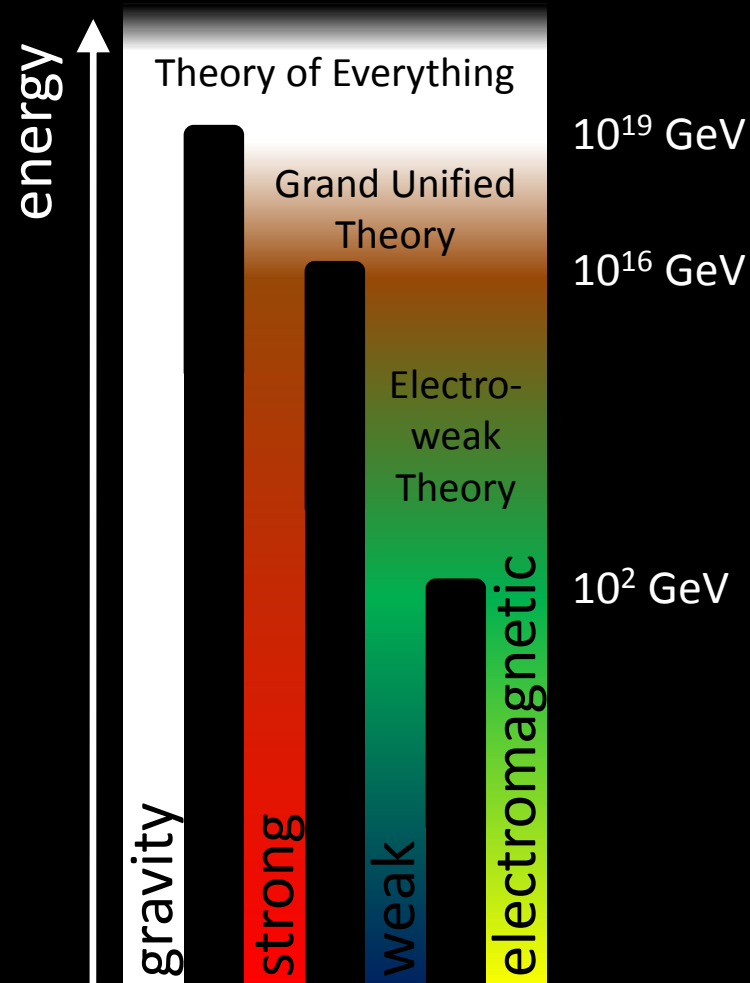
# GUT Monopoles

- ▶ Generic prediction of GUTs
- ▶ Mass typically at GUT scale  $M \sim 10^{17}$  GeV
- ▶ Also dyons with both electric and magnetic charge



# GUT Monopoles

- ▶ More complex GUTs, e.g.  $SO(10)$
- ▶ Monopoles with different charges
- ▶ Can be lighter e.g.  $\sim 10^7 \text{ GeV}$  in an  $SO(10)$  family unification model (Kephart et al)

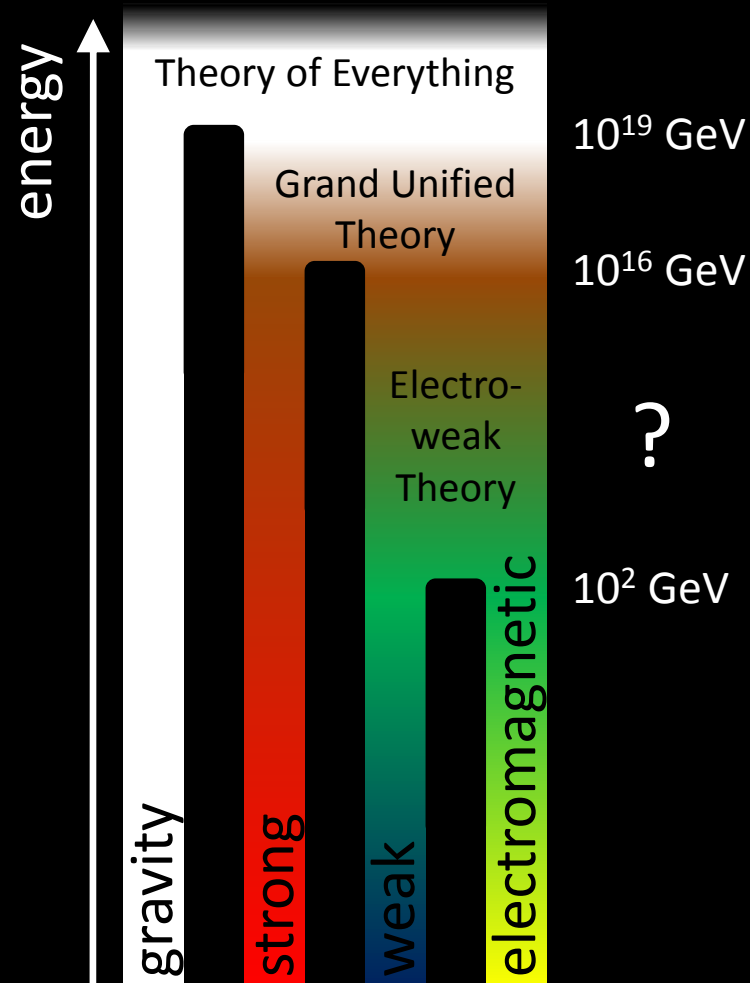


# String Theory Monopoles

- ▶ S-duality:  
Any superstring theory has magnetic monopoles
- ▶ Typical mass  $M \sim \frac{M_{\text{Pl}}}{e} \sim 10^{20} \text{ GeV}$
- ▶ Can be reduced by large extra dimensions or warped compactification, perhaps even to  $M \sim 10 \text{ TeV}$   
(Witten 2002)

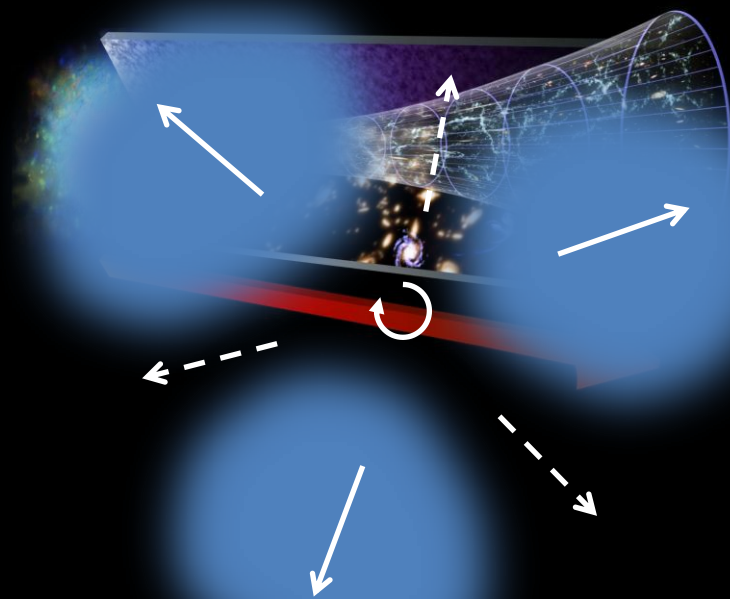
# Other Light Monopoles

- ▶ Massive range of energies between EW and GUT: Plenty of room for new physics
- ▶ Cho-Maison monopole (1996): Dirac solution generalised to electroweak theory
- ▶ Monopoles do not have to arise from unification



# Monopoles in Cosmology

- ▶ Hot Big Bang:  
GUT symmetry breaks  
in a phase transition
- ▶ The Higgs field chooses  
a direction randomly
- ▶ Kibble (1976):  
Monopoles form,  
at least one per horizon  
 $\rightarrow n_{\text{mon}} \sim H^{-3}$



# Monopole Problem

- ▶ Monopoles annihilate until they cannot find partners:  
Density decreases to

$$n_{\text{mon}} \sim 10^{-9} \left( \frac{M}{10^{16} \text{ GeV}} \right) T^3 \sim 10^{-2} \left( \frac{M}{10^{16} \text{ GeV}} \right) \text{m}^{-3}$$

(Zel'dovich & Khlopov 1979, Preskill 1979)

- ▶ Total energy density in the universe:  $\rho \sim 6 \text{ GeV m}^{-3}$
- ▶ Monopoles exceed this unless  $M \lesssim 10^{10} \text{ GeV}$
- ▶ Guth (1981):  
Period of inflation (accelerating expansion) dilutes  
monopoles away

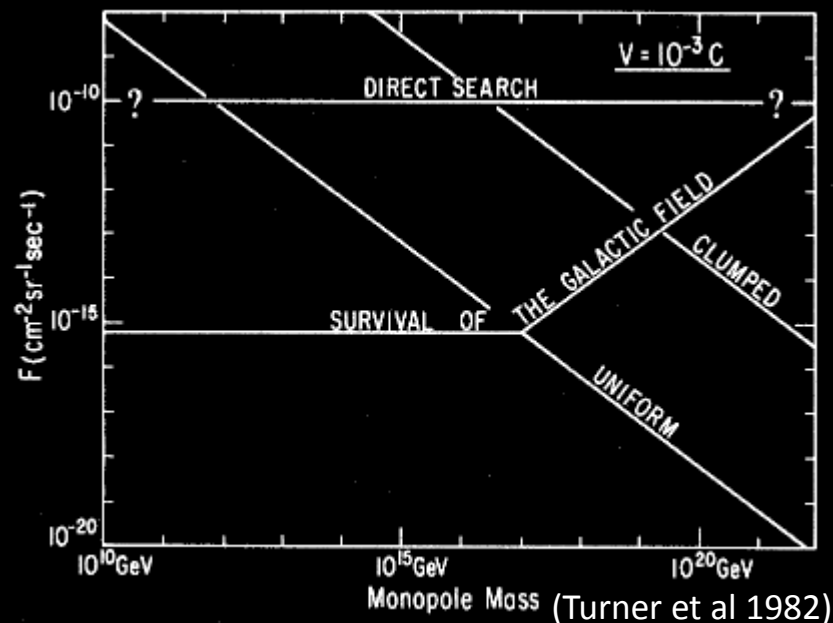


# Monopole Problem

- Density constraint in terms of flux  $F$ :  
Monopoles hitting unit area per unit time

- Uniform distribution:

$$F \approx \frac{n_{\text{mon}} v}{4\pi} \lesssim 10^{-15} \left( \frac{10^{16} \text{GeV}}{M} \right) \left( \frac{v}{10^{-3} c} \right) \text{cm}^{-2} \text{s}^{-1} \text{sr}^{-1}$$

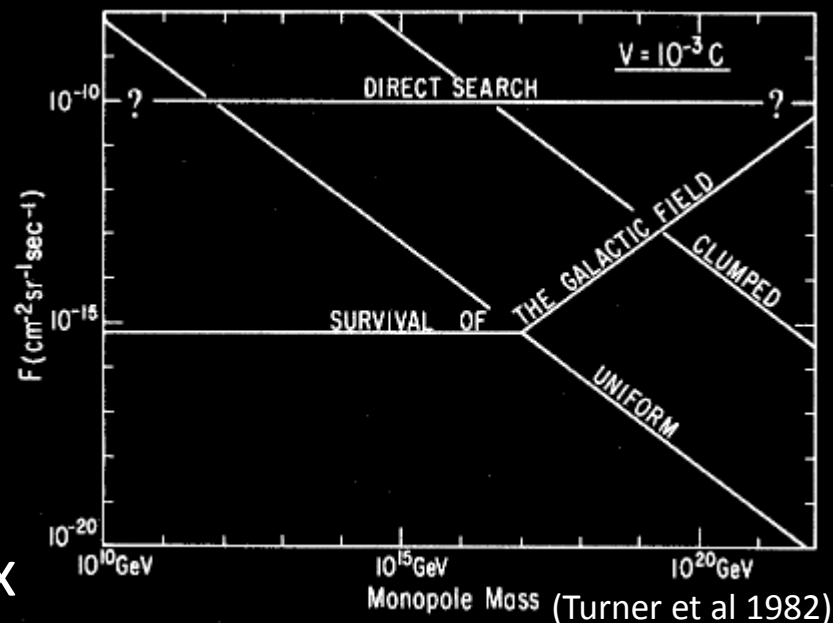


# Parker Bound (1970)

- Galactic magnetic fields  
 $B \sim 1\mu G$
- If  $M \lesssim 10^{17} \text{ GeV}$ , this creates a magnetic current, which dissipates the field
- Sets an upper bound on flux

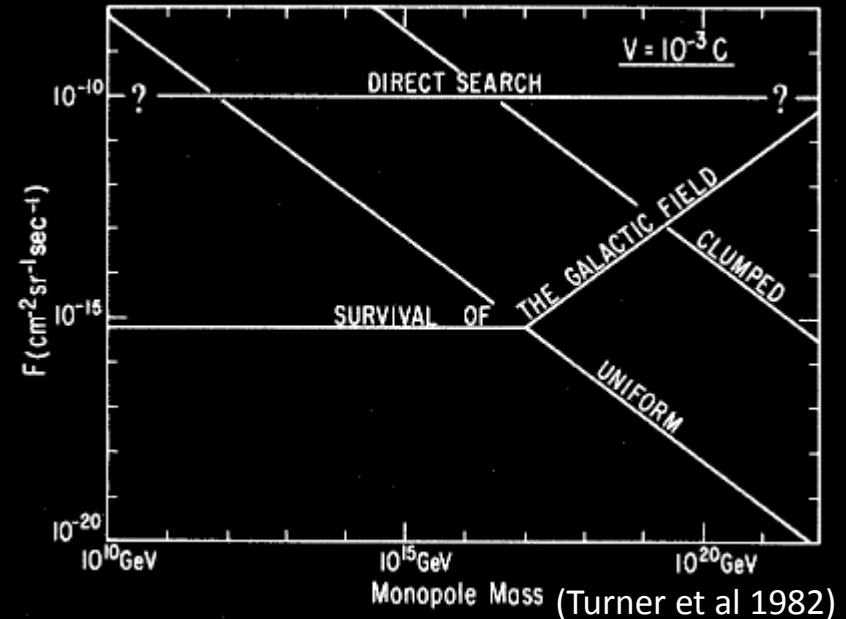
$$F = \frac{nv}{4\pi} \lesssim 10^{-15} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$$

- Bound gets weaker at higher  $M$



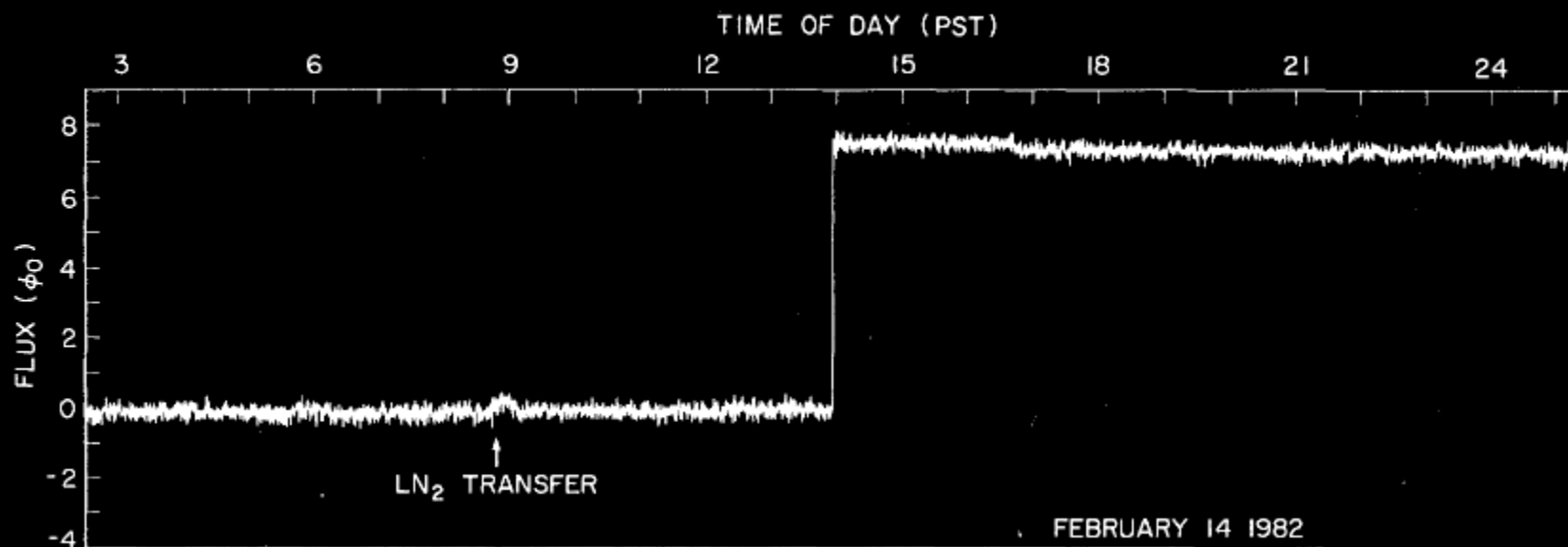
# Parker Bound (1970)

- ▶ If  $M \gtrsim 10^{17} \text{ GeV}$ ,  
monopoles remain bound  
to galaxies
- ▶ Constraint from the total  
mass of the Milky Way:



$$F \lesssim 10^{-13} \left( \frac{10^{16} \text{ GeV}}{M} \right) \left( \frac{v}{10^{-3} c} \right) \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$$

# Cosmic Rays

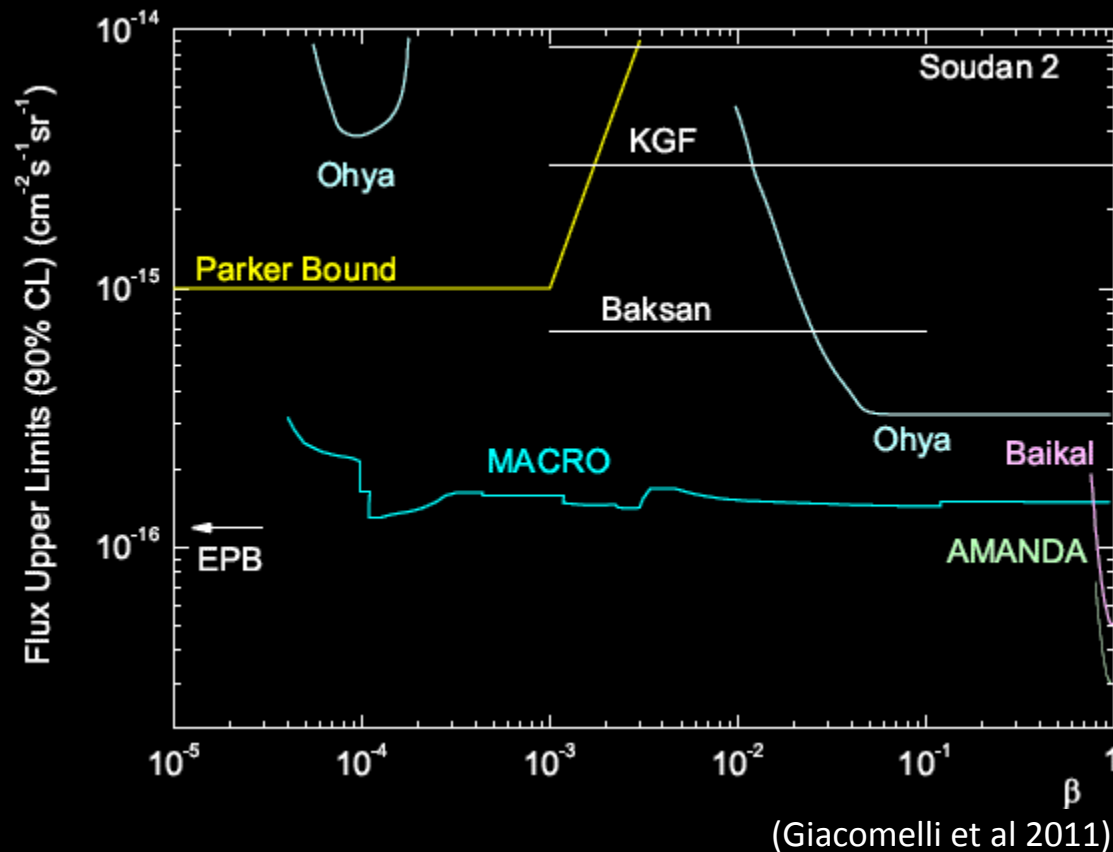


(Cabrera 1982)

## ► Early detections:

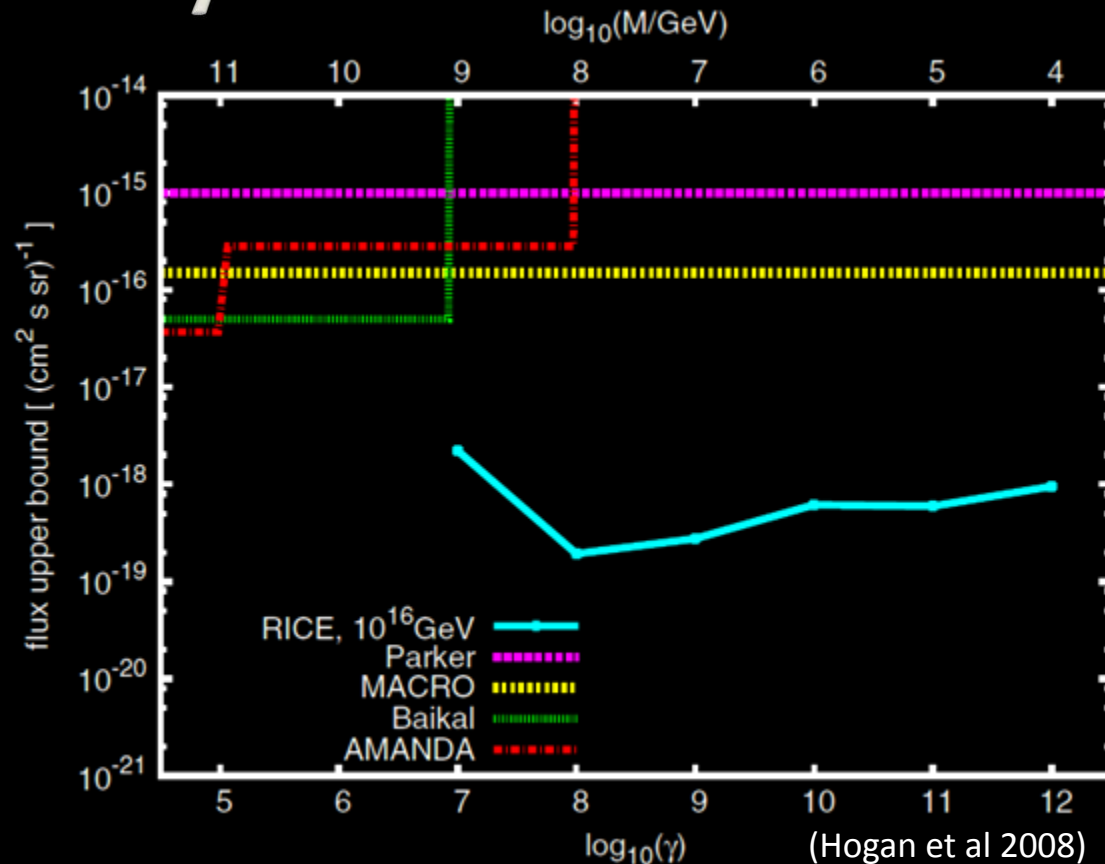
- Berkeley 1975, Stanford 1982, Imperial 1986
- All turned out to be false

# Cosmic Rays



- ▶ MACRO (Gran Sasso, Italy):
  - Upper bound  $F \lesssim 10^{-16} \text{ cm}^{-2} \text{s}^{-1} \text{sr}^{-1}$  over wide mass range

# Cosmic Rays



## ► RICE (South Pole):

- Intermediate mass monopoles  $F \lesssim 10^{-18} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$

# Monopole Problem?

- ▶ Preskill (1979):

Monopole density today  $n_{\text{mon}} \sim 10^{-2} \left( \frac{M}{10^{16} \text{ GeV}} \right) \text{ m}^{-3}$

- ▶ Light monopoles  $M \lesssim 10^{11} \text{ GeV}$  are relativistic, so

$$F \approx \frac{n_{\text{mon}} c}{4\pi} \approx 10 \left( \frac{M}{10^{16} \text{ GeV}} \right) \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$$

- ▶ Compatible with RICE bound only if  $M \lesssim 1 \text{ MeV}$
- ▶ Therefore we do need inflation to wipe out the monopoles!

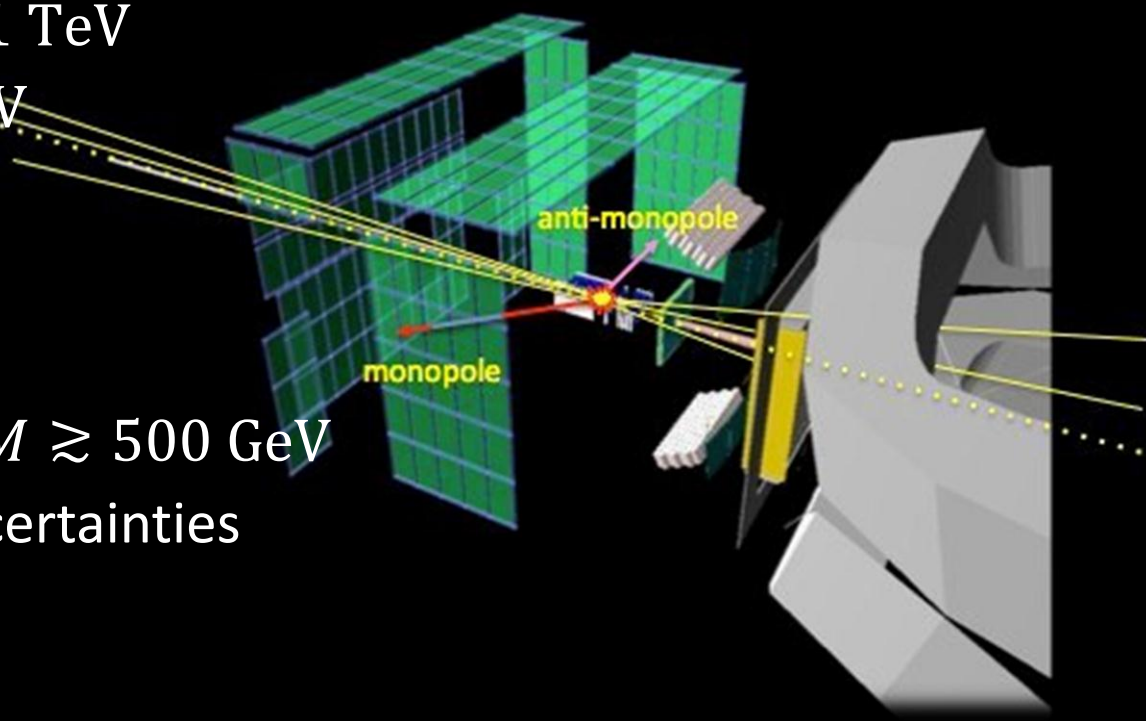
# Accelerator Searches

## ▶ Direct searches:

- Tevatron, LEP, HERA  
→ Lower bound  $M \gtrsim 1 \text{ TeV}$
- MoEDAL: Up to 7 TeV

## ▶ Indirect searches:

- Virtual monopoles  
 $M \gtrsim 500 \text{ GeV}$
- Large theoretical uncertainties





# TeV-scale monopoles

- ▶ Possible, but not really predicted by any theory
  - Perhaps large extra dimensions, or simply unrelated to unified theories
- ▶ Mass relation  $M \sim \frac{\Lambda}{e^2}$  means there would be lots of new exciting physics at accessible energies
- ▶ Monopoles would be a fantastic probe:
  - Absolutely stable
  - Strong EM interaction
  - Easy to handle

# Summary

- ▶ Monopoles are perhaps the best motivated new particles:
  - Explain charge quantisation
  - Exist in GUTs, string theory
- ▶ Produced copiously in the early universe, wiped away by inflation:
  - Stringent bounds from astrophysics, cosmic rays
- ▶ TeV-scale monopoles possible
  - Detectable in MoEDAL
  - Would open up a window to exciting new physics
- ▶ (For more: See AR, [Contemporary Physics 2012](#))