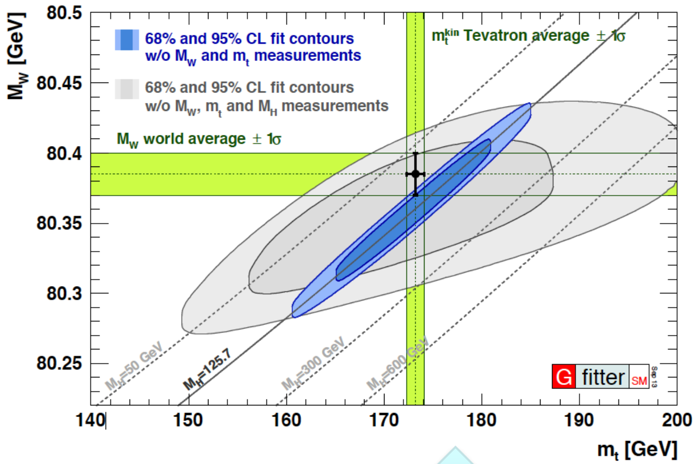
On the Theoretical Interpretation of Top Quark Mass Measurements

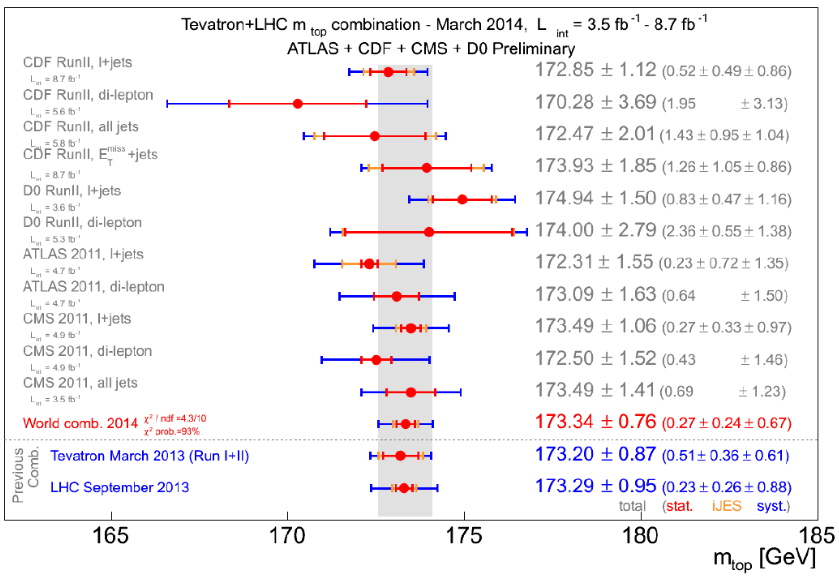
André H. Hoang

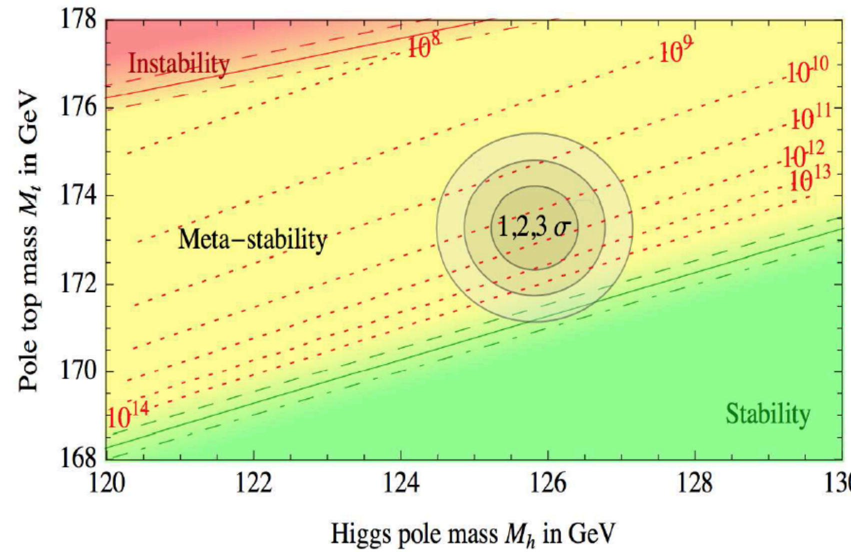
University of Vienna

CERN Theory Seminar, May 21, 2014

Motivation







Aims: m_{top} wanted !

- Reduce error in m_{top}(MC)
- Clarify mass scheme m_{top}(MC)
- Improve / understand better MC

Outline

Part 1: → Theoretical considerations on $m_t^{\rm MC}$

Why are other masses not mentioned?

> "How is the MC mass related to the pole mass?"
>
> → "What is the physics MC mass ?" ←

Same mass for different Monte-Carlos ?

- Why $m_t^{\rm MC}$ looks like being $m_t^{\rm pole}$, but is actually not.
- How to determine $m_t^{\rm MC}$ in terms of other masses.
- What if one sets $m_t^{\rm MC} = m_t^{\rm pole}$ anyway.
- Advertisement for the MSR mass: $m_t^{\rm MSR}(R)$

We need to distinguish between conceptual and practical views !

Part 2: → New tools concerning tools to measure $m_t^{\rm MC}$

- Variable Flavor Number Scheme for final state jets.
  Full massive event shape distribution

QCD Parameters

QCD Lagrangian:

$$\mathcal{L}_{\rm QCD} = \mathcal{L}_{\rm classic} + \mathcal{L}_{\rm gauge-fix} + \mathcal{L}_{\rm ghost}$$

$$\mathcal{L}_{\rm classic} = -\frac{1}{4} F^A_{\alpha\beta} F^{\alpha\beta}_A + \sum_{\text{flavors } q} \bar{q}_\alpha (i \not{D} - m_q)_{\alpha\beta}\, q_b$$

$$D^\mu = \partial^\mu + i g T^C A^{\mu C}$$

Formally $m_{\rm top}$ and α_s are couplings of the Lagrangian.

$m^0_{\rm top}\,,\quad \alpha^0_s$

→ bare UV-divergent
→ field theoretically unique
→ pure UV-object – NO IR dependence

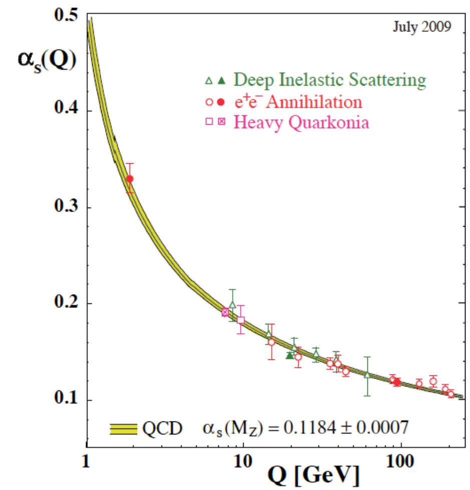
$m^R_{\rm top}\,,\quad \alpha^R_s$

→ renormalized UV-finite
→ renormalization scheme dependent
→ regularization scheme dependent

Strong Coupling

$\overline{\text{MS}}$ scheme:

$$\alpha_s^0 = \alpha_s(\mu)\,\mu^{2\epsilon}\left[1 - \underbrace{\frac{\alpha_s}{4\pi\epsilon}\beta_0 + \ldots}_{\text{pure UV-divergent}}\right] \qquad d = 4 - 2\epsilon \to 4$$



→ $\alpha_s(\mu)$ is pure UV-object without IR-sensitivity

→ Common consensus to use THIS scheme ONLY

→ Almost at the status of a "physical parameter"

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$$= N_c \sum_q e_q^2 \left\{1 + \frac{\alpha_s(\mu_0)}{\pi} + \frac{\alpha_s^2(\mu_0)}{\pi^2}\left[f_3 - \frac{\beta_0}{4}\ln\left(\frac{s}{\mu_0^2}\right)\right] + \ldots\right\}$$

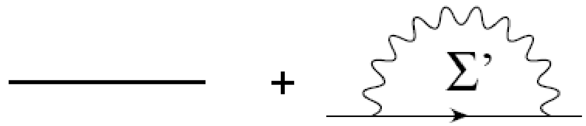
$$= N_c \sum_q e_q^2 \left\{1 + \frac{\alpha_s(\sqrt{s})}{\pi} + \ldots\right\}$$

Summation of (large) logarithms

→ "best" or "physical parameter": captures most of the quantum corrections in its definition

→ Common confidence: a badly behaved pert. series is considered a problem of the series and not of $\alpha_s(\mu)$.

Heavy Quark Mass



$$= p - m^0 - \Sigma(p, m^0, \mu)$$

$$\Sigma(m^0, m^0, \mu) = m^0 \left[\frac{\alpha_s}{\pi\epsilon} + \ldots \right] + \Sigma^{\text{fin}}(m^0, m^0, \mu)$$

$\overline{\text{MS}}$ scheme: $$m^0 = \overline{m}(\mu) \left[1 - \frac{\alpha_s}{\pi\epsilon} + \ldots \right]$$

→ $\overline{m}(\mu)$ is pure UV-object without IR-sensitivity

→ ONLY a useful scheme for $\mu > m$

→ No-one considers it a "physical parameter"
although it sums logarithms just as $\alpha_s(\mu)$

- Very energetic processes (E>>m)
- Total cross sections
- Off-shell massive quarks
- Away from thresholds/endpoints

Pole scheme: $$m^0 = m^{\text{pole}} \left[1 - \frac{\alpha_s}{\pi\epsilon} + \ldots \right] - \Sigma^{\text{fin}}(m^{\text{pole}}, m^{\text{pole}}, \mu)$$

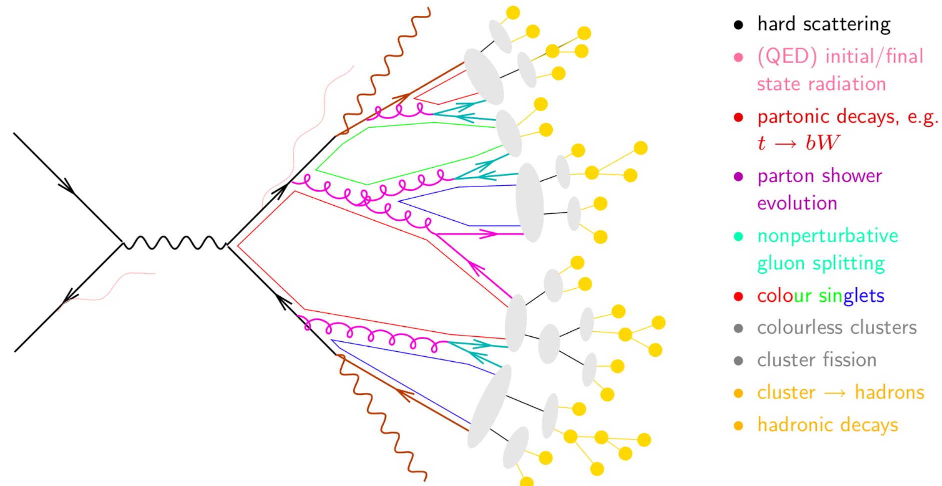
→ m^{pole} = perturbative single particle pole of perturbative S-matrix

→ Absorbes all self energy corrections into the mass parameter

→ Separation: self energy corrections $\longleftrightarrow$ inter quark/gluon interactions

→ Many consider it as a "physical parameter" due to the separation property.

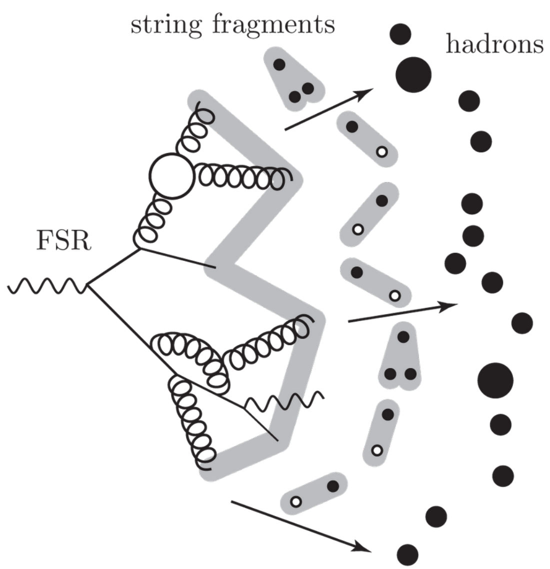
Heavy Quark Mass in the MC

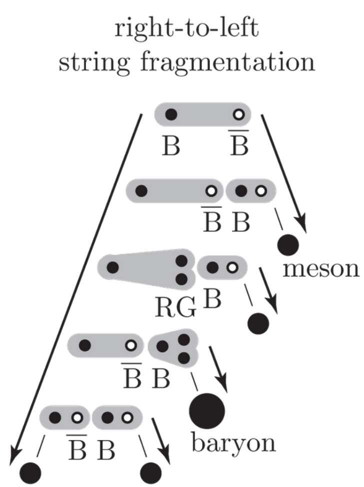


Monte-Carlo QCD Calculator:

- Computes all inter-quark/gluon and radiation processes
- Computes hadronization of partons
- Electroweak radiation effects

- Does NOT calculate self-energy processes





Intuition tells:
MC-mass IS the pole mass by design of the MC

BUT:
There is a subtlety related to the MC treats IR effects

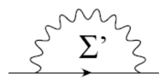
Caveat

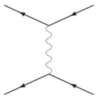
→ Let's step back from the MC and consider a system which is simpler to discuss, but has for the matters of this discussion the same features s the MC.

Well-defined short-distance quantity for R=1/r >> 1 GeV

Static energy of a heavy quark-antiquark pair:

$$E_{\rm stat} = 2m^0 + 2\Sigma(m,m) + V(R)$$





$$= 2m^{\rm pole} + V(R)$$

$$\Sigma^{\rm fin}(m,m) \sim m\left[\alpha_s + \ldots\right]$$

$$V(R) \sim -R\left[\alpha_s + \ldots\right]$$

Caveat

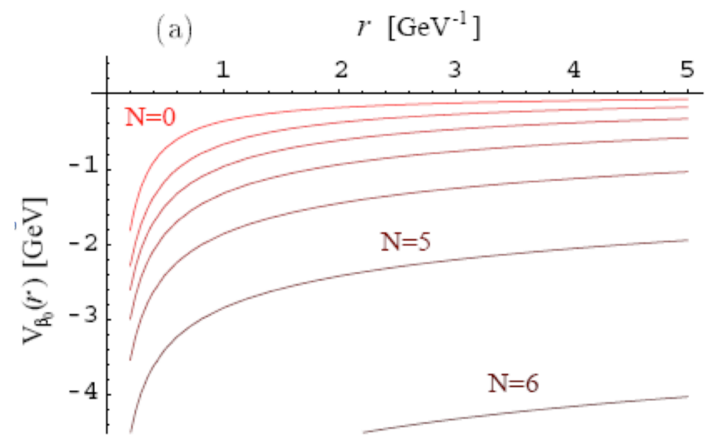
→ Let's step back from the MC and consider a system which is simpler to discuss, but has for the matters of this discussion the same features s the MC.

Static energy of a heavy quark-antiquark pair:

Well-defined short-distance quantity for R=1/r >> 1 GeV

$$E_{\rm stat} = 2m^0 + 2\Sigma(m,m) + V(R)$$

$$= 2m^{\rm pole} + V(R)$$



$$V_{\rm asym}(R) = -R \sum_{n=0} \left(\frac{\alpha_s(R)}{2\pi}\right)^{n+1} \beta_0^n \, n!$$

Static energy is not to be a short-distance quantity - in the pole mass scheme.

Pole mass is not a short-distance mass and has a badly behaved pert. expansion.

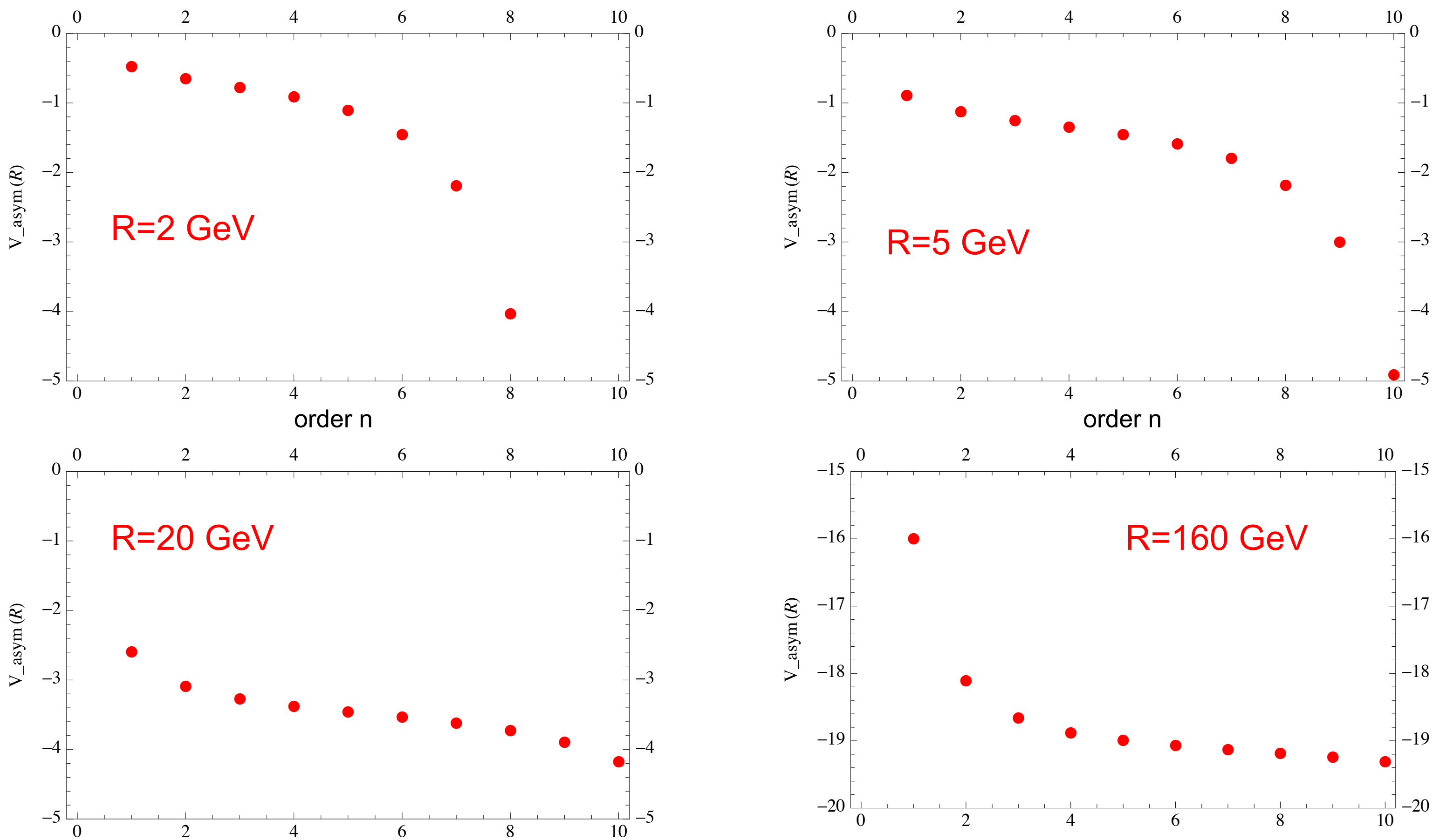
→ "Renormalons"

Caveat 1

→ How serious is the problem for a particular scale R ?

→ Series for large R converge longer, but size of corrections at lower order larger

→ Formal ambiguity: $\Lambda_{\rm QCD} \approx 0.5\ \rm GeV$



Caveat

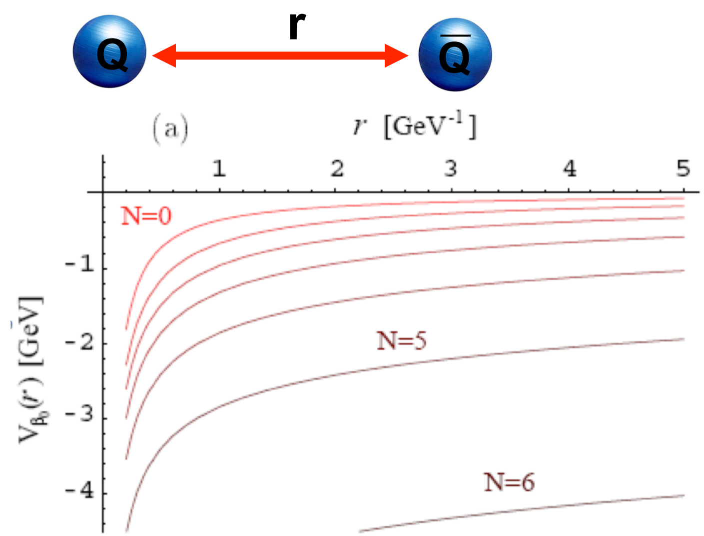
→ Let's step back from the MC and consider a system which is simpler to discuss, but has for the matters of this discussion the same features s the MC.

Static energy of a heavy quark-antiquark pair:

Well-defined short-distance quantity for R=1/r >> 1 GeV

$$E_{\rm stat} = 2m^0 + 2\Sigma(m,m) + V(R)$$

$$= 2m^{\rm pole} + V(R)$$



$$V_{\rm asym}(R) = -R \sum_{n=0} \left(\frac{\alpha_s(R)}{2\pi}\right)^{n+1} \beta_0^n \, n!$$

$$\Sigma^{\rm fin}_{\rm asym}(m,m) = \frac{1}{2}\, m \sum_{n=0} \left(\frac{\alpha_s(m)}{2\pi}\right)^{n+1} \beta_0^n \, n!$$

Bad behavior cancels in sum of self-energy and inter-quark effects.

Caveat 1

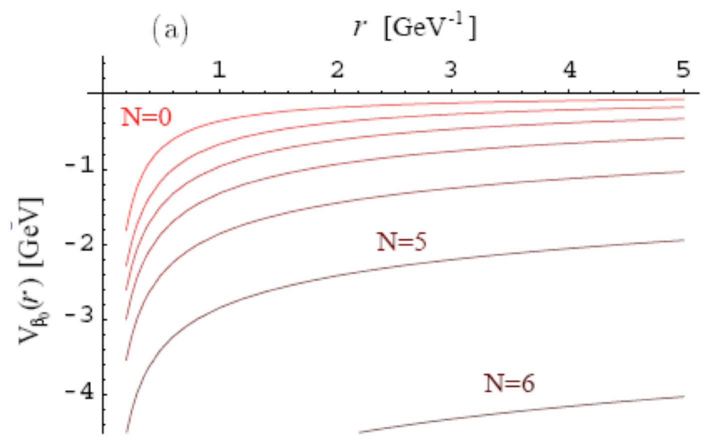
→ Let's step back from the MC and consider a system which is simpler to discuss, but has for the matters of this discussion has the same features as the MC.

Well-defined short-distance quantity for R=1/r >> 1 GeV

Static energy of a heavy quark-antiquark pair:

$$E_{\rm stat} = 2m^0 + 2\Sigma(m,m) + V(R)$$

$$= 2m^{\rm pole} + V(R)$$



$$= 2\overline{m}(\overline{m}) + \left[\, 2\Sigma^{\rm fin}(m,m) + V(R)\,\right]$$

$$V_{\rm asym}(R) = -R\sum_{n=0}\left(\frac{\alpha_s(R)}{2\pi}\right)^{n+1}\beta_0^n\, n!$$

$$\Sigma^{\rm fin}_{\rm asym}(m,m) = \frac{1}{2}\,m\sum_{n=0}\left(\frac{\alpha_s(m)}{2\pi}\right)^{n+1}\beta_0^n\, n!$$

Bad behavior does not fully cancel in the $\overline{\rm MS}$ scheme for R << m.

Caveat 1

→ Let's step back from the MC and consider a system which is simpler to discuss, but has for the matters of this discussion has the same features as the MC.

Well-defined short-distance quantity for R=1/r >> 1 GeV

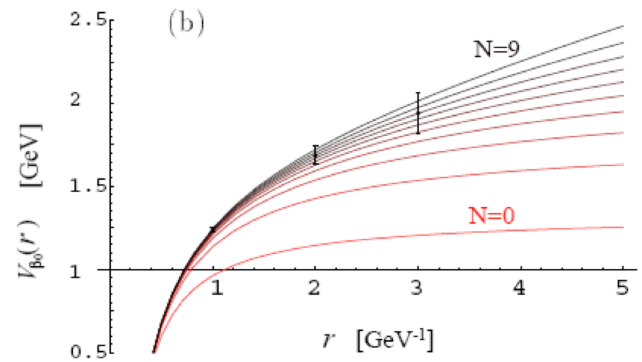
Static energy of a heavy quark-antiquark pair:

$$E_{\rm stat} = 2m^0 + 2\Sigma(m,m) + V(R)$$

$$= 2m^{\rm pole} + V(R)$$

$$= 2\overline{m}(\overline{m}) + [\, 2\Sigma^{\rm fin}(m,m) + V(R)\,]$$

$$= 2m^{\rm MSR}(R) + [\, 2\Sigma^{\rm fin}(R,R) + V(R)\,]$$



$$V_{\rm asym}(R) = -R\sum_{n=0}\left(\frac{\alpha_s(R)}{2\pi}\right)^{n+1}\beta_0^n\, n!$$

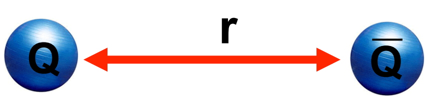
$$\Sigma^{\rm fin}_{\rm asym}(R,R) = \frac{1}{2}\,R\sum_{n=0}\left(\frac{\alpha_s(R)}{2\pi}\right)^{n+1}\beta_0^n\, n!$$

Cancellation of bad behavior in a low-scale short-distance mass: e.g. MSR mass.

Caveat 1

→ Let's step back from the MC and consider a system which is simpler to discuss, but has for the matters of this discussion has the same features as the MC.

Well-defined short-distance quantity for R=1/r >> 1 GeV



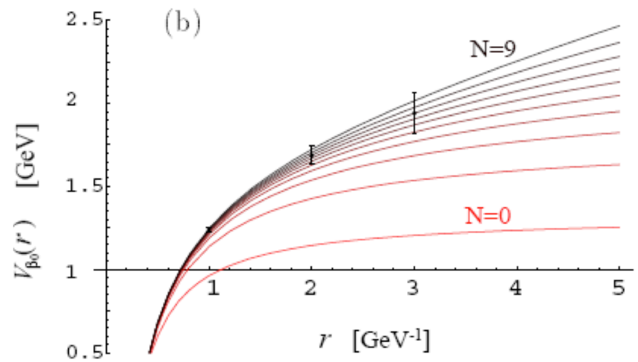
Static energy of a heavy quark-antiquark pair:

$$E_{\rm stat} = 2m^0 + 2\Sigma(m,m) + V(R)$$

$$= 2m^{\rm pole} + V(R)$$

$$= 2\overline{m}(\overline{m}) + [\,2\Sigma^{\rm fin}(m,m) + V(R)\,]$$

$$= 2m^{\rm MSR}(R) + [\,2\Sigma^{\rm fin}(R,R) + V(R)\,]$$



$V(R)$: → Interquark radiation in perturbation theory for all R

→ Uses partonic description to separate **mass** and **radiation**

→ **pole mass** → perturbation theory with instabilities

Generic for ALL short-distancer observables depending on the heavy quark mass !

$$V^R(R) \equiv 2\Sigma^{\rm fin}(R,R) + V(R)\,] : \; m_t^{\rm MSR}(R) = m_t^{\rm pole} - \Sigma^{\rm fin}(R,R)$$

→ Interquark parton radiation in perturbation theory with an IR subtraction / cutoff.

→ This implies a corresponding IR subtraction for the quark mass.

→ Separation between **mass** and **radiation** is scheme dependent

→ **scale-dep. short-dist mass** → perturbation theory stable

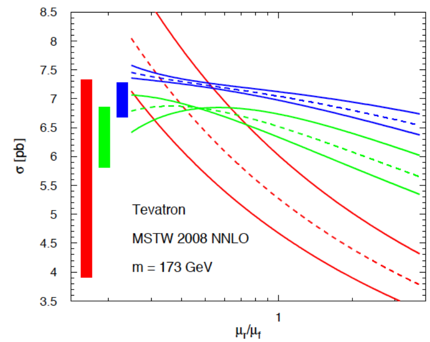
Top Quark Short-Distance Masses

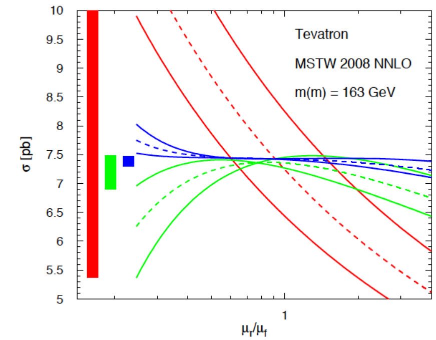
Total cross section (LHC/Tev):

$$m_t^{\mathrm{MSR}}(R = m_t) = \overline{m}_t(\overline{m}_t)$$

- This is the scheme that is used is many new physics studies (unification, vacuum stability, SUSY Higgs masses....)

Langenfeld, Moch, Uwer

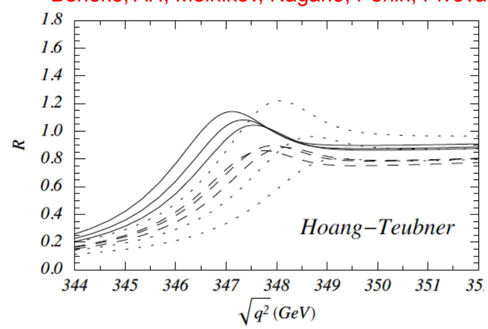


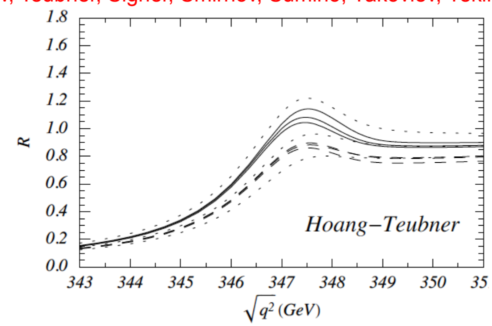


Threshold cross section (ILC):

$$m_t^{\mathrm{MSR}}(R \sim 20\ \mathrm{GeV})\,,\ m_t^{1\mathrm{S}}\,,\ m_t^{\mathrm{PS}}(R)$$

Beneke, AH, Melnikov, Nagano, Penin, Pivovarov, Teubner, Signer, Smirnov, Sumino, Yakovlev, Yeklkovski

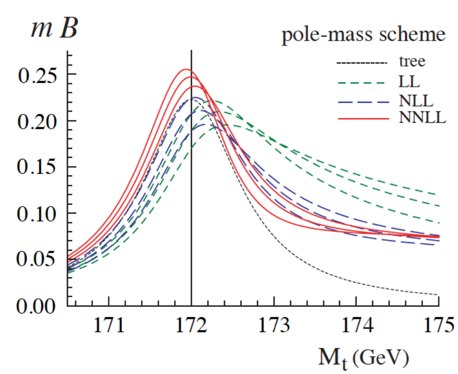


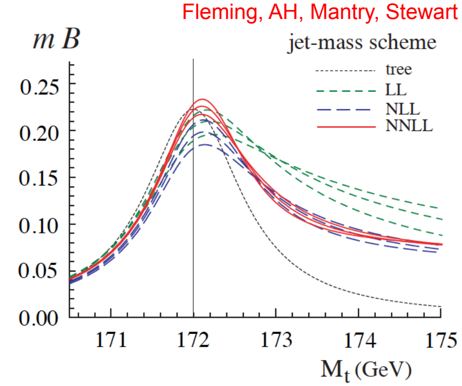


Inv. mass reconstruction (ILC/LHC):

$$m_t^{\mathrm{MSR}}(R \sim \Gamma_t)\,,\ m_t^{\mathrm{jet}}(R)$$

Fleming, AH, Mantry, Stewart





Lessons

Inter-quark/gluon radiation can only be separated from quark self-energy effects at the parton level.

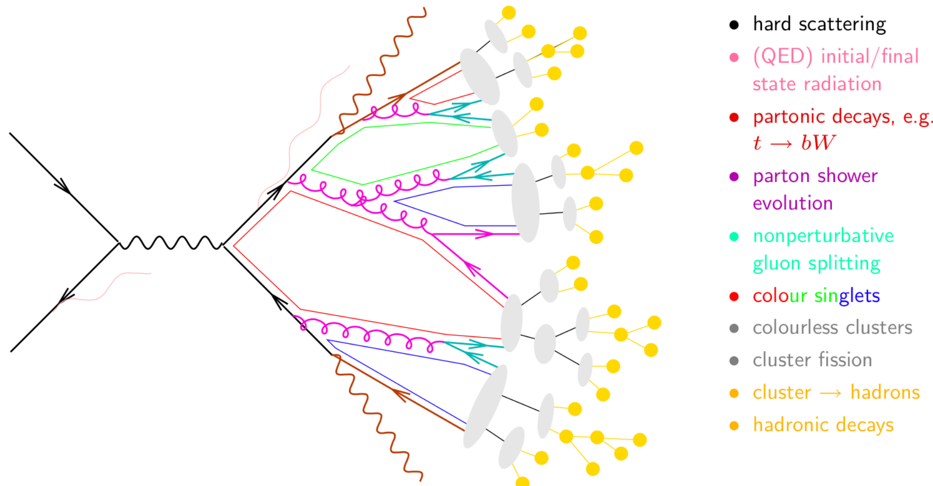
This separation can only be controlled as long as the parton description can be applied.

In the **pole mass** scheme, the parton description is imposed also for momenta at and smaller than the hadronization scale. The pole mass is therefore not physical.

The implementation of a an IR cutoff on the inter-quark/gluon radiation (and a hadronization model) implies a corresponding **short-distance mass** scheme that depends on details of the cutoff procedure.

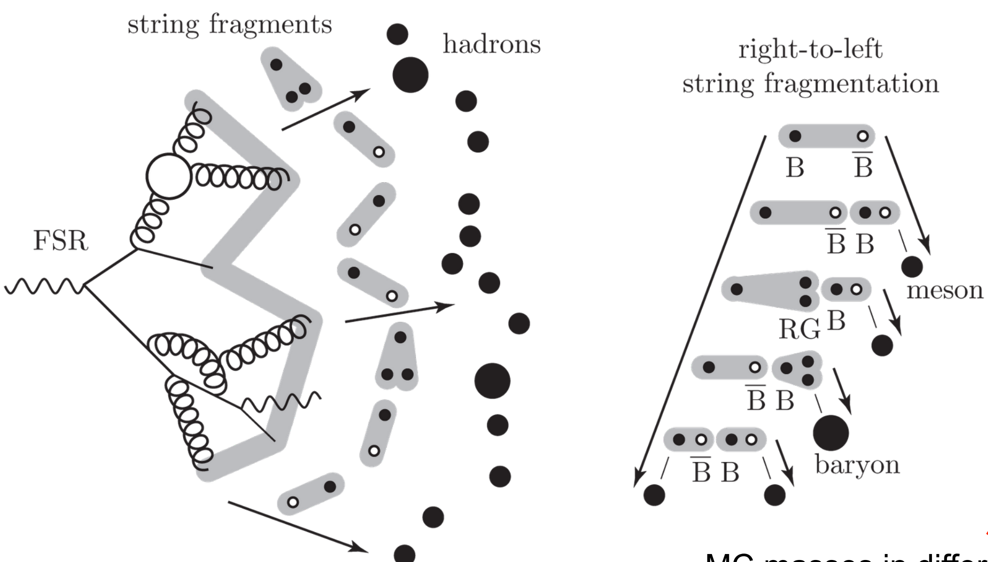
These physical issues are not at all tied to the renormalon problem. The role of the renormalon problem is that is makes the issue numerically relevant.

Heavy Quark Mass in the MC



Monte-Carlo QCD Calculator:

- Computes all inter-quark/gluon and radiation processes
- Computes hadronization of partons
- Electroweak radiation effects
- Does NOT calculate self-energy processes



Inter-quark/gluon radiation cut-off at $\Lambda_s = 1\ \text{GeV}$.

Hadronization model below.

Cut-off procedure (and model details) implicitly determine a short-distance top mass.

$$m_t^{\text{MC}} = m_t^{\text{MSR}}(R = \Lambda_s) + ?$$

MC masses in different MCs are a priori different masses.

How to measure the MC mass?

Static energy of a heavy quark-antiquark pair:

→ Let's assume that there is a lattice (or MC-QCD) calculation of the static energy:

$$E_{\rm stat}(R) \,=\, 2m_t^{\rm lat} \,+\, V^{\rm lat}(R)$$

- IR-stable
- non-perturbative

$$=\, 2m^{\rm MSR}(R) \,+\, \left[\,2\Sigma^{\rm fin}(R,R) + V(R)\,\right]$$

$$m_t^{\rm lat} \,=\, m_t^{\rm MSR}(R) \,+\, \underbrace{\left[\frac{1}{2}V(R) - \frac{1}{2}V^{\rm lat}(R) + \Sigma^{\rm fin}(R,R)\right]}$$

$$=\, \delta m_t(R) \,\sim\, \mathcal{O}(R\,\alpha_s(R), \Lambda_{\rm had})$$

- IR-stable
- perturbative
- non-perturbative

→ We can measure the lattice mass in terms of the MSR-mass at any scale R.

→ Highest precision achieved for smallest R value where pert.theory is still valid.

R-independence is important cross check.

$$m_t^{\rm lat} \,=\, m_t^{\rm MSR}(R\sim\Gamma_t) \,+\, \delta m_t(R\sim\Gamma_t)$$

$$\delta m_t(R\sim\Gamma_t) \,\lesssim\, \mathcal{O}(1\ {\rm GeV})$$

"Lattice mass is equal to the short-distance mass at a low scale up to a small correction."

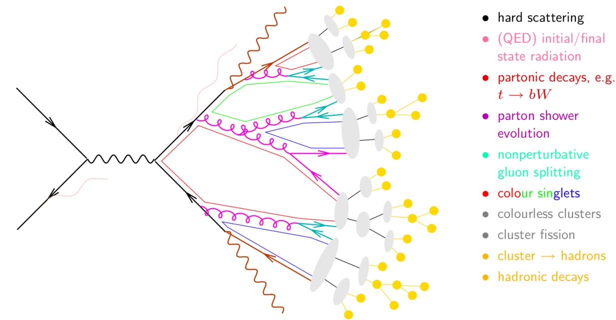
universität wien

How to measure the MC mass?

- $m_t^{\rm MC}$ can be related to $m_t^{\rm MSR}(R)$ by comparing its predictions to analytic calculations for any mass-dependent observable **at the hadron level**

  $\rightarrow$ R: typical physical scale of observable

  $\rightarrow$ $m_t^{\rm MC} - m_t^{\rm MSR}(R)$ can be large



Side-Remark:

This is also the way to check to which extend the MC masses of different MC generators agree (numerically).

$$m_t^{\rm MC-1} = m_t^{\rm MC-2} + \Delta m_t$$

Appears to be small.

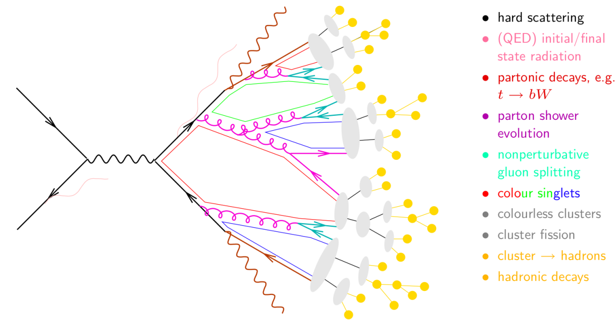
To have a more differentiated picture one should also do dedicated analyses for individual observables and not only check the outcome of different MC in the complete top mass analysis.

How to measure the MC mass?

- m_t^{MC} can be related to $m_t^{\mathrm{MSR}}(R)$ by comparing its predictions to analytic calculations for any mass-dependent observable **at the hadron level**

  → R: typical physical scale of observable

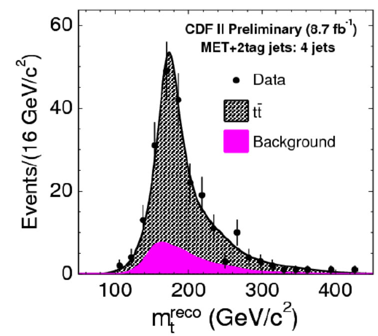
  → $m_t^{\mathrm{MC}} - m_t^{\mathrm{MSR}}(R)$ can be large

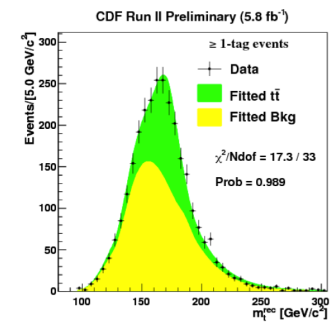


- Closest numerical relation between MC mass and the MSR mass happens for smallest possible R scale.

  → resonance / threshold / endpoint observables

  → $R \sim \Gamma_t \sim \Lambda_s$





$$m_t^{\mathrm{MC}} = m_t^{\mathrm{MSR}}(R \sim \Gamma_t) + \boxed{\delta m_t(R \sim \Gamma_t)}$$

→ $\lesssim \mathcal{O}(1\ \mathrm{GeV})$

→ Cannot be calculated!

→ Can only be measured

→ **It is a "conceputal" error at this time!**

AH, Stewart: arXive:0808.0222

$$m_t^{\mathrm{MC}} = m_t^{\mathrm{MSR}}(3^{+6}_{-2}\ \mathrm{GeV}) = m_t^{\mathrm{MSR}}(3\ \mathrm{GeV})^{+0.6}_{-0.3}$$

How to measure the MC mass?

Remark:

The mass $m_t^{\mathrm{MSR}}(R = \Lambda_s)$ is what comes closest to the concept of a "physical pole mass", but this concept itself is intrinsicly scheme-dependent as it is tied to the parton picture which looses meaning for quantum fluctuations below 1 GeV.

Reminder:

Everything that was said relies on the assumption that the MC is a reliable QCD calculator - and NOT JUST A MODEL.

Why did I not mention the top decay ?

The top decay does not affect anything said before. It adds a theoretical complication as makes measuring top properties dependent on the experimental procedure (and makes theory to describe this correctly more involved).

Measuring leptonic vs. hadronic decays (decay products) does not affect anything said before either. It affects other systematics.

What if you don't care about all this?

→ Let's set $m_t^{\mathrm{MC}} = m_t^{\mathrm{pole}}$

A. Relate MC mass to the wrong scheme (which has a renormalon)

B. Set $\delta m_t = 0$

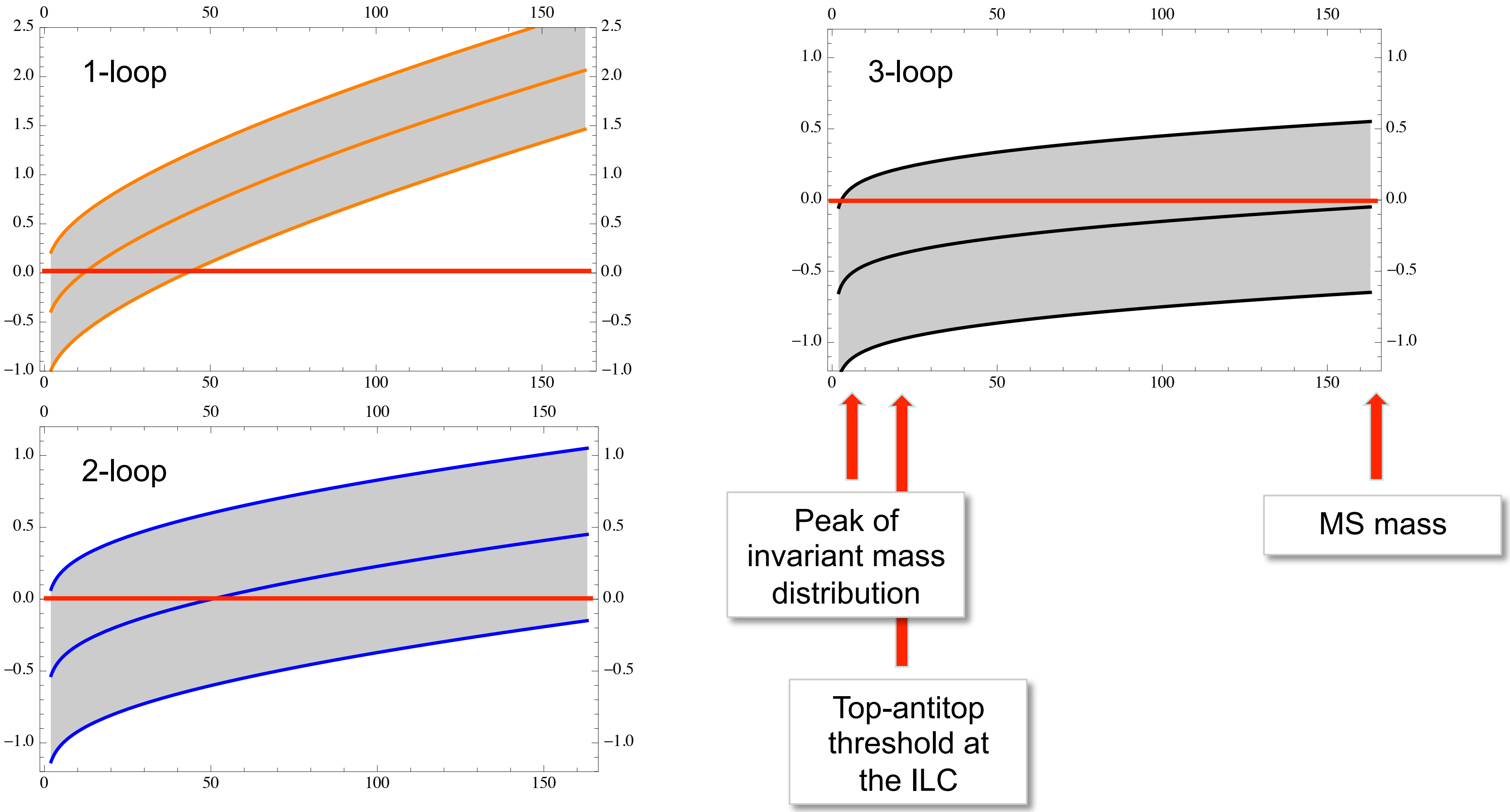
→ Two mistakes, which can – depending on what is done – add up or cancel.
The issue it more subtle than just the renormalon in the pole mass definition.

Exercise:

1) Set $m_t^{\mathrm{MSR}}(3) = 173.2 \pm 0.6 \text{ GeV}$ → compute@3-loop $m_t^{\mathrm{MSR}}(R)$
2) Set $m_t^{\mathrm{pole}} = 173.2 \text{ GeV}$ → compute $m_t^{\mathrm{MSR}}(R)$
3) Analyze $m_t^{\mathrm{MSR}}(R)|_{\mathrm{pole}} - m_t^{\mathrm{MSR}}(R)|$

What if you don't care about all this?

$$m_t^{\mathrm{MSR}}(R)|_{\mathrm{pole}} - m_t^{\mathrm{MSR}}(R)|$$



Summary

Part 1: $\rightarrow$ Theoretical considerations on m_t^{MC}

- Why m_t^{MC} looks like being m_t^{pole} , but is actually not.
- How to determine m_t^{MC} in terms of other masses.
- What if one sets $m_t^{\mathrm{MC}} = m_t^{\mathrm{pole}}$ anyway.
- Advertisement for the MSR mass: $m_t^{\mathrm{MSR}}(R)$

MSR Mass Definition

MSbar Scheme: $(\mu > \overline{m}(\overline{m}))$

$$\overline{m}(\overline{m}) - m^{\rm pole} = -\overline{m}(\overline{m})\,[\,0.42441\,\alpha_s(\overline{m}) + 0.8345\,\alpha_s^2(\overline{m}) + 2.368\,\alpha_s^3(\overline{m}) + \ldots]$$

MSR Scheme: $(R < \overline{m}(\overline{m}))$

$$m_{\rm MSR}(R) - m^{\rm pole} = -R\,[\,0.42441\,\alpha_s(R) + 0.8345\,\alpha_s^2(R) + 2.368\,\alpha_s^3(R) + \ldots]$$

$$m_{\rm MSR}(m_{\rm MSR}) = \overline{m}(\overline{m})$$

⟹ $m_{\rm MSR}(R)$ Short-distance mass that smoothly interpolates all R scales

- Excellent convergence of relation between MSR masses at different R values
- Excellent convergence of relation between MSR masses and other short-distance masses
- Smoothy interpolates to the MSbar mass.

MSR Mass Definition

R-Evolution of MSR mass:

$$m(R) = m_{\rm pole} - \delta m(R) \qquad \delta m(R) = R \sum_{n=1}^{\infty} \left(\frac{\alpha_s(R)}{4\pi}\right)^n a_n$$

$$R\frac{d}{dR}m(R) = -\frac{d}{d\ln R}\delta m(R) = R \sum_{n=0}^{\infty} \gamma_n^R \left[\frac{\alpha_s(R)}{4\pi}\right]^{n+1}$$

renormalon-free !

$$m(R_1) - m(R_0) = \int_{R_0}^{R_1} \frac{dR}{R} R\,\gamma^R[\alpha_s(R)]$$

can be calculated numerically

can be calculated analytically

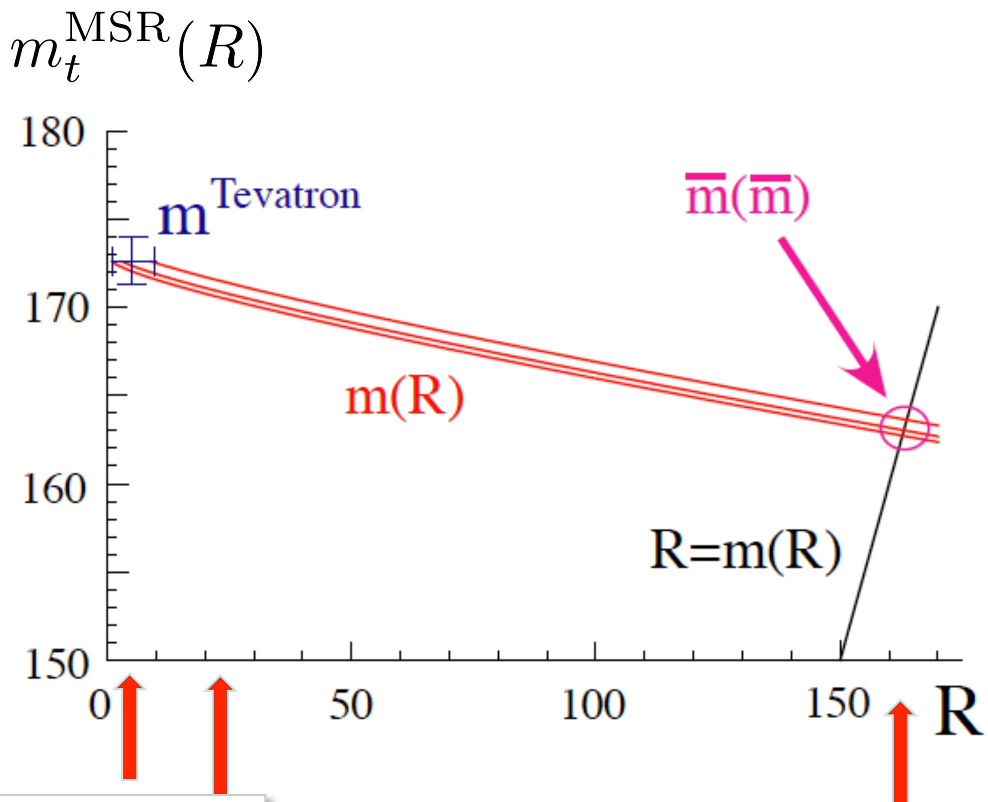
$$\overset{\rm N^kLL}{=} \Lambda_{\rm QCD}^{(k)} \sum_{j=0}^{k} S_j\,(-1)^j\, e^{i\pi\hat{b}_1}\left[\Gamma(-\hat{b}_1 - j, t_1) - \Gamma(-\hat{b}_1 - j, t_0)\right]$$

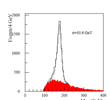
imaginary parts cancel

$$\Lambda_{\rm QCD}^{(0)} = Re^t \qquad S_0 = \frac{\gamma_0}{2\beta_0}$$

$$\hat{b}_1 = \frac{\beta_1}{2\beta_0^2} \qquad t_{0,1} = -\frac{2\pi}{\beta_0\alpha_s(R_{0,1})}$$

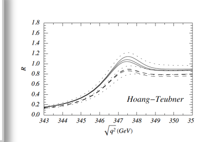
MSR Mass Definition





Peak of invariant mass distribution, endpoints

Top-antitop threshold at the ILC



Total cross section, e.w.precsion obs., Unification, MSbar mass

Theory Tools to Measure the MC mass

Part 2

Motivation:

- Accurate analytic QCD predictions beyond LL/LO with full control over the quark mass dependence
- Theoretical description at the hadron level

Here

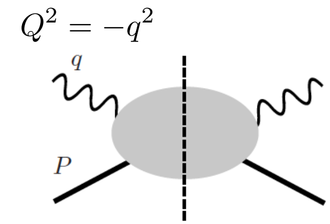
- Implementation of massive quarks into the SCET framework
- **VFNS for final state jets (with massive quarks)***

* In collaboration with: P. Pietrulewicz, I. Jemos, S. Gritschacher

arXiv:1302.4743 (PRD 88, 034021 (2013))
arXiv:1309.6251 (PRD 89, 014035 (2013))
arXiv:1405.4860

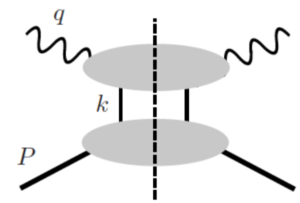
VFNS for Inclusive Hadron Collisions

e.g. Deep Inelastic Scattering: $\frac{d\sigma(e^- p \to e^- + X)}{dQ\, dx}$



→ consider all quarks as as light ($m_q < \Lambda$)

→ quark number operators with an anomalous dimension between proton states → DGLAP equations



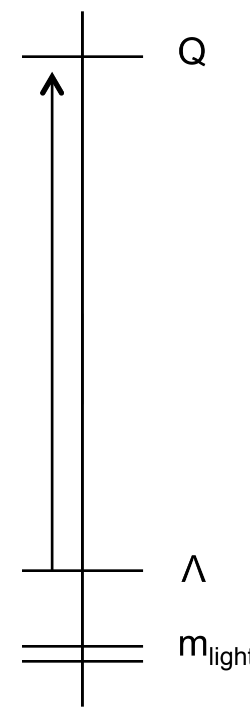
→ Hadronic tensor:

$$W_{\mu\nu}(Q,x) \sim \sum_{\text{partons a}} f_a(\mu) \otimes w_{\mu\nu}(Q,x,\mu)$$

→ μ-dependence with DGLAP equations for (light) parton distribution functions

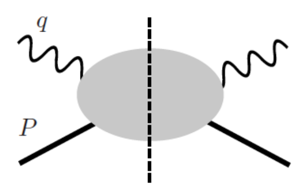
$$\frac{\partial}{\partial \ln Q^2}\begin{pmatrix} q_i(x,Q^2) \\ g(x,Q^2) \end{pmatrix} = \frac{\alpha_s(Q^2)}{2\pi}\sum_j \int_x^1 \frac{d\xi}{\xi} \times \begin{pmatrix} P_{q_i q_j}\left(\frac{x}{\xi},\alpha_s(Q^2)\right) & P_{q_i g}\left(\frac{x}{\xi},\alpha_s(Q^2)\right) \\ P_{g q_j}\left(\frac{x}{\xi},\alpha_s(Q^2)\right) & P_{gg}\left(\frac{x}{\xi},\alpha_s(Q^2)\right) \end{pmatrix} \begin{pmatrix} q_j(\xi,Q^2) \\ g(\xi,Q^2) \end{pmatrix}, \quad (11)$$

$$\frac{d\alpha_s(Q)}{d\ln Q^2} = -\beta_0 \frac{\alpha_s^2(Q)}{(4\pi)} + \ldots \qquad \beta_0 = 11 - \frac{2}{3} n_{\text{light}}$$



VFNS for Inclusive Hadron Collisions

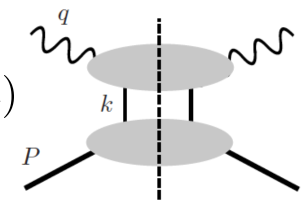
e.g. Deep Inelastic Scattering: $\frac{d\sigma(e^- p \to e^- + X)}{dQ\, dx}$



→ realistic case: massive quarks with Q > m > Λ
(charm, bottom [top])

→ Hadronic tensor:

$$W_{\mu\nu}(m,Q,x) \sim \sum_{a=q,g,Q} f_a^{(n_l+1)}(\mu) \otimes w_{\mu\nu}(m,Q,x,\mu)$$

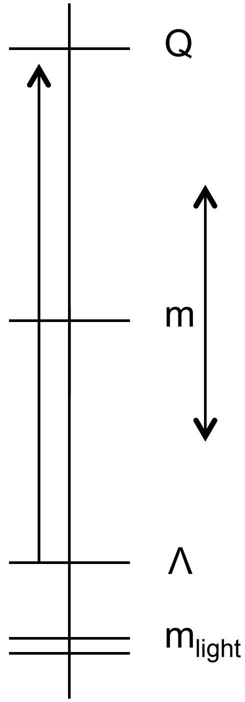


VFNS for pdf evolution:

- DGLAP evolution for n_l flavors for $\mu \lesssim m$ (only light quarks)
- DGLAP evolution for n_l+1 flavors for $\mu \gtrsim m$ (light quarks + massive quark)
- Flavor matching for α_s and the pdfs at $\mu_m \sim m$

$$f_{q,g,Q}^{(n_l+1)}(\mu_m) = \sum_{a=q,g} F_{q,g,Q|a}(m,\mu_m) \otimes f_a^{(n_l)}(\mu_m)$$

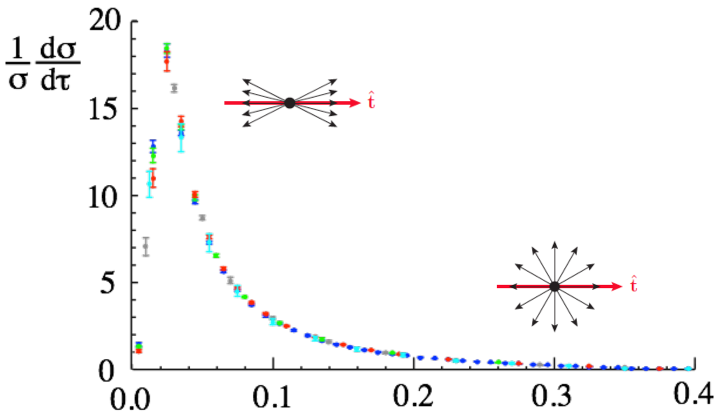
→ hard coefficient $w_{\mu\nu}(m,Q,x)$ approaches massless $w_{\mu\nu}(Q,x)$ for m→0
→ calculations of $w_{\mu\nu}(m,Q,x)$ involves subtraction of pdf IR mass singularities
→ full dependence on m/Q without any large logarithms

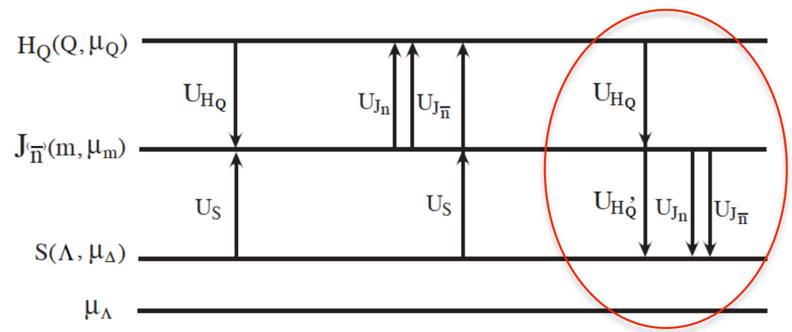


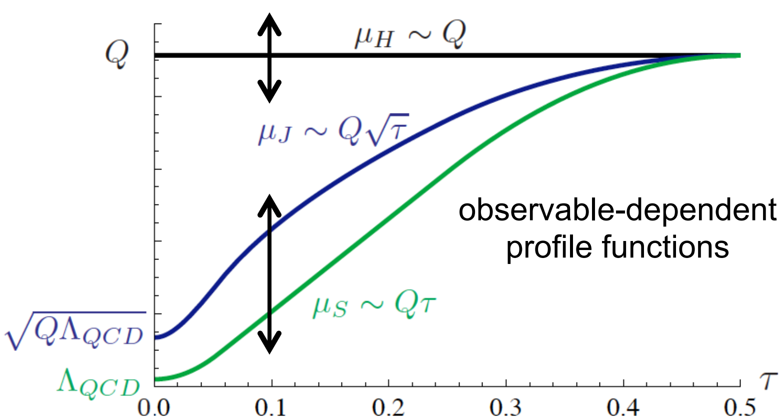
Factorization for Massless Quarks

$$\frac{d\sigma}{d\tau} = Q^2 \sigma_0 H_0(Q,\mu) \int d\ell \, J_0(Q\ell,\mu)\, S_0\left(Q\tau - \ell, \mu\right)$$

Schwartz
Fleming, AH, Mantry, Stewart
Bauer, Fleming, Lee, Sterman







→ evolution with n_l light quark flavors

→ consistency conditions w.r. to different evolution choices

→ top-down evolution considered in the following

$$\left(\frac{d\sigma}{d\tau}\right)^{\text{sing}}_{\text{part}} \sim \sigma_0\, H(Q,\mu_Q) U_H(Q,\mu_Q,\mu_s) \int d\ell d\ell' \, U_J(Q\tau - \ell - \ell', \mu_Q, \mu_s)\, J_T(Q\ell', \mu_j)\, S_T(\ell - \Delta, \mu_s)$$

VFN Scheme for Final State Jets

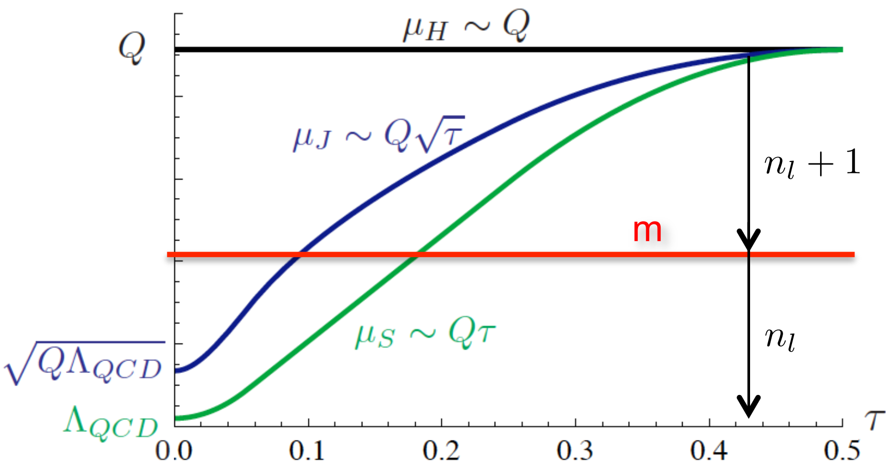
Gritschacher, AH, Jemos, Pietrulewicz

→ consider: dijet in e^+e^- annihilation, n_l light quarks ⊕ one massive quark

→ obvious: (n_l+1)-evolution for $\mu \gtrsim m$ and (n_l)-evolution for $\mu \lesssim m$

→ obvious: different EFT scenarios w.r. to mass vs. Q – J – S scales

"profile functions"



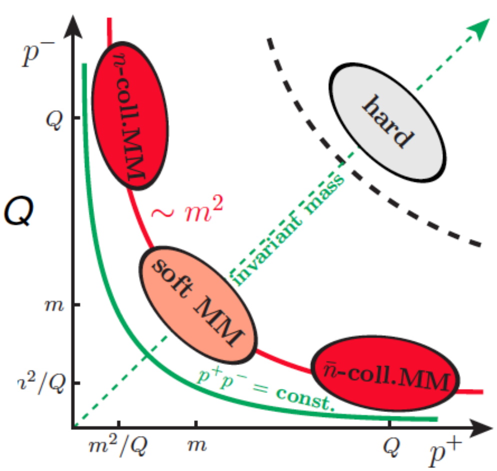
Aims:

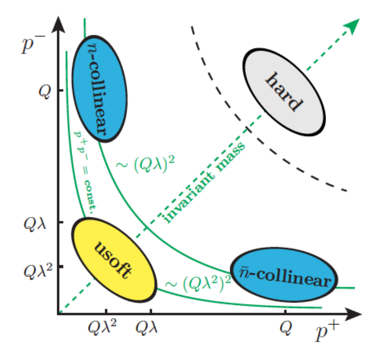
- Full mass dependence (little room for any strong hierarchies): decoupling, massless limit
- Smooth connections between different EFTs
- Determination of flavor matching for current-, jet- and soft-evolution
- Reconcile problem of $SCET_2$-type rapidity divergences

→ Deal with collinear and soft "mass modes"

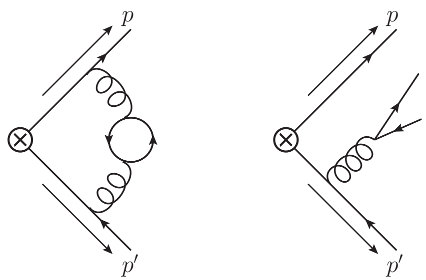
→ Additional power counting parameter $\lambda_m = m/Q$

mode	$p^\mu = (+,-,\perp)$	p^2
n-coll MM	$Q(\lambda_m^2, 1, \lambda_m)$	m^2
soft MM	$Q(\lambda_m, \lambda_m, \lambda_m)$	m^2





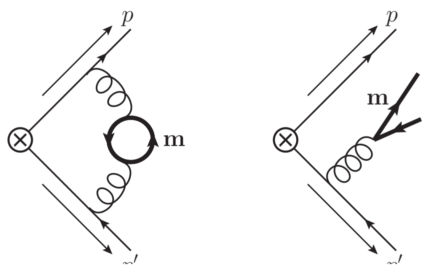
Fully Massive Thrust



→ fully massless

- Full N^3LL' (u.t. 4-loop cusp)+ 3-loop non-singular
- Gap scheme for soft function

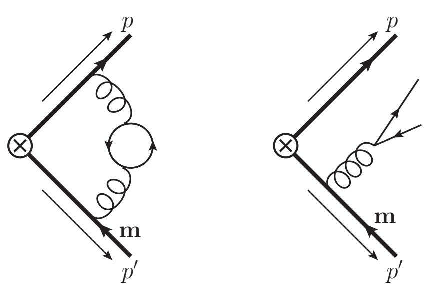
SCET authors: Becher, Schwartz,
Fleming, AH, Mantry, Stewart
Bauer, Fleming, Lee, Sterman
Fixed-order authors: Ge]hrmann etal, Weinzierl



→ secondary massive

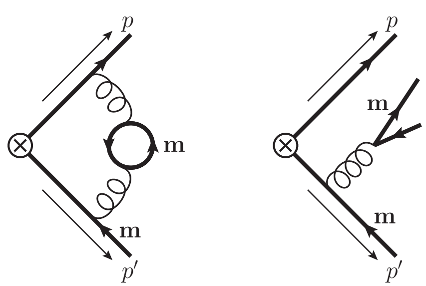
- Full N^2LL'/N^3LL
- Four different physical situations

Pietrulewicz, AH, Gritschacher, Jemos 2013+2014
→ paper with all details on the arXive today.



→ primary massive

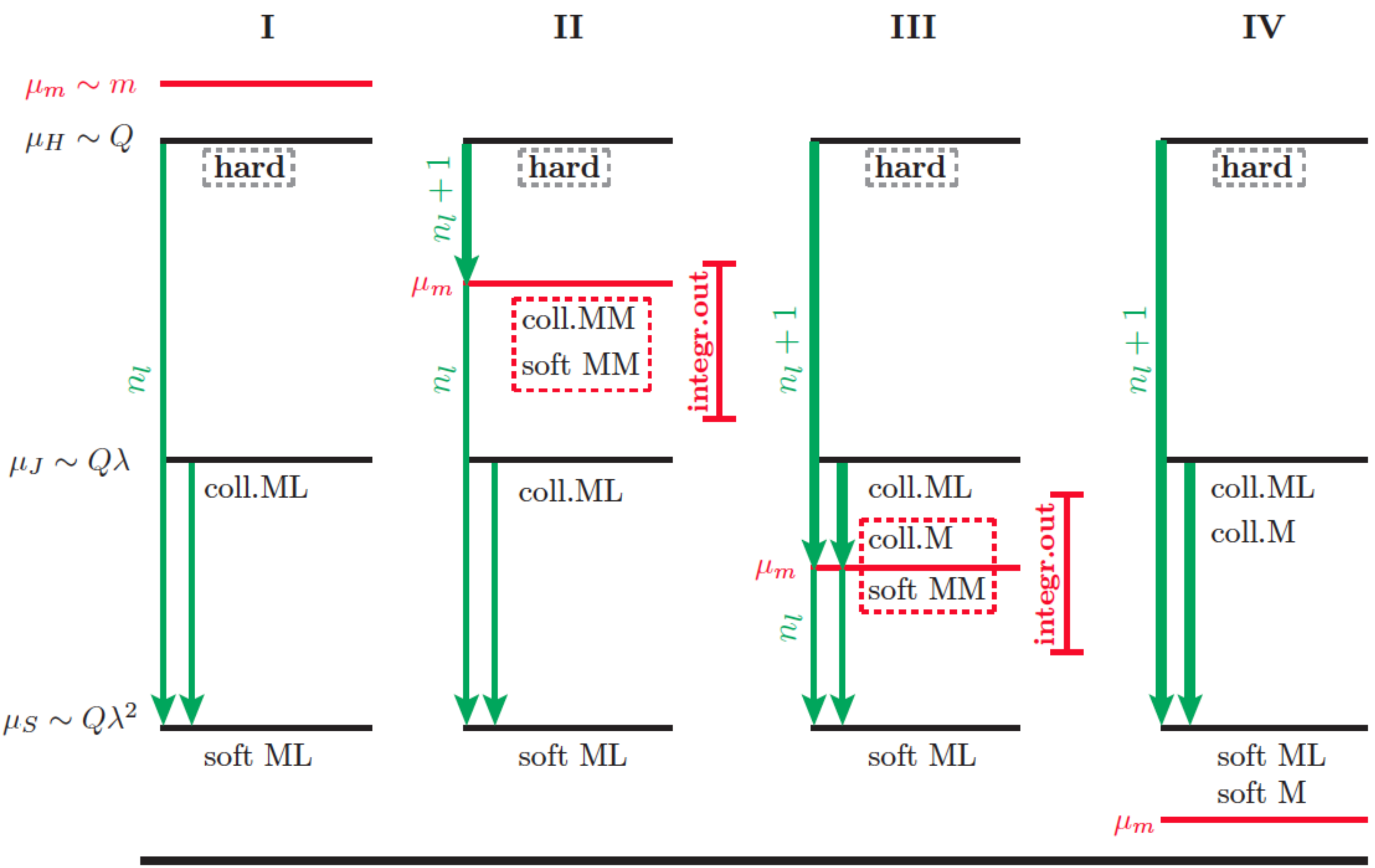
- Full N^2LL'/N^3LL on the way
- Three different physical situations



→ primary massive
secondary massive

No details in this talk!

VFN Scheme: Secondary Massive Quarks

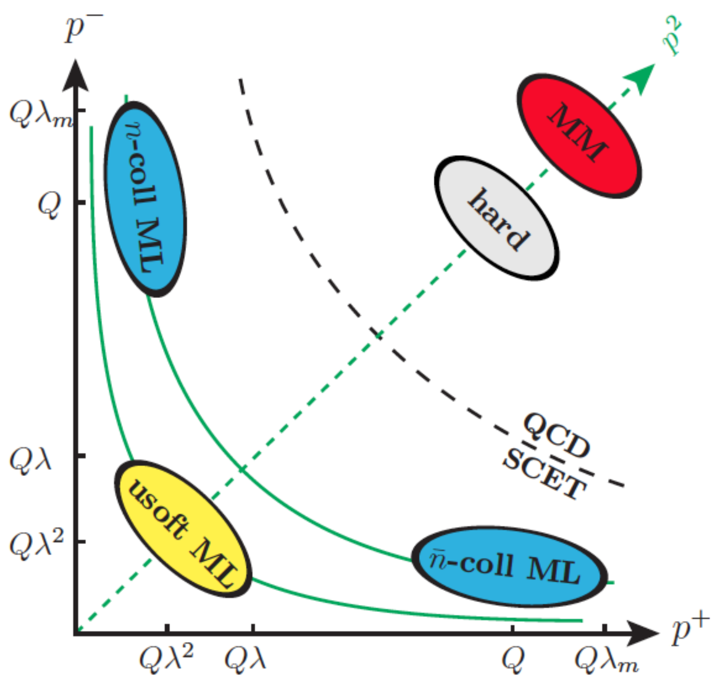


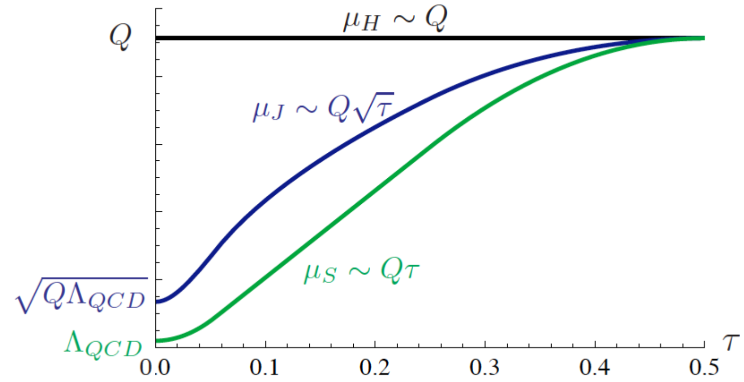
MM = mass-mode, ML = massless, M = massive

→ See Piotr's talk.

VFN Scheme: Secondary Massive Quarks

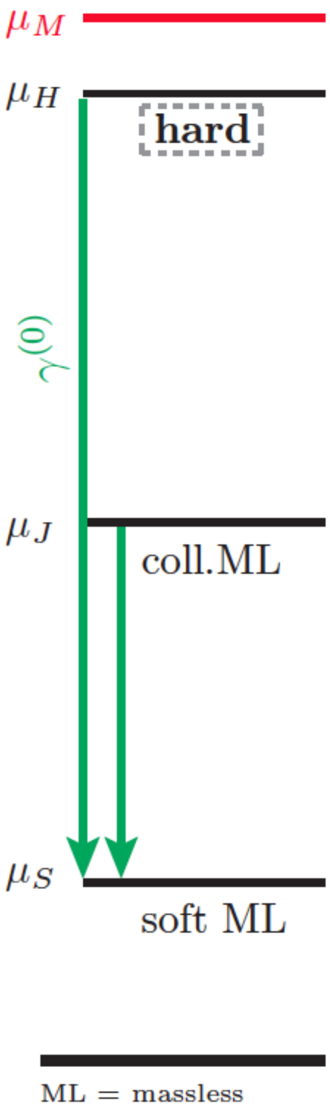
Scenario 1: $\lambda_m > 1 > \lambda > \lambda^2$ (m > Q > J > S)





- EFT only contains light quarks
- Massive quark only in current matching coeff.
- Decoupling for m/Q → ∞

VFN Scheme: Secondary Massive Quarks



integrate out mass modes at QCD level

$$\frac{d\sigma}{d\tau} \sim \left|C^I(\mu_H)\right|^2 U_H^{(0)}(\mu_H,\mu_S) \times \int d\ell \int ds\, J_0(s)\, U_J^{(0)}(Q\ell - s, \mu_S, \mu_J)\, S_0\,(Q\tau - \ell, \mu_S)$$

$U_H^{(0)}$, $U_J^{(0)}$: massless evolution factors

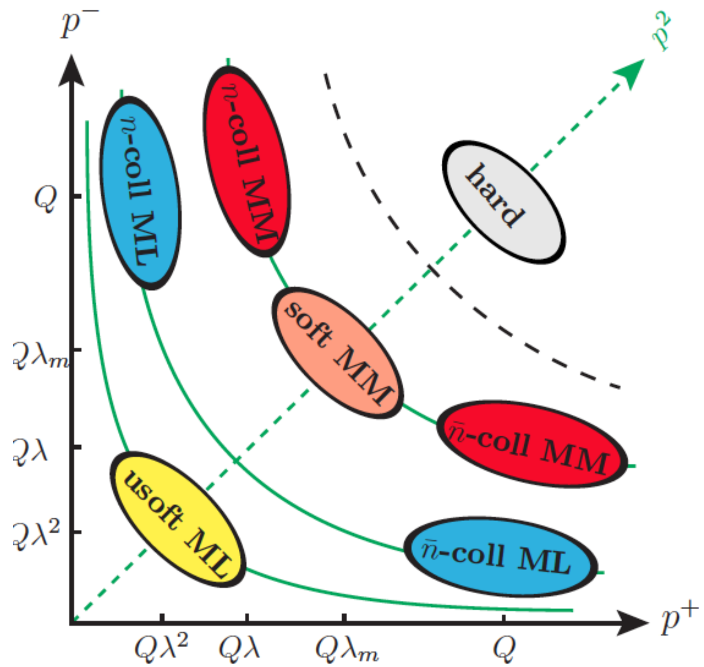
$$C^I(\mu_H) = C_0(\mu_H) + \delta F_m^{\mathrm{QCD}}$$

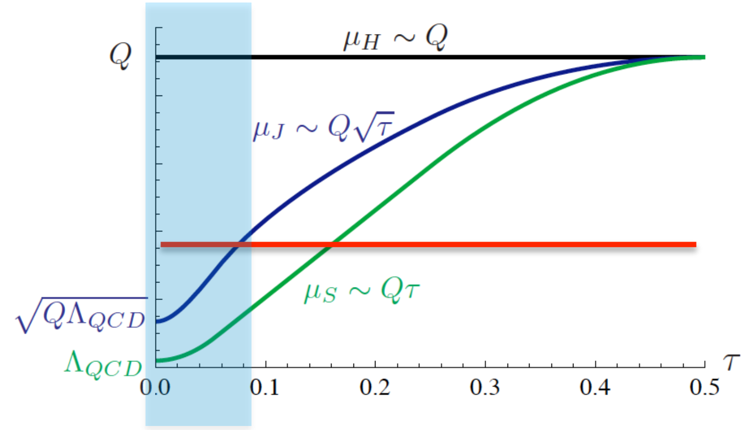
C_0: massless matching coefficient
$\delta F_m^{\mathrm{QCD}}$: massive full theory contribution (OS)
→ decoupling for $M/Q \to \infty$
→ IR-divergent expression for $M/Q \to 0$

$U_i^{(0)}$ stands for: (a) massive gluon integrated out
(b) (n_l)-evolution

VFN Scheme: Secondary Massive Quarks

Scenario 2: 1> λ_m > λ > λ^2 (Q > m > J > S)

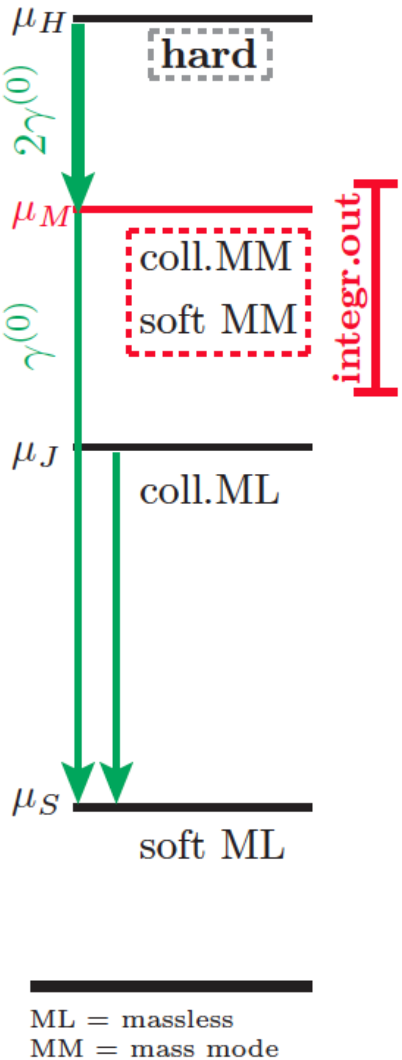




- Massive modes only virtual
- Jet and soft function as in massless case
- Hard coefficient must have massless limit
- Known Sudakov problem for massive gauge boson

Chiu, Golf, Kelley, Manohar
Chiu, Führer, Hoang, Kelley

VFN Scheme: Secondary Massive Quarks



mass modes enter SCET, but integrated out before the jet scale

$$\frac{d\sigma}{d\tau} \sim \left|\mathcal{C}^{II}(\mu_H)\right|^2 U_H^{(1)}(\mu_H,\mu_M)\left|\mathcal{M}_H(\mu_M)\right|^2 U_H^{(0)}(\mu_M,\mu_S) \times \int d\ell \int ds\, J_0(s,\mu_J)\, U_J^{(0)}(Q\ell - s,\mu_J,\mu_S)\, S_0(Q\tau-\ell,\mu_S)$$

$U_H^{(1)}$: evolution factor ($\gamma_H^{(1)} = 2\gamma_H^{(0)}$)

$\mathcal{C}^{II}(\mu_H) = \mathcal{C}^{I}(\mu_H) - \delta F_m^{\text{eff}}(\mu_H)$ ← Contains all mass-singularities

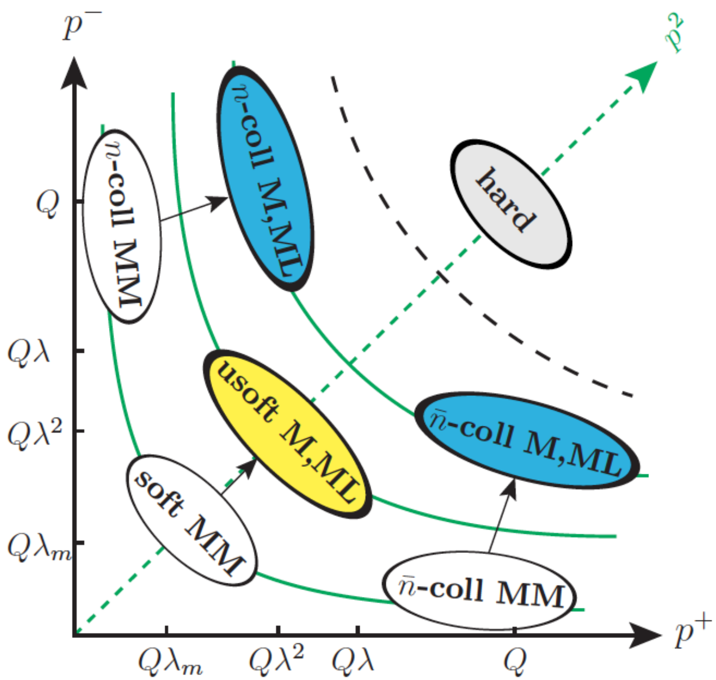
δF_m^{eff}: massive SCET contribution

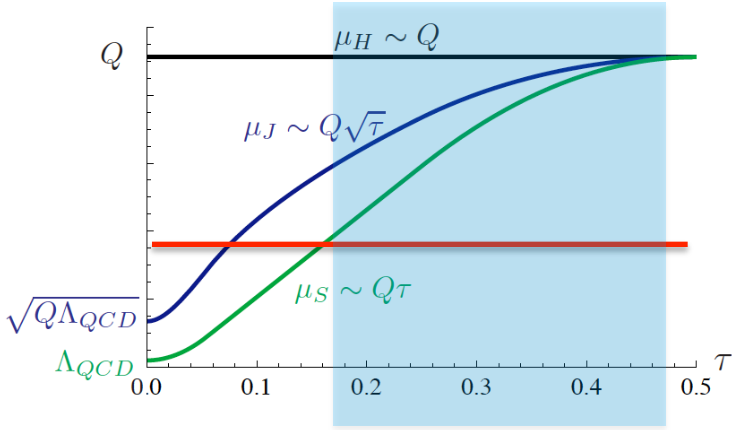
$U_i^{(0)}$ stands for: (a) massive gluon integrated out
(b) (n_l)-evolution

$U_i^{(1)}$ stands for: (a) massive gluon dynamical
(b) (n_l+1)-evolution

VFN Scheme: Secondary Massive Quarks

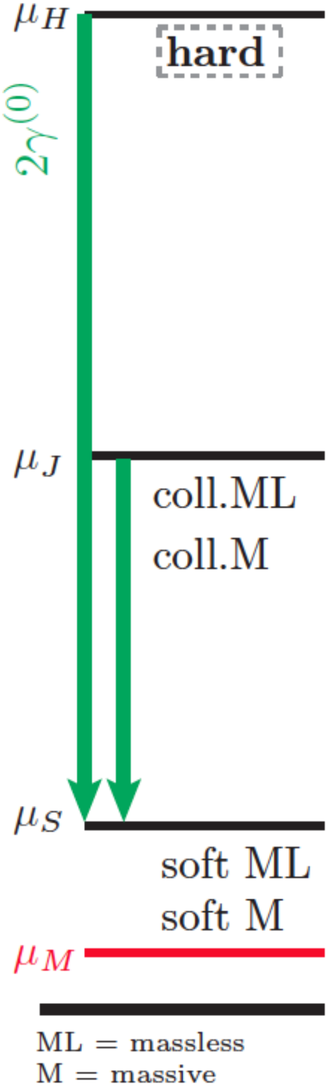
Scenario 4: 1 > λ > λ^2 > λ_m (Q > J > S > m)





- Current evolution unchanged w.r. to Scen. 2
- Jet function and evolution as in Scen. 2
- Massive and massless coll. modes same sector
- Massive and massless soft modes same sector
- Hard coefficient, jet and soft function must have massless limit
- All RG-evolution for (n_l+1) flavors

VFN Scheme: Secondary Massive Quarks



mass modes enter all sectors

$$\frac{d\sigma}{d\tau} \sim \left|C^{ll}(\mu_H)\right|^2 U_H^{(1)}(\mu_H, \mu_S) \times \int d\ell \int ds\, J_{0+m}(s, \mu_J)\, U_J^{(1)}(Q\ell - s, \mu_J, \mu_S)\, S_{0+m}(Q\tau - \ell, \mu_S)$$

$$S_{0+m}(\ell, \mu_S) = S_0(\ell, \mu_S) + \delta S_m^{\rm virt}(\ell, \mu_S) + \theta(\ell - M)\, \delta S_m^{\rm real}(\ell)$$

$\delta S_m^{\rm virt}$: virtual piece of massive soft function (distributive structure)

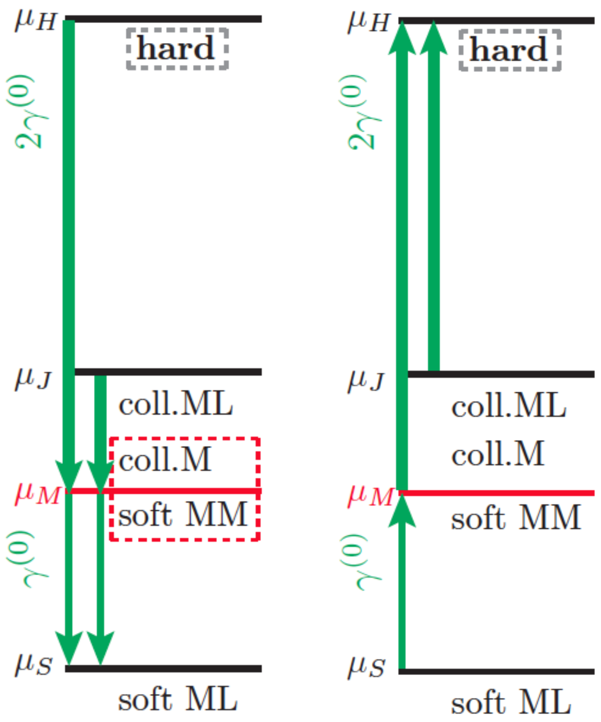
- Rapidity singularities cancel between contributions from both hemispheres (+,-)
- UV divergences agree with massless case

$\delta S_m^{\rm real}$: real radiation piece of massive soft function (function)

- finite
- sum of virtual and real: rapidity logs cancel
- sum of virtual and real: approaches massless soft function for m → 0

Consistency Conditions: Threshold Corrections

Important role of consistency relation: soft – jet – hard for scenario III



alternative description in bottom-up running ($\mu \sim \mu_H$):

$$\frac{d\sigma}{d\tau} \sim \left|C''(\mu_H)\right|^2 \int d\ell \int d\ell' \int d\ell'' \int ds \int ds' \\ \times U_J^{(1)}(s-s',\mu_J,\mu_H)\, J_0(s',\mu_J)\, U_S^{(1)}(\ell''-s/Q,\mu_M,\mu_H) \\ \times \mathcal{M}_S(\ell'-\ell'',\mu_M)\, U_S^{(0)}(\ell-\ell',\mu_S,\mu_M)\, S_0\left(Q\tau-\ell,\mu_S\right)$$

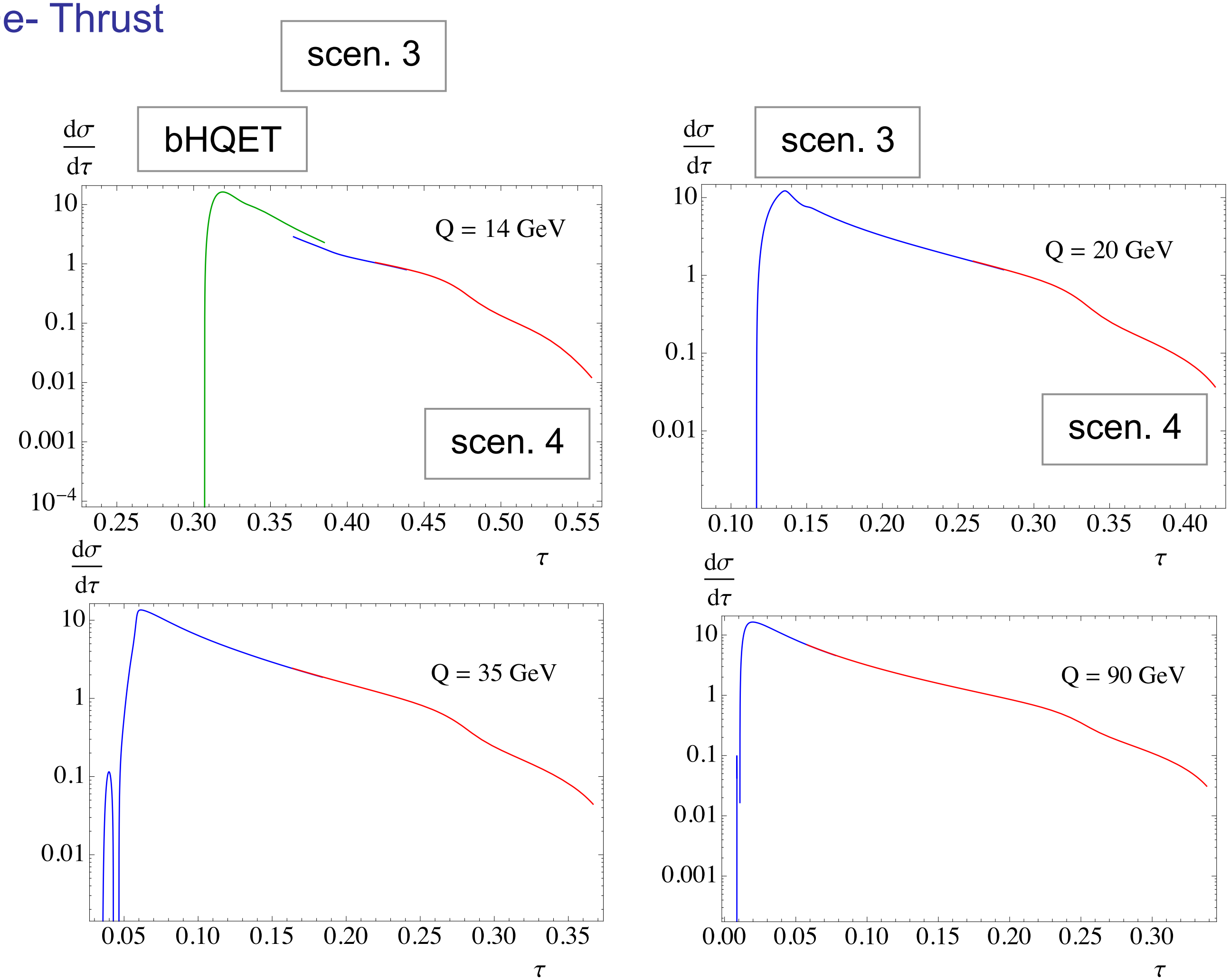
$$\mathcal{M}_S(\ell,\mu_M) = \delta(\ell) + \delta S_m^{\rm virt}(\ell,\mu_M)$$

consistency relation: $\mathcal{M}_S(\ell,\mu_M) = Q\,\left|\mathcal{M}_H(\mu_M)\right|^2 \mathcal{M}_J(Q\ell,\mu_M)$

similarly: $U_S^{(1)}(\ell,\mu_S,\mu_M) = Q\,U_H^{(1)}(\mu_M,\mu_S)\,U_J^{(1)}(Q\ell,\mu_M,\mu_S)$

VFN Scheme: Bottom Production

First prelim. analysis: m=4.5, Q= 14, 22, 35, 91 GeV ($\text{NNLL}_{\text{resum}}$ + $\text{NLO}_{\text{fixed-order}}$)
for e+e- Thrust



"Best" MSR mass depends on tau !

$$m_t^{\text{MSR}}(R(\tau))$$

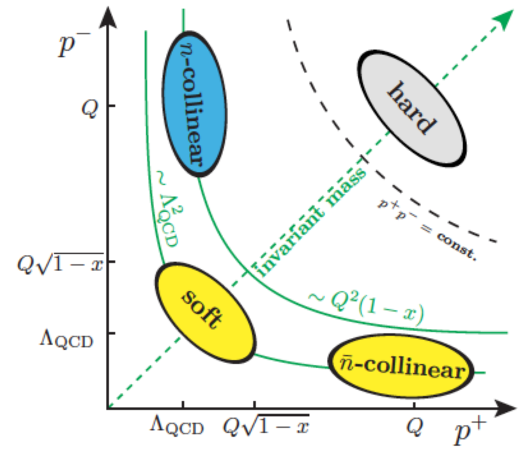
Consistency with VFNS in DIS ($x \to 1$)

P. Pietrulewicz, AH, in preparation

- $x \to 1$: experimentally barely accessible (small pdfs!)
  but: nontrivial factorization setup $\to$ interesting as a showcase for concepts
- quite a lot of SCET literature
  Manohar (2003), Becher, Neubert, Pecjak (2006),
  Chay, Kim (2006, 2010, 2013), Fleming, Zhang (2013), ...
- here: $1 - x \sim \Lambda_{\rm QCD}/Q$, conveniently: Breit frame

Factorization theorem:

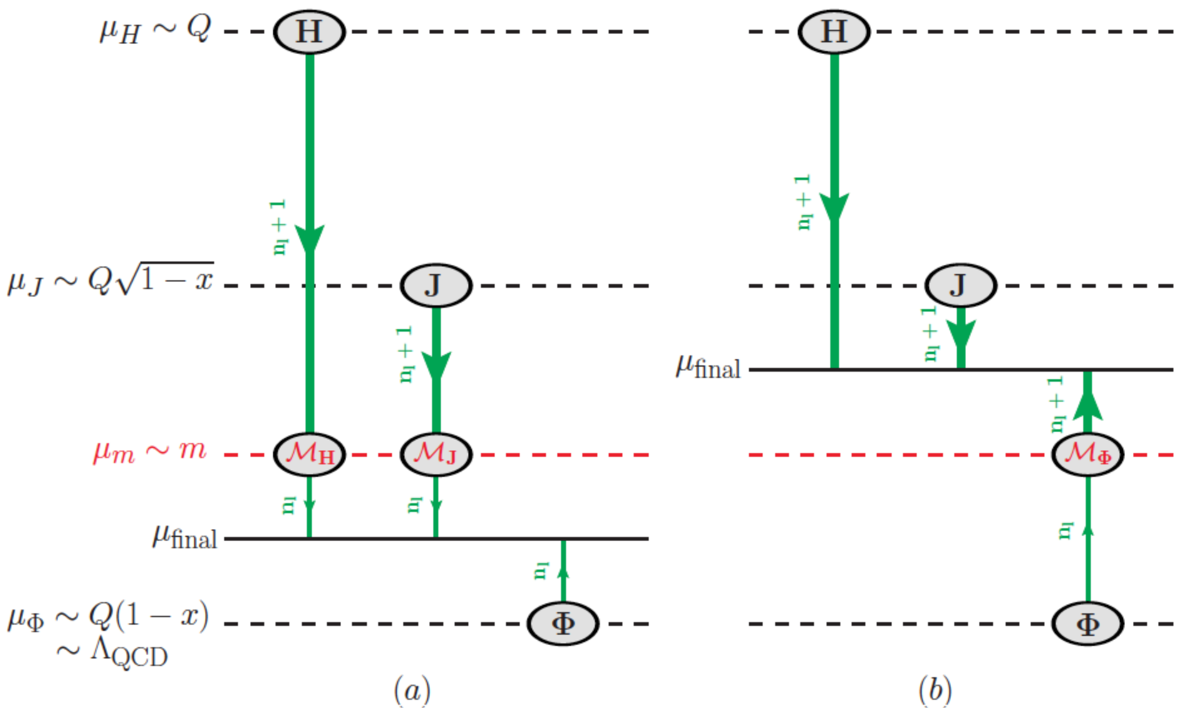
$$F_1 \sim \sum_{i=q} H_{\rm DIS}(\mu_H) J_{\rm DIS}(\mu_J) \otimes \underbrace{S_{\rm DIS}(\mu_\Phi) \otimes f_{i/P}(\mu_\Phi)}_{=\Phi_{i/P}(\mu_\Phi)}$$



Ingredients:

- at $\mu_H \sim Q$: hard function $H_{\rm DIS}(\mu_H) = |C(\mu_H)|^2$
- at $\mu_J \sim Q\sqrt{1-x}$: final state jet function $J_{\rm DIS}(\mu_J)$
- at $\mu_\Phi \sim \Lambda_{\rm QCD}$: pdf $\Phi_{q/P}(\mu_\Phi)$
  $\leftrightarrow$ in SCET II: collinear initial state function $f_{q/P}(\mu_\Phi) \otimes$ soft function $S_{\rm DIS}(\mu_\Phi)$

Consistency with VFNS in DIS (x→1)



physical cross section independent of μ_{final} → (a) and (b) equivalent
→ relation between evolution factors

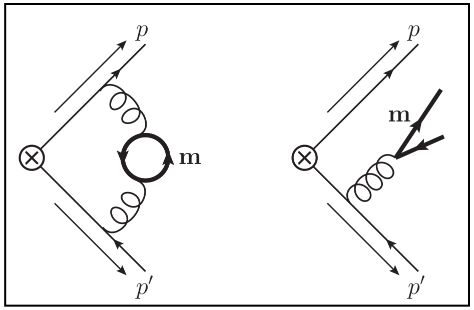
$$U_H^{(n_f)} \times U_J^{(n_f)} = \left(U_\Phi^{(n_f)}\right)^{-1} \quad \text{for } n_f = n_l, n_l + 1$$

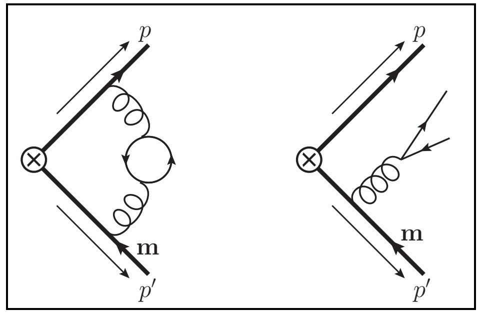
→ relation between matching conditions

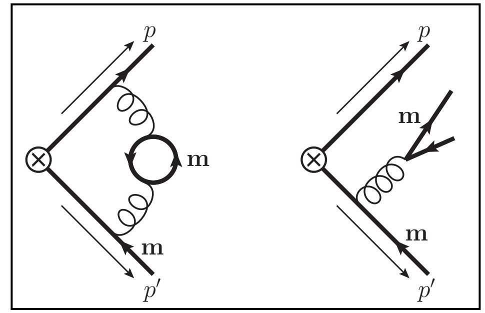
$$\mathcal{M}_H \times \mathcal{M}_J = \mathcal{M}_\Phi$$

Summary of Part 2

→ VFN Scheme for final state jets with massive quarks







→ Sums all large logarithms involving m (if they exist)

→ Keeps full mass dependence of singular terms

$$Q \gg J \gg S$$
$$\longleftarrow \longleftarrow \quad m \quad \longrightarrow \longrightarrow$$

→ Fully consistent and integrable with VFNS scheme for PDFs, beam fcts, …

→ Allows ZVNS applications for "minimalistic" quark mass implementation

(ONLY in case if large mass logs exist !)

→ Needs non-trivial mass-dependent ME calculations if mass is of order of another scale

→ Treatment for pp collisions very soon….